PHYSICAL REVIEW D **85,** 052008 (2012)

Angular correlations in three-jet events in ep collisions at HERA

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Three-jet production in deep inelastic ep scattering and photoproduction was investigated with the ZEUS detector at HERA using an integrated luminosity of up to 127 pb^{-1}. Measurements of differential cross sections are presented as functions of angular correlations between the three jets in the final state and the proton-beam direction. These correlations provide a stringent test of perturbative QCD and show sensitivity to the contributions from different color configurations. Fixed-order perturbative calculations assuming the values of the color factors C_F, C_A, and T_F as derived from a variety of gauge groups were compared to the measurements to study the underlying gauge group symmetry. The measured angular correlations in the deep inelastic ep scattering and photoproduction regimes are consistent with the admixture of color configurations as predicted by SU(3) and disfavour other symmetry groups, such as SU(N) in the limit of large N.

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I. INTRODUCTION

Quantum chromodynamics (QCD) is based on the non-Abelian group SU(3) which induces the self-coupling of the gluons. Investigations of the triple-gluon vertex (TGV) were carried out at LEP [1,2] using angular correlations in four-jet events from Z^0 hadronic decays. At HERA, the effects of the different color configurations arising from the underlying gauge structure can be studied in a clean way in three-jet production in neutral current (NC) deep inelastic scattering (DIS) and photoproduction (γp). These measurements provide complementary information to that already obtained in e^+e^- annihilation since they are probing the gauge structure in a different environment, a hadron-induced reaction, and are sensitive to new color configurations.

Neutral current DIS at high Q^2 ($\gg \Lambda^2_{\mathrm{QCD}}$, where Q^2 is the virtuality of the exchanged photon) up to leading order (LO) in the strong coupling constant, α_s, proceeds as in the quark-parton model ($Vq \to q$, where $V = \gamma^*$ or Z^0) or via the boson-gluon fusion ($Vg \to q\bar{q}$) and QCD-Compton ($Vq \to qg$) processes. Photoproduction is studied at HERA by means of ep scattering at low four-momentum transfers ($Q^2 \approx 0$). In γp reactions, two types of QCD processes contribute to jet production at LO [3,4]: either the photon interacts directly with a parton in the proton (the direct process) or the photon acts as a source of partons which scatter off those in the proton (the resolved process).

A subset of resolved subprocesses with two jets in the final state is described by diagrams with a TGV; however, such events are difficult to distinguish from two-jet events without such a contribution. Three-jet final states in direct γp processes also contain contributions from TGVs and are easier to identify. Since three-jet production in NC DIS proceeds via the same diagrams as in direct γp, such processes can also be used to investigate the underlying gauge symmetry. Examples of diagrams contributing to the four color configurations are shown in Fig. 1: (A) double-gluon bremsstrahlung from a quark line, (B) the splitting of a virtual gluon into a pair of final-state gluons, (C) the production of a $q\bar{q}$ pair through the exchange of a virtual

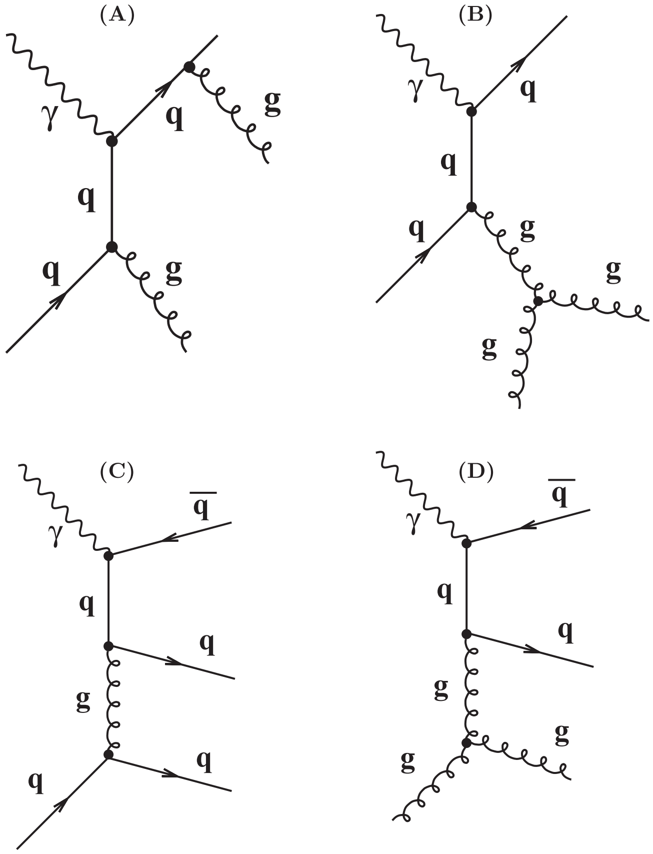

FIG. 1. Examples of diagrams for the photoproduction of three-jet events through direct-photon processes and in NC DIS three-jet events in each color configuration: (A) double-gluon bremsstrahlung from a quark line; (B) the splitting of a virtual gluon into a pair of final-state gluons; (C) the production of a $q\bar{q}$ pair through the exchange of a virtual gluon emitted by an incoming quark; (D) the production of $q\bar{q}$ pair through the exchange of a virtual gluon arising from the splitting of an incoming gluon.

gluon emitted by an incoming quark, and (D) the production of a $q\bar{q}$ pair through the exchange of a virtual gluon arising from the splitting of an incoming gluon.

Other possible diagrams and interferences correspond to one of the four configurations. The production rate of all contributions is proportional to the so-called color factors, C_F, C_A, and T_F, which are a physical manifestation of the underlying group structure. For QCD, these factors represent the relative strengths of the processes $q \to qg$, $g \to gg$, and $g \to q\bar{q}$. The contributions of the diagrams of Fig. 1 are proportional to C_F^2, $C_F C_A$, $C_F T_F$, and $T_F C_A$, respectively, independent of the underlying gauge symmetry. It should be noted that the $T_F C_A$ contribution, which arises from gluon-induced processes, is not present in e^+e^- annihilation and is investigated here for the first time.

Three-jet cross sections were previously measured in γp [5] and in NC DIS [6,7]. The shape of the measured cross sections was well reproduced by perturbative QCD (pQCD) calculations and a value of α_s was extracted [6]. In this paper, measurements of angular correlations in three-jet events in γp and NC DIS are presented. The comparison between the measurements and fixed-order $\mathcal{O}(\alpha_s^2)$ and $\mathcal{O}(\alpha_s^3)$ perturbative calculations based on different color configurations provides a stringent test of pQCD predictions directly beyond LO and gives insight into the underlying group symmetry. Phase-space regions where the angular correlations show potential sensitivity to the presence of the TGV were identified.

II. THEORETICAL FRAMEWORK

The dynamics of a gauge theory such as QCD are completely defined by the commutation relations between its group generators T^i,

$$[T^i, T^j] = i\sum_k f^{ijk} \cdot T^k,$$

where f^{ijk} are the structure constants. The generators T^i can be represented as matrices. In perturbative calculations, the average (sum) over all possible color configurations in the initial (final) states leads to the appearance of combinatoric factors C_F, C_A, and T_F, which are defined by the relations

$$\sum_{k,\eta} T^k_{\alpha\eta} T^k_{\eta\beta} = \delta_{\alpha\beta} C_F, \sum_{j,k} f^{jkm} f^{jkn} = \delta^{mn} C_A,$$

$$\sum_{\alpha,\beta} T^m_{\alpha\beta} T^n_{\beta\alpha} = \delta^{mn} T_F.$$

Measurements of the ratios between the color factors allow the experimental determination of the underlying gauge symmetry of the strong interactions. For SU(N), the predicted values of the color factors are

$$C_A = N, \qquad C_F = \frac{N^2 - 1}{2N} \quad \text{and} \quad T_F = 1/2,$$

where N is the number of color charges. In particular, SU(3) predicts $C_A/C_F = 9/4$ and $T_F/C_F = 3/8$. In contrast, an Abelian gluon theory based on U(1)^3 would predict $C_A/C_F = 0$ and $T_F/C_F = 3$. A non-Abelian theory based on SO(3) predicts $C_A/C_F = 1$ and $T_F/C_F = 1$.

The $\mathcal{O}(\alpha_s^2)$ calculations of three-jet cross sections for direct γp and NC DIS processes can be expressed in terms of C_A, C_F, and T_F as [8]:

$$\sigma_{ep\to 3\text{jets}} = C_F^2 \cdot \sigma_A + C_F C_A \cdot \sigma_B + C_F T_F \cdot \sigma_C + T_F C_A \cdot \sigma_D, \tag{1}$$

where $\sigma_A, \ldots, \sigma_D$ are the partonic cross sections for the different contributions (see Fig. 1).

III. DEFINITION OF THE ANGULAR CORRELATIONS

Angular-correlation observables were devised to distinguish the contributions from the different color configurations. They are defined in terms of the three jets with highest transverse energy in an event and the beam direction as:

(i) θ_H, the angle between the plane determined by the highest-transverse-energy jet and the beam and the plane determined by the two jets with the second-highest and third-highest transverse energy [9]. For three-jet events in ep collisions, the variable θ_H was designed [9] to be sensitive to the TGV in quark-induced processes [see Fig. 1(b)];

(ii) α_{23}, the angle between the two lowest-transverse-energy jets; the jets are ordered according to decreasing transverse energy. This variable is based on the angle $\alpha_{34}^{e^+e^-}$ for $e^+e^- \rightarrow 4\,\text{jets}$ [2], which distinguishes between contributions from double-bremsstrahlung diagrams and diagrams involving the TGV;

(iii) β_{KSW}, the angle defined via the equation $\cos(\beta_{\text{KSW}}) = \cos[\frac{1}{2}(\angle[(\vec{p}_1 \times \vec{p}_3), (\vec{p}_2 \times \vec{p}_B)] + \angle[(\vec{p}_1 \times \vec{p}_B), (\vec{p}_2 \times \vec{p}_3)])]$, where $\vec{p}_i, i = 1, \ldots, 3$ is the momentum of jet i and $\vec{p}_B$ is a unit vector in the direction of the proton beam. This variable is based on the Körner-Schierholz-Willrodt angle $\Phi_{\text{KSW}}^{e^+e^-}$ for $e^+e^- \rightarrow 4\,\text{jets}$ [10], which is sensitive to the differences between $q\bar{q}gg$ and $q\bar{q}q\bar{q}$ final states;

(iv) $\eta_{\max}^{\text{jet}}$, the maximum pseudorapidity[1] of the three jets.

IV. EXPERIMENTAL SET-UP

The data samples used in this analysis were collected with the ZEUS detector at HERA and correspond to an integrated luminosity of $44.9 \pm 0.8(65.1 \pm 1.5)\ \text{pb}^{-1}$ for e^+p collisions taken during 1995–97 (1999–2000) and $16.7 \pm 0.3\ \text{pb}^{-1}$ for e^-p collisions taken during 1998–99. During 1995–97 (1998–2000), HERA operated with protons of energy $E_p = 820(920)$ GeV and positrons or electrons[2] of energy $E_e = 27.5$ GeV, yielding a center-of-mass energy of $\sqrt{s} = 300(318)$ GeV.

A detailed description of the ZEUS detector can be found elsewhere [11,12]. A brief outline of the components that are most relevant for this analysis is given below. Charged particles were tracked in the central tracking detector (CTD) [13], which operated in a magnetic field of 1.43 T provided by a thin superconducting solenoid. The CTD consisted of 72 cylindrical drift-chamber layers, organized in nine superlayers covering the polar-angle region $15° < \theta < 164°$. The transverse-momentum resolution for full-length tracks was parametrized as $\sigma(p_T)/p_T = 0.0058 p_T \oplus 0.0065 \oplus 0.0014/p_T$, with p_T in GeV. The tracking system was used to measure the interaction vertex with a typical resolution along (transverse to) the beam direction of 0.4 (0.1) cm and to cross-check the energy scale of the calorimeter.

The high-resolution uranium–scintillator calorimeter (CAL) [14] covered 99.7% of the total solid angle and consisted of three parts: the forward (FCAL), the barrel (BCAL), and the rear (RCAL) calorimeters. Each part was subdivided transversely into towers and longitudinally into one electromagnetic section (EMC) and either one (in RCAL) or two (in BCAL and FCAL) hadronic sections (HAC). The smallest subdivision of the calorimeter was called a cell. Under test-beam conditions, the CAL single-particle relative energy resolutions were $\sigma(E)/E = 0.18/\sqrt{E}$ for electrons and $\sigma(E)/E = 0.35/\sqrt{E}$ for hadrons, with E in GeV.

The luminosity was measured from the rate of the bremsstrahlung process $ep \rightarrow e\gamma p$. The resulting small-angle energetic photons were measured by the luminosity monitor [15], a lead–scintillator calorimeter placed in the HERA tunnel at $Z = -107$ m.

V. DATA SELECTION AND JET SEARCH

A three-level trigger system was used to select events online [12,16]. At the third level, jets were reconstructed using the energies and positions of the CAL cells. Events with at least one (two) jet(s) with transverse energy in excess of 10(6) GeV and pseudorapidity below 2.5 were accepted. For trigger-efficiency studies, no jet algorithm was applied and events with a total transverse energy, excluding the energy in the eight CAL towers immediately surrounding the forward beampipe, of at least 25 GeV were selected in the γp sample; for the NC DIS sample, events were selected in which the scattered-electron candidate was identified using localized energy depositions in the CAL.

In the offline selection, a reconstructed event vertex consistent with the nominal interaction position was required and cuts based on tracking information were applied to reduce the contamination from beam-induced and cosmic-ray background events. The selection criteria of the γp and NC DIS samples were analogous to previous publications [17,18].

The selected γp sample, based on the 1995–2000 data, consisted of events from ep interactions with $Q^2 < 1\ \text{GeV}^2$ and a median $Q^2 \approx 10^{-3}\ \text{GeV}^2$. The event sample was restricted to the kinematic range $0.2 < y < 0.85$, where y is the inelasticity.

The k_T cluster algorithm [19] was used in the longitudinally invariant inclusive mode [20] to reconstruct jets in the measured hadronic final state from the energy deposits in

[1] The ZEUS coordinate system is a right-handed Cartesian system, with the Z axis pointing in the proton-beam direction, referred to as the "forward direction," and the X axis pointing towards the center of HERA. The coordinate origin is at the nominal interaction point. The pseudorapidity is defined as $\eta = -\ln(\tan\frac{\theta}{2})$, where the polar angle θ is taken with respect to the proton-beam direction.

[2] Here and in the following, the term "electron" denotes generically both the electron (e^-) and the positron (e^+).

the CAL cells (calorimetric jets). The axis of the jet was defined according to the Snowmass convention [21].

For γp events, the jet search was performed in the $\eta - \phi$ plane of the laboratory frame. Corrections [17,22] to the jet transverse energy, $E_T^{\rm jet}$, were applied to the calorimetric jets as a function of the jet pseudorapidity, $\eta^{\rm jet}$, and $E_T^{\rm jet}$ and averaged over the jet azimuthal angle. Events with at least three jets of $E_T^{\rm jet} > 14$ GeV and $-1 < \eta^{\rm jet} < 2.5$ were retained. Direct γp events were further selected by requiring $x_\gamma^{\rm obs} > 0.8$, where $x_\gamma^{\rm obs}$, the fraction of the photon momentum participating in the production of the three jets with highest $E_T^{\rm jet}$, is defined as

$$x_\gamma^{\rm obs} = \frac{1}{2yE_e}(E_T^{\rm jet1}e^{-\eta^{\rm jet1}} + E_T^{\rm jet2}e^{-\eta^{\rm jet2}} + E_T^{\rm jet3}e^{-\eta^{\rm jet3}}).$$

The final γp data sample contained 1888 events.

Events from NC DIS interactions were selected from the 1998–2000 data. Two samples were studied: $Q^2 > 125\ {\rm GeV}^2$ and $500 < Q^2 < 5000\ {\rm GeV}^2$. For both samples, $|\cos\gamma_h|$ was restricted to be below 0.65, where γ_h, which corresponds to the angle of the scattered quark in the quark-parton model, is defined as

$$\cos\gamma_h = \frac{(1-y)xE_p - yE_e}{(1-y)xE_p + yE_e}$$

and x is the Bjorken variable.

For NC DIS events, the k_T jet algorithm was applied after excluding those cells associated with the scattered-electron candidate and the search was conducted in the Breit frame. The Breit frame [23] is the frame in which the exchanged virtual boson is purely spacelike, with 3-momentum $\mathbf{q} = (0, 0, -Q)$, providing a maximal separation between the products of the beam fragmentation and the hard interaction. Jet transverse-energy corrections were computed using the method developed in a previous analysis [18,24]. Events were required to have at least three jets satisfying $E_{T,\rm B}^{\rm jet1} > 8$ GeV, $E_{T,\rm B}^{\rm jet2,3} > 5$ GeV, and $-2 < \eta_{\rm B}^{\rm jet} < 1.5$, where $E_{T,\rm B}^{\rm jet}$ and $\eta_{\rm B}^{\rm jet}$ are the jet transverse energy and pseudorapidity in the Breit frame, respectively. The final NC DIS data sample with $Q^2 > 125(500 < Q^2 < 5000)\ {\rm GeV}^2$ contained 1095 (492) events.

VI. MONTE CARLO SIMULATION

Samples of Monte Carlo (MC) events were generated to determine the response of the detector to jets of hadrons and the correction factors necessary to obtain the hadron-level jet cross sections. The hadron level is defined by those hadrons with lifetime $\tau \geq 10$ ps. For the NC DIS sample, the MC events were also used to correct the measured cross sections for QED radiative effects and the running of $\alpha_{\rm em}$.

The generated events were passed through the GEANT 3.13-based [25] ZEUS detector- and trigger-simulation programs [12]. They were reconstructed and analyzed by the same program chain as the data. The k_T jet algorithm was applied to the MC simulated events using the CAL cells in the same way as for the data. The jet algorithm was also applied to the final-state particles (hadron level) and the partons available after the parton shower (parton level).

The programs PYTHIA 6.1 [26] and HERWIG 6.1 [27] were used to generate γp events for resolved and direct processes. Events were generated using GRV-HO [28] for the photon and CTEQ4M [29] for the proton parton distribution functions (PDFs). In both generators, the partonic processes are simulated using LO matrix elements, with the inclusion of initial- and final-state parton showers. Fragmentation into hadrons is performed using the Lund string model [30] as implemented in JETSET [26,31] in the case of PYTHIA, and a cluster model [32] in the case of HERWIG.

Neutral current DIS events including radiative effects were simulated using the HERACLES 4.6.1 [33] program with the DJANGOH 1.1 [34] interface to the hadronization programs. HERACLES includes corrections for initial- and final-state radiation, vertex and propagator terms, and two-boson exchange. The QCD cascade is simulated using the color-dipole model (CDM) [35] including the LO QCD diagrams as implemented in ARIADNE 4.08 [36]; additional samples were generated with the MEPS model of LEPTO 6.5 [37]. Both MC programs use the Lund string model for the hadronization. The CTEQ5D [38] proton PDFs were used for these simulations.

VII. FIXED-ORDER CALCULATIONS

The calculations of direct γp processes used in this analysis are based on the program by Klasen, Kleinwort, and Kramer (KKK) [39]. The number of flavors was set to five; the renormalization, μ_R, and factorization scales, μ_F, were set to $\mu_R = \mu_F = E_T^{\rm max}$, where $E_T^{\rm max}$ is the highest $E_T^{\rm jet}$ in an event. The calculations were performed using the ZEUS-S [40] parameterizations of the proton PDFs; α_s was calculated at two loops using $\Lambda_{\overline{\rm MS}}^{(5)} = 226$ MeV, which corresponds to $\alpha_s(M_Z) = 0.118$. These calculations are $\mathcal{O}(\alpha_s^2)$ and represent the lowest-order contribution to three-jet γp. Full $\mathcal{O}(\alpha_s^3)$ corrections are not yet available for three-jet cross sections in γp.

The calculations of NC DIS processes used in this analysis are based on the program NLOJET++ [41], which provides $\mathcal{O}(\alpha_s^2)$ and $\mathcal{O}(\alpha_s^3)$ predictions for three-jet cross sections. The scales were chosen to be $\mu_R = \mu_F = Q$. Other parameters were set as for the γp program.

In general, the programs mentioned above are very flexible and provide observable-independent computations that allow a complete analytical cancellation of the soft and collinear singularities encountered in the calculations of jet cross sections. However, these programs were written assuming the SU(3) gauge group and the different ingredients necessary to perform a calculation according to

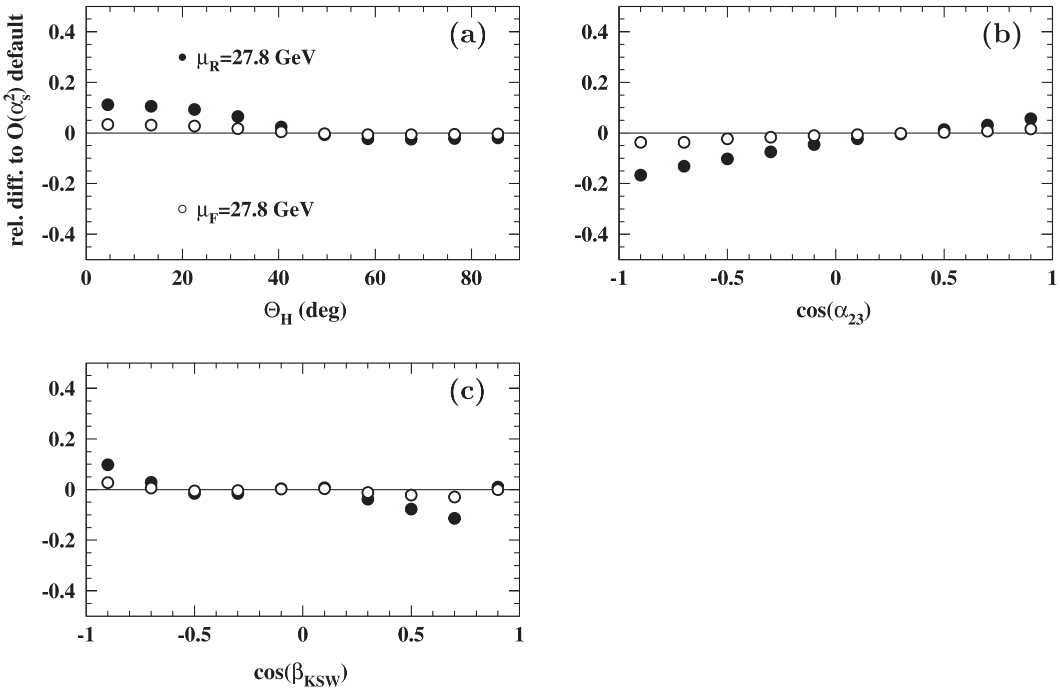

FIG. 2. Relative difference between the $\mathcal{O}(\alpha_s^2)$ normalized cross-section calculations with $\mu_R = 27.8$ GeV and the calculations with $\mu_R = E_T^{\max}$ (dots) and between the $\mathcal{O}(\alpha_s^2)$ calculations with $\mu_F = 27.8$ GeV and the calculations with $\mu_F = E_T^{\max}$ (open circles) in γp as functions of (a) θ_H, (b) $\cos(\alpha_{23})$ and (c) $\cos(\beta_{\rm KSW})$. These calculations do not include corrections for hadronization effects.

Eq. (1) were not readily available. The programs were rewritten in order to disentangle the color components to make separate predictions for $\sigma_A, \ldots, \sigma_D$.

The k_T jet algorithm was applied to the partons in the events generated by KKK and NLOJET++ in order to compute the jet cross-section predictions. Thus, these predictions refer to jets of partons. Since the measurements refer to jets of hadrons, the calculations were corrected to the hadron level. The multiplicative correction factors, defined as the ratios between the cross section for jets of hadrons and that for jets of partons, were estimated using the MC samples described in Sec. VI. The normalized cross-section calculations (see Sec. VIII for the definition of the cross sections) changed typically by less than $\pm 5(10\%)$ for the predictions in γp (NC DIS) upon application of the parton-to-hadron corrections. Therefore, the effect of the parton-to-hadron corrections on the angular distributions is small. In NC DIS processes, other effects not accounted for in the calculations, namely Z^0 exchange, were also corrected for using the MC samples.

The predictions for jet cross sections are expressed as the convolution of the PDFs and the matrix elements, which depend on α_s. Both the PDFs and α_s evolve with the energy scale. In the calculations performed for this analysis, QCD evolution via the DGLAP and the renormalization group equations, respectively, were used. These evolution equations also depend on the color factors. This procedure introduces an additional dependence on the color factors with respect to that shown in Eq. (1); this dependence is suppressed by considering normalized cross sections. The remaining dependence was estimated by comparing to calculations with fixed μ_F or μ_R, i.e. no evolution of the PDFs or α_s was allowed. The values chosen for μ_F and μ_R were the mean values of the data distributions, $\langle E_T^{\max}\rangle_{\rm data} = 27.8$ GeV for γp and $\sqrt{\langle Q^2\rangle_{\rm data}} = 31.3(36.6)$ GeV for NC DIS with $Q^2 > 125(500 < Q^2 < 5000)$ GeV^2.

Figure 2 shows the relative difference of the $\mathcal{O}(\alpha_s^2)$ γp normalized cross-section calculations with μ_F (μ_R) fixed[3] to those in which $\mu_F = E_T^{\max}$ ($\mu_R = E_T^{\max}$) as a function of the angular variables studied. Figs. 3(a) and 3(b) show the same relative difference for the $\mathcal{O}(\alpha_s^2)$ NLOJET++ calculations for $Q^2 > 125$ GeV^2.

Very small differences are observed for the μ_F variation. Sizeable differences for the μ_R variation are seen in some regions; in particular, a trend is observed for the relative difference as a function of $\eta_{\max}^{\rm jet}$: this trend is due to the fact that the mean values of Q^2 in each bin of $\eta_{\max}^{\rm jet}$ increase as $\eta_{\max}^{\rm jet}$ decreases.

These studies demonstrate that the normalized cross sections have little sensitivity to the evolution of the PDFs. It should be noted that there is a remaining dependence on the color factors through the relative

[3]When μ_F was fixed, μ_R was allowed to vary with the scale, and vice-versa.

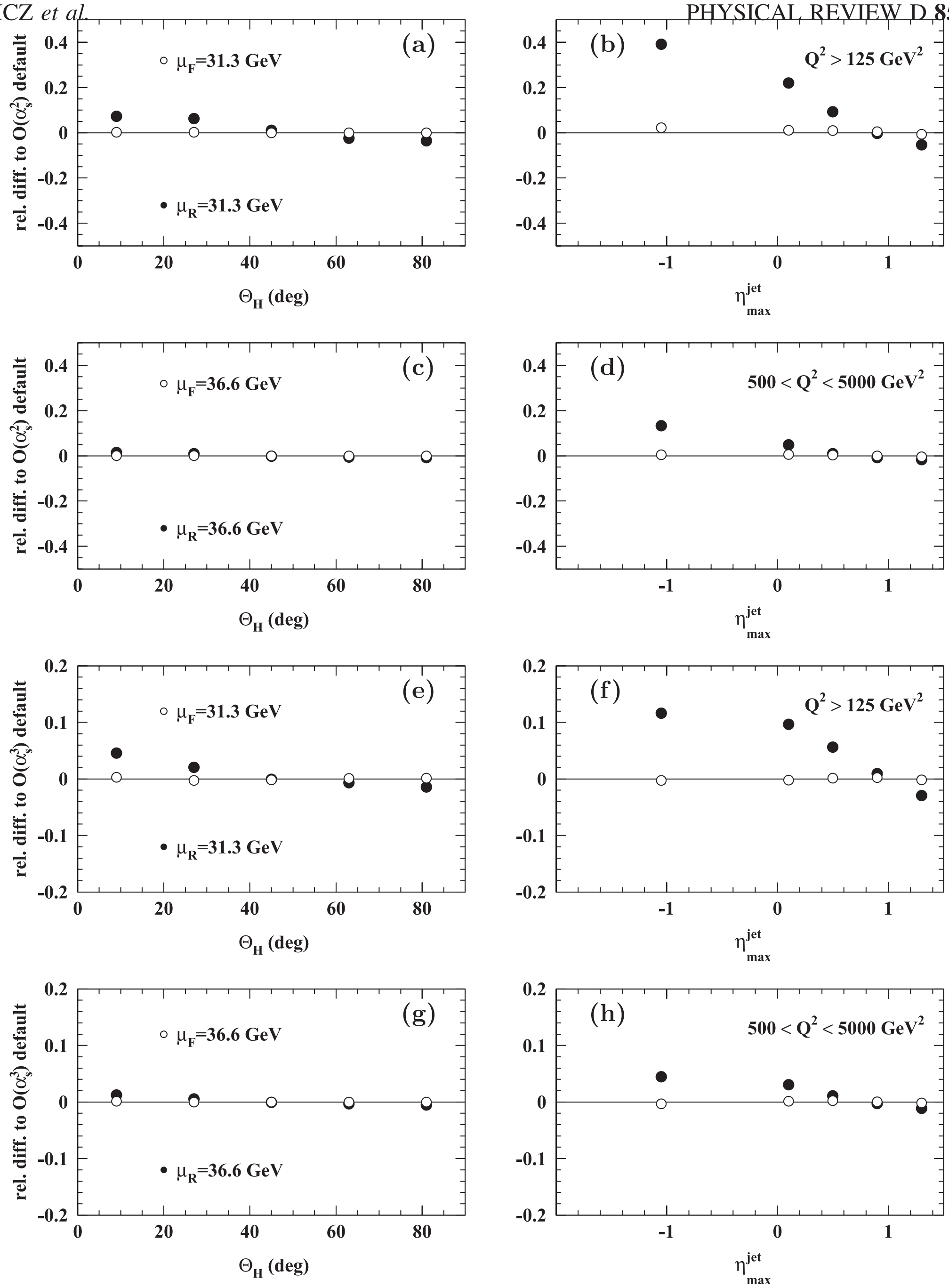

FIG. 3. Relative difference between the $\mathcal{O}(\alpha_s^2)$ normalized cross-section calculations with fixed μ_R and the calculations with $\mu_R = Q$ (dots) and between the $\mathcal{O}(\alpha_s^2)$ calculations with fixed μ_F and the calculations with $\mu_F = Q$ (open circles) in NC DIS for $Q^2 > 125\ \mathrm{GeV}^2$ as functions of (a) θ_H and (b) $\eta_{\max}^{\mathrm{jet}}$; (c) and (d) show the corresponding relative differences for $500 < Q^2 < 5000\ \mathrm{GeV}^2$. (e), (f), (g) and (h) show the corresponding relative differences using the $\mathcal{O}(\alpha_s^3)$ calculations. All these calculations do not include corrections for hadronization effects.

contributions of quark- and gluon-induced processes as obtained in the extraction of the PDFs, in which the values of $SU(3)$ were assumed.[4] There is still some sensitivity to the running of α_s. Figures 3(c) and 3(d) show the relative difference for $500 < Q^2 < 5000\ \mathrm{GeV}^2$. The restriction of the phase space further reduces the dependence on the running of α_s; thus, this region is more suitable to extract the color factors in NC DIS at $\mathcal{O}(\alpha_s^2)$. At $\mathcal{O}(\alpha_s^3)$ [see Figs. 3(e) to 3(h)], the effect due to the running of α_s is already very small for $Q^2 > 125\ \mathrm{GeV}^2$. Therefore, the wider phase-space region can be kept in an extraction of the color factors at $\mathcal{O}(\alpha_s^3)$.

[4]In order to consider that correlation, an extraction of the PDFs leaving the color factors as free parameters would be necessary, a task which is beyond the scope of the present paper.

The following theoretical uncertainties were considered (as an example of the size of the uncertainties, an average value of the effect of each uncertainty on the normalized cross section as a function of θ_H is shown in parentheses for γp, NC DIS with $Q^2 > 125$ GeV^2 and NC DIS with $500 < Q^2 < 5000$ GeV^2):

(i) the uncertainty in the modelling of the parton shower was estimated by using different models (see Sec. VI) to calculate the parton-to-hadron correction factors ($\pm$ 2.8%, $\pm$2.9% and $\pm$5.8%);

(ii) the uncertainty on the calculations due to higher-order terms was estimated by varying μ_R by a factor of 2 up and down ($^{+0.6}_{-0.8}$%, $\pm$1.6% and $\pm$2.2%);

(iii) the uncertainty on the calculations due to those on the proton PDFs was estimated by repeating the calculations using 22 additional sets from the ZEUS analysis [40]; this analysis takes into account the statistical and correlated systematic experimental uncertainties of each data set used in the determination of the proton PDFs ($\pm$ 0.7%, $\pm$0.2% and $\pm$0.1%);

(iv) the uncertainty on the calculations due to that on $\alpha_s(M_Z)$ was estimated by repeating the calculations using two additional sets of proton PDFs, for which different values of $\alpha_s(M_Z)$ were assumed in the fits. The difference between the calculations using

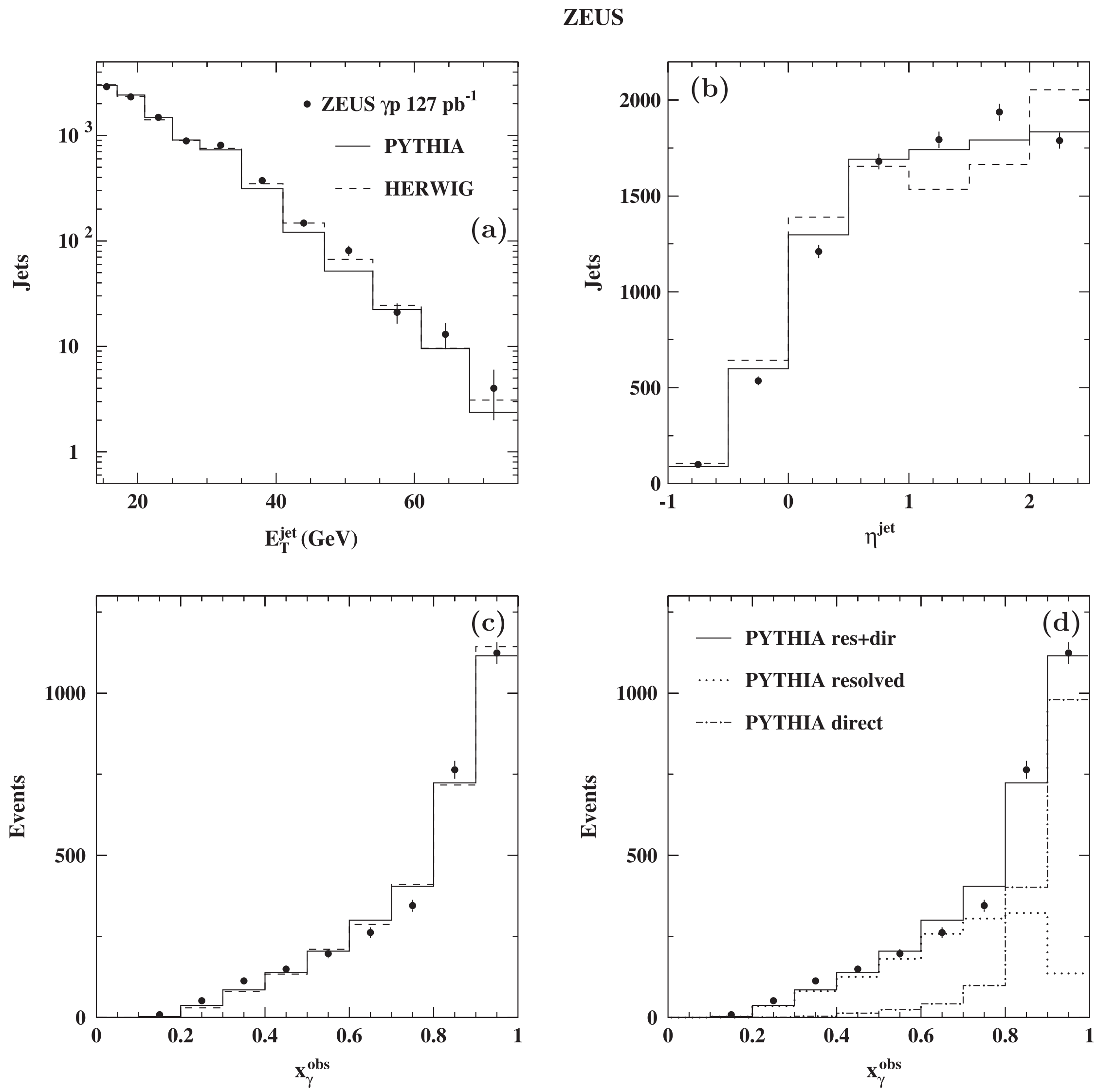

FIG. 4. Detector-level data distributions for three-jet photoproduction (dots) with $E_T^{\rm jet} > 14$ GeV and $-1 < \eta^{\rm jet} < 2.5$ in the kinematic region given by $Q^2 < 1$ GeV^2 and $0.2 < y < 0.85$ as functions of (a) $E_T^{\rm jet}$, (b) $\eta^{\rm jet}$ and (c, d) $x_\gamma^{\rm obs}$. For comparison, the distributions of the PYTHIA (solid histograms) and Herwig (dashed histograms) MC models for resolved plus direct processes normalized to the data are included. In (d), the contributions for resolved (dotted histogram) and direct (dot-dashed histogram) processes from PYTHIA MC are shown separately.

these various sets was scaled to reflect the uncertainty on the current world average of α_s [42] (negligible in all cases);

(v) the uncertainty of the calculations due to the choice of μ_F was estimated by varying μ_F by a factor of 2 up and down (negligible in all cases).

The total theoretical uncertainty was obtained by adding in quadrature the individual uncertainties listed above. The dominant source of theoretical uncertainty is that on the modelling of the parton shower, which is to a large extent correlated bin to bin.

VIII. DEFINITION OF THE CROSS SECTIONS

Normalized differential three-jet cross sections were measured as functions of θ_H, α_{23}, and β_{KSW} using the selected data samples in γp and NC DIS. For NC DIS, the normalized differential three-jet cross section as a function of $\eta^{\mathrm{jet}}_{\mathrm{max}}$ was also measured. The normalized differential three-jet cross section in bin i for an observable A was obtained using

$$\frac{1}{\sigma}\frac{d\sigma_i}{dA} = \frac{1}{\sigma}\frac{N_{\mathrm{data},i}}{\mathcal{L}\cdot\Delta A_i}\cdot\frac{N^{\mathrm{had}}_{\mathrm{MC},i}}{N^{\mathrm{det}}_{\mathrm{MC},i}},$$

where $N_{\mathrm{data},i}$ is the number of data events in bin i, $N^{\mathrm{had}}_{\mathrm{MC},i}(N^{\mathrm{det}}_{\mathrm{MC},i})$ is the number of MC events at hadron (detector) level, $\mathcal{L}$ is the integrated luminosity, and ΔA_i is the bin width. The integrated three-jet cross section, σ, was computed using the formula:

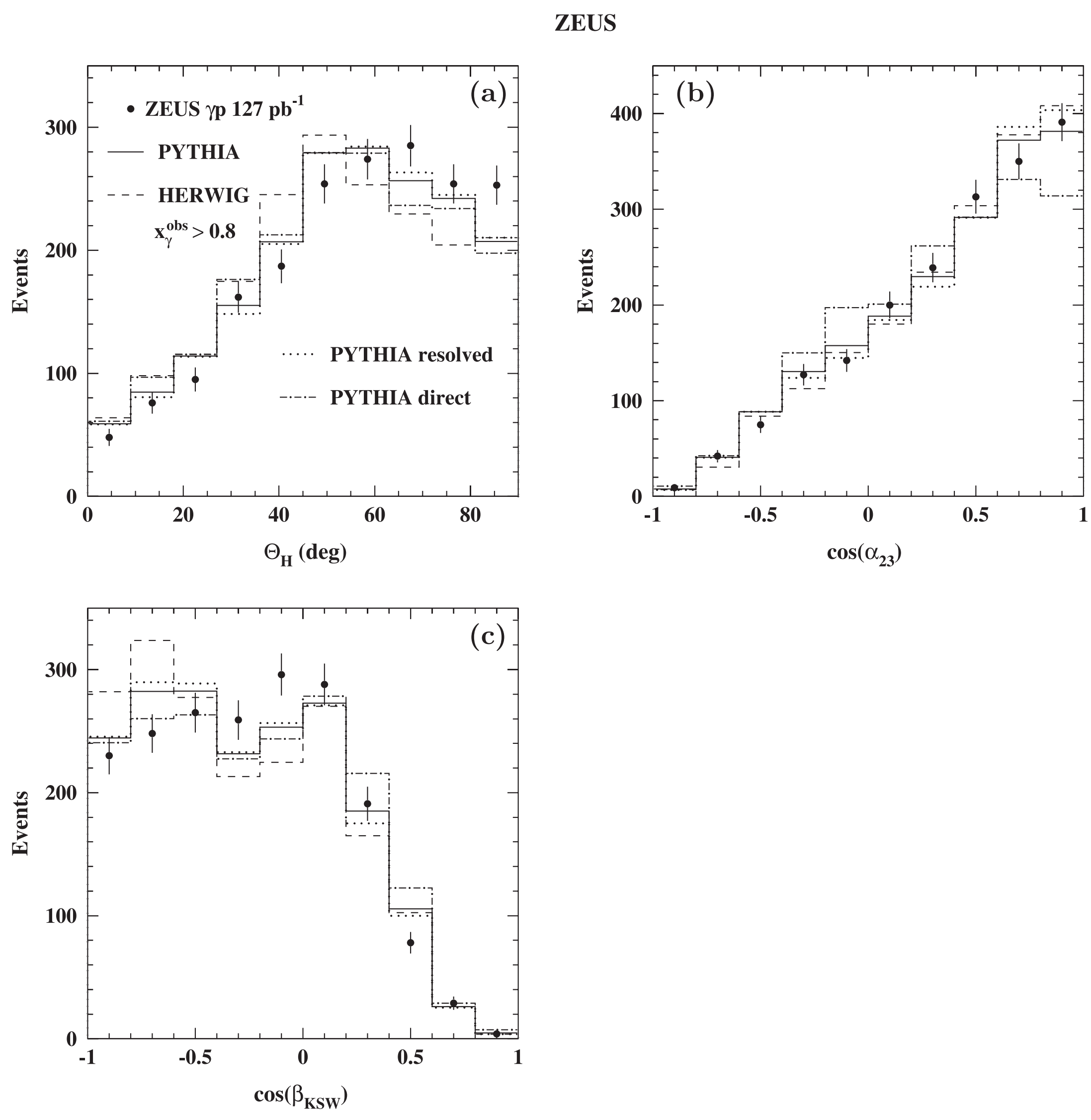

FIG. 5. Detector-level data distributions for three-jet photoproduction (dots) with $E_T^{\mathrm{jet}} > 14$ GeV and $-1 < \eta^{\mathrm{jet}} < 2.5$ in the kinematic region given by $Q^2 < 1$ GeV^2, $0.2 < y < 0.85$, and $x_\gamma^{\mathrm{obs}} > 0.8$ as functions of (a) θ_H, (b) $\cos(\alpha_{23})$, and (c) $\cos(\beta_{\mathrm{KSW}})$ Other details as in the caption to Fig. 4.

$$\sigma = \sum_i \frac{N_{\mathrm{data},i}}{\mathcal{L}} \cdot \frac{N^{\mathrm{had}}_{\mathrm{MC},i}}{N^{\mathrm{det}}_{\mathrm{MC},i}},$$

where the sum runs over all bins.

For the γp sample, due to the different center-of-mass energies of the two data sets used in the analysis, the measured normalized differential three-jet cross sections were combined using

$$\sigma^{\mathrm{comb}} = \frac{\sigma_{300} \cdot \mathcal{L}_{300} + \sigma_{318} \cdot \mathcal{L}_{318}}{\mathcal{L}_{300} + \mathcal{L}_{318}},$$

where $\mathcal{L}_{\sqrt{s}}$ is the luminosity and $\sigma_{\sqrt{s}}$ is the measured cross section corresponding to $\sqrt{s} = 300$ or 318 GeV. This formula was applied for combining the differential and total cross sections. The same formula was used for computing the $\mathcal{O}(\alpha_s^2)$ predictions in γp.

IX. ACCEPTANCE CORRECTIONS AND EXPERIMENTAL UNCERTAINTIES

The PYTHIA (MEPS) MC samples were used to compute the acceptance corrections to the angular distributions of the γp (NC DIS) data. These correction factors took into

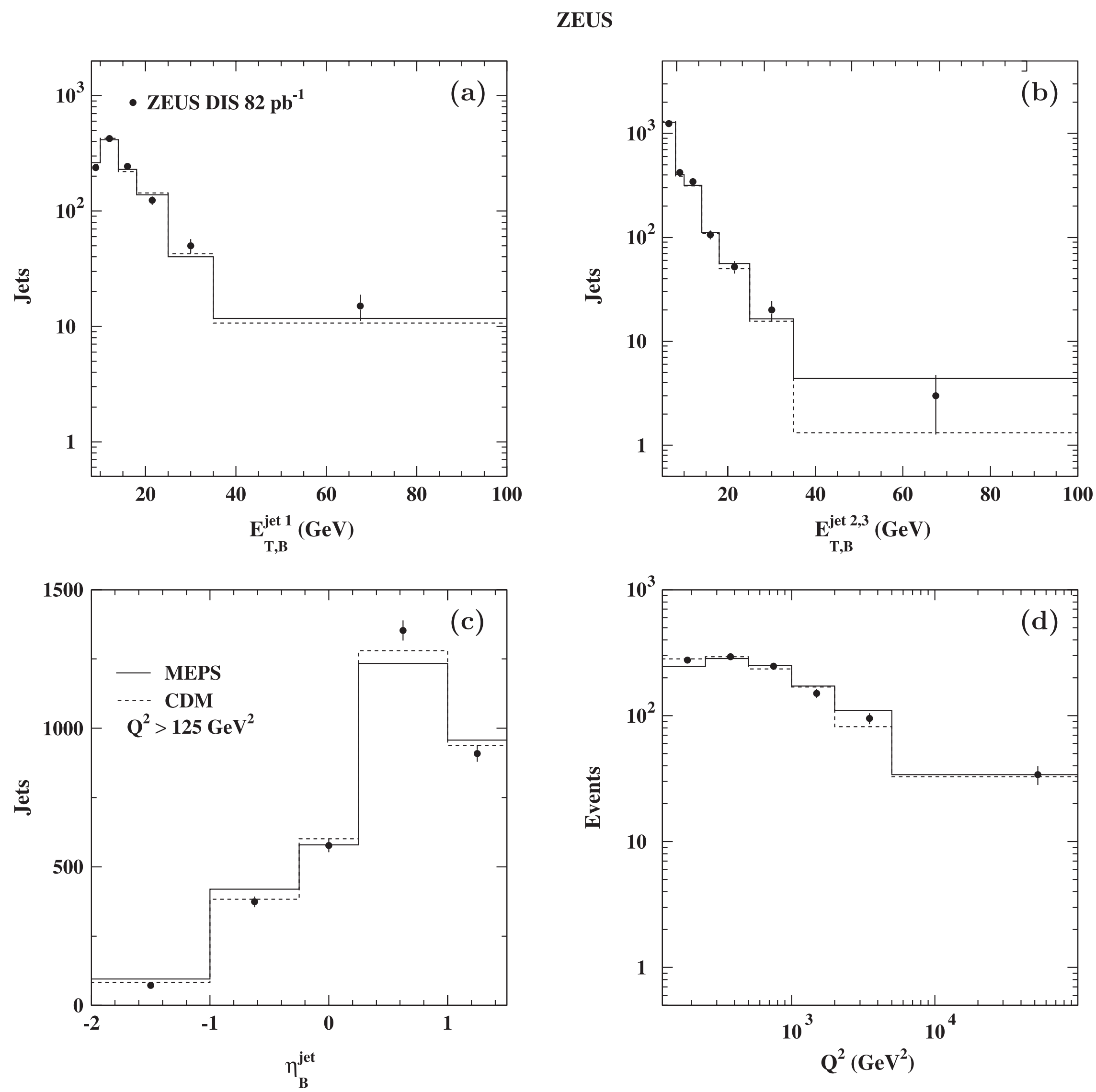

FIG. 6. Detector-level data distributions for three-jet production in NC DIS (dots) with $E_{T,\mathrm{B}}^{\mathrm{jet1}} > 8$ GeV, $E_{T,\mathrm{B}}^{\mathrm{jet2,3}} > 5$ GeV, and $-2 < \eta_{\mathrm{B}}^{\mathrm{jet}} < 1.5$ in the kinematic region given by $Q^2 > 125\ \mathrm{GeV}^2$ and $|\cos\gamma_h| < 0.65$ as functions of (a) $E_{T,\mathrm{B}}^{\mathrm{jet1}}$, (b) $E_{T,\mathrm{B}}^{\mathrm{jet2,3}}$, (c) $\eta_{\mathrm{B}}^{\mathrm{jet}}$, and (d) Q^2 For comparison, the distributions of the MEPS (solid histograms) and CDM (dashed histograms) MC models normalized to the data are included.

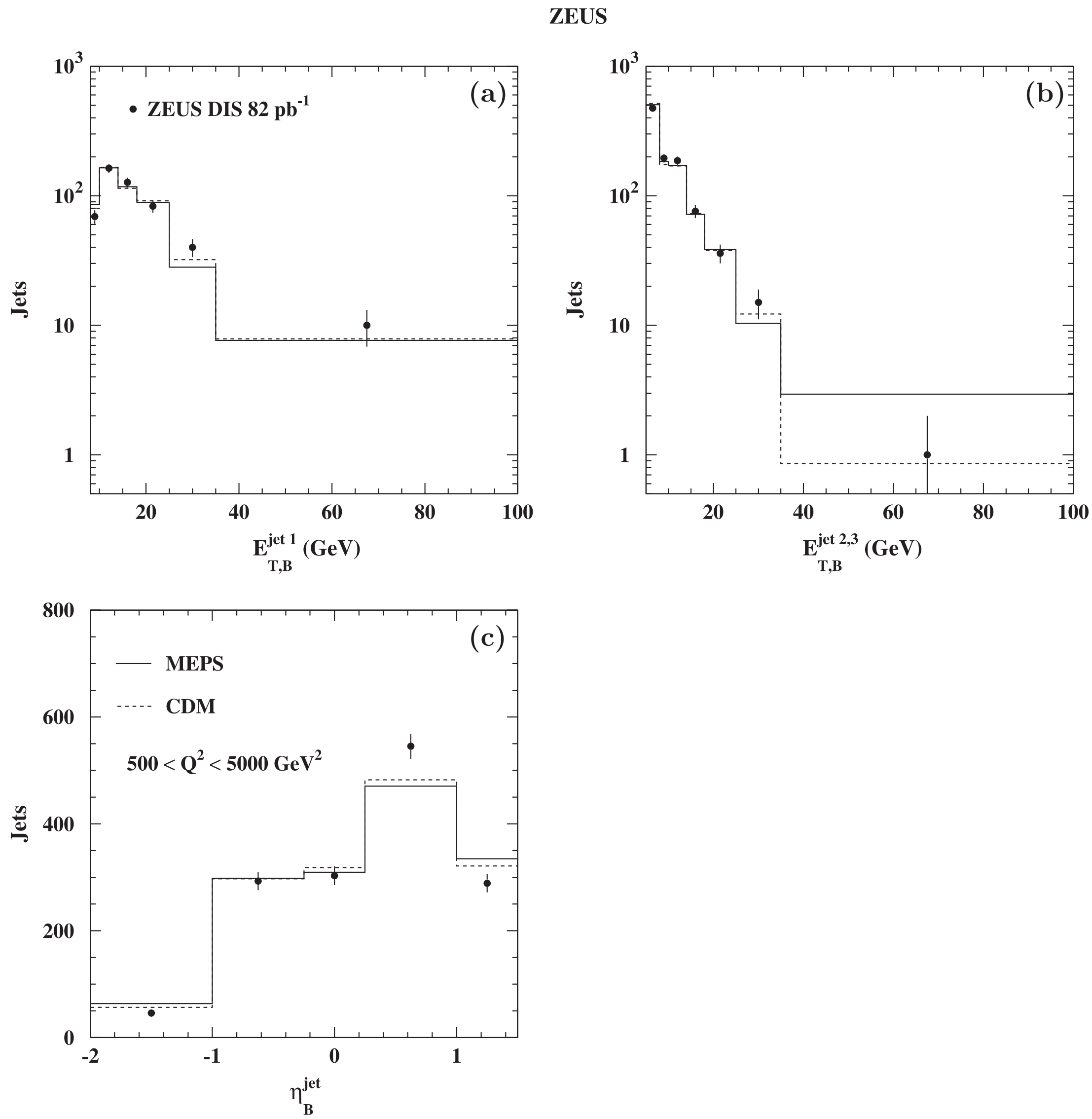

FIG. 7. Detector-level data distributions for three-jet production in NC DIS (dots) with $E_{T,\mathrm{B}}^{\mathrm{jet1}} > 8$ GeV, $E_{T,\mathrm{B}}^{\mathrm{jet2,3}} > 5$ GeV, and $-2 < \eta_{\mathrm{B}}^{\mathrm{jet}} < 1.5$ in the kinematic region given by $500 < Q^2 < 5000$ GeV^2 and $|\cos\gamma_h| < 0.65$ as functions of (a) $E_{T,\mathrm{B}}^{\mathrm{jet1}}$, (b) $E_{T,\mathrm{B}}^{\mathrm{jet2,3}}$, and (c) $\eta_{\mathrm{B}}^{\mathrm{jet}}$. Other details as in the caption to Fig. 6.

account the efficiency of the trigger, the selection criteria, and the purity and efficiency of the jet reconstruction. The samples of HERWIG and CDM were used to compute the systematic uncertainties coming from the fragmentation and parton-shower models in γp and NC DIS, respectively.

The data E_T^{jet}, η^{jet}, and x_γ^{obs} distributions of the γp sample, before the $x_\gamma^{\mathrm{obs}} > 0.8$ requirement, are shown in Fig. 4 together with the MC simulations of PYTHIA and HERWIG. Considering that three-jet events in the MC arise only from the parton-shower approximation, the description of the data is reasonable. Figure 4(d) shows the resolved and direct contributions for the PYTHIA MC separately. It is observed that the region of $x_\gamma^{\mathrm{obs}} > 0.8$ is dominated by direct γp events. The remaining contribution in this region from resolved-photon events was estimated using PYTHIA (HERWIG) simulated events to be $\approx 25(31)\%$.

Figure 5 shows the data distributions as functions of θ_H, α_{23}, and β_{KSW} together with the simulations of PYTHIA and HERWIG for $x_\gamma^{\mathrm{obs}} > 0.8$. The PYTHIA MC predictions describe the data distributions well, whereas the description given by HERWIG is somewhat poorer. It was checked that the angular distributions of the events from resolved processes with $x_\gamma^{\mathrm{obs}} > 0.8$ were similar to those from direct processes (see Fig. 5) and, therefore, no subtraction of the

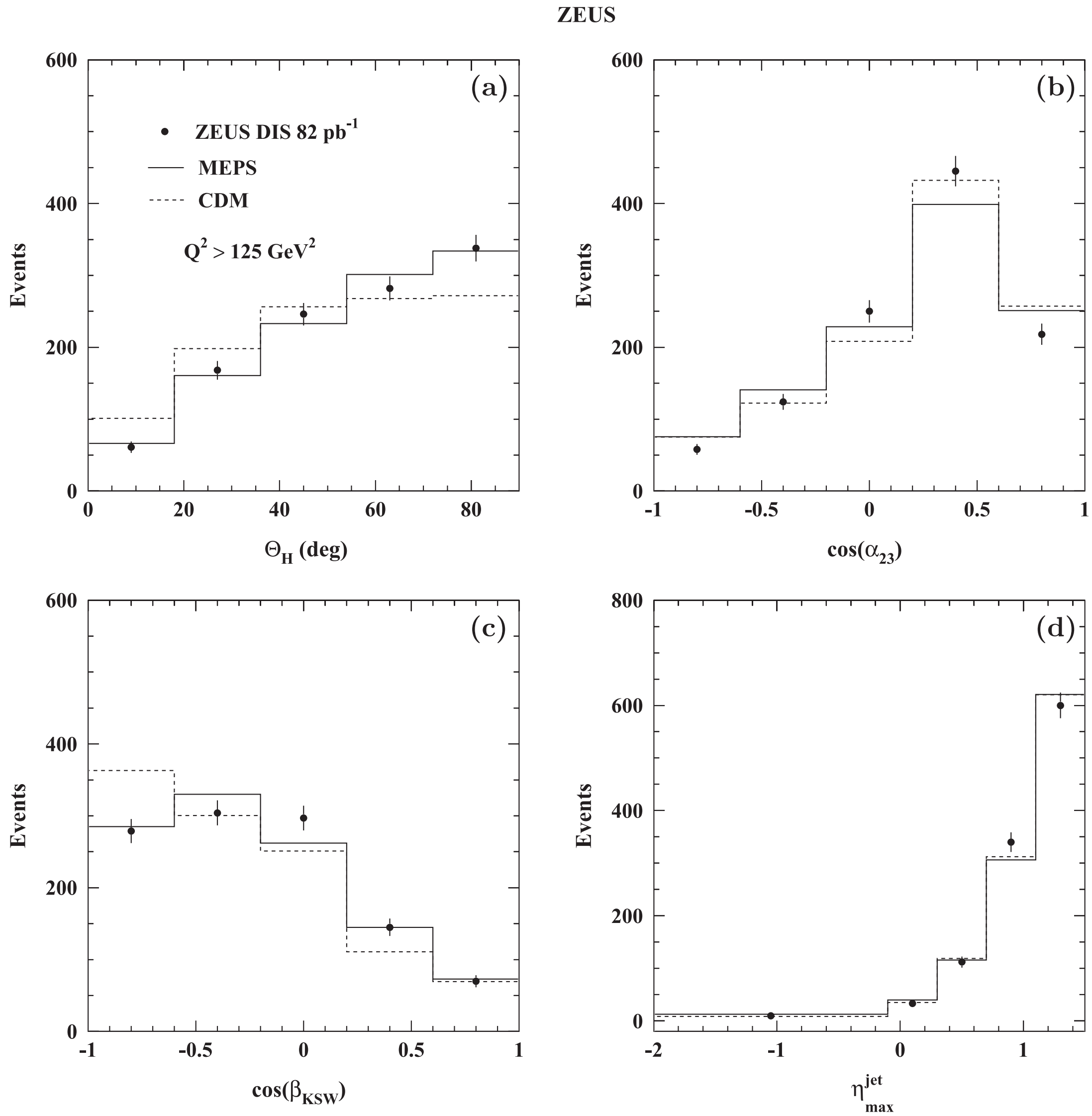

FIG. 8. Detector-level data distributions for three-jet production in NC DIS (dots) with $E_{T,\mathrm{B}}^{\mathrm{jet1}} > 8$ GeV, $E_{T,\mathrm{B}}^{\mathrm{jet2,3}} > 5$ GeV, and $-2 < \eta_{\mathrm{B}}^{\mathrm{jet}} < 1.5$ in the kinematic region given by $Q^2 > 125\ \mathrm{GeV}^2$ and $|\cos\gamma_h| < 0.65$ as functions of (a) θ_H, (b) $\cos(\alpha_{23})$, (c) $\cos(\beta_{\mathrm{KSW}})$, and (d) $\eta_{\mathrm{max}}^{\mathrm{jet}}$. For comparison, the distributions of the MEPS (solid histograms) and CDM (dashed histograms) MC models normalized to the data are included.

resolved processes was performed when comparing to the fixed-order calculations described in Sec. VII.

The data $E_{T,\mathrm{B}}^{\mathrm{jet1}}$, $E_{T,\mathrm{B}}^{\mathrm{jet2,3}}$, $\eta_{\mathrm{B}}^{\mathrm{jet}}$, and Q^2 distributions of the NC DIS samples are shown in Figs. 6 and 7 for $Q^2 > 125(500 < Q^2 < 5000)\ \mathrm{GeV}^2$ together with the MC simulations from the MEPS and CDM models. Both models give a reasonably good description of the data in both kinematic regions. The data distributions of θ_H, α_{23}, β_{KSW}, and $\eta_{\mathrm{max}}^{\mathrm{jet}}$ are shown in Figs. 8 and 9 for $Q^2 > 125(500 < Q^2 < 5000)\ \mathrm{GeV}^2$. The MEPS MC predictions describe the data distributions well, whereas the description given by CDM is somewhat poorer.

A detailed study of the sources contributing to the experimental uncertainties was performed [43]. The following experimental uncertainties were considered for γp (as an example of the size of the uncertainties, an average value of the effect of each uncertainty on the cross-section as a function of θ_H is shown in parentheses):

(i) the effect of the modelling of the parton-shower and hadronization was estimated by using HERWIG instead of PYTHIA to evaluate the correction factors ($\pm$ 6.1%);

(ii) the effect of the uncertainty on the absolute energy scale of the calorimetric jets was estimated by varying E_T^{jet} in simulated events by its uncertainty of

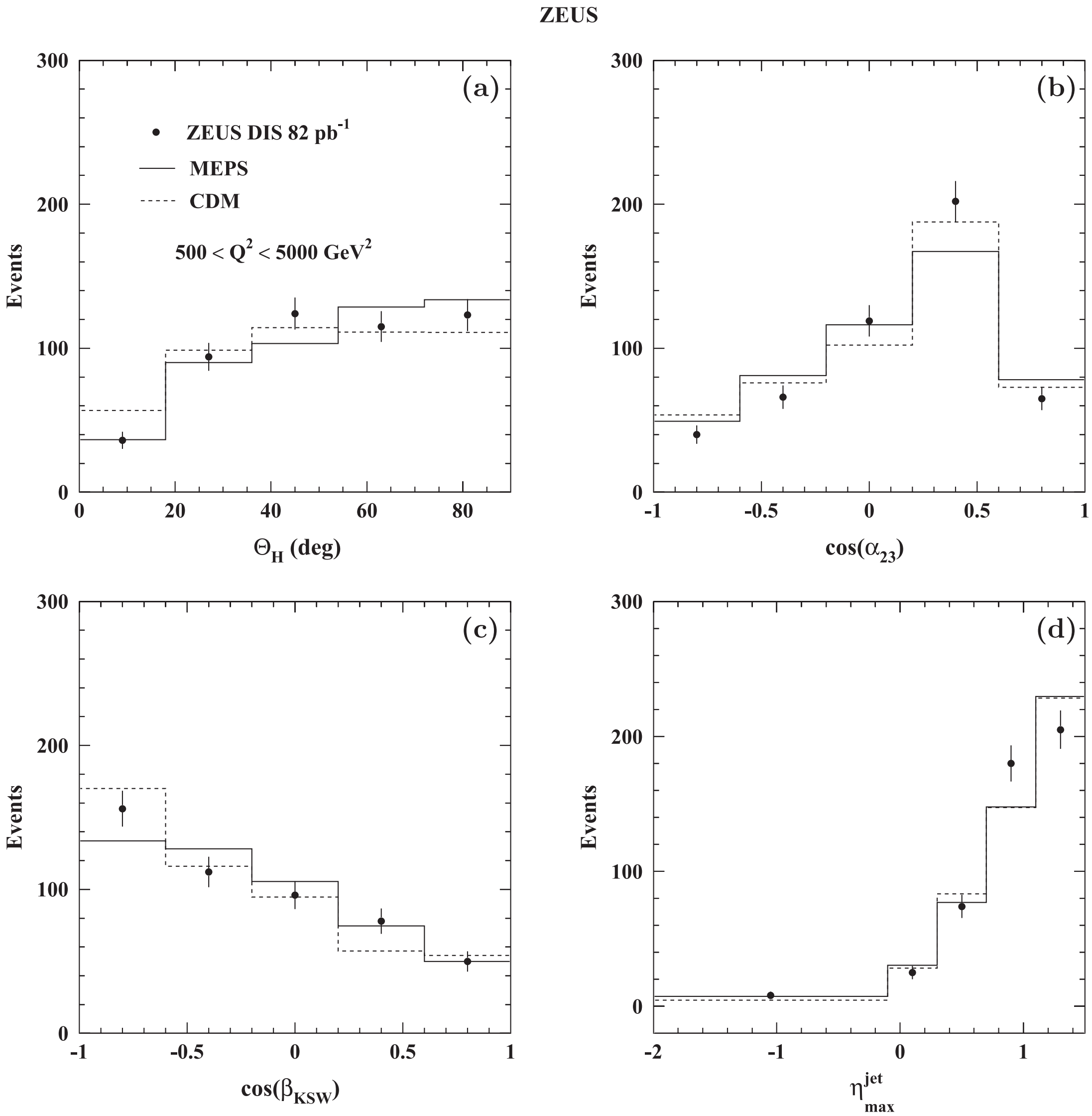

FIG. 9. Detector-level data distributions for three-jet production in NC DIS (dots) with $E_{T,B}^{\rm jet1} > 8$ GeV, $E_{T,B}^{\rm jet2,3} > 5$ GeV, and $-2 < \eta_B^{\rm jet} < 1.5$ in the kinematic region given by $500 < Q^2 < 5000\ {\rm GeV}^2$ and $|\cos\gamma_h| < 0.65$ as functions of (a) θ_H, (b) $\cos(\alpha_{23})$, (c) $\cos(\beta_{\rm KSW})$, and (d) $\eta_{\rm max}^{\rm jet}$. Other details as in the caption to Fig. 8.

$\pm 1\%$. The method used was the same as in earlier publications [17,18,44] ($\pm$ 1.6%);

(iii) the effect of the uncertainty on the reconstruction of y was estimated by varying its value in simulated events by the estimated uncertainty of $\pm 1\%$ ($\pm$ 1.0%);

(iv) the effect of the uncertainty on the parametrizations of the proton and photon PDFs was estimated by using alternative sets of PDFs in the MC simulation to calculate the correction factors ($\pm$ 0.4% and $\pm$2.0%, respectively);

(v) the uncertainty in the cross sections due to that in the simulation of the trigger ($\pm$ 0.4%).

For NC DIS events, the following experimental uncertainties were considered (as an example of the size of the uncertainties, an average value of the effect of each uncertainty on the cross section as a function of θ_H is shown in parentheses for the $Q^2 > 125\ {\rm GeV}^2$ and $500 < Q^2 < 5000\ {\rm GeV}^2$ kinematic regions):

(i) the effect of the modelling of the parton shower was estimated by using CDM instead of MEPS to evaluate the correction factors ($\pm$ 5.6% and $\pm$9.1%);

(ii) the effect of the uncertainty on the absolute energy scale of the calorimetric jets was estimated by varying $E_T^{\rm jet}$ in simulated events by its uncertainty of $\pm 1\%$ for $E_T^{\rm jet} > 10$ GeV and $\pm 3\%$ for lower $E_T^{\rm jet}$ values ($\pm$ 2.3% and $\pm$1.7%);

(iii) the uncertainties due to the selection cuts were estimated by varying the values of the cuts within

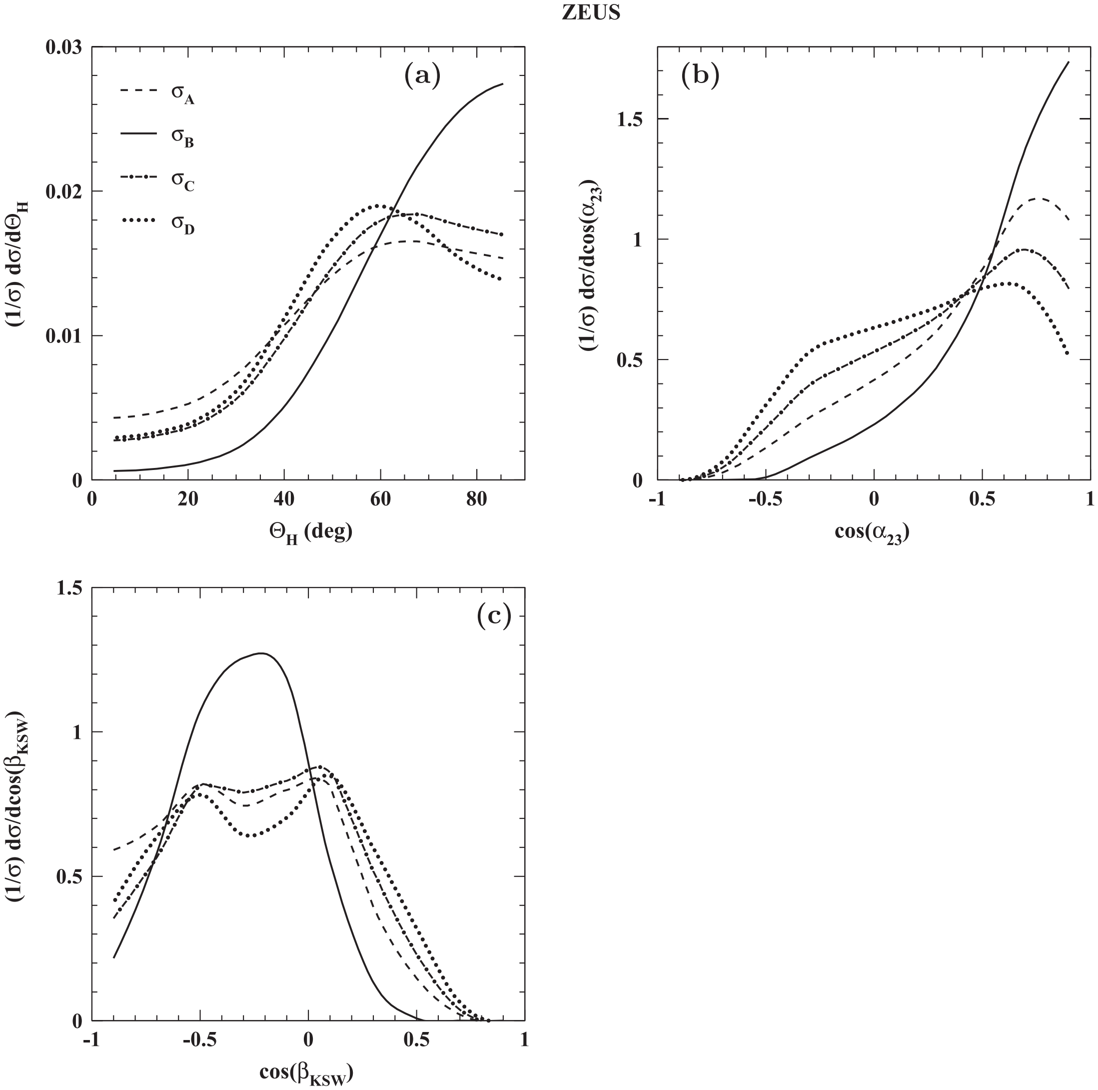

FIG. 10. Predicted normalized differential ep cross sections for three-jet direct-photon processes at $\mathcal{O}(\alpha_s^2)$ integrated over $E_T^{\rm jet} > 14$ GeV and $-1 < \eta^{\rm jet} < 2.5$ in the kinematic region defined by $Q^2 < 1$ GeV^2 and $0.2 < y < 0.85$ as functions of (a) θ_H, (b) $\cos(\alpha_{23})$, and (c) $\cos(\beta_{\rm KSW})$. In each figure, the predictions for the color components are shown: σ_A (dashed lines), σ_B (solid lines), σ_C (dot-dashed lines), and σ_D (dotted lines). These calculations do not include corrections for hadronization effects.

the resolution of each variable (less than $\pm 1.6\%$ and less than $\pm 4.2\%$ in all cases);

(iv) the uncertainty on the reconstruction of the boost to the Breit frame was estimated by using the direction of the track associated with the scattered electron instead of that derived from the impact position as determined from the energy depositions in the CAL ($\pm$ 1.6% and $\pm$1.6%);

(v) the uncertainty in the absolute energy scale of the electron candidate was estimated to be $\pm 1\%$ [45] ($\pm$ 0.2% and $\pm$0.3%);

(vi) the uncertainty in the cross sections due to that in the simulation of the trigger ($\pm$ 0.5% and $\pm$0.5%).

The dominant systematic effect comes from the modelling of the parton shower and hadronization, which is to a large extent correlated bin to bin. Nevertheless, the effect of these uncertainties on the normalized differential three-jet cross sections is small compared to the statistical uncertainties for the measurements presented in Sec. X. The systematic uncertainties were added in quadrature to the statistical uncertainties.

X. RESULTS

Normalized differential three-jet cross sections were measured in γp in the kinematic region $Q^2 < 1$ GeV^2, $0.2 < y < 0.85$, and $x_\gamma^{\rm obs} > 0.8$. The cross sections were determined for jets of hadrons with $E_T^{\rm jet} > 14$ GeV and

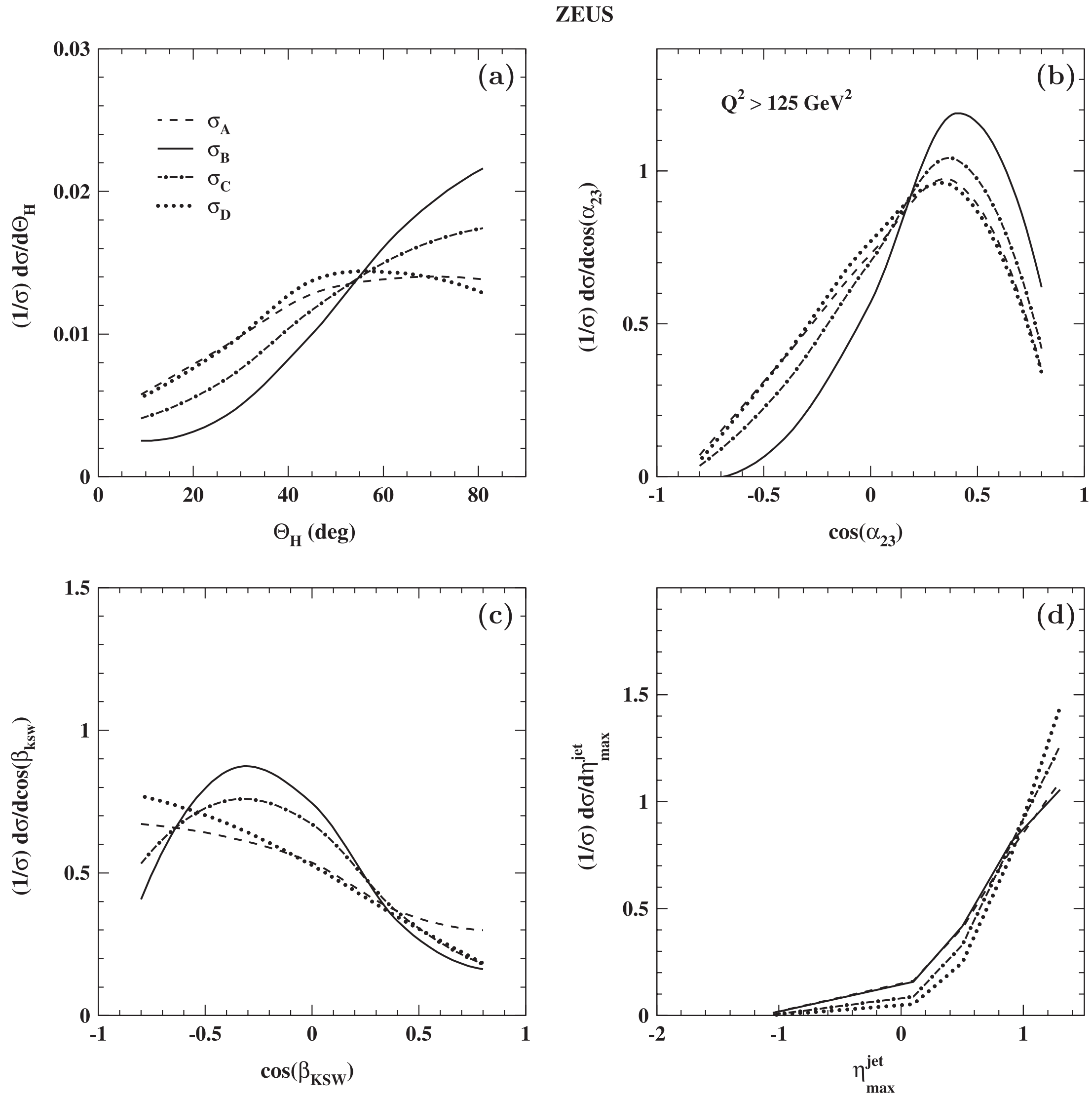

FIG. 11. Predicted normalized differential ep cross sections for three-jet production in NC DIS at $\mathcal{O}(\alpha_s^2)$ integrated over $E_{T,\mathrm{B}}^{\mathrm{jet1}} > 8$ GeV, $E_{T,\mathrm{B}}^{\mathrm{jet2,3}} > 5$ GeV, and $-2 < \eta_{\mathrm{B}}^{\mathrm{jet}} < 1.5$ in the kinematic region given by $Q^2 > 125$ GeV^2 and $|\cos\gamma_h| < 0.65$ as functions of (a) θ_H, (b) $\cos(\alpha_{23})$, (c) $\cos(\beta_{\mathrm{KSW}})$, and (d) $\eta_{\mathrm{max}}^{\mathrm{jet}}$. Other details are as in the caption to Fig. 10. These calculations do not include corrections for hadronization effects.

$-1 < \eta^{\mathrm{jet}} < 2.5$. In NC DIS, the cross sections were measured in two kinematic regimes: $Q^2 > 125$ GeV^2 and $500 < Q^2 < 5000$ GeV^2. In both cases, it was required that $|\cos\gamma_h| < 0.65$. The cross sections correspond to jets of hadrons with $E_{T,\mathrm{B}}^{\mathrm{jet1}} > 8$ GeV, $E_{T,\mathrm{B}}^{\mathrm{jet2,3}} > 5$ GeV and $-2 < \eta_{\mathrm{B}}^{\mathrm{jet}} < 1.5$.

A. Color components and the triple-gluon vertex

Normalized differential three-jet cross sections at $\mathcal{O}(\alpha_s^2)$ of the individual color components from Eq. (1), $\sigma_A, \ldots, \sigma_D$, were calculated using the programs described in Sec. VII and are shown separately in Fig. 10 for γp and in Figs. 11 and 12 for NC DIS with $Q^2 > 125(500 < Q^2 < 5000)$ GeV^2 as functions of the angular variables. In these and subsequent figures, the predictions were obtained by integrating over the same bins as for the data. The curves shown are a result of a cubic spline interpolation, except in the case of $\eta_{\mathrm{max}}^{\mathrm{jet}}$, for which a linear interpolation was used.

The component which contains the contribution from the TGV in quark-induced processes, σ_B, has a very different shape than the other components for all the angular variables considered, except for $\eta_{\mathrm{max}}^{\mathrm{jet}}$. The other components have distributions in β_{KSW} and θ_H that are similar and are best separated by the distribution of α_{23} in γp. In NC DIS with $500 < Q^2 < 5000$ GeV^2, the different color

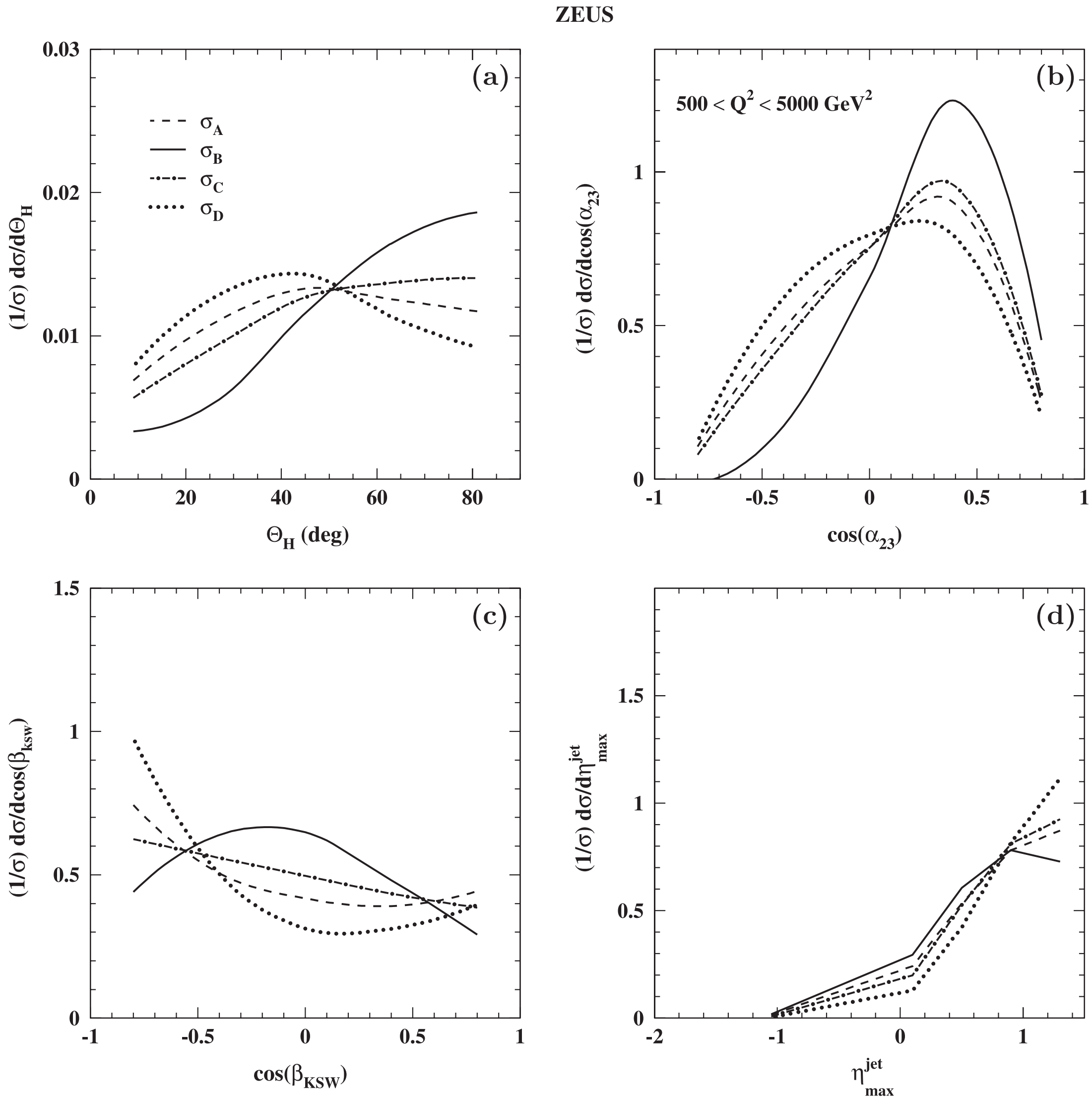

FIG. 12. Predicted normalized differential ep cross sections for three-jet production in NC DIS at $\mathcal{O}(\alpha_s^2)$ integrated over $E_{T,\mathrm{B}}^{\mathrm{jet1}} > 8$ GeV, $E_{T,\mathrm{B}}^{\mathrm{jet2,3}} > 5$ GeV, and $-2 < \eta_{\mathrm{B}}^{\mathrm{jet}} < 1.5$ in the kinematic region given by $500 < Q^2 < 5000$ GeV^2 and $|\cos\gamma_h| < 0.65$ as functions of (a) θ_H, (b) $\cos(\alpha_{23})$, (c) $\cos(\beta_{\mathrm{KSW}})$, and (d) $\eta_{\mathrm{max}}^{\mathrm{jet}}$. Other details are as in the caption to Fig. 10. These calculations do not include corrections for hadronization effects.

components as functions of θ_H and β_{KSW} also display different shapes. In particular, the σ_D component, which also contains a TGV, shows a distinct shape for these distributions. This demonstrates that the three-jet angular correlations studied show sensitivity to the different color components.

In γp (NC DIS: $Q^2 > 125$ GeV^2, $500 < Q^2 < 5000$ GeV^2), the SU(3)-based predictions for the relative contribution of each color component to the total cross section are: (A) 0.13(0.23, 0.30), (B) 0.10 (0.13, 0.14), (C) 0.45(0.39, 0.35) and (D) 0.32(0.25, 0.21). Therefore, the overall contribution from the diagrams that involve a TGV, B and D, amounts to 42(38, 35)% in SU(3).

B. Three-jet cross sections in γp

The integrated three-jet cross section in γp in the kinematic range considered was measured to be

$$\sigma_{ep\to 3\mathrm{jets}} = 14.59 \pm 0.34(\mathrm{stat.})^{+1.25}_{-1.31}(\mathrm{syst.})\mathrm{pb}.$$

The predicted $\mathcal{O}(\alpha_s^2)$ integrated cross section, which is the lowest order for this process and contains only direct processes, is $8.90^{+2.01}_{-2.92}$ pb.

The measured normalized differential three-jet cross sections are presented in Fig. 13 and Tables I, II, and III as functions of θ_H, $\cos(\alpha_{23})$, and $\cos(\beta_{\mathrm{KSW}})$. The measured cross section shows a peak at $\theta_H \approx 60°$, increases as $\cos(\alpha_{23})$ increases, and shows a broad peak in the range of $\cos(\beta_{\mathrm{KSW}})$ between -0.5 to 0.1.

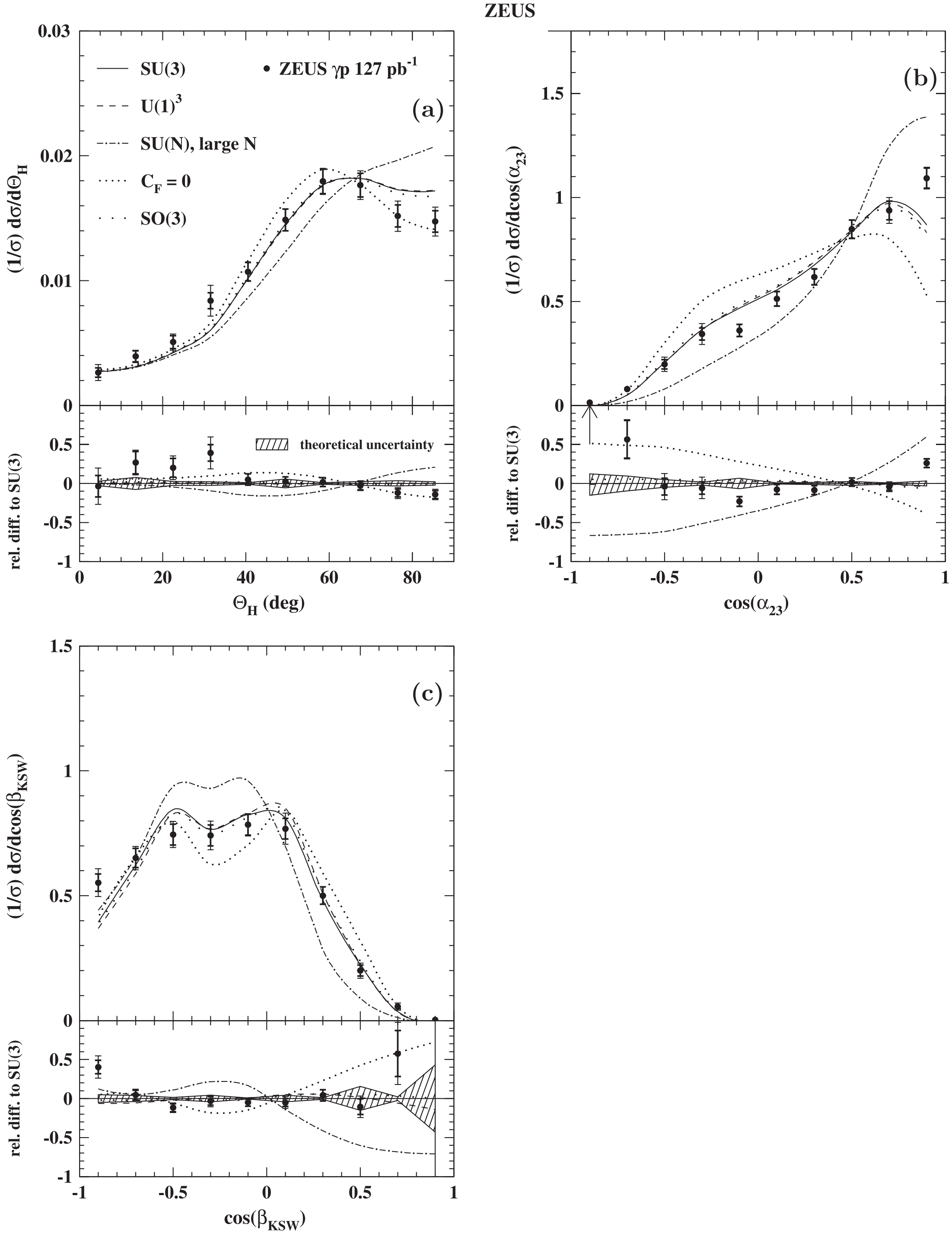

FIG. 13. Measured normalized differential ep cross sections for three-jet photoproduction (dots) integrated over $E_T^{\rm jet} > 14$ GeV and $-1 < \eta^{\rm jet} < 2.5$ in the kinematic region defined by $Q^2 < 1$ GeV^2, $0.2 < y < 0.85$, and $x_\gamma^{\rm obs} > 0.8$ as functions of (a) θ_H, (b) $\cos(\alpha_{23})$, and (c) $\cos(\beta_{\rm KSW})$. The data points are plotted at the bin centers. The inner error bars represent the statistical uncertainties of the data, and the outer error bars show the statistical and systematic uncertainties added in quadrature. For comparison, the $\mathcal{O}(\alpha_s^2)$ calculations for direct-photon processes based on $SU(3)$ (solid lines), $U(1)^3$ (dashed lines), $SU(N)$ in the limit of large N (dot-dashed lines), $C_F = 0$ (short-spaced dotted lines), and $SO(3)$ (long-spaced dotted lines) are included. The lower part of the figures displays the relative difference to the calculations based on $SU(3)$ and the hatched band shows the relative uncertainty of this calculation.

TABLE I. Normalized differential ep cross section for three-jet photoproduction integrated over $E_T^{\rm jet} > 14$ GeV and $-1 < \eta^{\rm jet} < 2.5$ in the kinematic region defined by $Q^2 < 1$ GeV^2, $0.2 < y < 0.85$, and $x_\gamma^{\rm obs} > 0.8$ as a function of θ_H. The statistical and systematic uncertainties are shown separately. The multiplicative corrections for hadronization effects to be applied to the parton-level QCD differential cross section, $C_{\rm had}$, are shown in the last column.

θ_H bin (deg)	$(1/\sigma)d\sigma/d\theta_H$	$\delta_{\rm stat}$	$\delta_{\rm syst}$	$C_{\rm had}$
0, 9	0.00264	0.00038	± 0.00052	0.93
9, 18	0.00393	0.00044	± 0.00021	0.94
18, 27	0.00507	0.00051	$^{+0.00040}_{-0.00039}$	1.00
27, 36	0.00838	0.00064	$^{+0.00105}_{-0.00104}$	0.93
36, 45	0.01071	0.00075	± 0.00023	0.96
45, 54	0.01486	0.00087	$^{+0.00021}_{-0.00016}$	0.94
54, 63	0.01795	0.00098	$^{+0.00036}_{-0.00035}$	0.95
63, 72	0.01765	0.00095	± 0.00062	0.94
72, 81	0.01517	0.00088	$^{+0.00081}_{-0.00084}$	0.94
81, 90	0.01473	0.00086	$^{+0.00075}_{-0.00077}$	0.96

TABLE II. Normalized differential ep cross section for three-jet photoproduction integrated over $E_T^{\rm jet} > 14$ GeV and $-1 < \eta^{\rm jet} < 2.5$ in the kinematic region defined by $Q^2 < 1$ GeV^2, $0.2 < y < 0.85$, and $x_\gamma^{\rm obs} > 0.8$ as a function of $\cos(\alpha_{23})$. Other details as in the caption to Table I.

$\cos(\alpha_{23})$ bin	$(1/\sigma)d\sigma/d\cos(\alpha_{23})$	$\delta_{\rm stat}$	$\delta_{\rm syst}$	$C_{\rm had}$
-1, -0.8	0.0138	0.0046	± 0.00042	1.04
-0.8, -0.6	0.078	0.012	$^{+0.004}_{-0.003}$	0.96
-0.6, -0.4	0.198	0.022	$^{+0.026}_{-0.027}$	0.95
-0.4, -0.2	0.343	0.029	$^{+0.041}_{-0.040}$	0.93
-0.2, 0	0.360	0.029	± 0.010	0.97
0, 0.2	0.512	0.034	$^{+0.014}_{-0.013}$	0.98
0.2, 0.4	0.618	0.037	$^{+0.015}_{-0.016}$	1.00
0.4, 0.6	0.847	0.044	± 0.013	0.99
0.6, 0.8	0.937	0.045	$^{+0.043}_{-0.042}$	0.99
0.8, 1	1.092	0.049	$^{+0.019}_{-0.018}$	1.02

TABLE III. Normalized differential ep cross section for three-jet photoproduction integrated over $E_T^{\rm jet} > 14$ GeV and $-1 < \eta^{\rm jet} < 2.5$ in the kinematic region defined by $Q^2 < 1$ GeV^2, $0.2 < y < 0.85$ and $x_\gamma^{\rm obs} > 0.8$ as a function of $\cos(\beta_{\rm KSW})$. Other details as in the caption to Table I.

$\cos(\beta_{\rm KSW})$ bin	$(1/\sigma)d\sigma/d\cos(\beta_{\rm KSW})$	$\delta_{\rm stat}$	$\delta_{\rm syst}$	$C_{\rm had}$
-1, -0.8	0.552	0.035	± 0.044	0.97
-0.8, -0.6	0.651	0.039	± 0.026	0.99
-0.6, -0.4	0.745	0.042	$^{+0.032}_{-0.031}$	0.97
-0.4, -0.2	0.741	0.042	± 0.039	0.93
-0.2, 0	0.784	0.042	$^{+0.014}_{-0.016}$	0.96
0, 0.2	0.768	0.042	± 0.046	0.95
0.2, 0.4	0.500	0.034	± 0.005	0.94
0.4, 0.6	0.200	0.022	± 0.021	0.95
0.6, 0.8	0.056	0.010	$^{+0.010}_{-0.009}$	0.85
0.8, 1	0.0029	0.0015	± 0.0037	0.74

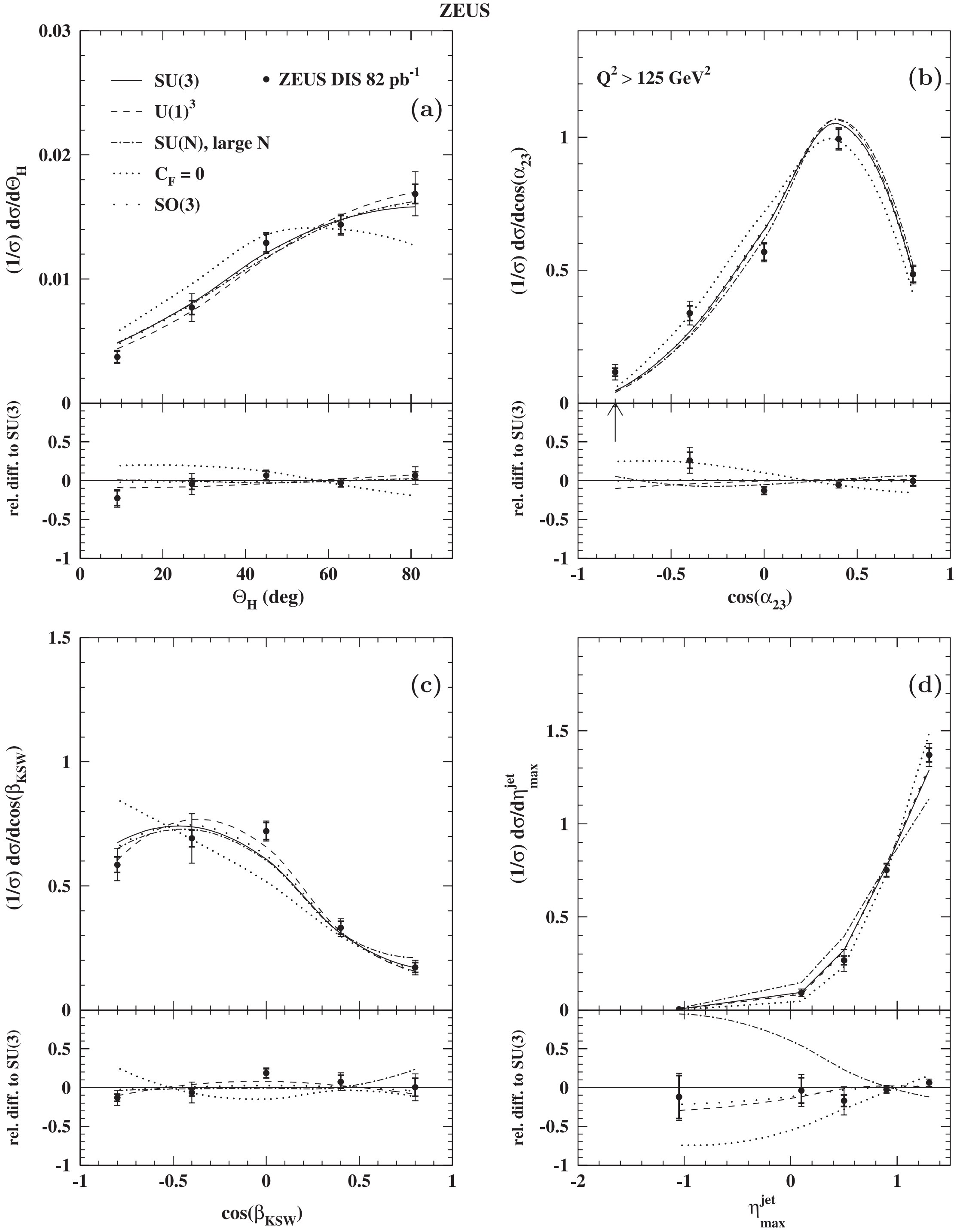

FIG. 14. Measured normalized differential ep cross sections for three-jet production in NC DIS (dots) integrated over $E_{T,B}^{jet1} > 8$ GeV, $E_{T,B}^{jet2,3} > 5$ GeV, and $-2 < \eta_B^{jet} < 1.5$ in the kinematic region given by $Q^2 > 125$ GeV^2 and $|\cos\gamma_h| < 0.65$ as functions of (a) θ_H, (b) $\cos(\alpha_{23})$, (c) $\cos(\beta_{KSW})$, and (d) η_{max}^{jet}. Other details as in the caption to Fig. 13.

C. Three-jet cross sections in NC DIS

The integrated three-jet cross sections in NC DIS for $Q^2 > 125$ GeV^2 and $500 < Q^2 < 5000$ GeV^2 were measured to be

$$\sigma_{ep\to 3\text{jets}} = 11.48 \pm 0.35(\text{stat.}) \pm 1.98(\text{syst.})\text{pb}$$

and

$$\sigma_{ep\to 3\text{jets}} = 5.73 \pm 0.26(\text{stat.}) \pm 0.60(\text{syst.})\text{pb}.$$

The predicted $\mathcal{O}(\alpha_s^3)$ integrated cross sections are 14.14 ± 3.40 pb and 6.86 ± 1.77 pb for the two kinematic regions, respectively.

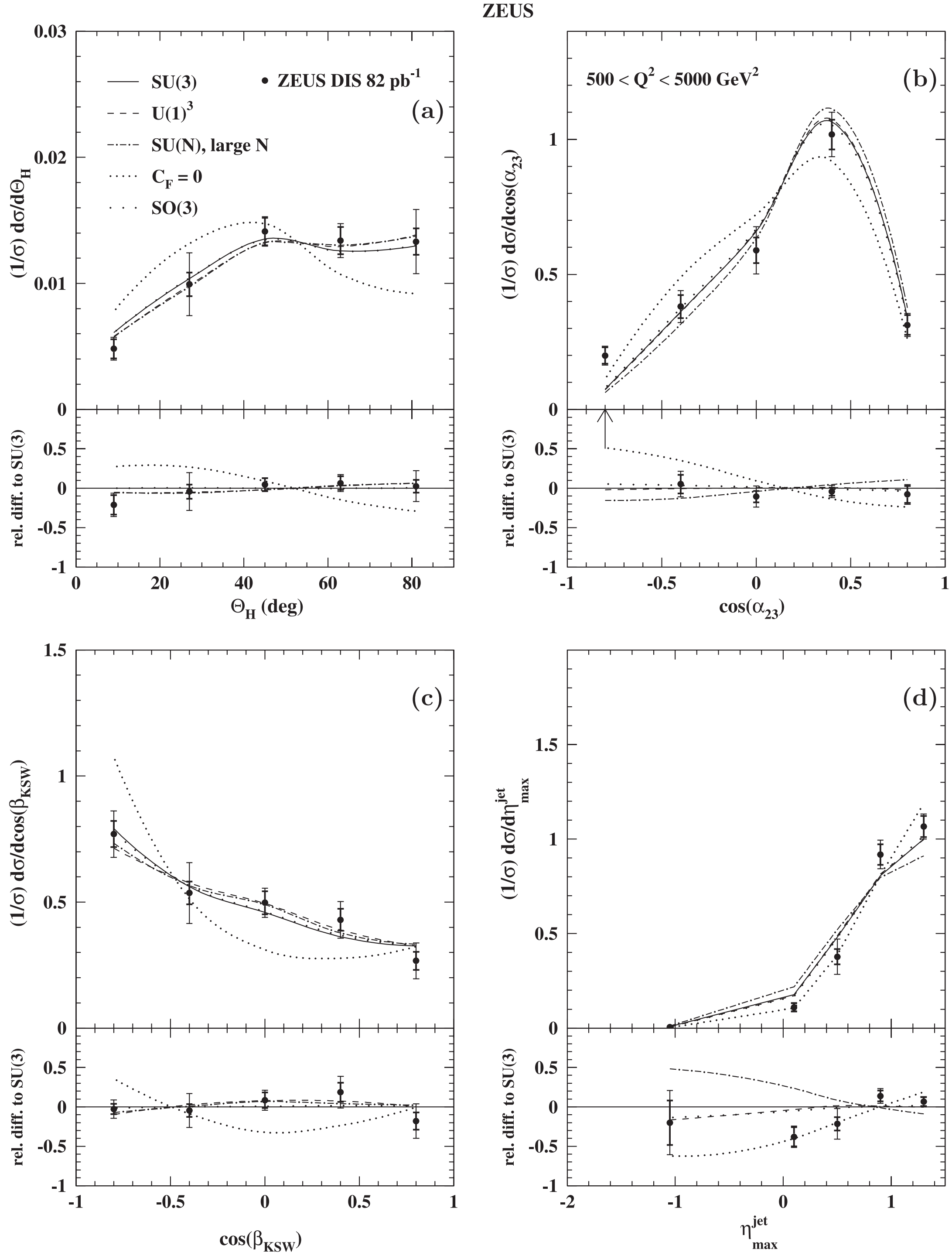

FIG. 15. Measured normalized differential ep cross sections for three-jet production in NC DIS (dots) integrated over $E_{T,\mathrm{B}}^{\mathrm{jet1}} > 8$ GeV, $E_{T,\mathrm{B}}^{\mathrm{jet2,3}} > 5$ GeV, and $-2 < \eta_{\mathrm{B}}^{\mathrm{jet}} < 1.5$ in the kinematic region given by $500 < Q^2 < 5000$ GeV^2 and $|\cos\gamma_h| < 0.65$ as functions of (a) θ_H, (b) $\cos(\alpha_{23})$, (c) $\cos(\beta_{\mathrm{KSW}})$, and (d) $\eta_{\max}^{\mathrm{jet}}$. Other details are as in the caption to Fig. 13.

The measured normalized differential three-jet cross sections in NC DIS for $Q^2 > 125$ GeV^2 and $500 < Q^2 < 5000$ GeV^2 are presented in Figs. 14 and 15, respectively, as functions of θ_H, $\cos(\alpha_{23})$, $\cos(\beta_{\mathrm{KSW}})$ and $\eta_{\max}^{\mathrm{jet}}$ (see Tables IV, V, VI, and VII). The measured cross sections have similar shapes in the two kinematic regions considered, except for the distribution as a function of $\cos(\beta_{\mathrm{KSW}})$: the cross section decreases as $\cos(\beta_{\mathrm{KSW}})$

TABLE IV. Normalized differential ep cross section for three-jet production in NC DIS integrated over $E_{T,\mathrm{B}}^{\mathrm{jet1}} > 8$ GeV, $E_{T,\mathrm{B}}^{\mathrm{jet2,3}} > 5$ GeV, and $-2 < \eta_{\mathrm{B}}^{\mathrm{jet}} < 1.5$ in the kinematic region given by $|\cos\gamma_h| < 0.65$ and $Q^2 > 125$ GeV^2 or $500 < Q^2 < 5000$ GeV^2 as a function of θ_H. The multiplicative corrections applied to the differential measured cross section to correct for QED radiative effects, C_{QED}, is also shown. The multiplicative corrections for hadronization effects and the Z^0-exchange contribution to be applied to the parton-level QCD differential cross section, C_{had}, are shown in the last column. Other details as in the caption to Table I.

θ_H bin (deg)	$(1/\sigma)d\sigma/d\theta_H$	δ_{stat}	δ_{syst}	C_{QED}	C_{had}
		$Q^2 > 125$ GeV^2			
0, 18	0.00372	0.00046	±0.00031	0.92	0.89
18, 36	0.00770	0.00056	±0.00095	0.88	0.90
36, 54	0.01291	0.00072	±0.00045	0.96	0.84
54, 72	0.01438	0.00074	±0.00042	1.00	0.84
72, 90	0.01686	0.00077	±0.00160	0.99	0.84
		$500 < Q^2 < 5000$ GeV^2			
0, 18	0.00481	0.00076	±0.00048	0.88	0.92
18, 36	0.00993	0.00094	±0.00231	0.95	0.96
36, 54	0.0141	0.0011	±0.0004	0.92	0.97
54, 72	0.0134	0.0011	±0.0008	1.03	0.89
72, 90	0.0133	0.0011	±0.0023	0.96	0.94

TABLE V. Normalized differential ep cross section for three-jet production in NC DIS integrated over $E_{T,\mathrm{B}}^{\mathrm{jet1}} > 8$ GeV, $E_{T,\mathrm{B}}^{\mathrm{jet2,3}} > 5$ GeV, and $-2 < \eta_{\mathrm{B}}^{\mathrm{jet}} < 1.5$ in the kinematic region given by $|\cos\gamma_h| < 0.65$ and $Q^2 > 125$ GeV^2 or $500 < Q^2 < 5000$ GeV^2 as a function of $\cos(\alpha_{23})$. Other details as in the caption to Table IV.

$\cos(\alpha_{23})$ bin	$(1/\sigma)d\sigma/d\cos(\alpha_{23})$	δ_{stat}	δ_{syst}	C_{QED}	C_{had}
		$Q^2 > 125$ GeV^2			
−1, −0.6	0.117	0.015	±0.025	0.96	0.90
−0.6, −0.2	0.338	0.028	±0.035	1.01	0.70
−0.2, 0.2	0.568	0.032	±0.018	0.90	0.78
0.2, 0.6	0.993	0.037	±0.021	0.95	0.88
0.6, 1	0.484	0.030	±0.020	1.02	1.01
		$500 < Q^2 < 5000$ GeV^2			
−1, −0.6	0.199	0.030	±0.018	1.04	0.83
−0.6, −0.2	0.381	0.043	±0.041	0.97	0.75
−0.2, 0.2	0.589	0.047	±0.074	0.92	0.83
0.2, 0.6	1.018	0.055	±0.061	0.95	1.07
0.6, 1	0.313	0.036	±0.022	0.97	1.16

TABLE VI. Normalized differential ep cross section for three-jet production in NC DIS integrated over $E_{T,\mathrm{B}}^{\mathrm{jet1}} > 8$ GeV, $E_{T,\mathrm{B}}^{\mathrm{jet2,3}} > 5$ GeV, and $-2 < \eta_{\mathrm{B}}^{\mathrm{jet}} < 1.5$ in the kinematic region given by $|\cos\gamma_h| < 0.65$ and $Q^2 > 125$ GeV^2 or $500 < Q^2 < 5000$ GeV^2 as a function of $\cos(\beta_{\mathrm{KSW}})$. Other details as in the caption to Table IV.

$\cos(\beta_{\mathrm{KSW}})$ bin	$(1/\sigma)d\sigma/d\cos(\beta_{\mathrm{KSW}})$	δ_{stat}	δ_{syst}	C_{QED}	C_{had}
		$Q^2 > 125$ GeV^2			
−1, −0.6	0.585	0.031	±0.057	0.92	0.95
−0.6, −0.2	0.691	0.034	±0.094	0.99	0.88
−0.2, 0.2	0.721	0.035	±0.020	1.01	0.85
0.2, 0.6	0.332	0.026	±0.025	0.92	0.74
0.6, 1	0.171	0.020	±0.022	0.93	0.71
		$500 < Q^2 < 5000$ GeV^2			
−1, −0.6	0.770	0.052	±0.076	0.94	1.04
−0.6, −0.2	0.536	0.045	±0.112	0.93	0.97
−0.2, 0.2	0.497	0.045	±0.037	1.01	0.94
0.2, 0.6	0.430	0.044	±0.058	1.01	0.84
0.6, 1	0.267	0.036	±0.061	0.89	0.78

TABLE VII. Normalized differential ep cross section for three-jet production in NC DIS integrated over $E_{T,\mathrm{B}}^{\mathrm{jet1}} > 8$ GeV, $E_{T,\mathrm{B}}^{\mathrm{jet2,3}} > 5$ GeV, and $-2 < \eta_{\mathrm{B}}^{\mathrm{jet}} < 1.5$ in the kinematic region given by $|\cos\gamma_h| < 0.65$ and $Q^2 > 125$ GeV^2 or $500 < Q^2 < 5000$ GeV^2 as a function of $\eta_{\mathrm{max}}^{\mathrm{jet}}$. Other details as in the caption to Table IV.

$\eta_{\mathrm{max}}^{\mathrm{jet}}$ bin	$(1/\sigma)d\sigma/d\eta_{\mathrm{max}}^{\mathrm{jet}}$	δ_{stat}	δ_{syst}	C_{QED}	C_{had}
		$Q^2 > 125$ GeV^2			
−2, −0.1	0.0042	0.0013	±0.0006	1.07	0.61
−0.1, 0.3	0.092	0.016	±0.012	1.17	0.77
0.3, 0.7	0.267	0.024	±0.054	0.96	0.81
0.7, 1.1	0.751	0.034	±0.016	0.93	0.83
1.1, 1.5	1.370	0.038	±0.048	0.96	0.88
		$500 < Q^2 < 5000$ GeV^2			
−2, −0.1	0.0059	0.0021	±0.0022	1.14	0.62
−0.1, 0.3	0.110	0.022	±0.011	0.96	0.77
0.3, 0.7	0.378	0.040	±0.084	0.96	0.86
0.7, 1.1	0.918	0.054	±0.052	0.93	0.93
1.1, 1.5	1.066	0.056	±0.035	0.98	1.00

increases for $500 < Q^2 < 5000$ GeV^2 whereas for $Q^2 > 125$ GeV^2 it shows an approximately constant behavior for $-1 < \cos(\beta_{\mathrm{KSW}}) < 0.25$. The measured cross section as a function of $\cos(\alpha_{23})$ peaks around 0.5 and increases as θ_H and $\eta_{\mathrm{max}}^{\mathrm{jet}}$ increase.

D. Comparison to fixed-order calculations

Calculations at $\mathcal{O}(\alpha_s^2)$ in which each color contribution in Eq. (1) was weighted according to the color factors predicted by SU(3) ($C_F = 4/3$, $C_A = 3$ and $T_F = 1/2$) are compared to the measurements in Figs. 13–17. The theoretical uncertainties are shown in Figs. 13, 16, and 17, as hatched bands. Since the calculations are normalized to unity, the uncertainties are correlated among the points; this correlation is partially responsible for the pulsating pattern exhibited by the theoretical uncertainties. The predictions based on SU(3) give a reasonable description of the data for all angular correlations. For γp, the predictions do not include resolved processes (see Sec. VII), as calculations separated according to the different color factors are not available. Monte Carlo simulations of such processes show that their contribution is most likely to be different from that of direct processes in the fifth and last bin of $(1/\sigma)(d\sigma/d\cos(\alpha_{23}))$ [see Figs. 5(b) and 13(b)].

To illustrate the sensitivity of the measurements to the color factors, calculations based on different symmetry groups are also compared to the data in Figs. 13 to 15. In these calculations, the color components were combined in such a way as to reproduce the color structure of a theory based on the non-Abelian group SU(N) in the limit of large N ($C_F = 1$, $C_A = 2$ and $T_F = 0$), the Abelian group U(1)^3 ($C_F = 1$, $C_A = 0$ and $T_F = 3$), the non-Abelian group SO (3) ($C_F = 1/3$, $C_A = 3$ and $T_F = 1/3$) and, as an extreme choice, a calculation with $C_F = 0$. The shapes of the distributions predicted by U(1)^3 in γp are very similar to those by SU(3) due to the smallness of the component σ_B and the difficulty to distinguish the component σ_D. In NC DIS, the predictions of U(1)^3 show differences of around 10% with respect to those of SU(3), which are of the same order as the statistical uncertainties. In both regimes, the data clearly disfavor a theory based on SU(N) in the limit of large N or on $C_F = 0$.

Figs. 16 and 17 show the measurements in NC DIS compared to the predictions of QCD at $\mathcal{O}(\alpha_s^2)$ and $\mathcal{O}(\alpha_s^3)$. This comparison provides a very stringent test of pQCD. The $\mathcal{O}(\alpha_s^3)$ calculations give a very good description of the data. In particular, a significant improvement in the description of the data can be observed for the first bin of the α_{23} distribution [Figs. 16(b) and 17(b)].

XI. SUMMARY AND CONCLUSIONS

Measurements of angular correlations in three-jet γp and NC DIS were performed in ep collisions at HERA using up to 127 pb^{-1} of data collected with the ZEUS detector. The cross sections refer to jets identified with the k_T cluster algorithm in the longitudinally invariant inclusive mode and selected with $E_T^{\mathrm{jet}} > 14$ GeV and $-1 < \eta^{\mathrm{jet}} < 2.5$ (γp) and $E_{T,\mathrm{B}}^{\mathrm{jet1}} > 8$ GeV, $E_{T,\mathrm{B}}^{\mathrm{jet2,3}} > 5$ GeV and $-2 < \eta_{\mathrm{B}}^{\mathrm{jet}} < 1.5$ (NC DIS). The measurements were made in the kinematic regions defined by $Q^2 < 1$ GeV^2, $0.2 < y < 0.85$ and $x_\gamma^{\mathrm{obs}} > 0.8$ (γp) and $Q^2 > 125$ GeV^2 or $500 < Q^2 < 5000$ GeV^2 and $|\cos\gamma_h| < 0.65$ (NC DIS). Normalized differential three-jet cross sections were measured as functions of θ_H, α_{23}, β_{KSW} and $\eta_{\mathrm{max}}^{\mathrm{jet}}$.

The color configuration of the strong interaction was studied for the first time in ep collisions using the angular correlations in three-jet events. While the

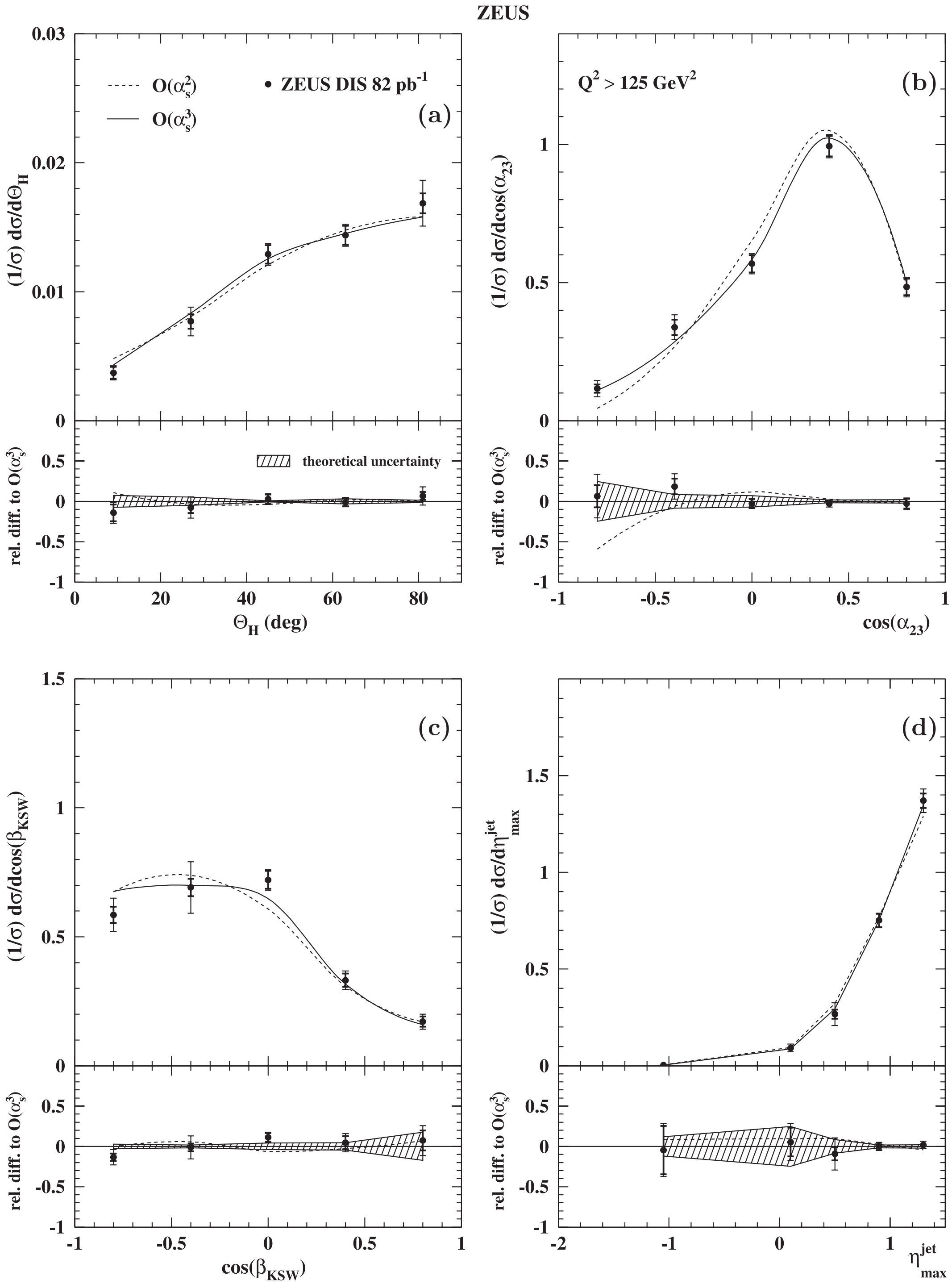

FIG. 16. Measured normalized differential ep cross sections for three-jet production in NC DIS (dots) integrated over $E_{T,\mathrm{B}}^{\mathrm{jet1}} > 8$ GeV, $E_{T,\mathrm{B}}^{\mathrm{jet2,3}} > 5$ GeV, and $-2 < \eta_{\mathrm{B}}^{\mathrm{jet}} < 1.5$ in the kinematic region given by $Q^2 > 125$ GeV^2 and $|\cos\gamma_h| < 0.65$ as functions of (a) θ_H, (b) $\cos(\alpha_{23})$, (c) $\cos(\beta_{\mathrm{KSW}})$, and (d) $\eta_{\mathrm{max}}^{\mathrm{jet}}$. For comparison, the $\mathcal{O}(\alpha_s^2)$ (dashed lines) and $\mathcal{O}(\alpha_s^3)$ (solid lines) QCD calculations are also included. The hatched band displays the relative theoretical uncertainty of the $\mathcal{O}(\alpha_s^3)$ calculation. Other details are as in the caption to Fig. 13.

extraction of the color factors will require the full analysis of all HERA data and complete $\mathcal{O}(\alpha_s^3)$ calculations, the studies presented in this paper demonstrate the potential of the method.

Fixed-order calculations separated according to the color configurations were used to study the sensitivity of the angular correlations to the underlying gauge structure. The predicted distributions of θ_H, α_{23}, and

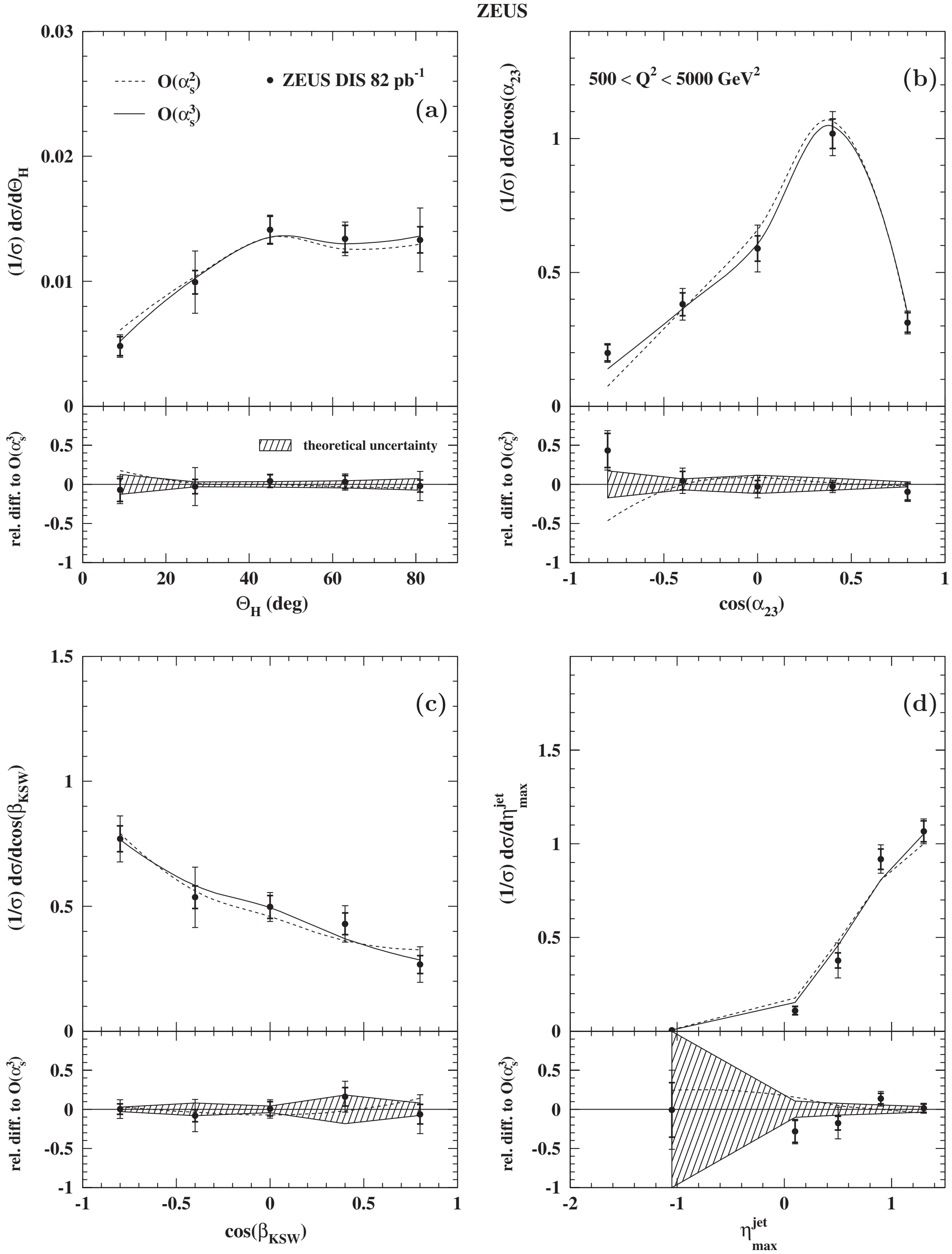

FIG. 17. Measured normalized differential ep cross sections for three-jet production in NC DIS (dots) integrated over $E_{T,B}^{jet1} > 8$ GeV, $E_{T,B}^{jet2,3} > 5$ GeV, and $-2 < \eta_B^{jet} < 1.5$ in the kinematic region given by $500 < Q^2 < 5000$ GeV^2 and $|\cos\gamma_h| < 0.65$ as functions of (a) θ_H, (b) $\cos(\alpha_{23})$, (c) $\cos(\beta_{KSW})$, and (d) η_{max}^{jet}. For comparison, the $\mathcal{O}(\alpha_s^2)$ (dashed lines) and $\mathcal{O}(\alpha_s^3)$ (solid lines) QCD calculations are also included. The hatched band displays the relative theoretical uncertainty of the $\mathcal{O}(\alpha_s^3)$ calculation. Other details are as in the caption to Fig. 13.

β_{KSW} clearly isolate the contribution from the triple-gluon coupling in quark-induced processes while η_{max}^{jet} isolates the contribution from gluon-induced processes. The variable α_{23} provides additional separation for the other contributions. Furthermore, the studies performed demonstrate that normalized cross sections in three-jet ep collisions have reduced sensitivity to the assumed evolution of the PDFs and the running of α_s.

The data clearly disfavor theories based on SU(N) in the limit of large N or $C_F = 0$. Differences between SU(3) and $U(1)^3$ are smaller than the current statistical uncertainties. The measurements are found to be consistent with the admixture of color configurations as predicted by SU(3). The $\mathcal{O}(\alpha_s^3)$ calculations give a very good description of the NC DIS data.

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