New Essays on Leibniz Reception

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Peano and His School
Between Leibniz and Couturat:
The Influence in Mathematics and
in International Language

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1. The influence of Leibniz on Giuseppe Peano

There are many significant testimonies to the legacy of Leibniz's thinking on Peano: at least two hundred textual references and quotations from Leibnizian writings, which can be identified in his output.[1] The influence of some of these suggestions in various mathematical disciplines can be recognised, above all, where Peano and his students consider and present their work as being designed to complete or to extend some of the lines of research indicated by Leibniz. Thus certain *Leitmotive* emerge: the desire to situate the developments of ideography in a historical context, finding in Leibniz a precursor of their work; the sharing of Leibniz's philanthropic ideals and, in particular, of his conviction of the usefulness of an international language to facilitate collaboration and communication among peoples; the inheritance of an interest in the history of mathematics, with a view to establishing the paternity of concepts, methods and symbols and lastly, the desire to take from Leibniz's work cues for the development of new results of pure and applied mathematics.

Nonetheless, if until 1894 the references are fragmentary and sporadic, from that date they become frequent in the works of Peano and of his collaborators Giovanni Vacca and Giovanni Vailati. The reason for this can be traced to the 'discovery' of algorithmic logic, to its use in mathematics and to the beginning of the plan of the *Formulaire*.

In fact, one feature of Peano's mathematical and linguistic studies is his resorting to the original sources, numerous excerpts from which he carefully transcribed. Attention to the philological aspect and preference for the faithful critical editions are aspects that are

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[1] The *corpus* of quotations is organized around a single conceptual axis, which can be recognized in the semantic link between sign, algorithm and language.

continually repeated by Peano. It is important, then, to ask what were the sources of Leibnizian literature that he consulted. Peano uses three different editions of Leibniz's works. Specifically, the results of Logic, Geometry and Arithmetic are taken from the *Opera philosophica* edited by Erdmann (1840) and, after 1898, are gathered from unpublished manuscripts examined by Vacca in Hanover in summer 1899. For the studies on Binary Arithmetic Peano makes reference to the oldest edition of Leibniz, the one edited by Louis Dutens (1778). Lastly, he makes systematic use of the volumes of the *Mathematische* and of the *Philosophische Schriften* published by Karl Immanuel Gerhardt. Peano does not carry out a philosophical and philological critique of Leibniz's works, but constantly updates his knowledge of the contemporary literature about the German mathematician and philosopher. For example, he follows Couturat's studies with interest, appreciating the two volumes *La Logique de Leibniz* and *Opuscules et fragments inédits de Leibniz*.[2] A further source of information must finally be identified in the writings of Ernst Schröder, whom Peano met at the International Congress of Mathematicians in Zurich in 1897 and who, on that occasion, defined Leibniz as a precursor of pasigraphy.[3] As regards the bibliography of Leibniz, in Peano's Library[4] the edition of the works by Dutens, the volumes by Couturat and a number of little-known extracts, among them the *Esquisse d'une grammaire de langue conforme aux idées de Leibniz* by V. Hely, have come to light.[5]

2. The *Formulaire* and the 'Leibnizian dream'

In 1894 Peano and a team of colleagues, collaborators and students launched an ambitious project, the *Formulaire de mathématiques*. At first it was simply a *Collection of Formulas* (*Raccolta di Formule*), but it soon became the engine of Peano's entire research activity, and of that of his School. Conceived in the threefold nature of encyclopedia of the elementary mathematics of earlier centuries, translated into logical language, as a teaching handbook, and as a repertoire of research programs, the *Formulaire* soon became an object of fierce debate, at national and international level.

It is in the *Formulaire* above all that Peano's debt to Leibniz's ideas about the *Encyclopedia generale* and the *Characteristica universalis* can be perceived. From 1669, Leibniz describes in numerous notes his plan for a universal dictionary of all knowledge, the compilation of which is made possible by the use of the *characteristica*:

[2] L. COUTURAT 1901, 1903.

[3] E. SCHRÖDER 1898, *Über Pasigraphie, ihren gegenwärtigen Stand und die pasigraphische Bewegung in Italien*, *Verhandlungen des Ersten Internationalen Mathematiker-Kongresses in Zürich vom 9 bis 11 August* 1897, Leipzig, Teubner, p. 147–162, in particular p. 147–148.

[4] It is possible today to have very exact knowledge of the sources consulted by Peano thanks to the discovery, in February 2007, of his personal Library, which had been believed lost. Cf. website http://www.peano2008.unito.it/ *Catalogo della Biblioteca di Peano* and E. LUCIANO 2007, *La biblioteca "ritrovata" di Giuseppe Peano*, in L. BONO, S. CHIAVERO, D. DAMIANO (eds.), *Rendiconti Cuneo*, Cuneo, Nerosubianco, p. 184–188. This is an important heritage, comprising approximately 1300 volumes and more than 2500 documents. Many of the volumes are peppered with notes handwritten by Peano with observations, corrections, notes on mathematics, history and bibliographic cross-references.

[5] V. HELY, *Esquisse d'une grammaire de la langue internationale*, Langre, M. Berret, 1905.

"Consilium de Encyclopaedia condenda, velut Inventario cognitionis humanae condendo in quod referantur utiliora, certiora, universaliora et magis sufficientia pro reliquis omnibus determinandis; additis semper rationibus eorum quae fiunt originibusque inventionibus. Quod opus non nimis erit prolixum [...]. Hujus operas usus erit ut occurratur confusioni librorum eadem repetentium, paucaque interdum utilia sub magna farragine obruentium, si sit Basis aliqua ad quam omnia imposterum nova per modum supplementorum referri possint."[6]

The distinctive features of such an encyclopedia are taken up by Peano, who liked to present the *Formulaire* as the fulfillment of Leibniz's dream of the construction of the universal encyclopedia, saying that:

«après avoir tombé depuis longtemps dans l'oubli, est maintenant réalisé, grâce a la logique mathématique, la nouvelle science qu'a pour objet les propriétés des opérations et des relations et donc les résultats sont merveilleux, et bien dignes des éloges de Leibniz à la science qu'il avait deviné».[7]

The *Formulaire* is, in any case, a work that created a rift in the publishing panorama of the time, so it is not surprising that Peano and his collaborators should look for its natural 'archetype', with a view to situating this treatise in a correct historical, mathematical and philosophical perspective.

In his prefaces to the works on logic, Peano repeatedly states that Leibniz is the mathematician who has understood the most general problem ever posed and faced in the course of the centuries, namely the problem of developing

«pendant toute sa vie, depuis son premier travail jusqu'à ses dernières lettres, une spécieuse générale ou une manière de langue ou d'écriture universelle, où toutes les vérités de raison seraient réduites à une façon de calcul.»[8]

The *Spécieuse générale* is conceived by Leibniz as a support for the mind and the memory and it made possible the discovery of the gaps and inaccuracies in the deductive procedures, since «sophismes et paralogismes ne sont rien d'autre que solécismes et barbarismes».[9]

Peano's ideography, analogously, is as expressive and economical as possible and serves, so to speak, as a 'filo d'Arianna' for thinking. Algorithmic logic is a set of procedures which transform reasoning into calculus and, at the same time, it is a tool that brings reflection closer to writing. Symbols, as Leibniz had already recommended, are not just abbreviations but represent ideas, hence «ideography is not tachigraphy».[10] Peano rediscovered Leibniz's reflections on Chinese ideography and made them once more of

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[6]G.W. Leibniz 1679, *Initia et Specimina Scientiae Generalis de instauratione et augmentis scientiarum*, GP, p. 58.

[7]G. Peano 1896b, *Introduction au tome II du "Formulaire...*, RdM, 6, p. 2. Cf. also G. Peano 1891c, *Principii di logica matematica*, RdM, 1, p. 9; 1894g*, *Notations de Logique Mathématique*, p. 3, 52; 1894e, *Un precursore della logica matematica*, RdM, 4, p. 120; 1896b, *Introduction au tome II du "Formulaire de mathématiques"*, RdM, 6, p. 1, 3; 1901b, *Formulaire de Mathématiques*, p. IX.

[8]G. Peano 1896i, *Réponse n.* 719. (*Lausbrachter*), L'intermédiaire des mathématiciens, 3, p. 169.

[9]G.W. Leibniz, A I, 2, p. 240.

[10]G. Peano 1896j, *Studii di logica matematica*, Atti della Reale Accademia delle Scienze di Torino, 32, p. 566.

current interest, for this ideography allows the reciprocity between characters and ideas to be preserved at best. Moreover, ideography not only enables the statement of theorems and definitions in a clear form, but is in general the indispensable tool for analyzing the principles of a theory and identifying its primitive and derived ideas, the axioms and the theorems. For Peano and for Leibniz, the *Characteristica* has consequently a sort of amphibious nature, for it records and at the same time it organizes and produces knowledge. This last aspect is the most problematic. In fact Peano several times states, under the influence of Leibniz, that symbols make the invention of new, elegant theories easy[11] and that:

> «l'utilità principale dei simboli di logica si è che essi facilitano il ragionamento. [...] Perciò il simbolismo è più chiaro; permette di costruire serie di ragionamenti quando l'immaginazione sarebbe interamente inabile a sostenere se stessa senza aiuto simbolico.»[12]

Nevertheless, in Italy and in France, many mathematicians were to continue to maintain that original results cannot be concretely obtained only by means of symbolism and without recourse to intuition and other forms of synthetic or *a priori* reasoning.

In the light of these quotations, the conviction emerges, largely shared by C. Burali-Forti, A. Padoa, G. Vacca, G. Vailati, M. Pieri and U. Cassina, that the new researches on pasigraphy fit harmoniously into a tradition which, after Leibniz, continued with the studies on the algebra of logic, to culminate in the birth of a new discipline.[13] In fact, this conviction would fade markedly with the passing of time. The tumultuous development of mathematics at the end of the 19th century had aroused the wish to found and contextualize in a specific historical perspective a vast mass of results of recent acquisition and for Peano "suddenly, what Leibniz had said about the *Characteristica Universalis* assumed the value of a prophecy".[14] His attempt to present the modern directions of logico-foundational researches as a repercussion of Leibniz's intuitions can, and must, today be broadly revaluated, but it seems to be the result of a carefully thought-out choice.

3. Leibniz's manuscripts in the *Formulaire des Mathématiques*

The *Formulaire* was from the very beginning conceived as a work of collaboration, carried out by a team of mathematicians, historians and secondary-school teachers.

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[11]G.W. LEIBNIZ, *Linguae philosophicae Specimen in geometria edendum*, 1680, A VI, 4 (1677–1690), Teil A, Band 1, Berlin, p. 384–385.

[12]G. PEANO 1915j, *Importanza dei simboli in matematica*, Scientia, 18, p. 170, 172.

[13]Cf. for example C. BURALI-FORTI 1897, *Introduction à la Géométrie différentielle suivant la méthode de H. Grassmann*, Paris, Gauthier-Villars, p. VI; C. BURALI-FORTI 1919, *Logica matematica*, Milano, Hoepli, p. XVII-XVIII; U. CASSINA 1933, *L'oeuvre philosophique de G. Peano*, Revue de Métaphysique et de Morale, 40, p. 489–491; A. PADOA 1933, *Il contributo di G. Peano all'ideografia logica*, Periodico di Matematiche, 4, 13, p. 15–18; M. PIERI 1906, *Uno sguardo al nuovo indirizzo logico-matematico delle scienze deduttive*, Annuario della Università di Catania, p. 394–396; G. VACCA 1946, *Origini della Scienza*, Roma, Partenia, p. 31.

[14]M. MUGNAI 1973, *Leibniz e la logica simbolica*, Firenze, Sansoni, p. 3.

The first edition was printed in 1894–1895, but work for a new edition began immediately, as Peano wrote to F. Klein: «every day is a new part that is translated into symbols».[15]

However, the first edition lacks virtually all the historical indications, which are essential for readers to perceive the origins, developments and links between propositions and theories.

At this time Peano happened to read an article by Vacca, a young student of mathematics at the University of Genoa.[16] In this text, by means of the correspondence between Leibniz and Johann Bernoulli, and thanks to the analysis of the *Nova Algebrae promotio* published by Gerhardt, Vacca shows that the German mathematician was the first to have established the formula which gives the coefficient of any term in the development of the power of a polynomial. Appreciating the norms of the historical research carried out by Vacca – distinguished as it was by the literal transcription of excerpts and precise references to the sources consulted – Peano did not hesitate to write to him, proposing that he collaborate on the *Formulaire*.[17] Thus, in 1894 a relationship was formed between Peano and Vacca, a very solid and long-lasting working and human alliance which is testified to by an intense correspondence. Vacca answered Peano a few days later, accepting his invitation, and commenting on his article:

> «Giunsi a trovare la dimostrazione di Leibniz leggendo le opere matematiche di Leibniz, delle quali è sperabile si faccia una edizione più facilmente consultabile che non quella del Gerhardt, la quale oltre ad essere priva di ogni indice analitico, necessario allorchè si vuole iniziare una qualche ricerca, è poco ordinata e confusa. Forse molte altre utili cognizioni si possono trovare in Leibniz, oggi trascurato e poco letto. Alle note storiche alla parte II § 10 si potrebbe aggiungere in relazione alla P.4 Leibniz 1695 lettere X e XII Joh. Bernoulli, ove si trova l'espressione form[ale] di una potenza qualunque di un polinomio».[18]

In fact, from this time on, the *Formulaire* is enriched by a myriad of references to the works of Leibniz on Logic, Geometry, Arithmetic, Algebra and Analysis.[19] Vacca

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[15]G. Peano to F. Klein, 29.8.1894, M. Segre 1997, *Le lettere di Giuseppe Peano a Felix Klein*, Nuncius. Annali di Storia della Scienza, 12, p. 119–121.

[16]G. Vacca 1894, *Intorno alla prima dimostrazione di un teorema di Fermat,* Bibliotheca Mathematica, 2, p. 46–48.

[17]G. Peano to G. Vacca, 15.5.1894, c. 1r-v, E. Luciano, C.S. Roero (eds.) to appear.

[18][«I came upon Leibniz's proof when reading his mathematical works, of which it is to be hoped there will be made an edition easier to consult than Gerhardt's, which in addition to being quite without an analytical index, necessary when one wants to undertake any research, is rather disorganized and confused. Perhaps much more useful knowledge can be found in Leibniz, today neglected and not much read. To the historical notes on part II § 10 there might be added in relation to P.4 Leibniz 1695 letters X and XII Joh. Bernoulli, where the form[al] expression of any power of a polynomial can be found».] G. Vacca to G. Peano, 31.5.1894, c. 1r-v, E. Luciano, C.S. Roero (eds.) to appear. Cf. also G. Vacca to G. Loria, [may 1894] and G. Loria to G. Vacca, 18.6.1894, P. Nastasi, A. Scimone (eds.) 1995, p. 90–91.

[19]Cf. G. Peano 1895aa, *Formulaire de mathématiques*, p. 1, 2, 3, 4, 25, 127, 128, 129, 132; G. Peano 1899b, *Formulaire des mathématiques*, p. 5, 6, 8, 9, 10, 11, 12, 14, 15, 16, 18, 20, 40, 42, 55, 66, 71, 72, 76, 80, 83, 85, 86, 99, 118, 119, 126, 144, 149, 165, 167, 169, 195; G. Peano 1903f, *Formulaire mathématique*, p. 3, 5, 8–10, 18, 21, 27, 50, 65, 71, 72, 91, 93, 142, 155, 156, 159, 169, 170, 174, 179, 189, 191, 206, 207, 234, 239, 264,

makes a close examination of both Gerhardt's collections and Leibniz's correspondence, progressively annotating the results of his studies.[20] The items gathered were first inserted in the *Additions et corrections* published in the *Revue de mathématiques* of which Peano was director[21]; they are recorded by Peano and by Vacca themselves in their autograph notes made on their personal copies of the *Formulaire*[22] and on the proofs of the treatise and, finally, they were brought together in subsequent editions. Vacca's researches are meticulous and very careful, but they lack orderliness. His collaboration on the edition of the historical notes to the *Formulaire* was greatly appreciated by Peano and further intensified when Vacca was staying in Turin, in the years 1897–1902, becoming Peano's assistant at the University. He was to remain in Turin until 1905, editing most of the historical and bibliographical apparatus of the *Formulaire*.

At first, in collaboration with Vailati, Vacca was occupied with historical notes to the chapter on Logic; subsequently, in 1898, he began to write sections on Arithmetic and, as a result, he studied Leibniz's contributions on the theory of numbers and on binary arithmetic, contributions which at the time were entirely forgotten and neglected by historiography.[23] Both Vacca and Peano gave particular importance to the article *Explication de l'arithmétique binaire*, in which Leibniz provides an interpretation of Fohy's system of hexagrams, as well as illustrating the laws of periodicity in many numerical dyadic progressions. Vacca's interest in this field continued until 1903, when he presented at the Second Congress of Historical Sciences in Rome a talk on the history of binary arithmetic, elaborated thanks to the information gathered in view of the historical notes to the *Formulaire*. In this note he goes back over the history of the dyadic system, starting from Fohy's hexagrams and arriving at the most recent developments of E. Lucas, also mentioning the applications to the technique of the calculating engines.[24] Once again it was to Leibniz that Vacca attributed the most brilliant and fruitful intuitions on the binary system.

265, 291, 306, 318, 358, 375; G. Peano 1908a, *Formulario Mathematico*, p. VII, 3, 4, 16, 17, 56, 61, 62, 92, 94, 129, 146, 223, 224, 256, 258, 263, 264, 277, 336, 342, 343, 395, 431.

[20]Cf. Vacca's manuscripts in the Library of the Dep. of Mathematics of the Turin University, *Fondo Peano-Vacca*, envelopes nn. 19, 25, 32, 33, 34, 35, 36.

[21]G. Peano et alii, 1898d, *Additions et corrections à F_2*, RdM, 6, p. 69, 70, 71, 73. Cf. also G. Vailati 1903, *Aggiunte alle note storiche del Formulario*, RdM, 8, p. 60–63.

[22]Cf. G. Peano 1894g*, *marginalia* p. 3; 1895aa*, *marginalia* p. 1, 2, 3, 88, 127, 128, 129, 141; 1897b*, *marginalia* p. 5, 18, 27, 32, 35, 39; 1899b*, *marginalia* p. 9, 10, 14, 15, 17, 18, 71, 135, 195; 1900a*, *marginalia* p. 19; 1901b*, *marginalia* p. 13, 14; 1903f*, *marginalia* p. 5, 7, 21; 1906g*, *marginalia* p. 32, 52, 124, 223; 1908a*, *marginalia* p. 276, 277, 401, 438.

[23]Under the influence of suggestions from Leibniz, in this period Peano planned and made a shorthand machine which worked on the basis of the binary system. Cf. G. Peano 1898m, *La numerazione binaria applicata alla stenografia*, Atti della Reale Accademia delle Scienze di Torino, 34, p. 47–49. Cf. also E. Luciano, C.S. Roero 2004, *La macchina stenografica di Giuseppe Peano*, Le Culture della Tecnica, AMMA, 15, p. 5–28 and E. Luciano, C.S. Roero 2004, *Dagli esagrammi di Fo-hy all'aritmetica binaria: Leibniz e Peano*, in E. Gallo, L. Giacardi, O. Robutti (eds.), *Conferenze e Seminari* 2003–2004, Torino, Ass. Sub. Mathesis, p. 49–69. The prototype has unfortunately been lost, but three postcards survive, written by Peano to Vacca in binary shorthand. Cf. G. Vacca to G. Peano, 2.11.1898, 28.10.1899 and 20.5.1903, E. Luciano, C.S. Roero, *La macchina stenografica di Giuseppe Peano*, 2004 cit., p. 20–22 and E. Luciano, C.S. Roero 2008, *Giuseppe Peano Matematico e Maestro*, Turin, Dep. of Mathematics, 2008, p. 167.

[24]G. Vacca 1904, *Sulla storia della numerazione binaria*, Atti del II Congresso Internazionale di Scienze Storiche, Roma 1–9.4.1903, v. 12, Roma, Tip. della R. Accademia dei Lincei, p. 63–67.

The second edition of the *Formulaire de Mathématiques*, published in 1898–99, is a key moment for the study of Leibniz's writings in the School of Peano because, when he came to write its historical notes, Vacca realized that there were some gaps in Gerhardt's *Schriften*. The ruling for the compilation of the apparatus to the *Formulaire* was very strict[25]. Peano had established that it was not sufficient to simply state that a certain result is found in a particular author or work. The exact place had to be specified and, where possible, the statement of every proposition was to be transcribed. It is not surprising, then, that in order to fulfil the task with which he had been entrusted, Vacca decided to go to Hanover to study Leibniz's manuscripts, which had been 'buried'[26] in the city library there for almost 200 years.

In 1903 Peano recalled that, after two editions of Leibniz's works, it was the general opinion at the time that the manuscripts no longer offered great novelties, but rather «suo importantia magis pate».[27] Vacca stayed in Hanover for only a matter of days in the month of August and immediately sent Peano the transcriptions of some unpublished papers so that they could be inserted in the *Formulaire*. Leibniz's were to be the only manuscripts extensively quoted in the historical notes to this treatise. Vacca's attention was directed above all to some *brouillons* that had been neglected till that moment: the ms. *Philosophie* VII B which comprises reflections on mathematical logic and the international language and the ms. *Mathematik* III A and III B, which are interesting for the results of the theory of numbers and of combinatorial mathematics.

In anticipation of the new edition of the *Formulaire*, Vacca also gave a brief account of his discoveries in the *Bollettino di Bibliografia e Storia delle Scienze Matematiche*,[28] a specialist journal on the history of mathematics, which had been founded in Turin in 1898 and was edited by the Genovese Gino Loria. Vacca here expressed his gratitude to E. Bodemann, who had made available to him the catalogues of Leibniz's unpublished works, allowing him to consult the manuscripts on the Fermat-Wilson theorem and thus to reconstruct the route followed by Leibniz in order to arrive at the proof of this theorem. At the end of his article Vacca wrote:

> «Un preciso esame delle scoperte di Leibniz relative alla logica matematica, che uscirebbe dall'indole di questa nota, sarà tra breve pubblicato nel *Formulaire de mathématiques* N° 3».[29]

In fact the second edition of this treatise is very different from the first, from the point of view of the history of mathematics, above all in its publishing Leibniz's fragments. These quotations were taken up several times by Peano and his students in the

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[25]Cf. G. Peano 1898e, *Sul § 2 del Formulario, t. II: Aritmetica*, RdM, 6, p. 83, 89.

[26]G. Peano 1903c, *De latino sine flexione*, Cavoretto (Torino), Tip. Cooperativa, p. 8. Cf. also G. Peano, 1914f, *Prof. Louis Couturat*, Revista universale (U. Basso), a. 4, 40, october 1914, p. 79.

[27]G. Peano 1903c, *De latino sine flexione*, Cavoretto, Tip. Cooperativa, p. 8.

[28]G. Vacca 1899, *Sui manoscritti inediti di Leibniz*, Bollettino di Bibliografia e Storia delle Scienze Matematiche (G. Loria), 2, p. 113–116.

[29][«An exact examination of Leibniz's discoveries regarding mathematical logic, which would go beyond the nature of this note, will shortly be published in the *Formulaire de mathématiques* N° 3».] G. Vacca 1899 cit., p. 115.

years that followed, to the point that they became a sort of 'shared patrimony of knowledge', a *topos* of their works on the history of mathematics and of socialization of mathematical culture.

The expression «an exact examination of Leibniz's discoveries regarding mathematical logic will be published in the *Formulaire de mathématiques*» might be perplexing since, on closer inspection, what is included in the *Formulaire* are simply a number of transcriptions of passages from Leibniz's manuscripts, without any kind of critical reflection *a latere*. Nevertheless, this is a typical aspect of the historiographic methodology adopted in the School of Peano, as Vacca himself made clear to Couturat:

> «Je veux vous faire encore une petite description relative au F[ormulaire], pour les indications historiques. Lorsque j'ai commencé à ajouter des notes, je l'ai fait presque au hasard. En avançant dans le travail j'ai vu qu'il y avait là une nouvelle méthode historiques. Qu'est que c'est l'histoire d'une science ? On peut penser que ce soit l'exposé impartial des idées scientifiques de ceux qui nous ont précédé. Mais on ne peut pas les exposer toutes, si l'on veut les exposer toutes avec impartialité il faut les reproduire presque en entier. Ce travail prépare l'histoire, ce n'est pas encore l'histoire. La seule conception qui permettre de choisir dans les travaux des anciens c'est de se mettre à notre point de vue. Faire l'histoire des vérités d'une science, c'est chercher et exposer dans le passé tous les essais qui ont produit successivement les vérités que nous connaissons. [...] L'histoire d'une science est alors l'exposition ordonnée des vérités de cette science suivie d'un nome ou d'un date.»[30]

4. The collaboration with Louis Couturat

In 1900, in Paris, two important international congresses were held, one on philosophy and the other on mathematics. The philosophy congress in particular proved to be a fertile opportunity for cultural exchanges for the School of Peano and, for Vacca, there were very stimulating conversations with the French philosopher Louis Couturat.[31] This is what he said on the subject, in a lecture dedicated to the memory of Peano, given in Rome in 1946:

> «In quei due congressi, ai quali partecipavo, ebbi occasione di conoscere Bertrand Russell, il quale aveva pubblicato allora un volume su Leibniz (tradotto più tardi in francese nel 1908). Io stesso già da più di un decennio avevo studiato gli scritti di Leibniz, e da questo studio era sorta la mia amicizia per Giuseppe Peano. Feci quindi da intermediario tra Peano e Russell, il quale era allora in relazione con Louis Couturat [...]. Meravigliai Couturat quando

[30] G. Vacca to L. Couturat, december 1901, P. Nastasi, A. Scimone (eds.) 1995, p. 51–52.
[31] Couturat was one of the most active promoters in France of Peano's logic and of his *Formulaire*, to which he devoted a number of articles in the *Bulletin des Sciences Mathématiques* and in the *Revue de Métaphysique et de Morale*. Cf. L. Couturat 1901, *Peano G.*, *professeur à l'Université de Turin. Formulaire de Mathématiques ... Turin*, *Bocca et Clausen*, Bulletin des Sciences Mathématiques, 2, 25, p. 159 et E. Luciano, C.S. Roero (eds.) 2005, p. IX–LX.

gli descrissi rapidamente la massa dei manoscritti inediti di logica matemati-
ca di Leibniz esistenti ad Hannover, che io avevo studiato colà nel 1899 e di
cui avevo già dato alcuni saggi nella *Rivista di Matematica*, nel *Formulario* di
Peano e nel *Bollettino di Storia della Matematica* del prof. Gino Loria.»[32]

Vacca then urged Couturat to take up systematic studies of Leibniz's manuscripts,
and Couturat – who had no wish to return to teaching after the sabbatical year he had been
granted to collaborate on the organization of the philosophy congress – asked Louis Liard
to authorize him to go to Hanover, with a ministerial mission, in order to carry out this
research.

In this way the contacts between Couturat and the School of Peano were intensified,
and they further consolidated on the occasion of his journey to Italy in 1902.[33] In addition
to Peano, Vacca e Vailati,[34] Couturat exchanged letters with R. Bettazzi, C. Burali- Forti
and M. Pieri, and all these mathematicians collaborated in various ways on the spread of
his works in Italy.[35]

When Couturat met Vacca in Paris, he was on the point of publishing the volume
La Logique de Leibniz, which appeared in 1901. Vacca's suggestions regarding Leibniz's
unpublished works were therefore of the greatest importance for him, and Couturat im-
mediately asked Peano for some clarifications regarding Vacca's articles,[36] telling him of
his intention to further his assistant's by editing a collection of leaflets and fragments by
Leibniz. At the same time, Couturat collaborated with the School of Turin on writing the
historical notes for the third edition of the *Formulaire*, adding further references to Leib-
niz's manuscripts.[37] Meanwhile, the preface to the *Formulaire* announced the impending
publication of his *Logique*,[38] the progress of which was attentively followed by Vacca,
Vailati and Peano.

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[32][«At these two congresses, in which I took part, I had the opportunity to meet Bertrand Russell, who had
then published a volume on Leibniz (later translated into French in 1908). I myself had already been studying
the writings of Leibniz for more than ten years, and out of this study had arisen my friendship with Giuseppe
Peano. I thus acted as intermediary between Peano and Russell, who at the time was in close contact with
Louis Couturat [. . .]. I astonished Couturat when I quickly described him the mass of unpublished manuscripts
on mathematical logic by Leibniz existing in Hanover, which I had studied there in 1899 and of which I had
already given samples in the *Rivista di Matematica*, in Peano's *Formulario* and in Prof. Gino Loria's *Bollettino
di Storia della Matematica*.»] G. Vacca 1946, *Origini della Scienza*, Roma, Partenia, p. 31.

[33]L. Couturat to G. Peano, 15.10.1902, E. Luciano, C.S. Roero (eds.) 2005, p. 48–49; L. Couturat to G. Vacca,
13.8.1902 and 15.10.1902, P. Nastasi, A. Scimone (eds.) 1995, p. 54–55, 56; G. Vailati to G. Vacca, 29.9.1902;
M. Calderoni to G. Vailati, 6.10.1902 and 22.10.1902, G. Lanaro (ed.) 1971, p. 210, 644, 645.

[34]Cf. E. Luciano, C.S. Roero (eds.) 2005, in particular p. VII, XIV–XVI, XXI–XXIII, LXI, LXV, 12–13, 15,
18, 21, 24, 26, 39, 46, 47, 49, 50, 51, 52, 54, 55, 58; P. Nastasi, A. Scimone (eds.) 1995, p. 48–57. Unfortunately,
the correspondence between Couturat and Vailati has been lost and only indirect traces remain, thanks to the
letters of Peano and Vacca.

[35]Cf. L. Couturat to R. Bettazzi, 5.3.1899, L. Couturat to C. Burali-Forti, 12.1.1906, 22.1.1906, E. Luciano,
C.S. Roero (eds.) 2005, p. 189–191, 221–224, 226; L. Couturat to M. Pieri, 15.6.1899, 26.7.1899, 24.4.1900,
30.5.1900, 8.2.1901, 7.7.1901, 29.3.1905, 2.3.1906, G. Arrighi (ed.) 1997, p. 42–50.

[36]L. Couturat to G. Peano, 15.1.1901, E. Luciano, C.S. Roero (eds.) 2005, p. 38–39.

[37]Cf. L. Couturat to G. Peano, 12.5.1899 and L. Couturat to G. Peano, 18.3.1902, E. Luciano, C.S. Roero (eds.)
2005, p. 15, 47.

[38]G. Peano 1901b, *Formulaire de Mathématiques*, t. III, Turin, Bocca-Clausen, p. IV.

The French philosopher constantly kept Peano and his collaborators up to date on his studies,[39] several times showing them his conviction that the new research on logic could profit from the reading of these fragments. For example, he wrote to Pieri in February 1901:

> «. . . ma *Logique de Leibnitz*, en ce moment sous presse, contient tout un chap. consacré à la Characteristica geometrica (par suite beaucoup plus étendu et détaillé que les 4 ou 5 pages de M. Cantor). Vous savez peut-être que, suivant l'exemple de M. Vacca je suis allé à Hanovre fouiller les mss. de Leibnitz et que j'en ai rapporté des inédits fort importants, notamment pour son Calcul logique. J'espère que leur publication contribuera à faire valoir les travaux modernes sur le même sujet, d'autant plus que presque toutes les formules fondamentales de la Logique algorithmique se trouvent déjà dans Leibniz»[40]

and also to Peano in 1903

> «Comme je le dis à Vacca, je suis heureux de voir que ma publication des *Inédits* de Leibniz profite aux travaux de Logique mathématique ; c'est d'ailleurs, je puis l'avouer, dans cette intention que je l'ai entreprise.»[41]

There was a very intensive exchange of opinions between Vacca, Vailati and Couturat in 1901–1902.[42] For example, while in Hanover, Couturat asked Vacca his opinion of the most important of Leibniz's unpublished works regarding algorithmic logic and mathematics.[43] Vacca urged him to examine the correspondence with the Jesuit missionaries in China, in which he had been particularly struck by the reflections on the universal characteristic and on binary arithmetic.[44]

Couturat sent Peano,[45] Vacca and Vailati the proofs of his *Opuscules* and, in 1902, when the editing was at an advanced stage, he asked Vacca to compare the first printed pages with the notes he had taken in 1899:

> «Si par hasard vous pensiez que dans vos notes de Hanovre se trouvent quelques passages que je n'aie pas, vous pourriez me les envoier, et je publierais sous votre nom ceux qui je n'aurais pas et que je vous emprunterai. De toute façon, d'ailleurs, votre nom figurera dans la *Préface*, pour la raison que vous

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[39]L. Couturat to G. Peano, 15.1.1901, 22.12.1901, 2.1.1903, 13.9.1903, 6.12.1903, E. Luciano, C.S. Roero (eds.) 2005, p. 38–39, 46, 49, 51–52, 54.

[40]L. Couturat to M. Pieri, 8.2.1901, G. Arrighi (ed.) 1997, p. 45. Cf. also L. Couturat to G. Peano, 15.1.1901, E. Luciano, C.S. Roero (eds.) 2005, p. 39.

[41]L. Couturat to G. Peano, 6.12.1903, E. Luciano, C.S. Roero (eds.) 2005, p. 54.

[42]L. Couturat to G. Vacca, 18.1.1901, G. Vacca to L. Couturat, 23.8.1901, L. Couturat to G. Vacca, 23.10.1901, G. Vacca to L. Couturat, december 1901, L. Couturat to G. Vacca, 16.1.1902, L. Couturat to G. Vacca, 20.7.1902, L. Couturat to G. Vacca, 13.8.1902, G. Vacca to L. Couturat, autumn 1902, P. Nastasi, A. Scimone (eds.) 1995, p. 48–56; G. Vailati to G. Vacca, 7.2.1901; G. Vailati to G. Vacca, spring 1901; G. Vailati to G. Vacca, 21.10.1901, G. Vailati to G. Vacca, 28.12.1901, G. Vailati to G. Vacca, 31.1.1902.

[43]L. Couturat to G. Vacca, 16.1.1902, P. Nastasi, A. Scimone (eds.) 1995, p. 52–53.

[44]G. Vacca to L. Couturat, 23.8.1901, P. Nastasi, A. Scimone (eds.) 1995, p. 48.

[45]Cf. L. Couturat to G. Peano, 22.12.1901, E. Luciano, C.S. Roero (eds.) 2005, p. 46.

savez. Oui, je crois qu'il y a encore aujourd'hui du profit à tirer de Leibniz pour s'orienter dans les doctrines philosophiques modernes.»[46]

Vacca and Vailati sent some comments on this second book by Couturat:

«J'avais aussi remarqué le système de lecture des nombres décimaux en syllabes. Mais ce procédé n'est pas du à Leibniz, mais à Ariabatta (voir le *Form.* T. 2). C'est pourquoi je ne l'avait cité dans le *Formulaire.*»[47]

Finally, thanks to their suggestions, Couturat revised *ex novo* a number of sections of this work. His gratitude to Vacca, who had opened up a new line of research for him, making him aware of the importance of Leibniz's manuscripts, is testified to by the very cordial acknowledgment at the beginning of the *Opuscules*:

«Notre ouvrage sur *La logique de Leibniz* était presque terminé (nous le croyions du moins) lorsque nous eûmes le plaisir de faire la connaissance de M. Giovanni Vacca qui avait compulsé, un an auparavant, les manuscrits de Leibniz conservés à Hanovre, et en avait extrait quelques formules de Logique insérées dans le *Formulaire de Mathématiques* de M. Peano. C'est lui qui nous révéla l'importance des œuvres inédites de Leibniz, et nous inspira le désir de les consulter à notre tour. [...]. C'est à ce concours de bonnes volontés, de conseils et de protections que notre ouvrage doit le jour ; nous nous faisons un plaisir et un devoir de le déclarer, et d'exprimer à MM. Liard, Bodemann et Vacca toute notre reconnaissance.»[48]

Throughout Europe, Couturat's books stirred interest and controversy with their theses and, especially, with the author's hypothesis according to which Leibniz's metaphysical system was mainly constructed starting from his logical system. Some critics considered this interpretation risky and debatable.[49] In contrast, Peano's disciples expressed enthusiastic approval. In spring 1902, for example, Vacca wrote to Couturat, thanking him for sending his volume:

«Votre splendide œuvre sur la *Logique de Leibniz* ! J'y avais pensé autrefois, vous avez réalisé mon songe. J'ai parcouru d'un seul trait tout le volume en y trouvant une clarté et une vision nette des idées de Leibniz que j'avais entrevues mais que je ne savais pas écrire. J'aime parfois à chercher des indices qui mesurent le mérite des ouvrages contemporaines et il me semble qu'on peut juger de la valeur d'un livre en s'imaginant l'intérêt qu'il pourrait présenter pour les savants célèbres des siècles passés. S'il pouvaient un moment revivre qu'il seraient contents de savoir que ... Après avoir vu votre livre j'ai tout de suite pensé : quels remerciements vous auriez de Leibniz s'il pouvait encore vivre pour vous connaitre !»[50]

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[46]L. Couturat to G. Vacca, 16.1.1902, P. Nastasi, A. Scimone (eds.) 1995, p. 53.

[47]G. Vacca to L. Couturat, autumn 1902, P. Nastasi, A. Scimone (eds.) 1995, p. 56. Cf. also G. Vailati to G. Vacca, 21.10.1901, G. Lanaro (ed.) 1971, p. 193.

[48]L. Couturat 1903, p. I, II.

[49]Cf. for example M. Ferrari 2006, p. 173, 181–183.

[50]G. Vacca to L. Couturat, autumn 1902, P. Nastasi, A. Scimone (eds.) 1995, p. 55.

Two reviews of Couturat's books, which Peano had been eager to have,[51] appeared in the *Revue de Mathématiques*.[52] Vailati also published an analysis of the *Logique de Leibniz* in Loria's *Bollettino*.[53] Vacca's and Vailati's reviews benefited from a continuous 'three-way dialog' with Couturat,[54] who gratefully wrote to Vacca:

> «Je viens de recevoir la Rev. de Math. et de lire votre article sur la Logique de Leibniz. Je vous remercie de toutes les choses flatteuses que vous dites de mon ouvrage. Mais ce qui me fait encore plus de plaisir, c'est de voir la partie que vous (et les autres collaborateurs de M. Peano) tirez de ma publication d'inédits [...]. Je souhaite qu'elle donne une nouvelle impulsion aux travaux de Logique mathématique et qu'elle attire sur eux l'attention des philosophes.»[55]

Concerning the volume *La Logique de Leibniz*, Vailati remarked that only a scholar who was master equally of mathematical and of philosophical culture could attempt to interpret Leibniz's writings, in which these two components are inextricably intertwined. The greatest merit of Couturat's work was therefore that it showed this interaction, in the light of which it was possible to fully appreciate the importance of the project of construction of the *characteristica*:

> «Ciò – said Vailati – rende anche, nello stesso tempo, ragione di un altro fatto notevole, messo chiaramente in luce dal presente volume del Couturat, che, cioè, perfino negli scritti del Leibniz, già da tempo pubblicati, le parti che toccano più davvicino gli argomenti a cui abbiamo sopra alluso, cioè in particolare i vari metodi di rappresentazione simbolica dei ragionamenti deduttivi e il concetto generale di un algoritmo operatorio (*calculus ratiocinator*), sembrano non aver quasi richiamato sopra di sé alcuna attenzione ed essere giaciute non meno neglette o ignorate di quelle altre parti, ad esse affini, che gli editori delle opere di Leibniz non avevano finora neppur stimate degne della pubblicazione.»[56]

In any case – concluded Vailati – it was precisely to Leibniz's reflections that reference should be made if the origins of and the links between his most original philosophical and mathematical ideas were to be understood.

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[51]G. Peano to G. Vacca, 9.1.1903, c.p., E. Luciano, C.S. Roero (eds.) to appear. Couturat had urged the publication of a review of his work in the RdM the previous week. Cf. L. Couturat to G. Peano, 2.1.1903, E. Luciano, C.S. Roero (eds.) 2005, p. 49.

[52]G. Vailati 1901a, p. 148–159; G. Vacca 1903, p. 64–74.

[53]G. Vailati 1901b, p. 103–110. The two reviews are very different one from each other.

[54]Cf. G. Vailati to G. Vacca, 7.2.1901, [february-march 1901], [may 1901], 21.10.1901, 28.12.1901, 31.1.1901, 19.2.1902, 10.6.1902, 7.7.1902, 7.11.1902, 8.11.1902, 12.2.1903, 20.4.1903; 7.12.1903, G. Vacca to G. Vailati, 15.2.1903, cc. 1r-2v, M. De Zan, E. Luciano, C.S. Roero, to appear.

[55]L. Couturat to G. Vacca, 6.12.1903, P. Nastasi, A. Scimone (eds.) 1995, p. 57.

[56][«This also, at the same time, confirms another important fact, clearly highlighted in this volume of Couturat, namely, that even in Leibniz's writings, already published some time ago, the parts that come closest to the subjects we have alluded to above, i.e., the various methods of symbolic representation of deductive reasoning and the general concept of an operating algorithm (*calculus ratiocinator*), seem hardly to have attracted any attention to themselves and to have lain no less neglected or ignored than those other parts, similar to them, which the publishers of Leibniz's works have so far failed even to consider worthy of publication.»] G. Vailati 1901a, p. 148–149.

Vacca, in his turn, identified the salient characteristics of Leibniz's logic starting from the autograph works published by Couturat. Peano's collaborator did not hesitate to affirm that Leibniz was in fact the first mathematician to have understood the importance of logic, attempting to build a complete, consistent symbolic system of it. Thus Vacca was able to conclude:

> «Da quanto precede appare quanto sia elevato il posto che Leibniz occupa nella storia della logica, e quanto da questo punto di vista sia stata utile la pubblicazione del Couturat. Egli dice, troppo modestamente, che la sua pubblicazione (p. III) «est un recueil de morceaux choisis, que parfois se réduit presque à un catalogue». Il volume edito dal Couturat di oltre 600 pagine supera di gran lunga le così dette edizioni complete di Gerhardt e di Erdmann. Se però questo volume è venuto in buon punto a soddisfare i più urgenti desideri degli studiosi di logica, non conviene però dimenticare che urge tuttavia la pubblicazione completa ed integrale delle opere di Leibniz. È un'opera colossale: si tratta di un centinaio di volumi in ottavo che occorrerà comporre, raccogliendo e decifrando con pazienza tutti i frammenti grandi e piccoli che Leibniz ha lasciato, e di cui il primo buon modello è quello ora datoci dal Couturat.»[57]

It was with interest that Couturat followed the work of promotion of his writings in Italy, several times thanking Peano and his collaborators.[58]

In addition to Peano's *Revue de Mathématiques* there was another scientific periodical, linked to the context of the University of Turin, which contributed to the spread of knowledge of Leibniz among mathematicians in Italy: this was the *Bollettino* under the direction of Loria. A friend of Peano since the time of his university studies, Loria contributed to the diffusion of the *Formulaire*, although he was perfectly aware of the criticisms that such a work would inevitably attract. He also shared the interest of Peano and his School in the history of logic, a subject to which he devoted the article *La logique mathématique avant Leibniz*, in which he wrote:

> «On voit, d'après ces lignes que nous avons empruntées à l'*Introduction au Formulaire de Mathématique* que vient de publier M. Peano dans sa qualité de directeur de la *Rivista di Matematica*, qu'un groupe de mathématiciens s'est proposé de rédiger une sorte de grande encyclopédie, une espèce de *repertorium* où l'on trouvera énoncés en symboles logiques les définitions et les théorèmes plus importants qui se rapportent aux différentes sciences exactes.

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[57][«From the above it emerges how elevated a place Leibniz occupies in the history of logic, and how useful from this point of view was the publication of Couturat. He says, too modestly that his publication (p. III) «est un recueil de morceaux choisis, que parfois se réduit presque à un catalogue». The volume published by Couturat, of more than 600 pages, is much greater than the so-called complete editions of Gerhardt and Erdmann. If, however, this volume has succeeded to a satisfactory degree in satisfying the most urgent wishes of logicians, we must nevertheless not forget that the complete and integral publication of the works of Leibniz is a pressing matter. It is a colossal work: it is a matter of about a hundred octavo volumes which will have to be put together, patiently gathering and deciphering all the fragments, small and large, that Leibniz has left, and the first good model of which is the one now given us by Couturat.»] G. Vacca 1903, p. 72–73.

[58]Cf. L. Couturat to G. Peano, 6.12.1903, E. Luciano, C.S. Roero (eds.) 2005, p. 54.

… Je ne veux pas discuter ici en détail ce projet, les critiques … mon but actuel n'est pas d'essayer de déterminer la valeur de la méthode dont M. Peano s'est fait un des avocats les plus actifs ; je veux, au contraire, seulement faire remarquer – ce qui semble avoir échappé même à Schröder – que l'entreprise que j'ai signalée a été essayée, il y a deux cent cinquante années (c'est-à-dire avant Leibniz), par le mathématicien français Pierre Hérigone.»[59]

As may be deduced, at this time Loria adopted the historiographic approach of Vacca and of the *Formulaire*, namely the search for precursors. Hence his work may reasonably be set beside that of Peano and of Vacca on the precursors of mathematical logic, or beside Vailati's essay on G. Saccheri's *Logica demonstrativa*.[60]

The *Bollettino* soon became the journal of reference for the spread in Italy of the initiatives on Leibniz and for the updating of the secondary literature regarding the German mathematician and philosopher.[61] In this journal – as we have said – Loria had accepted for publication Vacca's article on Leibniz's manuscripts and Vailati's review of Couturat's *Logique de Leibniz*. These were important contributions, to the point that in 1906 Loria was to remark, with regard to the editing of volume XXI of the *Abhandlungen zur Geschichte der Mathematischen Wissenschaften*:

«[esso] porge un notevole contributo alla letteratura ed è nuovo sintomo del salutare risveglio di studi intorno al sommo filosofo-matematico Leibniz, risveglio al quale il nostro *Bollettino* non è estraneo [. . .].»[62]

Furthermore, in the *News* section Loria published the announcements regarding the plan for an international edition of Leibniz's works, information about the initiatives for

[59]G. Loria 1894, *La logique mathématique avant Leibniz*, Bulletin des sciences mathématiques (Darboux), s. 2, 18, p. 107–112, quotation at p. 108–109.

[60]G. Peano 1894e, *Un precursore della logica matematica*, RdM, 4, p. 120; G. Vacca 1899, *Sui precursori della logica matematica*, RdM, 6, p. 121–125, 183–186; G. Vailati 1903, *Di un'opera dimenticata del P. Gerolamo Saccheri* (*Logica demonstrativa*), Rivista filosofica, 5, 6, 4, p. 528–540. Cf. also G. Peano to G. Vacca, 15.11.1906, E. Luciano, C.S. Roero (eds.) to appear. This research on the 'precursors', whose value must obviously be put in perspective today, nevertheless justifies the choice of quotations selected by Vacca starting from Leibniz's manuscripts. Vacca did not, in fact, use all the manuscripts he consulted, nor all those of Leibniz's manuscripts that he considered interesting or important. He transcribed and published only those manuscripts from which may be deduced a priority on Leibniz's part for anything regarding the propositions or proofs of logic and arithmetic.

[61]On the subject of Leibniz's acquaintances in Italy, the name of Federigo Enriques cannot be omitted: from 1907 he directed the journal *Scientia, Revue internationale de synthèse scientifique* and, in 1922, published a volume on the history of logic (*Per la storia della Logica. I principii e l'ordine della scienza nel concetto dei pensatori matematici*, Bologna, Zanichelli, 1922). He had contacts with the School of Peano, and above all with Vacca and Vailati, but his conception of logic and of historical research are very different. Enriques was interested above all in the philosophical aspects of Leibniz's writings, and gave space in *Scientia* to articles on and reviews of Leibniz's philosophical works. Cf. for example G. Loria 1922 *J.M. Child, The Early Mathematical Manuscripts of Leibniz* . . ., Scientia, 31, p. 237–238. A review of Child's volume also appeared in *Archeion*, another Italian journal on the history of sciences, contemporary with *Scientia* and published by Aldo Mieli. Cf. E. Rufini 1924, *J.M. Child, The Early Mathematical Manuscripts of Leibniz* . . ., Archeion, 5, 1, p. 68–69.

[62][«[it] offers a notable contribution to the literature and is a new symptom of the healthy revival of studies concerning the supreme philosopher-mathematician Leibniz, a revival to which our *Bollettino* is not unrelated [. . .].»] *Notizie. Abhandlungen zur Geschichte der Mathematischen Wissenschaften*, Bollettino di Bibliografia e Storia delle Scienze Matematiche (Loria), 9, 1906, p. 94.

the centenary of his death and clarifications on the publication of his scientific correspondence.[63] In this case too the activity of promotion carried out by Peano and his School seems clear: thanks to the correspondence between Couturat and Peano it may be supposed, for example, that it was Couturat himself who sent Loria the information about the agenda for the edition of Leibniz's works, undertaken by an International Commission.[64] In addition, the publisher of the correspondence between Leibniz and Kochanski, repeatedly announced in the *Bollettino*, was Samuel Dickstein, who in his turn was in close contact with Peano.

5. International language and teaching

In 1903 Peano published his first article in *latino sine-flexione*, in which he illustrates a plan for an international language deducible from classical Latin, with the suppression of declensions and inflexional endings.[65] Thus a new chapter was opened in his intellectual biography and in his scientific output. Peano's studies on the international language, which he himself conceived as an application of mathematical logic, undoubtedly arose from his reading of Leibniz's manuscripts on the *Langue rationelle*, published by Couturat in the *Opuscules*. Peano remembers:

> «L. Couturat publica *Opuscules et fragments inédits de Leibniz*, qui contine studio de Leibniz, summe praetioso per constructione de Vocabulario philosophico. Libro, nunc edito, L. Couturat et L. Leau *Histoire de la langue universelle*, Paris a. 1903 p. XXI+571, expone 56 projecto de lingua artificiale. Si in futuro analyse et synthese invicem conveni, ut duo exercito de minatore, qui labora tunnel ex duo extremitate, tunc Lingua rationale et Characteristica universale de Leibniz fore idem.»[66]

From 1903 Peano developed his studies on Leibniz's philosophical language; Couturat, in contrast, progressively distanced himself from research on Leibniz, to the point that in 1905, when E. Borel asked him for an article for the newly founded *Revue du mois*, he answered:

> «Je te félicite sincèrement de l'idée de créer la *Revue du Mois* ; mais, pour le moment, je ne puis rien te promettre comme collaboration. Je t'ai dit combien je suis occupé. Je ne puis plus revenir sur Leibniz, que j'ai quitté depuis plusieurs années. Quand j'aurai un moment de loisir, je penserai à ta Revue.

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[63]Cf. *Notizie. Carteggio di Leibniz*, Bollettino di Bibliografia e Storia delle Scienze Matematiche (Loria), 4, 1901, p. 127; *Notizie. Carteggio di Leibniz*; *Progetto d'un'edizione internazionale delle opere di Leibniz*, Bollettino di Bibliografia e Storia delle Scienze Matematiche (Loria), 6, 1903, p. 30, 126; AA.VV. 1906, *Notizie. Preparazione d'un'edizione completa delle opere di Leibniz*; *Abhandlungen zur Geschichte der Mathematischen Wissenschaften*, Bollettino di Bibliografia e Storia delle Scienze Matematiche (Loria), 9, 1906, p. 30, 94; *Notizie. Il II centenario della morte di Leibniz*, Bollettino di Bibliografia e Storia delle Scienze Matematiche (Loria), 19, 1917, p. 31–32.

[64]Cf. L. Couturat to G. Peano, 2.1.1903, E. Luciano, C.S. Roero (eds.) 2005, p. 49.

[65]G. Peano 1903d, *De latino sine- flexione*, RdM, 8, p. 74–83.

[66]G. Peano 1903d, *De latino sine- flexione*, RdM, 8, p. 80, 82. Cf. also L. Couturat to G. Peano, E. Luciano, C.S. Roero (eds.) 2005, p. 51–52.

Ce serait pour entretenir tes lecteurs des sujets qui m'occupent présentement,
à savoir la Logique formelle, d'une part, et la Langue internationale, d'autre
part.»[67]

Meanwhile, in 1908 Peano printed the definitive edition of the *Formulaire* which
for some time he had been using for his teaching of Analysis at the University of Turin.
In his lectures he liked to adopt the historical method, i.e., he proposed a reading of
passages from the classics of mathematics. So Peano devoted himself to going more
deeply into Leibniz's writings on differential and integral calculus, and transcribed parts
in his *marginalia* to the *Formulaire*, commenting them with his students.

The year 1910 was, perhaps, one of the most difficult for Peano, who was forced to
abandon the teaching of advanced Analysis, because of the criticisms of his colleagues
about his recourse to logic and to the *Formulaire* in his lectures. In February 1910 his
correspondence with Couturat was abruptly interrupted, but in any case Couturat was by
this time a fanatical interlinguist, devoting himself entirely to propagandizing the *Ido*. In
the same month, Peano and Vacca were involved in a controversy with Roderigo Biagini
in the journal of philology *Classici e Neolatini*.[68] The object of the dispute was whether
or not Peano's *latino sine-flexione* actually coincided with Leibniz's rational language,
though Biagini had recourse above all to the weapon of humor, trotting out the discovery
of Leibniz's unpublished works by Vacca in 1899 and Couturat's subsequent contribu-
tions. Both Peano and Vacca responded and Vacca, in particular, specified in irritated
tone:

«Pubblicai, in più luoghi, il risultato di questi miei studi ed altri ne pubblicherò
a tempo opportuno. In seguito a queste mie ricerche, che ebbero dagli studiosi
competenti l'accoglienza che si meritavano, apparvero evidenti le insufficien-
ze da me per primo segnalate. [. . .] Infine l'Associazione internazionale delle
Accademie, soddisfacendo ad un voto espresso prima da me, poi dal Couturat
e da altri, deliberò la pubblicazione integrale degli scritti di Leibniz [. . .]. Il
Sig. R.B. ha mal garbo a dire che io ho pubblicato alcuni manoscritti i quali, o
non erano stati veduti, o forse non voluti pubblicare. Erano stati veduti, perché
pubblicati in parte dal Gerhardt; e non erano stati pubblicati perché gli editori
precedenti non ne avevano capito l'importanza.»[69]

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[67]L. Couturat to E. Borel, 23.11.1905, Fonds Borel Paris, c.p., in E. Luciano 2008, *Giuseppe Peano docente
e ricercatore di Analisi* 1881–1919, PhD Thesis, Torino, Dep. of Mathematics, vol. 2, p. 128. In France,
meanwhile, a harsh controversy broke out in the *Revue de Métaphysique et de Morale* on rigor and intuition,
involving mathematicians and philosophers such as Couturat, Russell, Poincaré, Boutroux, Borel, Winter and,
in Italy, Peano, Vacca, Pieri, Croce. Cf. E. Luciano, C.S. Roero (eds.) 2005, p. XXV-LX.

[68]Cf. R. Biagini 1910, *La lingua internazionale in servizio delle scienze e degli scienziati*, Classici e Neolatini,
a. VI, 2–3, p. 1–24; G. Peano 1911a, *A proposito della lingua internazionale*, Classici e Neolatini, 4, p. 281–
285; G. Vacca 1910, *A proposito delle edizioni delle opere di Leibniz*, Classici e Neolatini, 4, p. 286. Cf. also
G. Peano to G. Vacca, 17.12.1910, c. 1r, E. Luciano, C.S. Roero (eds.) to appear.

[69][«I published, in several places, the result of my studies and published others at an appropriate time. Following
my research, which were welcomed as they deserved by competent scholars, the inadequacies first pointed out by
me appeared evident. [. . .] Finally the international Association of the Academies, satisfying a vote expressed
first by me, then by Couturat and by others, resolved on the publication in their entirety of Leibniz's works [. . .].
R.B. has the impertinence to say that I published some manuscripts which either the editors had not seen, or did

After 1910 Peano was forced to give up lecturing on Higher Analysis and turned to the world of secondary teaching and of the interlingua.[70] His activities in the area of didactics were many, including the publication of manuals for high schools and professional institutes, but also initiatives intended to foster contacts between the university and the community of teachers.

In the School of Peano particular importance was given to the critical editions of original sources, and he had some collaborators who worked on the preparation of anthologies and translations, which in effect proved to be very useful to teachers and students, by making them familiar with the classics of mathematics such as Leibniz's works. It is not surprising, then, that the first two Italian translations of writings by Leibniz on mathematics should be owed to scholars linked to the School of Peano. In 1920 Alpinolo Natucci, a teacher in the upper schools in Pinerolo and a friend of Peano who shared his opinions on the pedagogy of mathematics, translated Leibniz's manuscript *De solidorum elementis*.[71] This is a text of Descartes on the polyhedral geometry and on figurative numbers, copied by Leibniz and then developed by him in the years he spent in Paris. Then, in 1927, Ettore Carruccio, a student of Vacca, translated the famous *Nova Methodus*.[72] This research was developed in relation to courses on the history of mathematics given by Vacca at the University of Rome. The translation appeared in a journal expressly addressed to teachers: the *Periodico di Matematiche*. Nor does the choice of manuscripts seem accidental, if we bear in mind that both solid geometry and differential calculus were subjects typical of the curricula for secondary education.

In 1924–25, Peano left the chair of Infinitesimal Analysis, transferring to that of Complementary Mathematics, which was precisely designed for the training of future teachers. In his courses he developed subjects concerning the logical and historical foundations of mathematics . Information about his lectures may be gathered from some articles by the students who attended them and published extracts from them, at his invitation. In 1928, for instance, Ugo Cassina presented an article devoted to π in the *Periodico di Matematiche*.[73] In this paper there is a careful examination of Leibniz's contributions on the series of the *arcotang*, reconstructed thanks to an examination of the writings and correspondence of Leibniz. Cassina's results were subsequently taken up by Fausta Audisio, another student of Peano's courses of *Matematiche complementari*, who wrote her degree dissertation on π, under his supervision, and also published a number of papers on this subject.[74] Audisio wrote:

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not want to publish. They had been seen, because they were published in part by Gerhardt; and they had not been published because the earlier editors had not understood their importance.»] G. Vacca 1910, *A proposito delle edizioni delle opere di Leibniz*, Classici e Neolatini, 4, p. 286.

[70] Cf. E. Luciano, C.S. Roero 2008, *Giuseppe Peano Matematico e Maestro*, Turin, Dep. of Mathematics, p. 65–80.

[71] A. Natucci 1920, *Il De Solidorum Elementis di Leibniz*, Bollettino Mathesis, 12, 5–8, p. 117–127.

[72] E. Carruccio 1927, *Il «Nuovo Metodo» di Leibniz, con note storiche*, Periodico di Matematiche, s. 4, 7, p. 285–301.

[73] U. Cassina 1928, *Calcolo di π*, Periodico di Matematiche, s. 4, 8, p. 271–293.

[74] F. Audisio, *Il numero π*, Graduation Thesis in Mathematics, Turin, Historical Archives of the University, 1930, p. 1–48; *Calcolo di π in Archimede*, Atti della R. Accademia delle Scienze di Torino, 65, 1929–30, p. 101–108.

«La serie

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

fu pubblicata da Leibniz, in «Acta Eruditorum», pp. 41–46, l'anno 1682: *De vera proportione circuli ad quadratum circumpscriptum in numeris rationalibus* (Opera, III, p. 140). [...] Si sa che il resto di questa serie, dopo un termine qualunque, ha il segno ed è minore del primo termine trascurato. Quindi con questo criterio la serie è poco convergente e non atta al calcolo numerico. [...] Ma la regola precedente dà solo un limite superiore del resto. Il prof. Peano, in una sua lezione, indicò come si possa stimare il resto con maggiore approssimazione. Io espongo questo metodo e sviluppo il calcolo di $\pi/4$ con 5 decimali, servendomi di 14 termini della serie.»[75]

Cassina's and Audisio's works, like that of other students of Peano, which appeared in the last years of his life, are distinguished by the same structure: they arise from the reading of the classical sources, such as Leibniz; these sources are transcribed, translated and commented on, from a historical standpoint; finally, applying modern mathematical tools, among them the techniques of numerical calculus, these historical sources are the basis for the development of new results. The starting point is always the *Formulaire de Mathématiques*, so it is not surprising that the same passages from Leibniz's writings, both published and unpublished, should be repeated and taken up in exactly the same way by various students of Peano.

From the 1920s, Giovanni Vacca, for his part, interrupted his research activity on Leibniz. Friends and correspondents, Loria among them, contacted him several times, as the years went by, to ask for information about the existence of certain results or concepts in Leibniz and urged him to collate the notes he had made in Hanover, only partially published and divulged.[76] Nevertheless, Vacca had meanwhile become professor of History and Geography of Eastern Asia at the University of Rome, so he limited himself to a number of sporadic mentions, without taking up systematic studies on Leibniz. In an article on the geometry of folded paper, for example, he remarked incidentally:

«In un manoscritto di Leibniz abbiamo trovato la seguente lista: *Geometria est explicare figuras, quas natura et ars singulari quadam ratione producit, ita figurae cristallisationum, ecc.; Geometria sartorum; De artificio, puerorum,*

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[75]«The series

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

was published by Leibniz, in «Acta Eruditorum», pp. 41–46, in 1682: *De vera proportione circuli ad quadratum circumpscriptum in numeris rationalibus* (Opera, III, p. 140). [...] It is known that the rest of this series, after any term, has the sign and is less than the first term ignored. Hence with this criterion the series is slowly convergent and not suitable for numerical calculus. [...] But the preceding rule gives only a limsup for the rest. Prof. Peano, in one of his lectures, indicated how the rest can be estimated with greater approximation. I expound this method and develop the calculus of $\pi/4$ with 5 decimals, using 14 terms of the series.» F. Audisio 1930, *Calcolo di π colla serie di Leibniz*, Atti della R. Accademia dei Lincei, Rendiconti, 6, 11, p. 1077–1080; *Il numero π*, Periodico di Matematiche, 4, 3, 1931, p. 11–42; *Ancora sul numero π*, Periodico di Matematiche, 4, 20, 1931, p. 149–150.

[76]Cf. for example G. Loria to G. Vacca, 26.7.1901, G. Loria to G. Vacca, 14.1.1903, P. Nastasi, A. Scimone (eds.) 1995, p. 97, 103–104.

quo fila digitis implicata educunt; De textoria arte; De geometria apum et aranearum.»[77]

One year earlier Peano had addressed himself to Vacca to obtain information on the developments of the publication of Leibniz's works. Peano had actually proposed a research on Wilson's theorem to his assistant Maria Cibrario. Vacca, as we have said, had undertaken his Leibnizian studies on precisely this theorem but, in 1929, he was no longer up to date and merely replied:

«Non so a che punto sia la pubblicazione dei manoscritti di Leibniz. Credo che sia stato pubblicato, intorno al 1914, uno o due volumi, di un catalogo completo dei manoscritti di Leibniz, ma io non l'ho visto. Per quanto riguarda il teorema di Wilson, di cui avevo dato notizia per la prima volta nel Bollettino di Loria, 1899, T. II, pag. 113, è comparso nel 1912, Bibliotheca Mathematica, 1912, vol. XIII, III Folge, Leipzig, Teubner, p. 29, un articolo di Dietrich Mahnke, *Leibniz auf der Suche nach einer allgemeine Primzahlgleichung*, il quale contiene uno studio sui manoscritti di Leibniz, più diffuso, e completo del mio. Il Mahnke riproduce i passi ricopiati da me, ed alcuni altri. Io ho ancora l'impressione che una diligente ricerca tra i libri della biblioteca di Leibniz, che si conservano ancora nella sua casa, ridotta a museo in Hannover, potrebbe dare risultati assai interessanti, ma io non ho modo di andare laggiù, e non so se altri, oltre il Mahnke, se ne occupino. Forse potrebbe occuparsene il Wieleitner, il quale scrive interessanti studi di storia della matematica.»[78]

However, Peano's intention was implemented: in 1929, once more in the *Periodico di Matematiche*, Cibrario published an article entitled *Teorema di Leibniz-Wilson sui numeri primi*.[79] In this work she retraces the history of the proof of this proposition, starting from the studies of Vacca – which by this time had become classic references and were even quoted in the *Encyclopédie des sciences mathématiques* – down to the more recent works by Dietrich Mahnke, which Vacca himself had suggested to Peano. Cibrario's article takes its cue from the *Formulaire*, from which the historical information is drawn. The subject is yet again chosen on the basis of its possible didactic implications, for Wilson's

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[77]["In one of Leibniz's manuscripts we found the following list: Geometria est explicare figuras, quas natura et ars singulari quadam ratione producit, ita figurae cristallisationum, ecc.; Geometria sartorum; De artificio, puerorum, quo fila digitis implicata educunt; De textoria arte; De geometria apum et aranearum."] G. Vacca 1930, *Della piegatura della carta applicata alla geometria*, Periodico di Matematiche, 4, 10, p. 43–50.

[78]["I do not know what point the publication of Leibniz's manuscripts has reached. I believe that around 1914, one or two volumes were published, of a complete catalog of Leibniz's manuscripts, but I have not seen it. As to Wilson's theorem, which I first examined in Loria's *Bollettino*, 1899, T. II, pag. 113, there appeared in 1912, Bibliotheca Mathematica, 1912, vol. XIII, III Folge, Leipzig, Teubner, p. 29, an article by Dietrich Mahnke, *Leibniz auf der Suche nach einer allgemeine Primzahlgleichung*, which contains a study on Leibniz's manuscripts, more widespread and complete than mine. Mahnke reproduces the passages copied by me, and some others. I still have the impression that diligent research among the books in Leibniz's library, which are still kept in his house, now a museum in Hanover, might give very interesting results, but I have no way of going there, and I do not know whether others, besides Mahnke, are concerned with it. Perhaps Wieleitner, who writes interesting studies on the history of mathematics, could be interested in."] G. Vacca to G. Peano, 25.1.1929, C.S. Roero (ed.) 2001, *Giuseppe Peano Matematica, cultura e società*, Cuneo, L'Artistica Savigliano, p. 84.

[79]M. Cibrario 1929, *Teorema di Leibniz-Wilson sui numeri primi*, Periodico di Matematiche, s. 4, 9, p. 262–264.

theorem is included in the programs for competitive examinations for teachers of middle and secondary schools.

A tangible testimony to the success of the activity of making Leibniz's works known, carried on by the School of Peano, is found in the *Enciclopedia delle Matematiche Elementari*.[80] This encyclopedia, edited by L. Berzolari, D. Gigli and G. Vivanti, was a sort of *sussidiario* for students who were preparing for examinations for permanent teaching posts. There were numerous contributions from exponents of the School of Peano, which is why there are many cross-references in it to Leibniz's manuscripts and printed works on Logic, Arithmetic, *Analysis situs,* etc. These references often descend directly from the *Formulaire de Mathématiques* or from the publications of Peano and of Vacca; there are also, however, references to the works of Couturat. Leibniz's are almost the only manuscripts cited in the *Enciclopedia*.[81] Hence knowledge of them was considered an element for the cultural and historical training of future teachers of mathematics.

6. Conclusions

Today it seems almost inevitable to stand at some distance from a certain hagiographical literature on Peano which aims to give excessive emphasis to his links with Leibniz, in the end considering the *Formulaire* as the complete and immutable fulfillment of Leibniz's dream. In fact, the positions apropos this must be far more nuanced. Peano himself, as the years passed, became more prudent regarding the successes of algorithmic logic. His entourage – and above all those students who like Vacca had witnessed the fundamental developments of mathematical logic in the 1930s – did not hesitate either to make manifest a substantial skepticism, when faced with the enthusiastic initial declarations to the effect that, with the *Formulaire*, «Leibniz's dream had come true». Vacca, for instance, wrote in 1946:

> «É vero che in lavori precedenti G. Peano aveva considerato talvolta la logica matematica come la soluzione del desiderato di Leibniz [. . .]. Questo ideale di Leibniz, da lui non raggiunto, come non è raggiunto da nessun logistico moderno non era nemmeno raggiunto da chi lo aveva ingenuamente espresso nel Medioevo, da Raimondo Lullo, nella sua *Ars Magna*. Le citazioni di Leibniz negli scritti di Giuseppe Peano sono limitate effettivamente alle applicazioni della logica alla matematica.»[82]

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[80]Cf. L. Berzolari, D. Gigli, G. Vivanti (eds.), *Enciclopedia delle Matematiche Elementari*, Milano, Hoepli, I, 1, 1930, p. 6, 7, 23, 26, 28, 30, 33, 34, 35, 36, 38, 39, 43, 47, 48, 49, 52, 57, 60, 64, 65, 68, 69, 70, 71, 90, 91, 102, 105, 193, 219, 280, 287, 422; I, 2, 1932, p. 5, 13, 75, 76, 109, 111, 116, 407, 421, 455, 456, 457, 458, 459, 460, 463, 471, 474, 477, 478, 481, 486, 488, 491, 498, 500, 502, 504, 506, 507, 515, 521, 525, 526, 527, 530, 533; II, 1, 1937, p. 53, 63, 291, 392, 432, 536, 572; II, 2, 1938, p. 50, 128, 148, 153, 164, 202, 382, 413, 429, 431; III, 1, 1947, p. 210, 211, 267; III, 2, 1950, p. 45, 55, 56, 108, 423, 578, 654, 690, 703, 707, 714, 715, 724, 727, 734, 736, 739, 740, 741, 749, 755, 762, 894, 900, 941.

[81]Cf. L. Berzolari, D. Gigli, G. Vivanti (eds.), *Enciclopedia delle Matematiche Elementari*, Milano, Hoepli, I, 1, 1930, p. 6, 7, 23, 26, 60, 287; I, 2, 1932, p. 75; II, 1, 1937, p. 291; III, 2, 1950, p. 894.

[82][''It is true that in earlier works G. Peano had sometimes considered mathematical logic the solution to Leibniz's dream [. . .]. This ideal of Leibniz, which he never achieved, just as no modern logician has achieved it, was not even reached by the man who had naively expressed it in the Middle Ages, Raimondo Lullo, in his *Ars*

However, there are some very peculiar features of the reception of Leibniz by Peano and his School. It certainly seems risky to include these scholars among the protagonists of the *Leibniz Renaissance* in Italy, since they did not carry out philosophical, logical or philological analyses on Leibniz. The importance of the School of Peano must therefore be assessed bearing in mind that these scholars were the only ones who, *in the Italian scientific community*, spread an exact knowledge of Leibniz's results in Logic, Arithmetic, Analysis etc.[83] Nor is this all: this knowledge was recovered from study carried out *on the sources* (printed and manuscript), in contrast with what happened, for example, in F. Enriques' work on the history of logic. Besides, Peano and his students were personally committed to the divulgation of the works of Couturat, of which, in Italy, they were the genuine promoters.[84]

On examining Peano's and his collaborators' works on Leibniz, one may identify the epistemological arguments that frame the historiographical research carried out in relation to the *Formulaire*. History contributes to the 'founding' of every mature discipline as mathematics, identifying the paternity of the concepts and theorems, the links among the theories and the stages that gradually led to its modern, rigorous structure. Peano dedicated scrupulous attention to questions of priority and to precursors' research. Equal attention is devoted to transcriptions, critical editions and so on, for it is not considered appropriate that historical research be separated from philological and etymological criticism.

These arguments became concrete in specific didactic demands. The teaching of mathematics, according to Peano and his collaborators, must be developed with the historical method. Information on the origins of concepts, rules, symbols and proofs in fact help the student to understand their usefulness. On a closer look, one of the most characteristic aspects of study activities on Leibniz by the School of Peano is their promotion in the domain of teaching, thanks to the quotations from Leibniz's manuscripts in the manuals for secondary schools, by means of the translations into Italian of his most important works, etc. Knowledge of Leibniz's writings was considered, as we have seen, an important element in the cultural formation of teachers of mathematics. Moreover, the emphasis on didactics is also one of the principal characteristics of the editorial policy of Peano's *Revue de Mathématiques* which, according to the statement at the beginning of the first volume:

Magna. The quotations from Leibniz in the writings of Giuseppe Peano are in effect limited to the applications of logic to mathematics."] G. Vacca 1946, *Origini della Scienza*, Roma, Partenia, p. 32.

[83]Cf. A. Padoa, *Il contributo di G. Peano all'ideografia logica*, Periodico di Matematiche, 4, 13, 1933, p. 16: «nel 1889, cioè quando il Peano negli *Arithmetices principia, nova methodo exposita* [...] riuscì primo ad esporre con linguaggio simbolico una teoria deduttiva (postulati, definizioni, teoremi e dimostrazioni), il contributo di Leibniz alla sua *Ars characteristica* era ancora in gran parte ignoto» [«in 1889, i.e., when Peano in the *Arithmetices principia, nova methodo exposita* [...] first succeeded in expounding with symbolic language a deductive theory (postulates, definitions, theorems and proofs), Leibniz's contribution to his *Ars characteristica* was still largely unknown ».]

[84]In 1930 Vacca was given the task of writing the entry *Leibniz* for the *Enciclopedia Italiana Treccani.*

«ha scopo essenzialmente didattico, occupandosi specialmente di perfezionare i metodi di insegnamento. Essa conterrà pure articoli e discussioni riferentisi ai principi fondamentali della scienza e alla storia delle matematiche e vi avrà parte importante la recensione dei trattati, e di tutte le pubblicazioni che riguardano l'insegnamento.»[85]

Certainly it may seem surprising to consider the works of Couturat on Leibniz, reviews of which were published in the *Revue de Mathématiques*, as "treatises and publications having to do with teaching". Nevertheless, the desire to make research on Leibniz available to the world of students and teachers seems to be a constant feature of the work of Peano.

On examining Peano's and his students' articles on Leibniz, one notes an indisputable convergence of intentions. Even more: the interweaving of cross-references and reciprocal quotations makes it difficult to establish the exact paternity of individual reflections. Often the same passages from Leibniz are analyzed and quoted in several works, by different authors. At other times we are face to face with a mechanism of inter-crossed collaborations, as happens in the case of the *Formulaire*. Notwithstanding, the convergence of quotations is not a case of 'plagiarism' as one might be led to believe. It seems to me symptomatic of the existence of a patrimony of knowledge considered 'common' to a School. Peano's students thought of themselves as members of a School, and often seemed to consider the results of their historical, mathematical or philological research as results 'of the School', without *distinguo* by the name under which they had been published. Thus is expressed one of the finest aspects of the personality of Peano, a Master who knew how to make collaboration one of the cardinal features of his scientific activity.

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[85][«has essentially a didactic purpose and it occupies especially with perfecting teaching methods. It will also contain articles and discussions referring to the fundamental principles of science and to the history of mathematics and an important place will be given to reviews of treatises, and of all publications having to do with teaching.»] [G. PEANO], *Ai lettori*, RdM, 1, 1891, p. nn.

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