ARE WE TALKING ABOUT GRAPHS OR TRACKS? POTENTIALS AND LIMITS OF 'BLENDING SIGNS'

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Graphs of functions are often introduced to students by means of contexts involving perceptive and narrative features. This paper takes a Peircean semiotic approach to analyse learning processes related to graphs as models for tracks in the contexts of tramps in the mountains. Using Peirce's triadic model of sign, the new construct of 'blending sign' is introduced and illustrated with examples from a classroom teaching experiment at grade 11. Possible gains and limits of blending signs in teaching-learning processes are discussed, and the notion of 'semiotic control' is introduced to describe an important cognitive and didactic goal in the outlined perspective.

INTRODUCTION AND THEORETICAL BACKGROUND

Functions are one of the fundamental modelling tools and mathematical concepts of modern mathematics. From a didactical point of view, various contexts are used both to introduce functions that model them, and to approach the function concept itself. Over the last decade, research has deserved special attention to the contexts of motion and of change, especially from a graphical point of view and often using ICT technology (e.g. the SimCalc Project, Rochelle et al., 2000, and the studies of Arzarello & Robutti, 2001, and Nemirovsky et al., 1998)

Common assumptions of such approaches include that motions and change are meaningful contexts for the students and that their features (such as "embodied" aspects, Lakoff & Nùñez, 2000) can be exploited to bridge the gap between the worldly experiences and the abstract mathematical objects. In Bruner's psychological account (Bruner, 1986), didactical approaches grounded in everyday situations result helpful from a cognitive (and therefore didactic) point of view since they foster the dialectics between *narrative thinking* and *paradigmatic or logical-scientific thinking*. Narrative focuses on intentions and actions, and is strongly anchored to experience in context, as perceived by the subject. According to Bruner, it is the privileged form by which individuals construct meanings and interpret the world around. In fact, it creates structured sequences of events (involving people, objects, etc.), and meanings and understanding are determined by these structures. On the contrary, paradigmatic or logical-scientific thinking is based on proof and logical deduction, and develops towards abstraction in terms of formal and symbolic systems. Based on the essential principles of coherence and non-contradiction, it needs explicit teaching.

Mathematics is typically characterized by logical-scientific thinking. Furthermore, mathematical objects do not belong to our world and experience: they are not

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perceivable *per se* but only mediated by signs. It results therefore suitable to take a semiotic perspective to investigate learning processes in approaches to functions that start from contexts familiar to students, where narrative and embodied ways of thinking play important roles.

Following Peirce's theory (Peirce, 1931-1958), a *sign* is a triad composed by the sign or *representamen* (that which represents), the *object* (that which is represented), and the *interpretant*:

> It [The sign] addresses somebody, that is, creates in the mind of that person an equivalent sign or perhaps a more developed sign. That sign which it creates I call the interpretant of the first sign. The sign stands for something, its object. It stands for that object, not in all respects, but in reference to a sort of idea. (*ibid.*, vol. 2, par. 228)

Everything entering into a semiotic process is a sign, and therefore a large variety of phenomena, included body movements and gestures, can be considered as semiotic resources (Arzarello et al., 2009; Sabena, 2008).

The interpretant is the most delicate element in the definition. It can be an equivalent sign in another semiotic system (i.e. a drawing to explain a word meaning), a definition in the same semiotic system (i.e. salt for sodium chloride), or even a complex discourse, inferentially developing all the logical possibilities implicated in the sign. An innovative feature of the interpretant in the very definition of sign is that the sign is thereby intrinsically endowed with a dynamic character. In fact, the interpretant is a sign that translates and explains the previous one, and this other sign in its turn requires another sign as interpretant, and so on in a chain of infinite interpretation, establishing a process of dynamic unlimited semiosis.

Peirce's account provides also an insightful classification of signs, according to the manner in which they are capable of signification. A sign refers to its object in an *iconic* way if it resembles it (as a picture); in an *indexical* way if it is materially affected by it (as smoke with respect to fire); in a *symbolic* way if it refers to its object by means of a rule (such as language words or algebraic formulas). This characterization is not to be thought as splitting signs into separate categories, since these different features may coexist in the same sign.

This paper takes a Peircean semiotic approach to analyse learning processes related to the graph of functions as modelling tool in the contexts of tramps in the mountains. After outlining the research context and methodology, the construct of *blending signs* is introduced and illustrated through examples from classroom observations. Discussing implications at cognitive and didactic level the notion of semiotic control is described with reference to didactic context.

TEACHING EXPERIMENT

An on-going research project jointly involving the mathematics education teams of Auckland (New Zeland), Haifa (Israel), and Torino (Italy) is currently studying a new approach to the antiderivative function concept, focusing on its graphical aspects.

The first task of the teaching sequence is set in the context of tramps in the mountains, and is based on height-distance graphs. In line with the discussion above, such context is considered meaningful for the students (especially in Italy and New Zeland, rich in mountains) and rich in narrative and embodied elements that can be exploited from a cognitive and didactic point of view.

This paper is based on the teaching experiment carried out in Italy in an 11^{th} grade class of a tertiary school ('Professional Institute for Hotel Services and Refreshment'). In the previous year, students have been introduced to functions as modelling economic phenomena, and in particular to linear functions. Students are now organized in group-works (3-4 students each group) to solve the tasks and then to participate to classroom discussions co-ordinated by the teacher. Classroom observation is conducted by means of video-recording and the collection of students' written production. Collected data are analysed in an integrated way, providing the fabric for a semiotic analysis according to the perspective outlined above.

The next paragraph will show examples of semiotic processes appearing in students' solving processes of the first task of the teaching experiment. It shows a newspaper article reporting the tragedy of two trampers died in a steep ravine, and the critique of their friends about the lack of information about the tramp provided by authority.

> "The most useful information about a track is how steep it will get going uphill and downhill," say the trampers' friends, adding that few trampers know how to figure it out for themselves using contour maps.

Students are provided a Cartesian graph of a tramping track that goes up and down a hill (Figure 1) and are asked about: *the horizontal distance of the track, at what horizontal distance the track is the highest, and how high is the highest part of the track*. Then they are invited to find the steepness of the track by finding approximate values of the slope of the tangent line to the graph, using the formula: Slope = $\frac{y_2 - y_1}{x_2 - x_1}$.

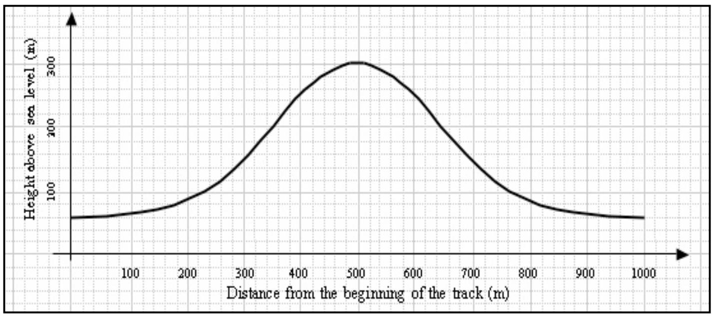

Figure 1: Distance-height graph of track

They are asked to find 1) *how much the track is steep at certain given points (e.g. at 400 metres)*; 2-3) *how the track looks like when the slope of the track is zero/negative*; 4-5) *where are the track steepest uphill/downhill*. They are also asked to *explain their answers*.

ANALYSIS

As confirmed by classroom observation, the story presented in the newspaper article is motivating and provides narrative elements which help making sense of the task (according to Bruner, 1986, stories involve narrative thinking) and linking it to lived or imagined bodily experiences.

On the other hand, the task provides itself complex mathematical tools modelling the situation: the graph in the Cartesian plane, and the slope formula. Let us focus on the graph. From a cognitive point of view, students have to interpret the given graph to get information about a certain context (regarding a track and its slope). The relationship between the graph and its object is not direct. In a Peircean semiotic perspective, the graph is a sign (or representamen) for the track (the object), and their relationship is given by an interpretant. In Figure 2, only some possible features of the interpretant are listed. The complexity of the sign is also due to its cognitive characterization. In fact, by its nature, a sign depends on the subject that is interpreting it.

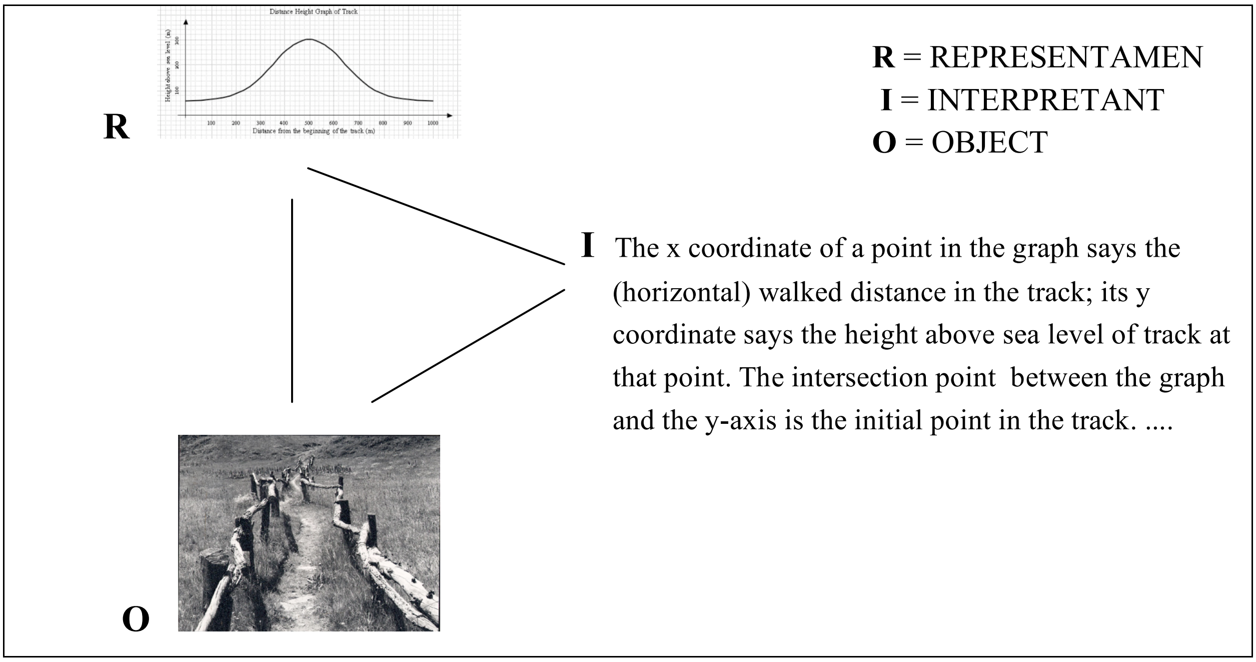

Figure 2: A Peircean semiotic interpretation of the graph in Figure 1.

The graph/representamen is linked to the track/object in a strongly *symbolic* way (in Peirce's classification of signs). For instance, every point of the graph corresponds to a certain position of the (real or imagined) track, its y coordinate telling the highness above the sea level of such corresponding point. An interesting analysis of the symbolic features of Cartesian graphs modelling motion and of their implications in teaching-learning context has been proposed by Radford (2009).

In our case—as in many cases in motion contexts— the graph (representamen) is also characterized by *iconic* relationships with the track (object). In fact, not only both are observable and perceivable (also a formula is like so), but locally the steepness of the graph at any given point looks like the steepness of the track at the corresponding position. For instance, where the graph looks like having no or little slope (e.g. at distance of 500 m), also the track appears with no or little slope, and vice versa. In this sense, a graph is semiotically very different from a formula, which is a symbolic sign. Coherently with those psychological perspectives assigning fundamental roles to embodied aspects of cognition and to narrative modes of thinking, it can be hypothesized that it is such iconic characterization of the graph-track relationship to support the students in facing and solving the task.

However, such iconic features of graphs may also have a negative counterpart, that emerged through the analysis the students' processes and written solutions to the task. Let us focus on a short excerpt from the video-recording of the solving process of a group of medium-achievers students. In the initial part of the activity, the students try to make sense of the given graph (Figure 1) in terms of the track. We observe that in many cases the students are using words and gestures that could refer both to the given graph, and to the corresponding imagined track. For instance:

Robert: *Well, let's say that it starts to uphill here* (he is pointing his index finger to the point with ordinate equal to 100, Figure 3 left) *at 100...It starts here* (repeating the same pointing) to uphill*, let us say 100... [...] Here it is the point of highest slope* (he is pointing to the flex, Figure 3 right)...

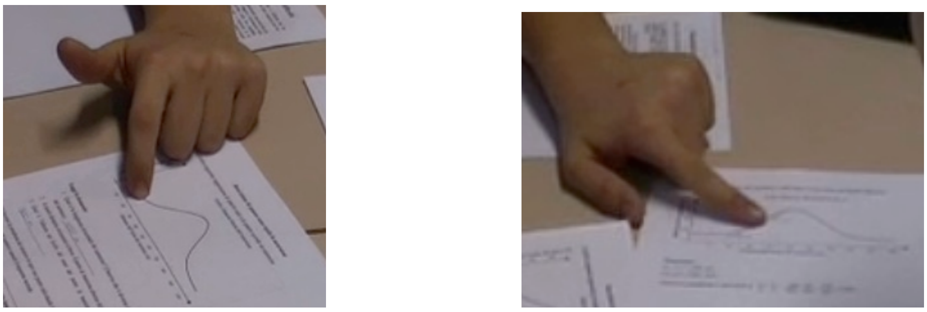

Figure 3: Gestures pointing to the graph of Figure 1

Because of their deictical nature, the word *'here'* and the co-timed pointing gestures are both indexically referring to the given graph. On the other hand, *'uphill'* (*'salita'* in Italian) is semantically related to the track, and the task itself is asking questions about the track. Hence, the student's utterance as a whole may be interpreted as referring either to the given graph or to the track. In Peircean terms, the object (O) of the semiotic act related to words and gestures (R) could be the modelling graph, the modelled track, or both of them. I call *blending signs* such kind of signs in which the representamen is linked or may be linked by the interpretant to two (or more) objects. In other words, blending signs refer to two distinct domains, by means of certain relationships between them. Notions of 'blended spaces' and 'conceptual blending' as fundamental cognitive mechanisms have been introduced in cognitive linguistics by Fauconnier and Turner (1996). They have been applied by Lakoff and Nùñez (2000) for describing the number line, and by Edwards (2009) to examine how gestures express meaning. Not necessarily incompatible with such cognitive accounts, the notion of blending signs introduced in this paper differs from them for its essential semiotic nature, illustrated in Figure 4.

The relationships between the objects of blending signs, oversimplified in Figure 4 by a dotted line, can be very complex. For instance, in our case one of the objects (the graph) may act as a representamen for the other one (the track), as discussed above and illustrated in Figure 2.

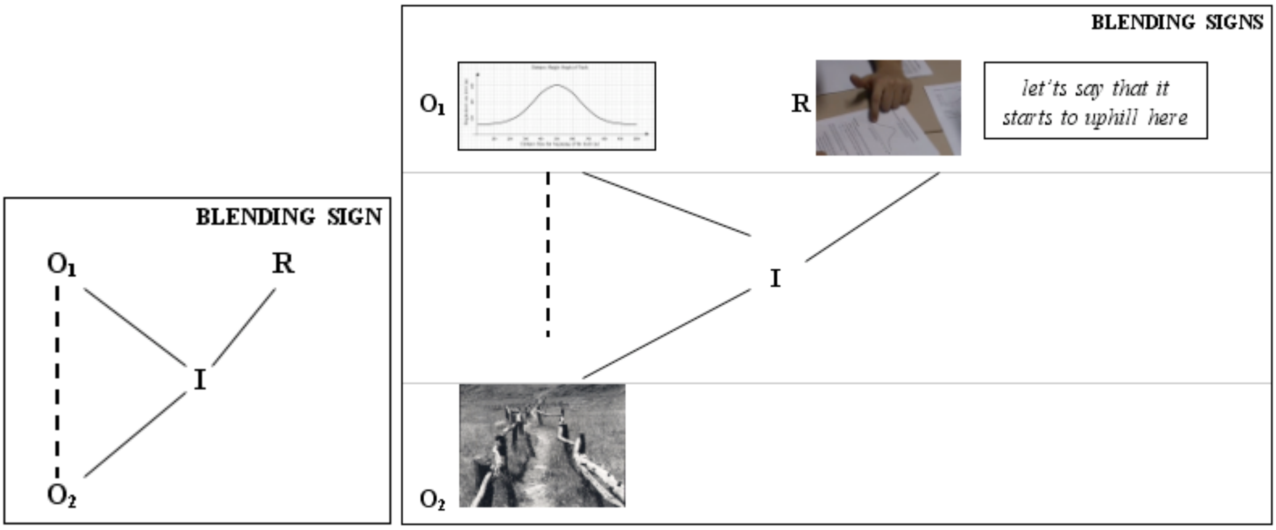

Figure 4: A Peircean diagram illustrating the blending signs

The subjects that are making use of blending signs may be not (fully) conscious of what they are referring to. And when blending signs intervene in the interaction with others, their objects may remain fuzzy. This important feature of blending signs will be discussed in next paragraph.

From the video-recording, other similar blending signs can be found in the students' semiotic activity. Their frequent use does not prevent the students to solve the task, and to correctly articulating the graphical domain with the requests about the track, as we can see from the written answers to questions 2 and 3:

Question 2: The track looks like flat if the slope of the tangent line is zero.

Question 3: The track looks like downhill if the slope of the tangent line is negative

Other groups, on the contrary, show blending signs also in their final answers, as in the following examples, where a track is described as being parallel to the x-axis, and a graph is told to go downhill:

Question 2: If the slope of the tangent line is zero the track is parallel to the x-axis.

Question 3: The graph goes downhill.

In both answers there is some element referring to the Cartesian plane ('parallel to the x-axis', 'graph') directly associated with some element referring to the track ('track', 'downhill'). Therefore, we can consider them as blending signs.

DISCUSSION

The semiotic analysis based on video-recordings and written protocol has allowed us to identify a special kind of signs that have been called 'blending signs'. From the overall analysis of the teaching-experiment classroom activities, many occurrence of blending signs can be pointed out. They appear strongly related to the *iconic* aspects of the mathematical semiotic activity, specifically to the iconic aspects of the function graphs with respect to their modelled objects (in this case, the track). Through such iconic characterization, the use of blending signs may play a certain role in supporting students in coping with the task. In her didactic action, the teacher can profit of this cognitive gain, both in the choice and design of tasks and in her direct intervention (for instance to help students with difficulties).

However, besides cognitive and didactic gains of blending signs, there are also corresponding cognitive and didactic risks. The subjects involved in semiotic processes with blending signs may be not (fully) conscious of what they are referring to. Typical cases may happen in classroom context, if we imagine a student that may be simply describing what he is seeing in a graph, and other students of even the teacher may interpret him as referring to the modelled context. Or vice versa: a teacher making use of strongly indexical words and gestures (similar to those shown in the above example), actually be referring to the corresponding objects in the modelled context. This phenomenon did happen in the observed classroom.

Blending signs, in fact, are not peculiar to novices. It would be very interesting to analyse with such lens a conversation between two working mathematicians. I guess I would find lots of examples of blending signs, whose objects are of a very abstract nature. In their work, in fact, mathematics experts use not only highly formal semiotic systems characterizing logical-scientific thinking, but also everyday language, rich in narrative and metaphoric thinking (Thurston, 1994).

What would then be the difference between experts' and novices' blending signs?

Such difference may be accounted in terms of *semiotic control*, that is to say in terms of how much a subject is able to use signs, included blending signs, as long as they are helpful with respect to the task at hand, and to abandon them in favour of more suitable signs. Recalling Schoenfeld's account of control (Schoenfeld, 1985), 'semiotic control' can be defined as the ability of choosing and mastering signs in order to solve a given problem. Semiotic control is what is missing in those cases of blind automatisms that often guide novices' actions, for instance when students are able to master (even complex) techniques to solve an equation but cannot interpret the found solution in terms of the problem at hand. Of course, semiotic automatisms are essential to reduce the cognitive load; semiotic control allows the subject to regulate them: to let them go as long as it is necessary, and then to stop and rule over them.

In the case of blending signs, semiotic control allows a subject to correctly use signs linking them exactly to the object that is useful in that certain moment in the problem: in the examples analysed above, to the graph when referring to characteristics of the graph (e.g. its coordinates), and to the track when referring to features of the track (e.g. a uphill).

This perspective has important didactic implications. In fact, if mastering of techniques is not always an easy goal in learning activity, developing a semiotic control is far away a more difficult endeavour, and it requires explicit teaching.

Teachers need to be conscious of the rich and complex semiotic features of certain kinds of tasks and to master suitable teaching methodologies to guide students towards an effective semiotic control.

Further research—included research on teacher formation with respect to semiotic issues—appear necessary to reach these ambitious aims.

References

Arzarello, F. & Robutti, O. (2001). From body motion to algebra through graphing. In H. Chick, K. Stacey & J. Vincent (Eds.), *Proceedings of the 12th ICMI Study Conference: The teaching and learning of algebra* (vol. 1, pp. 33-40). Melbourne, AU: The University of Melbourne.

Arzarello, F., Paola, D. Robutti, O., & Sabena, C. (2009). Gestures as semiotic resources in the mathematics classroom. *Educational Studies in Mathematics*, *70*(2), 97-109. New York: Springer.

Bruner, J. (1986). *Actual Minds, Possible Worlds*. Cambridge, MA/London, UK: Harvard Univ. Press.

Edwards, L.D. (2009). Gestures and conceptual integration in mathematical talk. *Educational Studies in Mathematics*, *70*(2), 127-141.

Fauconnier, G. & Turner, M. (1996). Blending as a central process of grammar. In A. Goldberg (Ed.), *Conceptual Structure, Discourse, and Grammar* (113-129). Stanford, CA: Center for the Study of Language and Information, Cambridge University Press.

Lakoff, G. & Nùñez, R. (2000). *Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being.* New York: Basic Books.

Nemirovsky, R., Tierney, C. & Wright, T. (1998). Body motion and graphing. *Cognition and Instruction, 16*(2), 119-172.

Peirce, C.S. (1931-1958). *Collected Papers, Vol. I-VIII*. Edited by C. Hartshorne, P. Weiss & A. Burks. Cambridge, MA: Harvard University Press. (CP, Volume number. Paragraph number).

Radford, L. (2009). Signifying Relative Motion: Time, Space and the Semiotics of Cartesian Graphs. In W.-M. Roth (Ed.), *Mathematical Representations at the Interface of the Body and Culture* (pp. 45-69). Charlotte, NC: Information Age Publishers.

Roschelle, J., Kaput, J., & Stroup, W. (2000). SimCalc: Accelerating student engagement with the mathematics of change. In M. J. Jacobson & R. B. Kozma (Eds.), *Learning the sciences of the 21st century: Research, design, and implementing advanced technology learning environments* (pp. 47–75). Mahwah, NJ: Lawrence Erlbaum Associates.

Sabena, C. (2008). On the semiotics of gestures. In L. Radford, G. Schumbring & F. Seeger (Eds.), *Semiotics in Mathematics Education: Epistemology, History, Classroom, and Culture* (pp. 19-38). Rotterdam, Netherlands: Sense Publishers.

Schoenfeld, A. H. (1985). *Mathematical problem-solving*. New York: Academic Press.

Thurston, W.P. (1994). On proof and progress in mathematics. *Bulletin of the American Mathematical Society*, *30*(2), 161-177.