IN FAVOUR OF A UNIFIED MODEL OF DEDUCTIVE REASONING

Bara B., Bucciarelli M. & Lombardo V. 2000. In favour of a unified model of deductive reasoning. García-Madruga, J., Carriedo, N.& González-Labra, M.J. (Ed), Mental Models in Reasoning. Madrid: UNED, 69-81.

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Entre los diferentes tipos de razonamiento deductivo están el razonamiento silogístico, el relacional y el proposicional; éstos implican, respectivamente, razonar a partir de enunciados cuantificados, de premisas que contienen relaciones y de proposiciones que incluyen conectivas. La teoría de los modelos mentales (MMT; Johnson-Laird, 1983; Johnson-Laird & Byrne, 1991) explica y predice la actuación humana en estos tres campos del razonamiento. En Bara, Bucciarelli and Lombardo (1999), hemos propuesto un modelo computacional unificado del razonamiento deductivo mediante modelos mentales, que abarca los campos arriba mencionados del razonamiento silogístico, relacional y proposicional (UNICORE: UNIfied COmputer REasoner).

El modelo supone que cualquier tipo de razonamiento consta de cinco procesos principales: construcción de modelos mentales de las premisas, integración de modelos, formulación de conclusiones consistentes con los modelos integrados, falsificación de conclusiones y producción de respuestas. Para lograr esto, UNICORE supone que existe un conjunto de procedimientos que es común a cualquier tipo de deducción, y que forma parte de la competencia del sistema humano. La validez del modelo en los diferentes campos del razonamiento, proviene del hecho de que las predicciones de las respuestas de los sujetos se basan en un único mecanismo básico, cuyo funcionamiento puede verse afectado por una serie de restricciones cognitivas.

En este capítulo presentamos un experimento cuyo propósito es analizar las habilidades básicas implicadas en las diferentes fases de la

deducción. En UNICORE algunas de las fases son dependientes de la tarea, mientras que otras son independientes. Los resultados del experimento apoyan la existencia de la serie de habilidades básicas supuestas por el modelo UNICORE de la deducción, contribuyendo así a reforzar su plausibilidad psicológica.

INTRODUCTION

The contribution of the different psychological theories to the understanding of deduction can be evaluated according to three criteria (see Bara *et al.*, 1999). First, the theory ought to scale up to explain multiple domains of reasoning: a global and parsimonious theory has to be preferred to a theory restricted to a single domain. Second, the theory ought to account for the differences in competence and performance. In particular, a theory should not only predict the correct responses, but also predict and explain, by means of its basic tenets, systematic errors. Third, the theory ought to explain developmental trends in deductive performance. This would ensure that the theory is better grounded than steady-state theories (Bara, 1995).

As for the first criterion, two relevant attempts for unified theories of deduction are those by Polk and Newell (1995) and Rips (1994). Polk and Newell (1995) propose, within the Soar unified architecture (Newell, 1990), the Verbal Reasoning Model. They argue that there are no specific mental processes devoted to reasoning: language comprehension and generation can give an account of the entire range of deductive phenomena. In particular, they analyze syllogistic reasoning and claim that it relies on the encoding and decoding of the verbal information given in the premises. Their proposal is an alternative theory within the model-based paradigm, but it states that the search for counter-examples plays little or no role in syllogistic reasoning. However, recent studies show that the search for counter-examples is a fundamental step of syllogistic reasoning (Bucciarelli & Johnson-Laird, 1999), and though SOAR might be extended in principle to incorporate developmental aspects of syllogistic reasoning (second and third requirements), current it does cover only the syllogistic domain.

Rips (1994) makes an attempt of unified theory of deduction inside the mental logic paradigm, and claims the existence of a central deductive mechanism devoted to formal reasoning. He reproduces the mental processes through a system named PSYCOP which, when presented with premise-conclusion pairs, uses a set of formal rules to construct the lines of a proof. Among the models based on formal rules of inference, this is the best account of both syllogistic and propositional reasoning. However, it has the major limitation of not accounting for consistent patterns of erroneous inferences (see Johnson-Laird, 1997), and developmental issues and relational inferences have not been considered.

Besides these relevant attempts for unified theories of deduction, there exist many microtheories developed to account for single domains. In particular, microtheories for syllogistic reasoning (e.g. Guyote & Sternberg, 1981, and Stenning & Oberlander, 1995), relational reasoning (e.g. Bar-Hillel, 1967, and Clark, 1969) and propositional reasoning (e.g. Braine, 1978, 1990, 1998; Braine & Rumain, 1983; Inhelder & Piaget, 1958; Piaget, 1953; Rips, 1990). Other microtheories have been formulated to account for a single phenomenon inside a single domain, as in the case of conditional reasoning (e.g. Cheng & Holyoak, 1985; Cosmides & Tooby, 1994; Gigerenzer & Hug, 1992; Griggs & Cox, 1982; Platt & Griggs, 1993; Pollard, 1981). A common property of all such microtheories is their limited scope: they lack a unified view of the deductive process. Also within mental model theory, the present state of the art does not offer a synthetic theory able to realize the paradigmatic claim of the mental model approach. Thus, one finds a constellation of microtheories, each devoted to analyze a specific sector of reasoning. Unfortunately, model microtheories are dishomogeneous among them, because they differ even in the crucial choices, like the primitive concepts and the basic abilities assumed as their foundations.

The debate whether the deductive capability is a unique machinery or a collection of micro-mechanisms needs further psychological evidence to be solved in either way. In this paper we present an experiment where results support the view of a unified machinery for deduction. In a previous work we have stressed the importance of unified models of deduction (Bara *et al.*, 1999), and we have presented UNICORE, a unified model of deductive reasoning which follows the basic tenets of MMT. The model is fully consistent with the microtheories developed within the mental model framework, and it does satisfy the three dimensions which assess the acceptability of a theory of deductive reasoning. The core of the system relies on a series of abilities that require some empirical evidence, which we are going to search in this paper.

THE REASONING PROCESS IN UNICORE

We assume that the reasoning process consists of the five phases illustrated in Figure 4.1.

The *Construction phase* takes as input the linguistic or perceptual premises of the task at hand and returns as output their model representations. In UNICORE, this phase is considered as a black box that translates the premises into their propositional representations, and then the latter into the mental models that are passed to the reasoning mechanism. As a consequence, UNICORE does not take into account possible incomplete representations of the input, due to the lack of general or linguistic knowledge.

Integration phase takes as input the models of the premises and returns as output an integrated model. This integrated model, which is the first result of the exploration in the space of the integrated models, is then passed to Conclusion, in order to extract the relevant information for a specific deductive task (e.g., syllogistic inferences, relational inferences, propositional inferences). If Integration fails to produce any integrated model, the reasoning process yields a failure, which interrupts the reasoning process.

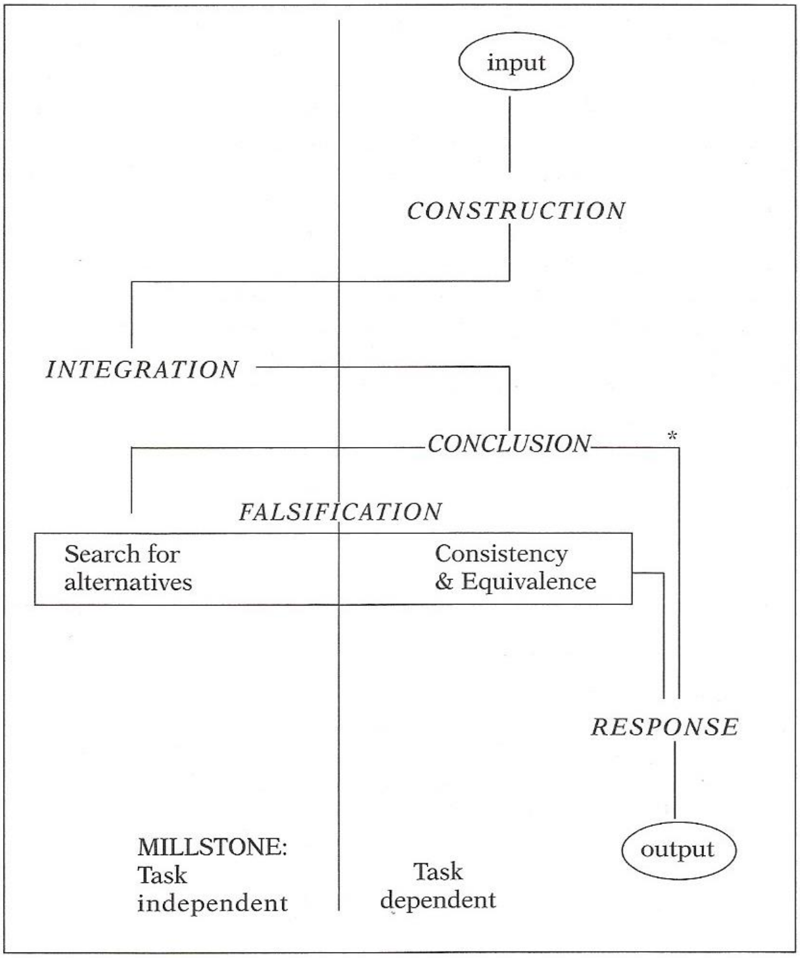

Figure 4.1. The reasoning process. * marks the erroneous control flow.

Conclusion phase takes as input an integrated model and produces a result model, which represents a first putative conclusion. For example, in the case of syllogistic reasoning, Conclusion marks the tokens and relations concerning the end terms. This result model becomes the (unique) current model in the working memory. In case there are no information relevant for the task, Conclusion exits with FAILURE.

The reasoning program maintains only one result model in the working memory, i.e. the current result model. The result model which is in the working memory after visiting the whole space of the integrated models will be passed to the Response phase.

After Conclusion, the control can flow in two directions: the correct one goes to Falsification; the erroneous one (which is marked by *) goes to Response, and shortens the reasoning process. This premature exit of the reasoning process, due either to the limited capacity of the working memory or to a poor degree of mastery of Falsification, can explain several data about subjects' erroneous conclusions.

The asterisk in the Figure (*) indicates that it is premature to exit at this point, because the conclusion which is based on the first integrated model may be falsified by a further integrated model of the premises. In case of failure, the control passes directly to the Response phase, which acts consequently (see below).

Falsification phase takes as input a putative conclusion (first result model), and gives as output a validated result model. It consists of *Search-for-alternatives* and *Consistency-&-Equivalence*. Each time it is invoked, Search-for-alternatives tries to produce a new integrated model (if any) of the premises. This integrated model is then passed to Consistency-&-Equivalence. Consistency-&-Equivalence first invokes Conclusion, which produces a new result model, and then detects the possible presence of contradictions between the current result model in the working memory and the new result model just yielded. If it does not detect any contradiction, but realizes that the new model supports the same conclusion as the current result model, Consistency-&-Equivalence leaves the working memory unaltered. Otherwise, if the new model does support a looser conclusion than the current result model, it replaces the current result model with the new result model. For instance, a model where 'Some A are C' supports a looser conclusion than a model where 'All A are C'. Finally, if it detects a contradiction, i.e. the two models support inconsistent conclusions, it generates INCONSISTENCY and passes it to the Response phase. When Search-for-alternatives exhausts the integrated models, the control goes to the Response phase.

In case of success of the previous phases, *Response phase* takes as input the current result model in the working memory and translates it into linguistic or motorial responses. Otherwise (either the flag FAILURE or INCONSISTENCY has been raised), it interprets the failure according to the task.

Detailed descriptions of the procedures which implement the five phases of the reasoning process are in Bara *et al.* (1999).

TASK DEPENDENT AND TASK INDEPENDENT PHASES OF THE REASONING PROCESS

Deduction occurs through some task dependent and some task independent procedures (see Figure 4.1).

The integration procedures and part of the falsification procedures (Search-for-alternatives) are task independent; they form a core system which we call *Millstone*. Construction, Conclusion, part of the falsification procedures (Consistency-&-Equivalence), and Response are task dependent.

The computational model UNICORE relies on the central role of the Millstone, which represents the core of the unified mechanisms for reasoning.

From an architectural point of view, however, deduction is conceived as the fruit of a continuous interaction between the Millstone and the task dependent procedures. As the latter behave in different ways depending on the task at hand, they link the core deductive processes and the context within which they operate. Moreover, the interface between the Millstone and the environment, represented by the task dependent phases, is sensible to the system's goals; in UNICORE the type of task counts as a pragmatic factor which influence the model manipulation process.

Thus, the context within which deduction occurs contributes to determine the sort of conclusion (either correct or erroneous) drawn by the reasoner. In particular, the experience of the subject in a specific reasoning domain will affect the reasoning process. We assume that the experience in a specific domain increases with increasing age.

In our view, the task independent phases of the reasoning process rely on some innate predispositions in the reasoner to make sense of the world; thus, the development of the ability to reason rely on the interaction between the mind and the environment. In particular, according to UNICORE, the innate predispositions are the ability to integrate information in a single model, and the ability to search for alternative models of such information.

The ability to integrate information is involved in our ability to make sense of the world. An unpublished experiment by Johnson-Laird and Anderson (cited in Johnson-Laird, 1993) shows that subjects —when invited to consider two sentences randomly chosen by two different stories— are very good in making sense of them, by constructing plausible scenarios.

The ability to search for alternative models is also involved in our ability to comprehend and predict events of the world. As Bucciarelli and Johnson-Laird claim in this volume, the ability to conceive of alternative models of reality characterizes our abilities to attribute emotional states, perceptions, and mental states.

BASIC ABILITIES NECESSARY TO REASON

The architecture of UNICORE is based on a series of assumptions about the abilities the system must possess, in order to be capable to make deductions. The initial choices on which are the basic abilities have been made according to the findings of a pioneering experiment on the development of syllogistic reasoning (Bara, Bucciarelli & Johnson-Laird, 1995). The basic abilities involved in the task dependent phases should not be necessarily involved in any sort of deductive reasoning, whereas the basic abilities involved in the task independent phases should be involved in any sort of deductive reasoning.

Abilities involved in the task dependent phases

The *knowledge of the deductive terms* (e.g. *all, some, if-then*) should be clearly involved in the construction phase of the reasoning process, which is task dependent. Currently, UNICORE does not take into account possible incomplete representations of the input, but it can actually occur that reasoners construct incomplete representations of the premises. In particular, their lack of knowledge of quantifiers and connectives might affect their reasoning performance. Relational terms, instead, should be quite easy to understand also for very young subjects. Thus, we might expect a correlation between the ability to solve syllogistic and propositional problems and the knowledge of the deductive terms involved. Reasoners who have a poor knowledge of the deductive terms should perform poorly in the reasoning task where the terms are involved.

The ability to *use the middle term* of two premises in order to draw a conclusion concerning the end terms (in syllogistic and relational reasoning) might be involved in the Conclusion phase, which is also task dependent. Indeed, this ability allows to obtain information concerning the relation between the end terms once the models of the premises have been integrated. We hypothesize that reasoners deficient in this ability often err by drawing a conclusion involving the middle term.

Abilities involved in the task independent phases

The ability to *grasp* the importance of *falsification* might be crucial in deciding whether it is necessary to attempt a falsification once one has reached a putative conclusion. Thus, this ability should be involved in the Falsification phase, and in particular in its activation, which is task independent. We might expect that reasoners who do not grasp the importance to falsify will tend to produce erroneous responses for all the problems requiring the construction of more than one model. Thus, we could expect that this ability is not in relation with the ability to solve one model problems.

The ability to *search for alternative models* allows to produce alternative integrated models of the premises, and it is task independent. If reasoners are poor in producing alternative models, they will base their conclusion on a subset of the possible models of the premises and, as a consequence, they will draw an erroneous conclusion. Again, this ability should be involved in multiple-model problems.

Eventually, the five reasoning phases should be affected by the *working memory capacity* required by the task at hand. UNICORE incorporates constraints on the number of models retained in order to draw a conclusion. To compare the results of our experiment with the constraints already incorporated in the program would allow us to better calculate the increasing working memory capacity of reasoners of increasing age.

THE EXPERIMENT

In a previous experiment Bara *et al.* (1995) explored the relation between the ability to draw syllogistic inferences and possible basic abilities. In particular, they found out that the working memory capacity and the ability to perceive identities in pairs of configurations account for variance in the ability to solve syllogisms.

We have carried out an experiment which investigates the basic abilities hypothesized on the basis of the functioning of UNICORE, rather than on theoretical speculations. We have explored the basic abilities involved in relational and propositional reasoning along with those involved in syllogistic reasoning.

The participants were children (7-to-8 year olds), pre-adolescents (11-to-12 year olds), adolescents (15-to-16 year olds), and adults (over 21 year olds), with twenty in each groups. Children and adolescents attended four primary and high schools in Torino respectively, and adults were Psychology students at the University of Torino: none of them had a previous training in logic.

The experiment was in two sessions; each of them was carried out individually, in a quiet room. In the first session, participants dealt with the basic abilities tasks and, in the second session —which occurred one week later—, with the deductive tasks. The order of presentation of the basic abilities tasks and of the deductive tasks was counterbalanced.

The deductive performance of the participants was measured throughout syllogistic, propositional and relational problems. Syllogistic problems consisted of four syllogisms in each of the following categories: one model, multiple model and multiple model with no valid conclusion. Propositional problems were four, involving one of the following connectives: exclusive disjunction (two models), bi-conditional (two models) and conditional (three models). Relational problems were six one-model problems and six multiple model problems with no valid conclusion.

The participants' basic abilities were measured through a series of tasks.

In the *knowledge of deductive terms* task, participants were invited to select pictures consistent with some utterances involving either a *quantifier* (all, some, none, some not) or a *connective* (exclusive or, if, only if).

In the *middle term* task, participants were invited to form chains of figures by overlapping the elements which were identical in the figures. The aim was to connect two specific elements which were represented in different figures.

In the *grasp of falsification* task, participants were presented with utterances, and they were invited to make one critical question for each of them in order to discover whether the utterance was true or false with respect to an hidden state of affairs.

In the *search for alternative models* task participants were presented with 64 cards, each representing different elements (either star, square, triangle or circle) of different colors (either red, yellow, green or blue). The experimenter invited the participants to form as many 'families' as possible.

Finally, the *working memory capacity* was measured through the digit span. Participants were invited to repeat numerical series uttered by the experimenter.

The results concerning the deductive problems can be interpreted in terms of number of models they required (see Table 4.1).

TABLE 4.1
Percentages of correct responses in the deductive problems

	Deductive problems							
Age groups (n=20)	**Syllogistic (n=4x3)**			**Propositional (n=4x3)**			**Relational (n=6x2)**	
	1-mod	**m-mod**	**m-nvc**	**2-mod (bi-con)**	**2-mod (disj)**	**3 mod (cond)**	**1-mod**	**m-mod**
7-8	23	13	34	64	66	40	40	39
11-12	44	16	29	80	78	60	54	56
15-16	59	30	30	88	95	65	78	70
>21	78	38	28	85	86	68	79	77
Global %	51	24	30	78	81	58	62	56

Model theory, in fact, predicts that the bigger is the number of models to construct, the harder is the reasoning problem. This prediction is confirmed for syllogisms: one model syllogisms were easier than multiple model syllogisms both valid (Wilcoxon test: $z = -5.965$, $p < .0001$) and invalid (Wilcoxon test: $z = -3.42$, $p = .0006$). Also, the prediction is confirmed for propositional problems: two-model problems, i.e. those involving bi-conditional and disjunction, were equally easy to deal with (Wilcoxon test: $z=-0.676$, $p=.49$). The three-model problems were more difficult: those involving the conditional connective were more difficult than those involving both the bi-conditional (Wilcoxon test: $z=-7.113$, $p<.0001$), and the disjunction (Wilcoxon test: $z=-0.673$, $p<.0001$). However, as regards relational problems, one-model and multiple-model problems with no valid conclusion did not significantly differ in difficulty (Wilcoxon test: $z=-.903$, $p=.37$). A possible explanation is the tendency of young subjects to conclude that 'no valid conclusion' follows from complex problems. As a consequence, they would draw the correct conclusion to the invalid problems, but for the wrong reason (see Johnson-Laird & Bara, 1984).

Further, mental model theory predicts that the ability to deal with multiple model problems increases with the increasing age. This prediction holds for all propositional problems (Jonckheere Trend test: $z=2.14$, $p=.016$) and for multiple model relational problems (Jonckheere Trend test: $z = 4.81$, $p<.00003$). Also, the performance of the different age groups with multiple model valid syllogisms increases with the age, although the improvement is not statistically significant (Jonckheere Trend test: $z= 0.46$, $p=.32$). However, the performance with multiple model invalid syllogisms does not increase with the age. Again, young subjects could perform well in invalid syllogisms for the wrong reason.

As regards the basic abilities, the results show that some of them increase with the age. In particular, the knowledge of connectives (Jonckheere Trend test: $z= 3.25$, $p<.0005$) —the knowledge of quantifiers did not improve with age—, the use of the middle term (Jonckheere Trend test: $z= 4.62$, $p<.00001$), the ability to search for alternative models (Jonckheere Trend test: $z=2.95$, $p <.002$), and the working memory capacity (Jonckheere Trend test: $z=4.69$, $p =.00001$).

Some of the basic abilities correlate (Fisher's correlation) with the ability to reason in a specific reasoning domain, others correlate with the ability to reason in all the three deductive domains we investigated (see Table 4.2). To stress the involvement of a specific ability in more than one sort of deductive reasoning, we present the correlation considering each ability separately.

TABLE 4.2
Significant correlations (Fisher's correlations) between the performance in the basic abilities tasks and the deductive problems

Abilities and deductive problems	Correlation	P - value
knowledge of disjunction - propositional probl.	.267	.016
knowledge of conditional - propositional probl.	.233	.037
grasp the importance to falsify - relational probl.	.226	.043
use the middle term		
— syllogistic 1model problems	–.44	<.0001
— syllogistic multiple model problems	–.267	.016
— syllogistic nvc problems	.327	.0029
— relational problems	–.473	<.0001
— propositional problems	–.306	.0055
search for alternative models		
— syllogistic problems	.364	.0008
— propositonal problems	.336	.002
— relational problems	.527	<.0001
working memory capacity		
— syllogistic	.278	.0122
— propositional	.255	.0224
— relational	.492	<.0001

The *knowledge of disjunction* and the *knowledge of conditional*, which are involved in a task dependent phase (Construction), correlate with the ability to solve propositional problems.

As regards the ability to *use the middle term*, which we considered as involved in a task dependent phase (Conclusion), we found a significant correlation with the general ability to solve syllogistic, propositional and relational problems. In the task devised to measure this ability, a high score corresponds to a poor performance, so the correlations are negative. Nevertheless, the correlation is positive for no valid conclusion syllogisms. This result is in line with our assumptions: when reasoners are poor in constructing the integrated models from complex problems, they tend to conclude that nothing follows from the premises. But, in fact, 'nothing follows' is the correct conclusion for invalid problems. Our hypothesis was that the ability to use the middle term is involved in a task dependent phase; the fact that it does correlate with the ability to deal with all sorts of deductive problems can be explained by the fact that it is also involved the Integration phase, which is task independent. In fact, it is through the overlapping of the identical tokens belonging to different models (i.e. middle terms) that Integration occurs. A significant improvement in using the middle term occurs around 11-12 years. Thus, we expect that children younger than 11 years may err by drawing conclusions involving the middle term. In fact, the 26% of errors in syllogistic reasoning and the 30% of errors in relational reasoning reflect this pattern. Such a pattern of errors is much more common in young children than in the other groups of participants. In propositional problems, where there is no middle term, this error is very rare.

The ability to *grasp the importance to falsify* correlates with the ability to solve relational problems. However, consistently with the fact that the ability to solve multiple model syllogisms does not significantly improve with the age, the grasping of the importance to falsify does not significantly improve with the age.

The ability to *search for alternative models* correlates with the general ability to solve syllogistic, propositional and relational problems. This ability improves with the age: as a matter of fact there is an improvement, with the increasing age, in the ability to solve multiple model problems.

The *working memory capacity* correlates with the ability to solve syllogistic, propositional and relational problems. The results concerning deductive problems show that a major improvement in the working memory capacity occurs around 11 years; such results are consistent with the mentioned correlation in that young children tend to base their responses on the first integrated model of the premises more often than the other participants.

CONCLUSIONS

In a preceding work, we substantiated the claim in favour of the existence of a unified mechanism for deduction, by devising a computer program able to reproduce the performance of human subjects in three different domains: syllogistic, propositional and relational reasoning. UNICORE is the proof that it is possible that such a mechanism does exist in the human mind.

Our approach differs from theories that are more concerned with the actual reasoning performances (see, e.g. Evans, 1989; Evans & Over, 1996); such theories, in fact, emphasize the understanding of performance constraints rather than reasoning competence. Also, our approach differs from theories claiming that deductive competence is part of the general thought processes (see e.g. Oaksford & Chater, 1994).

UNICORE is equivalent to a formal axiomatization of mental model theory; thus, it represents a computational evidence, but without a psychological validity. In the present paper we have tried to face a second question: does UNICORE possess a psychological plausibility? In other words, is our unified deductive mechanism not only theoretically possible, but also psychologically probable, in that people have something equivalent in their minds?

From the tenets of model theory, and from the architecture of UNICORE, we have devised some hypotheses on the basic abilities involved in deductive reasoning. The results of an experiment we carried out on subjects of different age confirm our predictions. In particular, we found that there exist basic abilities which are common to more than one sort of deductive reasoning, and some which are involved in a single reasoning domain. These results are in favour of the existence of a set of basic procedures involved in any kind of deductive reasoning, and the co-existence of contextualized abilities.

We regard our results as an important source of experimental evidence in favour of the plausibility of UNICORE. In fact, we have now an independent confirmation of the claim that humans possess in their brain/mind a unified mechanism for deductive reasoning. The weaknesses of UNICORE's predictions are many, and the next step of our research program is to incorporate into the computer model new constraints, in order to finesse its performance, especially in a developmental perspective.

ACKNOWLEDGEMENTS

This research was supported by the Ministero della Pubblica Istruzione of Italy (MPI ex-40%, 1999). The names of the authors are in alphabetic order.

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