The Geneva Papers on Risk and Insurance Theory, 26: 195-224, 2001
(c) 2001 The Geneva Association

# The Valuation of Insurance under Uncertainty: Does Information about Probability Matter? 

CARMELA DI MAURO<br>Dipartimento di Economia Politica, Università della Calabria, Arcavacata di Rende, Italy<br>ANNA MAFFIOLETTI<br>Dipartimento di Economia Politica, Università di Torino, Italy; School of Economic Studies, University of Hull, UK

Received June 16, 2000; Revised October 25, 2001


#### Abstract

In a laboratory experiment we test the hypothesis that consumers' valuation of insurance is sensitive to the amount of information available on the probability of a potential loss. In order to test this hypothesis we simulate a market in which we elicit individuals' willingness to pay to insure against a loss characterised either by known or else vague probabilities. We use two distinct treatments by providing subjects with different information over the vague probabilities of loss. In general we find that uncertainty about probabilities has a weak impact on consumers' valuation of insurance. However, additional information about probabilities tends to marginally increase the price individuals are willing to pay to insure themselves. Implications for the insurance market are derived.


Key words: insurance, probability, uncertainty, ambiguity, information

JEL Classification No.: D81

## 1. Introduction

Since the work of Knight [1921], economists have made the distinction between risk and uncertainty. While risk refers to a situation in which the probabilities assigned to outcomes are precisely stated, uncertainty concerns those situations in which probabilities are either unknown or vague. Keynes [1921] introduced a further distinction between judged probability and the "weight of evidence". While the former represents the evidence in favour of one proposition, the latter determines the decision-maker's degree of belief in the evidence supporting that proposition. A greater weight of evidence, which depends "on the amount, type and unanimity of information" [Ellsberg, 1961, p. 657], will increase subjects' willingness to bet on a proposition. Since Ellsberg's seminal paper [1961], the observed tendency by individuals to value prospects that are characterised by vague probabilities less than equivalent prospects characterised by exact probabilities has been called ambiguity aversion [Camerer and Weber, 1992], whereas the opposite tendency is referred to as ambiguity proneness.

Implications for economic behaviour of the existence of an individual reaction to uncertainty might be of paramount importance. Ambiguity affects many market and public
policy decisions: buying and selling insurance, the choice of new technology, investment in new industries or countries, stock market purchases and sales (see the recent theory by Epstein and Wang [1994] and Epstein [2000]).

The experimental literature has proved the robustness of ambiguity reaction under different experimental conditions ${ }^{1}$ but while the issue of comparing individual valuations under known probabilities (risk) versus unknown/vague probabilities (ambiguity) has been dealt with in several papers, little attention has been devoted to the analysis of the effect that alternative representations of ambiguity may have on individuals' reaction to uncertainty. One common strategy used by economists to describe uncertainty about probability is the use of a second-order probability distribution (SOP henceforward). This strategy draws its justification from the fact that, in many instances, the subjective probability of an event can be interpreted in terms of a SOP. For instance, in Ellsberg's two-urns experiment, subjects were asked to bet on a colour and draw a ball either from an urn containing 50 black and 50 red balls or from another urn containing 100 red and black balls in unknown proportion. The probability of drawing a red ball from the latter urn (the ambiguous one) could be taken to lie uniformly in the interval $[0,1]$.

The objective of this paper is to investigate whether and how individual valuations of insurance are affected by the way in which probability uncertainty is described and represented and whether reaction to ambiguity is sensitive to the amount and type of information about the probability of a loss. In particular, we test if reaction to ambiguity persists when uncertainty is represented as a second-order probability distribution, and whether different descriptions of probability uncertainty affect individuals' perception of ambiguity.

To illustrate how ambiguity may matter for insurance valuation, consider the following examples: (a) how much would you pay for flight insurance to fly Rome-New York with FlyItalia? Assume FlyItalia is an established company whose record of air accidents is low and publicly known. (b) Would you be willing to pay more/less/the same to purchase flight insurance to fly the same distance with AirItalia? AirItalia is a new company and little information is available about its safety standards. (c) Consider now an alternative situation to (b): assume you have access to expert information (at no cost) which allows you to get a more precise idea of the probability of a plane accident if you travel with AirItalia. Would you be willing to pay more/less/the same with respect to cases (a) and (b)?

If valuation of insurance protection by consumers and/or firms is sensitive to the presence of uncertainty then (b) is going to be valued differently from (a); moreover the pattern of preference between a) and b) could be affected when additional information about the probability of a plane accident is conveyed to the public.

To date, there is no conclusive experimental evidence of individual reaction to uncertainty in insurance contexts. Hogarth and Kunreuther [1985, 1989, 1992] find that valuation of insurance protection by consumers and/or firms is sensitive to the presence of uncertainty, but this result is not confirmed by the work of Camerer and Kunreuther [1989]. Moreover, empirical work which tests reaction to uncertainty in the domain of losses also gives mixed results (see for example Einhorn and Hogarth [1985, 1986, 1990], Cohen, Jaffray, and Said [1985], and Eisenberg and Weber [1995]). The results of these studies are difficult to compare given the use of different contexts, distinct procedures to elicit preferences, and
incentive mechanisms to reward subjects. In addition, these studies are heterogeneous with respect to the information experimental subjects receive concerning the stochastic process that determines the potential loss.
In order to pursue our investigation, we proceed in two steps: first, we build an experimental auction market in which consumers state their maximum willingness to pay to insure against both potential losses characterised by known probabilities and potential losses in which probabilities are ambiguous. Second, to investigate the differential impact of "more" and "less" information about the probability of the loss, ambiguous probabilities are made operational in two alternative fashions. Individuals involved in our experiment are either given a second-order probability distribution of the potential loss or just a vague description of the probability of the loss. In addition, by making uncertainty operational through three different SOPs, we investigate whether alternative second-order probability distributions matter for individual valuation. In designing and running this experiment, our main working hypothesis was that the impact of ambiguity on the valuation of insurance would have been stronger if the second-order distribution had not been clearly outlined. This because we thought that the lack of information would have made the chances of the event under consideration vaguer to the decision-maker (see Einhorn and Hogarth [1985] and Gardenfors and Sahlin [1983] for a theoretical interpretation). Moreover, the only studies (Yates and Zukowsky [1976] and Bernasconi and Loomes [1992]) which have performed a direct test of individual reaction to uncertainty as a SOP, show that in the domain of gains individual reaction to an SOP cannot account for all the violations of the Ellsberg paradox.

Our work departs from previous research because-by keeping task-relevant incentives and the outcome domain constant, we explicitly test for the difference between a situation in which subjects might interpret uncertainty through a second-order probability distribution and a situation in which subjects receive a detailed description of the second-order distribution used to represent the uncertain scenario.

We find that the presence of ambiguity has an impact on insurance valuation, although this impact is weaker than what is found in experiments such as those by Hogarth and Kunreuther [1989], that do not use incentive-compatible elicitation mechanisms. In addition, our results point out that specifying ambiguity as a precise second-order probability distribution does not reduce the impact of ambiguity on individual valuations.

The plan of the article is the following: Section 2 presents the theoretical background and the hypotheses tested. Section 3 explains the experimental design and procedures. Section 4 presents our results and, finally, Section 5 discusses the implications for experimental research in individual decision making under uncertainty, and for the functioning of insurance markets.

## 2. Theoretical background and hypotheses tested

How do people make decisions in the face of ambiguity? How does the amount and type of information about the chances of an event influence willingness to pay to insure?
Subjective Expected Utility (SEU) theory predicts that ambiguity should not matter to willingness to pay for insurance, provided that the expected probability of loss coincides
with the objective probability under risk. However, there is huge experimental evidence that people do not behave according to subjective expected utility.

Most of the theoretical models which allow for ambiguity reaction identify ambiguity either with a second-order probability distribution or with the incapability of subjects to form subjective probabilities that obey probability laws. Within the former class of models, uncertainty is either interpreted as the existence of a unique second-order probability distribution, as in Segal [1987], or as different sets of probability measures as in Ellsberg [1961], Gardenfors and Sahlin [1982, 1983], Levi [1974, 1986], and Gilboa and Schmeidler [1989]. Additional models which allow for a similar interpretation of uncertainty are the models that use adjusted probabilities such as Einhorn and Hogarth [1985], and Hogarth and Einhorn [1990]. Einhorn and Hogarth [1985], for example, state that "..ambiguity results from the uncertainty associated with specifying which of a set of distributions is appropriate in a given situation. Moreover, the amount of ambiguity is an increasing function of the number of distributions that are not ruled out by one's knowledge of the situation (p. S229)". All these models differ on the base of the decision rules adopted to "rule out" all but one possible probability measure; however, the common intuition behind them is that individuals identify uncertainty with the impossibility to assign a unique and precise probability to the occurrence of an event.
In the second group of theoretical models reaction to uncertainty is simply identified with the incapability of subjects to form subjective probabilities which obey to probability laws. In this latter class, ambiguity reaction is explained through the use of decision weights which might not be additive (among others Tversky and Kahnemann [1992], Fox and Tversky [1995], and Wakker and Tversky [1995]). The existence of these decision weights might or might not be associated with the existence of second-order probability distributions.
In this paper, the class of models which will guide our hypotheses about behaviour under ambiguity in the insurance context, is that based on the second-order probability interpretation of uncertainty.

The hypotheses that will be subject to test concern (i) individual attitudes towards ambiguity in an insurance context; (ii) the size of reaction to ambiguity according to the amount of information about the stochastic process determining the loss; and, finally, (iii) the effect of different representations of uncertainty on individual valuation of insurance.

### 2.1. WTP to insure under ambiguity versus WTP under risk

If we consider SEU as the base-line theory, we can state our null hypothesis about ambiguity reaction as follows:

Ambiguity Hypothesis 1A (SEU behaviour): The decision maker will be ambiguity neutral, i.e., his maximum willingness to pay (WTP) to insure against a loss whose probability is vague will be equal to his WTP for an equivalent loss whose probability is exactly known.

No conclusive test of this hypothesis exists so far. For instance, while Camerer and Kunreuther [1989] and Cohen, Jaffray, and Said [1985] find that in the domain of losses the
impact of ambiguity is insignificant, Einhorn and Hogarth [1985, 1986] and Hogarth and Kunreuther [1989, 1992], report a significant effect of ambiguity on consumers' willingness to pay for insurance. ${ }^{2}$

Theories alternative to SEU predict that ambiguity will affect WTP for insurance in various fashions. The "Anchoring and Adjustment" model by Einhorn and Hogarth [1985] assumes that people assess ambiguous probabilities, first, by anchoring on some reference value of the probability, called the anchor, ${ }^{3}$ and then adjusting this value upwards or downwards. The model predicts upward adjustments (with respect to the anchor) for low probability levels and downward adjustments for high probability levels. As a consequence, the ratio of WTP for insurance under ambiguity to WTP under risk will display a decreasing pattern as the reference probability used as "anchor" increases. Their model explicitly allows for ambiguity proneness when the anchor probability of loss is high, while it predicts ambiguity aversion at low probabilities. Therefore, as the likelihood of the loss increases from low to high, the individual will switch from being ambiguity averse to ambiguity prone. ${ }^{4}$

Thus, we can formulate one hypothesis alternative to 1 A concerning ambiguity attitude:
Ambiguity Hypothesis $1 B$ (Einhorn and Hogarth behaviour): The decision maker's maximum willingness to pay (WTP) to insure against a loss whose probability is vague will be greater than his WTP for an equivalent risky loss if the "reference" probability is low, while it will be smaller if the reference probability is high.

On the other hand, the models by Segal [1987] and Gardenfors and Sahlin [1982] do not explicitly allow for a switch in ambiguity attitudes according to the probability level. These models generally predict ambiguity aversion, i.e. that ambiguity will increase consumers' willingness to pay to insure. Hence, we should formulate a second alternative hypothesis as follows:

Ambiguity Hypothesis 1 (Segal and Gardenfors and Sahlin behaviour): The decision maker's maximum willingness to pay (WTP) to insure against a loss whose probability is vague will be always greater than his WTP for an equivalent risky loss at all "reference" probabilities.

### 2.2. The impact of probability information on WTP for insurance under ambiguity

The next set of hypotheses is related to the influence of information about the stochastic process that generates the loss on the evaluation of insurance by subjects.
According to the models by Einhorn and Hogarth [1985], Ellsberg [1961], and Gardenfors and Sahlin [1982], uncertainty is identified with the presence of more than one probability distribution or by more than one probability measure for each of the possible events.
In Einhorn and Hogarth [1985, 1986], the distributions mentally simulated, as well as the amount of simulation, depend on (a) the individual's attitude towards ambiguity and (b) the amount of perceived ambiguity. Ceteris paribus, individual perception of uncertainty will
decrease the more knowledge an individual has about the process that generates outcome probabilities [Curley and Yates, 1985]. The same process holds for Gardenfors and Sahlin [1982]: vague or unknown probabilities can be perceived as a set of reliable probability measures of which some might be more reliable than others. Information over the distribution is due to have a direct impact on the reliability of the probability measures that individuals include in their choice set.

For these models, ambiguity is "..the inability to rule out distributions of probability" [Camerer and Kunreuther, 1989, p. 287]; therefore, the more the information, the smaller the number of probability distributions the individual will simulate. In this light, giving individuals more information about the probability of a potential loss should reduce their reaction to ambiguity, i.e. reduce the differential between the valuation of known probabilities and uncertainty. In the limit, if the available information rules out all probability measures but one, ambiguity might disappear. ${ }^{5}$

Information Hypothesis 2A (Mental simulation models): The difference between the decision-maker's maximum WTP to insure against the loss under risk and under uncertainty will substantially decrease or disappear when uncertainty is explicitly represented with a precise second-order probability distribution.

However, this does not hold in Segal's model. That model in fact explains ambiguity as reaction to a precise second-order probability distribution. Moreover, according to Segal, probability distributions are ranked according to the notion "more ambiguous than", where the uniform distribution is considered the most ambiguous one. Consequently, according to Segal's model, the information hypothesis should be framed in the following way:

Information Hypothesis $2 B$ (Segal's hypothesis): The decision-maker's maximum WTP to insure against the loss will exhibit reaction to ambiguity even when he is provided with a precise second-order probability distribution. Moreover, her/his willingness to pay will be higher when uncertainty is represented through a uniform probability distribution.

In contrast with the above interpretations and hypotheses, more recently some studies have shown that the addition of information may intimidate decision makers by reminding them that in that specific issue they are "comparatively ignorant" [Fox and Tversky, 1995]. Fox and Weber [1999] show that the comparative ignorance condition can be induced in non explicitly comparative settings, by making the decision maker feel that he or she lacks essential information. Chow and Sarin [1998] have shown that although people prefer known to unknown probabilities, they also prefer "unknown" uncertainty to "known" uncertainty, whereby the former refers to a situation in which probabilities are not known to anyone.

Information Hypothesis 2C (Extended Comparative Ignorance Hypothesis): The decisionmaker's maximum WTP to insure against the loss will exhibit greater reaction to uncertainty if he knows exactly the precise second-order probability distribution. ${ }^{6}$

### 2.3. The effect of alternative probability distributions on the valuation of insurance under ambiguity

Using alternative descriptions of ambiguity has been a feature of several experiments (see Hogarth and Kunreuther [1989], Sarin and Weber [1993], and Di Mauro and Maffioletti [1996]), but no-one has explicitly explored the effect of the use of different descriptions or different SOPs on subjects' valuations. Hogarth and Kunreuther [1989], for instance, use a the "Best Estimate" representation of uncertainty (subjects are provided with a probability measure and they are told that this measure is the "best estimate" available), and a probability interval (subjects know that every probability measure inside a given range is equally likely). ${ }^{7}$ Gardenfors and Sahlin [1982] and Sarin and Weber [1993] use an ambiguity scenario involving a set of four probability measures.

These three representations of ambiguity can be mapped into as many theoretical models. The "Best Estimate" representation allows the experimenter to induce subjects to anchor on a probability and, consequently, it is the most intuitive way to test Einhorn and Hogarth model. On the other hand, according to Gardenfors and Sahlin theory, the agent's knowledge and beliefs about the relevant states are represented by a class II of probability measures. Different degrees of information will determine which probability measures, among all the possible distributions, are epistemically possible in a given context. Hence, using a set of four probability measures is one way to represent that model. Finally, Segal's model is best tested by the use of a uniform distribution, where the possible probability measures (all the epistemically possible probability measures) are equally likely.

In this experiment, we test explicitly whether the "Best Estimate" (B.E.), the "Interval of Probability" (I.P.), and the "Set of Probabilities" (S.P.) are perceived differently. Moreover, as the following section on the experimental design will make clear, we "translate" each of the three representations of ambiguity into a corresponding second-order probability distribution.
If, when facing uncertainty, the decision maker mentally simulates several probability distributions, one possible implication is that this process of mental simulation may "blur" the perceived differences among alternative representations of uncertainty. When alternative scenarios are made operational by means of an explicit second-order probability distribution, our intuition suggests that differences among these descriptions might become more "salient" and hence valuations will vary between representations.
Consider the case in which you are going to fly London-Milano Malpensa and you have to decide what is your WTP to buy flight insurance concerning baggage loss. The possible scenarios are the following: (a) the civil aviation authorities declare that the probability of baggage loss at Malpensa lies within $\underline{P}=1 \%$ and $\overline{\mathrm{P}}=5 \%$. Since the precise SOP is not known, the decision maker may mentally simulate several probability distributions. As figure 1(A) illustrates, pessimists may simulate distributions skewed towards the probability of loss of $5 \%$, whereas optimists may overestimate the low end of the interval, i.e. $1 \% .^{8}$ (b) The civil aviation authorities declare that every value inside the stated probability interval is equally likely. Since the SOP is clearly specified as a uniform distribution (case (c)), only one distribution (namely the one in figure $1(\mathrm{~B})$ is possible.


Figure 1. Interval of probability. (A) IP NO INFO. (B) IP NO INFO.
Let us consider now the "Best Estimate" definition of ambiguity in order to compare it with the "Interval of probability". Figure 2(A) shows some of the probability distributions that the decision maker may mentally simulate, all having the common feature of being centred on the "Best Estimate" (say, 3\%). Consider instead the case of making the "Best Estimate" operational by means of a precise second-order probability, for instance a discrete symmetric distribution, such as the one shown in figure 2(B). In this situation, the decision maker knows that three specific probabilites values, including the "Best estimate", are possible, $1 \%, 3 \%, 5 \%$, each of them occurring with different probabilities.



Figure 2. Best estimate. (A) Be NO INFO. (B) Be INFO.

In case no SOP is provided, mental accounting will lead an individual to simulate several probability distributions, so that the difference between "Best Estimate" and "Interval of Probability" may appear less clear. When specific SOPs are adopted (such as in figures 1(B) and 2(B)) the differences between the two cases are made explicit. Moreover, according to Segal's model, one should expect the highest degree of ambiguity aversion when subjects are provided with a uniform probability distribution representation of probability uncertainty.
We can therefore state our conjecture about the impact of alternative representations as follows:
Conjecture on Representation of Ambiguity-The difference in the valuation of insurance under the three alternative representations of uncertainty will be neater when ambiguity is made operational by means of a second-order probability distribution.

Table 1. Summary of experimental design.

| How much does valuation under ambiguity differ from valuation under risk? | - Risky scenarios (probability is exactly known) <br> - Ambiguous scenarios (probability is not known exactly) <br> Within subject factor |
| :---: | :---: |
| Does ambiguity reaction depend on the size of probability? | - Four probability levels $3 \%, 20 \%, 50 \%, 80 \%$ Within subject factor |
| What is the impact of different representations of ambiguity? | Three specifications of ambiguity: <br> - Best estimate <br> - Interval of probability <br> - Set of probabilities <br> Between subject factor |
| What is the impact of information on the perception of ambiguity? | Two groups of experiments: <br> - Ambiguity as a vague description of the probability of loss <br> - Ambiguity as second order probability distribution <br> Between subject factor |

## 3. The experimental design

The experimental data were obtained from a series of 12 experimental sessions run at the Centre for Experimental Economics of the University of York (EXEC). Forty-six subjects participated in the experiment using the vague characterisation of ambiguity (which shall be referred to as NO-INFO, 6 sessions), and forty-two subjects took part in the experiment using the SOP characterisation (which shall be referred to as INFO, 6 sessions). Subjects were graduate and undergraduate students recruited by mail-shot through the EXEC mail lists.

The design of the experiment is summarised in Table 1. In this experiment, the subjects' decision task parallels that of Di Mauro and Maffioletti [1996]: each subject was asked to state her/his maximum willingness to pay to reduce a potential loss of $£ 10$ to zero. Subjects were given eight different scenarios involving both risky and uncertain prospects. Four of the eight scenarios referred to a potential loss occurring with a known probability (risky scenarios), and four others to a loss having an uncertain probability (ambiguous scenarios). Risk and ambiguity were manipulated on a within subject basis. In each of the four risky scenarios we used a different reference probability. The probabilities of the potential loss were either $3 \%$, or $20 \%$, or $50 \%$, or $80 \%$. The expected probability of loss in the ambiguous scenario was the same as in the risky scenarios. To avoid order effects, in each session the scenarios were arranged in random order by means of the table of random numbers.

### 3.1. $\quad$ The incentive mechanism for value elicitation

In order to elicit subjects' preferences, we adopted a computerized auction mechanism, which is a variant of the classical second-price auction (see Harstad [1990] and Kagel [1995]). Subjects were asked to place a bid to purchase the right to insure themselves
against the potential loss. Before each bid, each subject was endowed with $£ 10$ and was told that he or she faced a potential loss of $£ 10$ with a given probability. With respect to the classical Vickrey auction, our variant adopted a clock mechanism: at the start of each auction, a clock was visualised on each subject's computer screen, with an ascending price going from zero British penny to ten pounds. Subjects were asked to press any key when the price reached the most they were willing to pay, i.e. when they wanted to leave the auction. ${ }^{9}$
The use of a second price auction to elicit preferences was motivated by the search for an incentive compatible system capable of inducing truthful revelation of private values [Vickrey, 1961]. Second price auctions have been used extensively to elicit individual willingness to pay (WTP) and willingness to accept (WTA) (see among many others McClelland, Schulze, and Coursey [1993] and Shogren et al. [1994]). Incentive compatible elicitation systems have not been used in previous experiments on insurance markets under uncertainty. Hogarth and Kunreuther [1989] used no incentive at all, while Camerer and Kunreuther [1989] adopted a double oral auction with markets of five trading periods. In the latter study, the authors point out that, if auction periods have to exercise market discipline, uncertainty should be resolved at the end of each period. This implies that, at each round, subjects learn more about the second-order probability distribution and this reduces the effect of ambiguity. In order to avoid that learning about the nature of the uncertainty could be confused with learning about the dominant strategy in auctions, we set up our experiment as a one-shot second price auction. Repeated auction periods would have made impossible to distinguish the effect of market discipline from the effect of learning about the second-order probability distribution. ${ }^{10}$

### 3.2. The description of ambiguity

The design considers three representations of ambiguity, "Best Estimate" probability, Interval of Probability, and Set of Probabilities. The representation of ambiguity was manipulated as a between subject factor, that is subjects evaluated scenarios referring to only one of these representations. Here below we provide examples, while the complete text of the ambiguous scenarios is reported in Appendix 1:

1. Best estimate probability (B.E.). Subjects were given a probability value which was an expert's estimate of the likelihood of the potential loss. However, they were told that "this is the first investigation ever carried out and consequently you experience considerable uncertainty about the precision of this estimate". By this, we tried to induce the subjects to anchor to the given probability value. We provided subjects with four different anchors $3 \%, 20 \%, 50 \%$ and $80 \%$.
2. Interval of probability (I.P.). Subjects were given a range ( $\mathrm{p}_{\mathrm{L}}, \mathrm{p}_{\mathrm{H}}$ ) within which the true probability lay: "an expert... estimates that the probability of the occurrence of such an event can be anywhere between $65 \%$ and $95 \%$ ". The probability intervals were ( $1 \%$, $5 \%$ ), $(5 \%, 35 \%),(35 \%, 65 \%),(65 \%, 95 \%)$.
3. Set of probability measures (S.P.). Subjects where given four probability estimates of the possible loss, $\left(p_{1}, p_{2}, p_{3}, p_{4}\right)$, for each scenario. However, the reliability of these estimates was not known: "four experts... have each provided estimates of the
probability of the occurrence of such an event. These four estimates of the probability are $65 \%, 75 \%, 85 \%$ and $95 \%$. All these estimates carry some reliability, although you do not know if any of them is more reliable than the others". The sets of probability provided were:
$(1 \%, 2 \%, 4 \%, 5 \%),(5 \%, 15 \%, 25 \%, 35 \%),(35 \%, 45 \%, 55 \%, 65 \%),(65 \%, 75 \%, 85 \%, 95 \%)$. In order to allow for scenarios which would all be equivalent to a decision-maker acting according to SEU, in the I.P. and S.P. representations, the mean value for each interval and for each set corresponds to the "best estimate" value provided in the corresponding scenario. These mean values, in turn, are used as the exact probabilities in the risky scenarios.

### 3.3. The information problem

To test whether attitude towards ambiguity is sensitive to the amount of information about the stochastic process determining the loss, we ran two sets of experiments. In the NOINFO experiments, subjects were given a vague description of the probability of loss. In the INFO experiments ambiguity was made operational by means of a second-order probability distribution. Subjects participated in only one type of experiment, either INFO or NO-INFO.
3.3.1. NO-INFO experiment. In these experiments subjects were shown eight scenarios of the types sketched in Section 3.2. and fully reported in Appendix 1. Before evaluating the scenarios, they were asked to evaluate two hypothetical scenarios in order to make themselves familiar with the auction procedure and the decision problem. Subjects were shown how the risky scenarios were going to be resolved, while the resolution of the ambiguous scenarios was left unexplained. In this treatment, subjects were told about the use of a SOP to resolve uncertainty only after they had completed the evaluation process.
3.3.2. INFO experiment. In this treatment, at the start, subjects received a written explanation of the resolution of uncertainty adopted in that particular session. The explanation contained the exact SOP that was going to be used to play out the lottery for real at the end of the experiment. (See Appendix 2 for the text.) Both risky and uncertain hypothetical lotteries were resolved before the start of the real experiment. Therefore, the INFO experiments were based explicitly on the SOP characterization of uncertainty. In particular, we interpreted the "Best Estimate" as a symmetric second-order distribution centred on the best estimate probability (figure 2(B)). The "Interval of probability" corresponded to a uniform probability distribution bounded by the extremes of the interval (figure $1(\mathrm{~B})$ ).

In the "Set of Probabilities" the operationalization of ambiguity was slightly more complex. We wanted to convey the idea that all the probability values given to subjects had some reliability (they were all epistemically possible in Gardenfors and Sahlin's terminology) but some of those values could be more reliable than others. With respect to B.E. and I.P., in S.P. several second probability distributions were explicitly allowed for. More specifically, the concept of set of probability measures was made operational in the following way: we told subject that the loss could occur with four different probabilities. To illustrate, consider
the following set:


In order to determine the weight of each of these measures, a die was drawn from a bag containing 10 dice. Numbers on each die went from 1 to 4 , with each number corresponding to the probability measures included in the set. Each number $1,2,3,4$, appeared on each of the 10 dice in a different proportion to simulate the fact that, for a given die, one probability measure could be more reliable than another. ${ }^{11}$ The die was played and the number that came out determined the exact probability of loss.

## 4. Experimental results

The results of the experiment will be presented as follows: in Section 4.1, we will analyse the impact of ambiguity on valuation of insurance decisions, in Section 4.2, we will discuss the effect of information about the stochastic process on the valuation of insurance; and, finally, in Section 4.3, we will address the issue of how the representations of ambiguity have been perceived by individuals.

### 4.1. Valuation of ambiguous losses versus risky losses

Table 2 shows the mean, median (in bold), and standard deviation (in parenthesis) of the individual ratios of ambiguous to risky bids for all the various experimental conditions. For both the INFO and the NO INFO treatments there are two rows corresponding to the "Set of Probabilities" representation of ambiguity. The reason for this is that-in both treatmentsone observation, which represents an extreme outlier, biased the means at the probability of loss of $80 \% .^{12}$ In the second row corresponding to the S.P. these observations have been removed. Henceforward, we shall comment the summary statistics in Table 2 by making reference to the S.P. row with the reduced sample size.

Table 2 allows us to carry out a test of Hypothesis 1. As can be observed, ambiguity has an impact on the valuation of insurance, which is stronger at the probability of loss of $3 \%$ and declines at higher probabilities. In particular, mean ratios are greater than one at the probabilities of $3 \%, 20 \%$, and $50 \%{ }^{13}$ Median ratios show the same declining pattern of means: however, since most of the cell distributions are non normal the median shows slightly different results. In particular, at "intermediate" probabilities ( $20 \%$ and $50 \%$ ) the median shows that ambiguity neutrality prevails in most experimental conditions. When the probability of loss is $80 \%$ both means and medians show the presence of ambiguity preference. ${ }^{14}$

The summary statistics, therefore, fully support the behavioural model by Einhorn and Hogarth [1985] which predicts a decline in the ratio of WTP to insure under ambiguity to WTP under risk as the probability of loss increases: ambiguity aversion prevails when

Table 2. Summary statistics of ratios of ambiguous to risky prices. Median (in bold), mean, and standard deviation (in parenthesis).

| Ambiguity definition | Probability level |  |  |  | Sample size |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 3\% | 20\% | 50\% | 80\% |  |
| No information |  |  |  |  |  |
| B.E. | $\begin{gathered} 16.03(35.20) \\ \mathbf{1 . 6 4} \end{gathered}$ | $\begin{gathered} 0.92(0.35) \\ \mathbf{1} \end{gathered}$ | $\begin{gathered} 1.09(0.69) \\ \mathbf{1} \end{gathered}$ | $\begin{gathered} 0.86(0.21) \\ \mathbf{0 . 9 1} \end{gathered}$ | 16 |
| I.P. | $\begin{gathered} 3.74(6.82) \\ \mathbf{1 . 1 6} \end{gathered}$ | $\begin{gathered} 1.10(0.22) \\ \mathbf{1 . 0 4} \end{gathered}$ | $\begin{gathered} 1.10(0.20) \\ \mathbf{1 . 0 1} \end{gathered}$ | $\begin{gathered} 1.10(1.55) \\ \mathbf{0 . 8 5} \end{gathered}$ | 15 |
| S.P. | $\begin{gathered} 2.4(4.16) \\ 1 \end{gathered}$ | $\begin{gathered} 1.16(0.25) \\ \mathbf{1 . 0 3} \end{gathered}$ | $\begin{gathered} 5.56(20.33) \\ \mathbf{1 . 0 4} \end{gathered}$ | $\begin{gathered} 24.15 \text { (90.14) } \\ 1 \end{gathered}$ | 15 |
| S.P. ${ }^{\text {a }}$ | $\begin{gathered} 2.51(4.3) \\ \mathbf{1} \end{gathered}$ | $\begin{gathered} 1.12(0.22) \\ \mathbf{1 . 0 2} \end{gathered}$ | $\begin{gathered} 1.32(0.75) \\ \mathbf{1 . 0 3} \end{gathered}$ | $\begin{gathered} 0.87 \text { (0.27) } \\ \mathbf{1} \end{gathered}$ | $14^{\text {a }}$ |
| Information |  |  |  |  |  |
| B.E. | $\begin{gathered} 3.8(6.67) \\ 2.04 \end{gathered}$ | $\begin{gathered} 1.25(0.70) \\ \mathbf{1 . 0 2} \end{gathered}$ | $\begin{gathered} 1.37(0.52) \\ \mathbf{1 . 3 1} \end{gathered}$ | $\begin{gathered} 0.92(0.11) \\ \mathbf{0 . 9 2} \end{gathered}$ | 13 |
| I.P. | $\begin{gathered} 6.18 \text { (14.02) } \\ 1 \end{gathered}$ | $\begin{gathered} 4.47 \text { (12.6) } \\ \mathbf{1 . 1 8} \end{gathered}$ | $\begin{gathered} 1.13(0.31) \\ \mathbf{1} \end{gathered}$ | $\begin{gathered} 0.82(0.26) \\ \mathbf{0 . 9 1} \end{gathered}$ | 15 |
| S.P. | $\begin{gathered} 4.7 \text { (11.12) } \\ \mathbf{1 . 4} \end{gathered}$ | $\begin{gathered} 1.43 \text { (0.67) } \\ \mathbf{1 . 3 5} \end{gathered}$ | $\begin{gathered} 1.33(1.36) \\ \mathbf{0 . 9 8} \end{gathered}$ | $\begin{gathered} 54.36(200.1) \\ \mathbf{0 . 9 0} \end{gathered}$ | 14 |
| S.P. ${ }^{\text {a }}$ | $\begin{gathered} 5(11.52) \\ \mathbf{1 . 7 5} \end{gathered}$ | $\begin{gathered} 1.49(0.65) \\ \mathbf{1 . 3 6} \end{gathered}$ | $\begin{gathered} 1.38(1.41) \\ \mathbf{0 . 9 8} \end{gathered}$ | $\begin{gathered} 0.85(0.21) \\ \mathbf{0 . 8 7} \end{gathered}$ | $13^{\text {a }}$ |

${ }^{a}$ With respect to the previous row sample size is reduced by one unit to eliminate one extreme outlier.
the likelihood of a loss is low, whereas ambiguity preference holds at high probabilities of loss. At low "anchors" of the probability, under ambiguity, the individuals over-insure themselves with respect to the risky situation, while, when the "anchor" probability of loss is high, individuals under-insure themselves with respect to the risky situation. When there are equal chances that the loss occurs or not, ambiguity has little impact on individuals' willingness to pay.

The observed pattern of valuation of insurance is not new: it is found for instance in Hogarth and Kunreuther [1989, 1992]. With respect to these experiments, ours uses an incentive compatible elicitation mechanism. Hence, the declining pattern of ambiguity aversion shows to be robust to the provision of incentives. However, in our experiment, the median ratios of ambiguous to risky bids are lower than the mean ratios reported in Hogarth and Kunreuther [1989], (medians are not presented in that study), signalling a weaker impact of ambiguity on valuation at low (3\%) and high probabilities ( $80 \%$ ). ${ }^{15}$ The different elicitation method is probably the explanation for this result.

To test the statistical significance of reaction to ambiguity we apply the Wilcoxon ranksum test for paired samples to compare the distribution of ambiguous bids with that of risky bids. The results of the test (reported in Table 3) can be reconciled with what is shown by Table 2 . We find that the impact of ambiguity is statistically significant only under some experimental conditions, and, hence, the impact of ambiguity is not pervasive at

Table 3. Wilcoxon rank-sum test between risky and ambiguous bids ( T values, two-tailed test, $95 \%$ significance level).

| Ambiguity <br> definition | $3 \%$ | $20 \%$ | $50 \%$ | $80 \%$ |  |  |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: |
|  | No information |  |  |  |  |  |  |
|  | $30^{\mathrm{a}}$ | 42.5 | 52 | $12^{\mathrm{a}}$ |  |  |
|  | 35 | $21^{\mathrm{a}}$ | 34 | $18^{\mathrm{a}}$ |  |  |
| S.P. | 19.5 | $9^{\mathrm{a}}$ | 30 | 39 |  |  |
|  | Information |  |  |  |  |  |
| B.E. | $16^{\mathrm{a}}$ | 24 | $9^{\mathrm{a}}$ | $13^{\mathrm{a}}$ |  |  |
| I.P. | 38 | $11^{\mathrm{a}}$ | 29.5 | $3^{\mathrm{a}}$ |  |  |
| S.P. | $10^{\mathrm{a}}$ | $17^{\mathrm{a}}$ | 46 | 23 |  |  |

${ }^{\text {a }}$ Significant.
all probabilities of loss. As Table 3 shows, in the NO-INFO design the value of the test statistic $T$ (two-tailed test) shows that only in 5 out of 12 experimental conditions the difference in the distribution of bids for risky and ambiguous lotteries is significant. In the INFO design, the number of experimental conditions in which the difference between risky and ambiguous bids is significant is slightly bigger (7 out of 12). On the whole, the effect of ambiguity (see Table 3) is not very strong. In particular, Table 3 shows that the effect of ambiguity appears to be weaker at the probability of $50 \%$. This result can be accounted for by the switch from ambiguity aversion to ambiguity preference as the probability of loss increases, as predicted by Einhorn and Hogarth model. ${ }^{16}$

Aggregate measures such as means and medians may be misleading since they might conceal heterogeneity among individuals. In order to address the problem of individual differences, we present in Table 4 an analysis of individual patterns of behaviour under ambiguity. We check whether individual reaction to ambiguity changes as the probability of loss increases. At the aggregate level, the observed pattern was a switch from ambiguity

Table 4. Individual patterns of response to ambiguity.

| Behavioral pattern | Subjects in "NO INFO" experiment | Subjects in "INFO" experiment |
| :--- | ---: | ---: |
| Decreasing ratio $A / R$ | $\mathbf{2 3}(\mathrm{BE}=11, \mathrm{IP}=8, \mathrm{SOP}=4)$ | $\mathbf{2 6}(\mathrm{BE}=9, \mathrm{IP}=8, \mathrm{SOP}=9)$ |
| Increasing ratio $A / R$ | $\mathbf{5}(\mathrm{BE}=2, \mathrm{IP}=1, \mathrm{SOP}=2)$ | $\mathbf{1}(\mathrm{BE}=0, \mathrm{IP}=1, \mathrm{SOP}=0)$ |
| Ambiguity proneness | $\mathbf{1}(\mathrm{BE}=0, \mathrm{IP}=1, \mathrm{SOP}=0)$ | $\mathbf{3}(\mathrm{BE}=0, \mathrm{IP}=2, \mathrm{SOP}=1)$ |
| Ambiguity neutrality | $\mathbf{4}(\mathrm{BE}=0, \mathrm{IP}=1, \mathrm{SOP}=3)$ | $\mathbf{1}(\mathrm{BE}=0, \mathrm{IP}=1, \mathrm{SOP}=0)$ |
| Ambiguity aversion | $\mathbf{2}(\mathrm{BE}=0, \mathrm{IP}=1, \mathrm{SOP}=1)$ | $\mathbf{0}$ |
| Unclear pattern | $\mathbf{1 1}(\mathrm{BE}=3, \mathrm{IP}=3, \mathrm{SOP}=5)$ | $\mathbf{1 1}(\mathrm{BE}=4, \mathrm{IP}=3, \mathrm{SOP}=4)$ |
| Total | $\mathbf{4 6}$ | $\mathbf{4 2}$ |

aversion to ambiguity preference as the probability of loss increases. Table 4 shows that a decreasing ratio of ambiguous to risky bids ("Decreasing ratio $A / R$ " in the Table) is also the modal pattern (about $50 \%$ of sample) at the individual level, both in the INFO and in the NO INFO treatment. We have identified other possible patterns of response: a switch from ambiguity preference to aversion as the probability of loss increases ("Increasing ratio $A / R$ " in the Table), constant ambiguity aversion or constant ambiguity proneness, and constant ambiguity neutrality. These patterns were chosen by a far lower number of subjects. In the category of behaviour we have denominated "unclear", a significant number ( 6 in the INFO treatment and 5 in the NO INFO) were made up of subjects who were ambiguity prone at the probabilities of loss of $3 \%$ and $80 \%$ and ambiguity averse in between, and who, therefore, seemed to prefer ambiguity when the expected probability of loss was either very low or very high.

Although the main focus of this paper is behaviour under conditions of ambiguity, in order to pursue the analysis of individual behaviour in more detail, we checked whether individual attitudes towards ambiguity are matched by an equivalent attitude towards risk, so that if an individual is ambiguity averse at a probability of loss of $3 \%$, he/she will likewise be risk averse, and so forth. Results of this analysis are summarised in figure 3(A) and (B) which refer to the NO INFO and INFO treatments respectively, and show


Figure 3. Individual attitudes towards risk and uncertainty. (A) NO INFO. (B) INFO.


Figure 3. (Continued).
the ratio of WTP under ambiguity to WTP under risk $(A / R)$ and the ratio of WTP under risk to the expected value of the lottery $(R / E V) .{ }^{17}$ Subjects are pooled across definitions of ambiguity, and are sorted in ascending degree of risk aversion (measured by the ratio of WTP under risk to the expected value of the lottery) in order to make the interpretation of the graph easier. Both figure $3(\mathrm{~A})$ and (B) show that there is no clear-cut relation between risk attitude and ambiguity attitude. This result is especially strong at the probability of loss of $3 \%$ : at this probability, attitude to risk and attitude to ambiguity seem to have opposite patterns. Also, comparison of the subject data points with the reference line of risk and ambiguity neutrality $(R / E V=A / R=1)$ shows that, at this probability, most subjects are either very prone or very averse to risk and ambiguity. As the probability of loss increases, the size of both reaction to uncertainty and to risk diminish. Yet the two ratios continue to go in opposite directions. At the highest probability of loss ( $80 \%$ ), many subjects are mildly ambiguity prone, but they behave as neutral towards risk.

The lack of a correspondence between attitude to risk and attitude to uncertainty is reported also by Camerer and Weber [1992], who warn that this result may be due to the presence of a measurement error in the elicitation of values under risk and under ambiguity. Although our study did not adopt any explicit manipulation check to establish a precise equivalence between risky and ambiguous probabilities, it is nevertheless true that the
divergence between risk and uncertainty attitudes has been observed also in the INFO design, in which subjects knew exactly the second-order distribution. ${ }^{18}$

### 4.2. The impact of information on the valuation of ambiguity

In Hypothesis 2A we stated that in the absence of a precisely specified second-order distribution, reaction to ambiguity would have been stronger. One of the reasons is that decisionmakers may mentally simulate several "plausible" or "reliable" probability distributions. Comparison between the mean $A / R$ ratios in Table 2 in the INFO and NO-INFO treatments shows that the values of ratios are further from the ambiguity neutrality value ( $A / R=1$ ) under the INFO treatment. ${ }^{19}$ Hence, contrary to expectations, ambiguity seems to be stronger in the INFO treatment rather than in the NO INFO one. Table 2 also shows that the median ratios of ambiguous to risky prices are likewise slightly higher in the INFO treatment in nearly all the cells.

Further, if the effect of ambiguity were stronger when individuals have vague information about the probability of the loss, we would expect the distribution of bids under risk and under uncertainty to be statistically different in more cells under the NO-INFO treatment. As we mentioned in the discussion of Table 3, this is not the case. On the contrary, ambiguity is marginally stronger in the INFO treatment; there are 7 cells out of 12 in which the impact of ambiguity is significant while this is not true in the NO INFO case where ambiguity is significant only in 5 cells. This result can be interpreted with Fox and Weber [1999] hypothesis of "extended induced comparative ignorance" (Hypothesis 2C). According to this hypothesis, when more information about the outcome-generating process is given to individuals, people may feel that they lack significant information, and hence may be induced to have a stronger reaction to the presence of uncertainty.

Table 5 provides a more direct test of Hypothesis 2A. The table reports the results of a Mann-Whitney U-test for independent samples which compares the distribution of ambiguity ratios $(A / R)$ in the two information treatments. The difference between the explicit second-order characterisation of ambiguity and the vaguer characterisation is never significant. As can be seen from the table, the value of the two-sided probability is always higher than the significance value. To conclude, our data does not support Hypothesis 2A: the provision of information on the second-order probability distribution does not

Table 5. Mann-Whitney U-test between ambiguity ratios $(A / R)$ under the INFO and the NO INFO treatments (two-tailed, $95 \%$ significance level).

|  | Probability of loss |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| Ambiguity <br> definition | $3 \%$ | $20 \%$ | $50 \%$ | $80 \%$ |
| B.E. | 0.87 | 0.14 | 0.11 | 0.84 |
| I.P. | 0.75 | 0.34 | 0.73 | 0.91 |
| S.P. | 0.15 | 0.16 | 0.22 | 0.14 |

reduce the impact of uncertainty on individual's willingness to pay to insure. On the contrary, the summary statistics and Table 3 show that there is slightly more ambiguity in the INFO treatment. However, this differential impact is not strong enough to be statistically significant. ${ }^{20}$

### 4.3. The impact of the representation of the second-order probability

In Section 2 we have made the conjecture that in the INFO design the information provided to the subjects about the three types of representations of ambiguity, should make the differences among them neater. In fact, in the NO-INFO design, the absence of exact information concerning the resolution of uncertainty may induce the subject to simulate a multiplicity of SOPs for each definition of uncertainty, and thus blur the differences among them. Conversely, in the INFO treatment, while the Set of probabilities representation implies ten alternative SOPs (see Appendix 2 below), the Best Estimate and the Interval of Probability representations entail only one SOP each.

Given this conjecture, our working hypothesis was that the difference between the distribution of WTP under ambiguity across the three representations would be greater in the INFO experiment rather than in the NO-INFO. ${ }^{21}$ In order to test this hypothesis, we compared the distribution of bids under the three representations of ambiguity by means of Kruskal-Wallis non-parametric one-way analysis. As can be seen from Table 6 which reports the results of the test, the differences among the three representations are very weak in both designs. They are never significant in the INFO design, and there is weak significance (at the $10 \%$ level) in the NO-INFO design at the probability level of $3 \% .^{22}$

Also, both the summary statistics and the individual patterns of behaviour do not betray any salient differentiation among the three representations. What this equivalence entails is probably that, knowing exactly the nature of uncertainty, induced subjects to anchor on the mean value of the probability distribution which, as it will be recalled, was the same for all three representations of ambiguity. Also, the weakly significant impact that we have found for ambiguity in the insurance framework, may be responsible for the small differences found in the valuation of different ambiguity representations.

Table 6. Impact of the representation of ambiguity in the INFO and in the NO INFO experiments. Kruskal-Wallis test for independent samples (chi-square distribution, 2 d.f.).

|  | INFO |  |  | NO INFO |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Probability | Chi-square | Significance |  | Chi-square | Significance |
| $3 \%$ | 3.9543 | .1385 |  | 5.5789 | .0615 |
| $20 \%$ | .5004 | .7786 |  | 4.57 | .1014 |
| $50 \%$ | 4.4592 | .1076 |  | .9837 | .6115 |
| $80 \%$ | 1.88643 | .3937 |  | 1.0754 | .5840 |

## 5. Discussion of results

### 5.1. Implications of results for theory

We can summarize the results of our experiment as follows:
(a) ambiguity aversion decreases with the reference probability of loss;
(b) ambiguity does affect the valuation of losses, but the impact is not statistically significant at all probabilities of loss;
(c) For the B.E. and S.P. representations of ambiguity, the impact of ambiguity is slightly stronger when more information is provided. ${ }^{23}$

Result (a) i.e. the switch of individual attitudes from ambiguity aversion to ambiguity proneness as the probability of the losses increases provides evidence in favour of Einhorn and Hogarth's model. We can conclude (rejecting Hypothesis 1A) that individuals exhibit reaction to ambiguity at the individual as well as at the aggregate level. However, (point b) this reaction to ambiguity is weak i.e. at intermediate probabilities of loss $(p=0.20$ and $p=0.50$ ) the ratio of WTP under ambiguity to WTP under risk is close to one. This can be explained allowing for a weighting function in which decision weights coincide with liner probabilities at intermediate probability values (see Fox and Tversky [1995]).

An alternative way to explain a weak reaction to ambiguity is the one used by Cohen, Jaffray, and Said [1985]. According to these authors-in the domain of losses-individuals do not treat ambiguity by simulating one or more probability distributions, but rather they use some coarser process of evaluation.

In addition, one should reflect on whether the observed disparity between the valuation of risky and ambiguous bets is influenced by the elicitation procedure. In particular, reaction to ambiguity-a violation of subjective expected utility-may be reduced when values are elicited through a market institution. In the experiment we have presented, the use of a the second-price auction to elicit individual willingness to pay gives rise to a perception of ambiguity weaker than what is found by other studies. Results similar to ours were in effect reported by Camerer and Kunreuther [1989] who used a market mechanism, namely the double oral auction, to elicit preferences. Further research, therefore, is needed to disentangle the contribution of alternative incentives (i.e. the use of an auction to elicit preferences instead of other monetary incentives like for example a flat payment) from that of the outcome domain (in our experiment outcomes were losses) in determining reactions to ambiguity.

Our finding (c) does not support the cognitive model according to which more information about the probability generating process reduces uncertainty, by reducing the amount of mental simulation carried out by the decision maker (Hypothesis 2A). On the contrary, we find that in the INFO treatment summary statistics indicate a stronger reaction to ambiguity and that ambiguity is statistically significant in more experimental conditions. Although no formal model of comparative ignorance exists, we think that Hypothesis 2C is the only one to receive some support from our results. Further investigation into the relevance and extension of the "comparative ignorance" model is necessary
to understand more clearly the effect of information on individual decision making under ambiguity.

To conclude, our experimental data show that: (i) ambiguity aversion can be observed even if uncertainty is made operational as an SOP and (ii) reaction to uncertainty defined as a precise SOP is no weaker than the reaction that was found when no information about the probability distribution was provided. Individuals might consider both imprecise probabilities and SOPs as equivalent manifestations of uncertainty and react to them in the same fashion.

### 5.2. Implications for insurance markets

In the past years there has been considerable interest in the exploration of the causes of the failure of some insurance markets (environmental liability, workplace risk, medical malpractice etc.). These failures have been attributed not only to the presence of asymmetries of information, but also to the presence of uncertainty concerning the likelihood of the potential loss. Although the perception of ambiguity can be reduced by the repetition of the experience (so, for instance, frequent travellers will likely have lower WTPs to pay for flight insurance), many potential losses (such as floods, earthquakes, nuclear and chemical disasters) are regarded as once in a lifetime experiences and hence considered highly ambiguous.

In the experiments we ran, the impact of ambiguity on individual WTP for insurance, although not pervasive, was found to be statistically significant in about half the experimental conditions. The probability levels at which this divergence was observed, although variable according to the definition of ambiguity used, tended to be the same across the INFO and NO-INFO experiments for any given definition of ambiguity. As can be seen from Table 3, in the case of the "Best estimate" definition of ambiguity, low (3\%) and high (80\%) probabilities of loss determined maximum willingness to pay to insure ambiguous losses that diverged from bids for risky losses. For the "interval of probability" the divergence occurred at the probability levels of $20 \%$ and $80 \%$. Table 2 shows that this divergence is due to ambiguity aversion at low probability levels and to ambiguity preference at high probability levels, a result that confirms those of Hogarth and Kunreuther [1989] and Di Mauro and Maffioletti [1996].

In order to establish whether probability ambiguity causes disruption to insurance markets one should be able to observe both sides of the market. Since our experiment refers only to the demand side of the market, we can conjecture that if firms increase prices in response to probability ambiguity, as in Hogarth and Kunreuther [1989], ${ }^{24}$ the market for insurance may be thin. However, a thin market will occur only in those situations in which ambiguity weakly affects consumers' valuations (for instance at the probability of $50 \%$ ), and a fortiori when the loss is highly likely (see the median ratio of ambiguous to risky prices at a probability of $80 \%$ ). In these cases, increased insurance prices set by companies will not be matched by increased willingness to pay on the consumers' side and little trade will occur.

Prescriptively, this implies that insurers should frame insurance contracts so that the probability of loss appears to be low, for instance by providing separate contracts for specific risks [Hogarth and Kunreuther, 1989, p. 30]. Alternatively, government policies or improved
risk-assessment procedures may be called in to lower premiums set by insurers. Camerer and Kunreuther [1989], however, find that, if multiple losses are possible, insurance firms may become risk-seeking: in such a case prices will drop and the number of losses insured will increase.

The new-and somehow surprising-result to come out of this experiment is that the provision of more information about the probability of the potential loss may increase reaction to ambiguity rather than reduce it. In particular, we find that for some probability levels and for some definitions of ambiguity, knowledge of the second order distribution increases the impact of ambiguity on individual WTPs.

To give a practical example of what this entails, consider again two alternative scenarios relating to flight insurance: (a) AirUS, is a newly born company and little information is available about its safety standards; estimates of the average rate of air accidents for new companies worldwide is .0003 ; (b) AirUS is a newly born company. Of the 5 new air carriers which entered the market last year, one has an estimated probability of air accident of .0001 , another an estimated probability of .0005 , and the remaining three an estimated probability of .0003 . How much would consumers be willing to pay for flight insurance in (a) and (b)?

On the basis of Tables 2 and 3 we would conclude that WTP (b) > WTP (a). The intuition behind this result is that-even if the expected probability is .0003 in all three cases-for a pessimist, the knowledge of the tails of the probability distribution may induce higher ambiguity aversion. The addition of information which can help diagnose the true probability has the opposite effect of inducing a sense of "comparative" ignorance. In our experiment this is a marginal and not widespread effect which is tied to two definitions of ambiguity (SP and BE). Also, given that INFO and NO-INFO are between subject treatments, we cannot be sure that these effects hold also on a within subject basis. However, given Fox and Weber's [1999] hypothesis that "competence" and "comparative ignorance" may have a larger meaning than that isolated by the original studies [Heath and Tversky, 1991] and Fox and Tversky [1995], context specific studies will be needed to assess the relevance of these effects.

If applied to the context of insurance markets, our results imply that information about low probability events may increase willingness to pay for insurance. For instance, an individual may be more willing to buy health insurance if-instead of being told that on average $3 \%$ of individuals in their thirties undergo surgery-he receives detailed estimates about the probabilities of undergoing various types of surgery. Thus, for low probability losses, the provision of more information about probability could lead to an increase in the amount of trade, and thus have beneficial effects which may counterbalance higher premiums determined by insurers' ambiguity.

## 6. Conclusions

This experiment has investigated the impact on individual valuation of insurance of ambiguous probabilities of loss. We have run a second-price auction market in order to elicit individuals' willingness to pay to insure against losses characterised either by known or else vague probabilities. We have further tested the hypothesis that consumers' valuation of insurance under ambiguity is sensitive to the amount of information available about the
probability of loss. To this end, we have used two distinct treatments by providing subjects with different information over the vague probabilities of loss.
We find that the characterisation of ambiguity in terms of SOPs does not alter valuation with respect to a situation in which no ex ante information about the resolution of ambiguity is provided. If anything, in some treatments the impact of ambiguity appears to be stronger when more information is provided. The fact that individuals react to uncertainty even when it is represented by a SOP does not have any implication on whether the same individuals might exhibit a stronger reaction to ambiguity when real events and not lottery events are concerned. Others factors can be involved when assessing event lotteries and estimating judgmental probabilities which might influence the perceived level of uncertainty (see for example Heath and Tversky [1991] and Fox and Weber [1999]). But this is a topic for further research.

Finally, we find that the use of a second price auction to elicit values leads to a weaker effect of ambiguity compared to other studies which have used no incentive-compatible mechanism. While this conclusion may lead us to dismiss the relevance of the ambiguity phenomenon for insurance markets, it must be underlined that our experiment does not consider reaction to ambiguity by sellers (i.e. insurers). Greater reaction by insurers, as found by other studies, would lead to a "thin" market.
As part of our future research agenda, it remains to be established whether it is the outcome domain which affects the impact of ambiguity on values, or rather the institutional setting.

## Appendix 1: Text of scenarios in the experiment

## A1.1. Example of "risky" scenario

Assume that there is a risk of $80 \%$ that some event occurs. If this event occurs you will suffer a loss of $£ 10$.

You are now asked to state which is the maximum amount of money that you would be willing to pay to reduce this potential loss to zero.

You will be asked to press any key when the price reaches the most that you are willing to pay; that is, when you want to leave the auction.

## A1.2. Example of scenario with "Best Estimate" ambiguity

Assume that there is a potential risk of the occurrence of some event. There is an estimate of the occurrence of this event; an expert, hired by a governmental agency, estimates that the probability of the occurrence of such an event is $80 \%$. However, this is the first investigation ever carried out and consequently you experience considerable uncertainty about the precision of this estimate. If this event occurs, you will suffer a loss of $£ 10$.
You are now asked to state which is the maximum amount of money that you would be willing to pay to reduce this potential loss to zero.

You will be asked to press any key when the price reaches the most that you are willing to pay; that is, when you want to leave the auction.

## A1.3. Example of scenario with "Interval of probability" of ambiguity

Assume that there is a potential risk of the occurrence of some event. There is an estimate of the possible occurrence of this event; an expert, hired by a governmental agency, estimates that the probability of the occurrence of such an event can be anywhere between $65 \%$ and $95 \%$. If this event occurs, you will suffer a loss of $£ 10$.

You are now asked to state which is the maximum amount of money that you would be willing to pay to reduce this potential loss to zero.

You will be asked to press any key when the price reaches the most that you are willing to pay; that is, when you want to leave the auction.

## A1.4. Example of scenario with "Set of probability measures" ambiguity

Assume that there is a potential risk of the occurrence of some event. There are estimates of the possible occurrence of this event; four experts, hired by a governmental agency, have each provided estimates of the probability of the occurrence of such an event. These four estimates of the probability are $65 \%, 75 \%, 85 \%$ and $95 \%$. All these estimates carry some reliability, although you do not know if any of them is more reliable than the others. If this event occurs, you will suffer a loss of $£ 10$.

You are now asked to state which is the maximum amount of money that you would be willing to pay to reduce this potential loss to zero.

You will be asked to press any key when the price reaches the most that you are willing to pay; that is, when you want to leave the auction

Appendix 2: Information about the resolution of ambiguous lotteries provided to the subjects in the "INFO" experiment

## A2.1. Risky scenarios ${ }^{25}$

Scenarios *, *, *, * will be played out in the following way: there will be an opaque bag containing 100 balls. One ball will be drawn from the bag with black representing loss and white representing no loss. The proportion of black balls is the probability of the loss and the proportion of white balls is the probability of no loss.

The selected scenario will be played out for each subject separately. Each one of the participants will be asked to draw a ball from the bag.

After every draw the ball will be replaced before the next subject draws another ball. A white ball results in no loss, i.e. a payoff of $£ 10$ for the participant who drew the ball. A black ball results in a loss of $£ 10$, i.e. a payoff of $£ 0$ for the participant who drew the ball.

## A2.2. "Best Estimate" scenarios

Scenarios *, *, *, * will be played in the following way: one of the participants will be asked to draw a ticket from a bag. The bag contains five tickets. Three out of the five tickets will bear on them the number given in the scenario as the Governmental Agency's best estimate. The remaining two tickets will bear one a number above the best estimate, and one a number below the best estimate. The numbers above and below the best estimate will be symmetrically distributed around it. If, say the best estimate given in the scenario is $80 \%$, the bag will contain five tickets, three bearing the number 80 on them, one with the number 65, and the other with the number 95 . Since three tickets out of five bear the number corresponding to the best estimate, the best estimate is the most likely probability of loss. The experimenter will then prepare one bag containing 100 balls. One ball will be drawn from the bag with black representing loss and white representing no loss. The proportion of black balls is the probability of the loss and the proportion of white balls is the probability of no loss.

The selected scenario will be played out for each subject separately. Each one of the participants will be asked to draw a ball from the bag. After every draw the ball will be replaced before the next subject draws another ball. A white ball results in no loss, i.e. a payoff of $£ 10$ for the participant who drew the ball. A black ball results in a loss of $£ 10$, i.e. a payoff of $£ 0$ for the participant who drew the ball.

After the lottery has been played out, you will be free to check whether the tickets in the bag and the combination of white and black balls in the bag correspond to the explanations given above.

## A2.3. "Interval of probability" scenarios

Scenarios *, *, *, * will be played in the following way: one of the participants will be asked to draw a ticket from a bag. The bag contains a number of tickets corresponding to the integers within the interval provided in the scenario (including the extremes of the interval). Numbers on the tickets are integers that go from the lower bound of the interval to the upper bound. So, if the interval presented in the scenario goes from .65 to .95 , the bag will contain 31 tickets, numbered from 65 to 95.

Each number represents a value of the probability of the loss that lies in the probability interval given in the scenario.

The experimenter will then prepare one bag containing 100 balls. One ball will be drawn from the bag with black representing loss and white representing no loss. The proportion of black balls is the probability of the loss and the proportion of white balls is the probability of no loss.

After every draw the ball will be replaced before the next subject draws another ball. A white ball results in no loss, i.e. a payoff of $£ 10$ for the participant who drew the ball. A black ball results in a loss of $£ 10$, i.e. a payoff of $£ 0$ for the participant who drew the ball.

After the lottery has been played out, you will be free to check whether the tickets in the bag and the combination of white and black balls in the bag correspond to the explanations given above.

## A2.4. "Set of probability measures" scenarios

Scenarios *, *, *, * will be played in the following way: one of the participants will be asked to draw a six-faced die out of a bag containing 10 dice. The number on each face of each die is either 1 or 2 or 3 or 4 , and stands for the 4 probabilities of loss given in the scenario. Number 1 corresponds to the lowest probability and number 4 corresponds to the highest. E.g. if in the scenario the 4 estimates of the probability of the loss are $.65, .75, .85, .95$, the numbers corresponding to each probability will be $1,2,3,4$ respectively. Each number appears on the dice at least once.

The chosen die will be rolled by one of the participants. The number uppermost will determine the probability of the event occurring. In turn the probability selected will determine the combination of black and white balls that will be put in an opaque bag containing a total of 100 balls. Each one of the participants will be asked to draw a ball from the bag. After every draw the ball will be replaced before the next subject draws another ball. A white ball results in no loss, i.e. a payoff of $£ 10$ for the participant who drew the ball. A black ball results in a loss of $£ 10$, i.e. a payoff of $£ 0$ for the participant who drew the ball.

After the lottery has been played out, you will be free to check whether the dice in the bag and the combination of white and black balls in the bag correspond to the explanations given above.
The following lists gives you the numbers that you find on the six faces of each of the 10 dice:

| A | $1,1,1,2,3,4$ |
| :--- | ---: |
| B | $2,2,2,1,3,4$ |
| C | $3,3,3,1,2,4$ |
| D | $4,4,4,1,2,3$ |
| E | $1,1,2,2,3,4$ |
| F | $1,1,3,3,2,4$ |
| G | $1,1,4,4,2,3$ |
| H | $2,2,3,3,1,4$ |
| I | $2,2,4,4,1,3$ |
| J | $3,3,4,4,1,2$ |

## Acknowledgments

We thank Norman Spivey for preparing the software used in the laboratory and helping us to run the experiment. We thank for their comments Michele Cohen, Louis Eeckhoudt, John Hey, John Kagel and two anonymous referees. The authors' names appear in alphabetical order and their respective contributions are equivalent. The revision of the paper was undertaken while the second author was at SFB504 University of Mannheim which she gratefully acknowledges for financial support. Financial support was also provided by CNR (Italy) and Exec (York).

## Notes

1. Among others Becker and Brownson [1964], MacCrimmon [1968], Slovic and Tversky [1974], Yates and Zukowski [1976], MacCrimmon and Larsson [1979], Cohen, Jaffray, and Said [1985], Curley and Yates [1985], Einhorn and Hogarth [1985], Curley, Yates, and Abraham [1986], Kahn and Sarin [1988], Curley and Yates [1989], Bernasconi and Loomes [1992], Sarin and Weber [1993], Keppe and Weber [1995], and Fox and Tversky [1995].
2. Operationalization of uncertainty as a second-order distribution can be found in Hogarth and Kunreuther [1989], Camerer and Kunreuther [1989], and Sarin and Weber [1993]. As far as reaction to ambiguity is concerned, the results of these studies are not clear cut: while Hogarth and Kunreuther and Sarin and Weber find a significant impact of ambiguity, Camerer and Kunreuther find a weak effect. Moreover, these experiments differ in the elicitation procedures and in the outcome domain. While Sarin and Weber [1993] use the gain domain, the other two studies refer to insurance; a market mechanism is adopted by Camerer and Kunreuther [1989] and Sarin and Weber [1993], while in Hogarth and Kunreuther subjects answer a questionnaire.
3. The "anchor" may be the expected value of the probability distribution, a value of probability obtained from experience or expert advice, etc.
4. Evidence of this switch from ambiguity aversion to ambiguity preference as the probability of loss increases can be found in Hogarth and Kunreuther [1989] and Di Mauro and Maffioletti [1996].
5. "A well-specified second-order probability distribution... needs not involve uncertainty about the process of outcome determination" [Curley and Yates, 1985, p. 278].
6. Hypothesis 2B and 2C give the same prediction when the uniform distribution is adopted but, contrary to 2B, 2 C predicts a stronger reaction to uncertainty when ambiguity is made operational by means of a second-order probability distribution in all the three representations.
7. Hogarth and Kunreuther, however, do not adopt a uniform treatment: while in the "Best Estimate" scenario subjects received only vague information about the probability of the loss, in the "interval of probability" scenario ambiguity was made operational by means of an explicit second-order distribution.
8. The two authors would surely be among the pessimists.
9. The rest of the auction procedure was standard. When all the eight markets were through, one scenario was chosen at random. For that scenario the first and the second highest bid were announced. The player who had made the highest bid acquired the right to completely insure himself and had to pay the second highest bid. The rest of the participants played the lottery and they were paid according to the resolution of uncertainty.
10. We are aware of the fact that several auction periods are generally adopted in the literature as solution of the problem of overbidding/underbidding. However, we use the second price auction to elicit subjects' valuations of potential losses under risk and under uncertainty. Since there are no reasons to believe that overbidding/underbidding should be stronger or weaker under risk than under uncertainty, this problem should not affect the comparison between the two decision tasks and the test of our hypotheses. This idea has been confirmed to us by John Kagel in a personal communication. Also, Cox and Grether [1996] show that in the preference reversal framework the behaviour observed under the standard Becker, De Groot, Marshak device coincides with that obtained with a one-shot second price auction.
11. Let us consider as an example the case of the set of probabilities ( $0.05,0.15,0.25,0.35$ ); consider further that the chosen die has three sides with number 1 on them, and the other sides with the numbers 2,3 , and 4 respectively. Hence, the die gives a weight of $1 / 2$ to the probability .05 , while it gives a weight of $1 / 6$ to each of the other values contained in the set.
12. In the INFO treatment the outlier displayed a ratio of the ambiguous to risky WTP equal to 750 , whereas in the NO INFO, the ratio of WTPs of the outlier was equal to 350 .
13. Except in the Best Estimate-NO INFO subsample at $p=20 \%$.
14. Except the mean in the Interval of Probability-NOINFO subsample.
15. In Hogarth and Kunreuther the low probability was 0.01 and the high one was 0.90 .
16. This last result is in line also with the studies by Cohen, Jaffray, and Said [1985, 1987] which found no effect for ambiguity when the probability of loss was $50 \%$.
17. The scale used is logarithmic.
18. A manipulation check adopted by Heath and Tversky [1991] is to ask subjects whether they think the ambiguous probability is equal, lower or higher than the objective probability used in the risky scenario. This check-as
observed by Camerer and Weber [1992]—may be misleading, as the elicited probability judgement may in effect be akin to a decision weight and thus be itself a measure of ambiguity aversion (see also Keppe and Weber [1995])
19. This happens in all cells except in the Best estimate definition of ambiguity at the reference probabilities of $3 \%$ and $80 \%$.
20. It can be useful to notice that the impact of ambiguity is statistically significant at two probability levels in both INFO and NOINFO in the IP representation. Segal's hypothesis is partially supported in the sense that people can be averse to a uniform second-order distribution. Moreover, the fact that information does not influence valuation may suggest that a probability interval is easily perceived as a uniform distribution in any case.
21. Note, however, that since each subject evaluated eight scenarios referring to only one type of ambiguity, subjects were not given a chance to compare the differences among the probability distributions given.
22. The representations of ambiguity were also compared on a one to one basis by means of the Mann Whitney test. Results confirm those given by the Kruskal-Wallis test. In particular, no significant difference emerged between the BE and the IP definitions and the SOP representation in the INFO design, as we had assumed.
23. As far as IP is concerned the effect of ambiguity is the same in both the INFO and NOINFO case.
24. Results of this paper show that ambiguity aversion for firms is greater than ambiguity aversion of consumers in the insurance market.
25. In the experiment, the asterisks were replaced by the numbers of the scenarios that belonged to that given category. These numbers varied according to the experimental session, given that they were chosen randomly.

## References

BECKER, S.W. and BROWNSON, F.O. [1964]: "What Price Ambiguity? Or the Role of Ambiguity in Decision Making," Journal of Political Economy, 72, 62-73.
BECKER, G., DEGROOT, M.H., and MARSHACK, J. [1964]: "Measuring Utility by a Single-Response Sequential Method," Behavioural Science, 9, 226-232.
BERNASCONI, M. and LOOMES, G. [1993]: "Failure in the Reduction Principle in an Ellsberg Type Problem," Theory and Decision, 32, 77-100.
CAMERER, C. and KUNREUTHER, H. [1989]: "Experimental Markets for Insurance," Journal of Risk and Uncertainty, 2, 265-300.
CAMERER, C. and WEBER, M. [1992]: "Recent Developments in Modelling Preferences: Uncertainty and Ambiguity," Journal of Risk and Uncertainty, 5, 325-370.
CHOW, C. and SARIN, R. [1998]: "Choice under Uncertainty: Known, Unknown, Unknowable," UCLA, unpublished manuscript.
COHEN, M., JAFFRAY, J.-Y., and SAID, T. [1985]: "Individual Behaviour under Risk and Under Uncertainty: An Experimental Study," Theory and Decision, 18, 203-228.
COHEN, M., JAFFRAY, J.-Y., and SAID, T. [1987]: "Experimental Comparison of Individual Behaviour under Risk and under Uncertainty for Gains and for Losses," Organizational Behaviour and Human Decision Processes, 36, 272-287.
COX, J.C. and GRETHER, D.M. [1996]: "The Preference Reversal Phenomenon: Response Mode, Markets and Incentives," Economic Theory, 7, 381-405
COURSEY, D., HOVIS, J., and SCHULTZE, W. [1987]: "On the Supposed Disparity Between Willingness to Accept and Willingness to Pay Measures of Value," Quarterly Journal of Economics, 102, 679-690.
CURLEY, S. and YATES, F. [1985]: "The Centre and Range of the Probability Interval as Factor Affecting Ambiguity Preferences," Organizational Behaviour and Human Decision Processes, 36, 272-287.
CURLEY, S. and YATES, F. [1989]: "An Empirical Evaluation of Descriptive Models of Ambiguity Reactions in Choice Situations," Journal of Mathematical Psychology, 33, 397-427.
CURLEY, S., YATES, F., and RICHARD, A. [1986]: "Psychological Sources of Ambiguity Avoidance," Organisational Behaviour and Human Decision Process, 38, 230-256.
Di MAURO, C. and MAFFIOLETTI, A. [1996]: "An Experimental Investigation of the Impact of Ambiguity on the Evaluation of Self-insurance and Self-protection," Journal of Risk and Uncertainty, 12, 359-377.

EINHORN, H. and HOGARTH, R. [1985]: "Ambiguity and Uncertainty in Probabilistic Inference," Psychological Review, 92, 433-459.
EINHORN, H. and HOGARTH, R. [1986]: "Decision Making under Ambiguity," Journal of Business, 59, 224249
EISENBERGER, R. and WEBER, M. [1995]: "Willingness to Pay and Willingness to Accept for Risky and Ambiguos Lotteries," Journal of Risk and Uncertainty, 10, 223-223.
ELLSBERG, D. [1961]: "Risk, Ambiguity, and the Savage Axiom," Quarterly Journal of Economics, 75, 643669.

EPSTEIN, L. [2000]: "Are Probabilities used in Markets?" Journal of Economic Theory, 91, 86-90.
EPSTEIN, L. and WANG, T. [1994]: "Intertemporal Asset Pricing under Knightian Uncertainty," Econometrica, 62(3), 283-322.
FOX, C. and TVERSKY, A. [1995]: "Ambiguity Aversion and Comparative Ignorance," Quarterly Journal of Economics, 110, 585-603.
FOX, C. and WEBER, M. [1999]: "Ambiguity Aversion, Comparative Ignorance, and the Role of Context," Universitat Mannheim, No. 99-47.
GARDENFORS, P. and SAHLIN, N.E. [1982]: "Unreliable Probabilities, Risk Taking, and Decision Making," Synthese, 53, 361-386.
GARDENFORS, P. and SAHLIN, N.E. [1983]: "Decision Making with Unreliable Probabilities," British Journal of Mathematical and Statistical Psychology, 36, 240-251.
GILBOA, I. and SCHMEIDLER, D. [1989]: "Maximin Expected Utility with a Non Unique Prior," Journal of Mathematical Economics, 16, 65-88.
HARSTAD, R. [1990]: "Dominant Strategy Adoption Efficiency and Bidders. Experience with Pricing Rules," Mimeo, Virginia Commonwealth University.
HEATH, C. and TVERSKY, A. [1991]: "Preference and Belief: Ambiguity and Competence in Choice Under Uncertainty," Journal of Risk and Uncertainty, 4, 5-28.
HOGARTH, R. [1989]: "Ambiguity and Competitive Decision Making," Annals of Operations Research, 19, 31-50.
HOGARTH, R. and EINHORN, H. [1990]: "Venture Theory: A Model of Decision Weights," Management Science, 36, 780-803.
HOGARTH, R. and KUNREUTHER, H. [1985]: "Ambiguity and Insurance Decisions," American Economic Review, Paper and Procedings, 386-391.
HOGARTH, R. and KUNREUTHER, H. [1989]: "Risk, Ambiguity, and Insurance," Journal of Risk and Uncertainty, 2, 5-35.
HOGARTH, R. and KUNREUTHER, H. [1992]: "Pricing Insurance and Warranties: Ambiguity and Correlated Risks," Geneva Papers on Risk and Insurance Theory, 17, 35-60.
KAGEL, J. [1995]: "Auctions: A Survey of Experimental Research," in Handbook of Experimental Economics, J. Kagel and A. Roth (Eds.), Princeton University Press, Princeton, Ch. 7, 501-536.
KAHN, B. and SARIN, R. [1988]: "Modelling Ambiguity in Decision Making Under Uncertainty," Journal of Consumer Research, 15, 267-272.
KEYNES, M.J. [1921]: A Treatise on Probability. London: MacMillan.
KEPPE, H. and WEBER, M. [1995]: "Judged Knowledge and Ambiguity Aversion," Theory and Decision, 39, 51-77.
KNIGHT, F. [1921]: Risk Uncertainty and Profit. Houghton-Mifflin: Boston.
LEVI, I. [1974]: "On Indeterminate Probabilities," Journal of Philosophy, 71, 391-418.
LEVI, I. [1989]: "Reply to Maher," Economics and Philosophy, 5, 79-90.
MACCRIMMON, K.R. [1968]: "Descriptive and Normative Implication of Decision Theory Postulates," in Risk and Uncertainty, K. Borch and J. Mossin (Eds.), London: MacMillan.
MACCRIMMON, K.R. and LARSSON, S. [1979]: "Utility Axioms Versus Paradoxes," in Expected Utility and the Allais Paradox, M. Allais and O. Hagen (Eds.), D. Reidel Publishing: Dordrecht, Holland.
MAFFIOLETTI, A. [1995]: "Evaluating Lotteries with Unreliable Probabilities: An Experimental Test of Explanations for the Ellsberg Paradox," Discussion Papers in Economics No. 1, University of York.
MCLELLAND, G.H., SCHULZE, W., and COURSEY, D.L. [1993]: "Insurance for Low-Probability Hazards: A Bimodal Response to Unlikely," Journal of Risk \& Uncertainty, 7(1), 95-116.

SARIN, R.K. and WEBER, M. [1993]: "Effects of Ambiguity in Market Experiments," Management Science, 39, 609-615.
SEGAL, U. [1987]: "The Ellsberg Paradox and Risk Aversion: An Anticipated Utility Approach," International Economic Review, 28, 175-202.
SHOGREN, J., SHIN, S., HAYES, D., and KLIEBENSTEIN, J. [1994]: "Resolving Differences in Willingness to Pay and Willingness to Accept," American Economic Review, 84, 255-270.
SLOVIC, P. and TVERSKY, A. [1974]: "Who Accept Savage's Axioms?" Behavioural Science, 18, 368-373.
TVESKY, A. and FOX, G. [1995]: "Weighting Risk and Uncertainty," Psychological Review, 102(2), 269-283.
TVERSKY, A. and KAHNEMAN, D. [1992]: "Advances in Prospect Theory: Cumulative Representation of Uncertainty," Journal of Risk and Uncertainty, 5, 297-323.
TVERSKY, A. and WAKKER, P. [1995]: "Risk Attitudes and Decision Weights," Econometrica, 63, 1255-1280.
VICKREY, W. [1961]: "Counter Speculation, Auction and Competitive Sealed Tenders," Journal of Finance, 16, 8-37.
YATES, F. and ZUKOWSKI, L. [1976]: "Characterization of Ambiguity in Decision Making," Behavioral Science, 21, 19-25.

