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Fuzzy Ranking Methods and Their Applications

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Fuzzy Ranking Methods and Their Applications

Chai Kok Chin

A thesis submitted
In fulfillment of the requirement for the Doctor of Philosophy

Faculty of Engineering
UNIVERSITI MALAYSIA SARAWAK
2016

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ABSTRACT

Fuzzy ranking is a procedure used to compare and order a sequence of fuzzy sets (FSs). It is an essential step in fuzzy decision making problems before a final decision can be drawn. While many fuzzy ranking methods are available in the literature, a generic method that can provide appropriate and satisfactory solutions across a variety of situations has yet to be developed. Many existing methods are limited to rank either type-1 fuzzy sets (T1FSs) or interval type-2 fuzzy sets (IT2FSs), and only few methods can flexibility handle both types of FSs. In particular, fuzzy ranking becomes complicated when FSs are represented by possibility distributions, which can overlap with one another. In this thesis, two new fuzzy ranking methods with different purposes are proposed. The first method ranks both T1FSs and IT2FSs by considering ranking and weighting issues, while the second ranks both T1FSs and IT2FSs by integrating decision makers' viewpoints. Besides that, it is important for a fuzzy ranking method to satisfy a set of reasonable fuzzy ordering properties. As a result, the capability of the proposed fuzzy ranking methods in fulfilling the relevant properties is analyzed and discussed. The usefulness of both methods is demonstrated using real-world applications. The results positively indicate efficacy of the proposed fuzzy ranking methods in solving fuzzy ranking problem as well as complex decision making problems in practical environments.

Keywords: Fuzzy ranking, decision viewpoint, peer assessment, failure mode and effect analysis, ordering properties, real-world applications

Kaedah Pengaturan Set Kabur dan Aplikasinya

ABSTRAK

Kedudukan kabur adalah prosedur yang digunakan untuk membandingkan dan mengaturkan urutan set kabur. Ia adalah satu langkah penting dalam masalah membuat keputusan dalam keadaan kabur sebelum keputusan muktamad yang boleh didapati. Walaupun banyak kaedah kedudukan kabur boleh didapati dalam kesusasteraan, kaedah generik yang boleh memberikan penyelesaian yang memuaskan dalam pelbagai situasi masih belum dibangunkan. Banyak kaedah yang sedia ada adalah terhad untuk mengendalikan set kabur jenis-1 atau set kabur jenis-2, dan hanya beberapa kaedah yang boleh mengendalikan kedua-dua jenis set kabur secara fleksibel. Khususnya kedudukan kabur menjadi rumit apabila set kabur diwakili oleh pengagihan kemungkinan, yang boleh bertindih dengan satu sama lain. Dalam tesis ini, dua kaedah kedudukan kabur baru dengan tujuan yang berbeza dicadangkan. Kaedah pertama boleh mengendalikan kedua-dua jenis set kabur dengan mempertimbangkan kedudukan dan isu pemberat, manakala alternatif kedua mengendalikan kedua-dua jenis set kabur dengan mengintegrasikan pandangan pembuat keputusan. Selain itu, ia adalah penting untuk kaedah kedudukan kabur untuk memenuhi ciri-ciri yang munasabah. Oleh itu, keupayaan kedua-dua kaedah yang dicadangkan dalam memenuhi ciri-ciri tersebut juga dianalisis dan dibincangkan. Kebergunaan kedua-dua kaedah ini juga ditunjukkan menggunakan aplikasi dunia sebenar. Keputusan positif menunjukkan keberkesanan yang dicadangkan kaedah kedudukan kabur dalam menyelesaikan masalah yang kompleks membuat keputusan dalam persekitaran yang praktikal.

Kata kunci: *Kedudukan set kabur, keputusan pandangan, penilaian rakan sebaya, Analisis kesan-kesan dan potensi kegagalan mod, aplikasi dunia sebenar*

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LIST OF ABBREVIATIONS

Notations	Descriptions
AHP	Analytical hierarchy Process
AI	Artificial intelligence
ANP	Analytic network process
ARAS	Additive Ratio Assessmen
ART	Adaptive resonance theory
BPA	Basic probability assignment
COPRAS	Complex PRoportional ASsessment method
CWW	Computing with words
DEMATEL	Decision Making Trial and Evaluation Laboratory
D	Detection
DST	Dempster-Shafer theory
EBN	Edible bird nest
EIA	Enhanced Interval Approach
EKM	Enhanced Karnik Mendel
ELECTRE	ELimination and Choice Expressing Reality
FMEA	Failure mode and effect analysis
FOU	Footprint of uncertainty
FWA	Fuzzy weighted averaging
GFS	Generalized fuzzy set
GM	Group mark
IM	Individual mark
IT2FS	Interval type-2 fuzzy set
IWF	Individual Weighting Factor
JS	Jaccard similarity
LINMAP	Linear Programming Technique for Multidimensional Analysis of Preference

LMF	Lower membership function
LWA	Linguistic weighted average
MP	Mathematical programming
MCDM	Multi-criteria decision-making
O	Occurrence
PA	Peer assessment score
Per-C	Perceptual computing
PROMETHEE	Preference Ranking Organization METhod for Enrichment Evaluations
QUALIFLEX	QUALItative FLEXible multiple criteria
RPN	Risk Priority Numbers
S	Severity
T1FS	Type-1 fuzzy set
TBM	Transferable Belief Model
TM	Team member
TOPSIS	Technique of Order of Preference by Similarity to Ideal Solution
UMF	Upper membership function
VIKOR	VIseKriterijumska Optimizacija I Kompromisno Resenje

LIST OF NOTATIONS

Notations	Descriptions
$\tilde{A}$	An IT2FS
C	Set for risk factors, where $C = \{S, O, D\}$
$core$	core of a T1FS
$\hat{C}_{\tilde{A},l}$	Left centroids of $\tilde{A}$, $\forall \hat{C}_{\tilde{A}} \in U$
$\hat{C}_{\tilde{A},r}$	Right centroids of $\tilde{A}$, $\forall \hat{C}_{\tilde{A}} \in U$
F_i	Label of failure mode, where $i = 1,2, \dots, n$
FOU	FOU of an IT2FS in the universe of discourse U
$\underline{FOU}$	Lower bound of FOU
$\overline{FOU}$	Upper bound of FOU
L	Left switch point
R	Right switch point
$supp$	support of a 1FS
T	Fuzzy targets
T_n	Neutral fuzzy target
T_o	Optimistic fuzzy target
T_p	Pessimistic fuzzy target
TM_k	An FMEA user, where $k = 1,2, \dots, K$
U	Universe of discourse
μ	membership function of a fuzzy sets
$\tilde{Y}_{F_i}$	Per-C –RPN score for $F_{i\|i=1,2,\dots,n}$ on D_Y
$\tilde{Y}_{R_g}$	Risk words in risk rating scale, where $R_{g\|g=1,2,\dots,G}$
$[\overline{a_1}, \overline{a_2}, \overline{a_3}, \overline{a_4}, \overline{h}]$	Parameters in $\overline{FOU(\tilde{U})}$ where $\overline{a_1}, \overline{a_2}, \overline{a_3}, \overline{a_4} \in U$ and $\overline{h} \in [0,1]$
$[\underline{a_1}, \underline{a_2}, \underline{a_3}, \underline{a_4}, \underline{h}]$	Parameters in $\underline{FOU(\tilde{U})}$ where $\underline{a_1}, \underline{a_2}, \underline{a_3}, \underline{a_4} \in U$ and $\underline{h} \in [0,1]$

CHAPTER 1

INTRODUCTION

1.1 Background

Decision making is an essential part of everyday life. In practice, many decision making tasks take place in an environment in which the goals, constraints, and consequences of feasible options (alternatives and actions) are not known precisely (Bellman & Zadeh, 1970). As an example, "investing in stock market is *very high risk*" and "student #B is *excellent* in collaborative work", in which words such as "*very high risk*" and "*excellent*" are imprecise and vague terms that describe the risk of investment and student's contribution, respectively. To cope with the challenges of decision making, fuzzy set theory was introduced more than fifty years ago to model uncertain, imprecise, and vague information (Zadeh, 1965). Since then, the concepts of fuzzy set theory have received prominent attention and are well-studied across different disciplines, e.g., economy (Tiryaki & Ahlatcioglu, 2005), engineering (Wang & Elhag, 2006), and education (Chang & Chen, 2009).

When fuzzy set theory is incorporated into decision models for solving decision making problems, the overall performance measures of decision alternatives often are represented with fuzzy sets (Bellman & Zadeh, 1970). As an example, a set of decision alternatives with a common type of fuzzy set, i.e., alpha-level set, is presented in (Wang, Chin, Poon & Yang, 2009, Wang & Elhag, 2006) . In a fuzzy environment, ranking of fuzzy sets (hereafter named as fuzzy ranking) is an essential step for fuzzy decision making problem in order to reach a final decision (Chen and Klein, 1997, Chen & Tang, 2008). Specifically, fuzzy ranking is a procedure used to compare and order a sequence of fuzzy sets. It is a complex and challenging task because fuzzy sets are represented by possibility distributions, which can overlap with one another (Chang & Lee, 1994, Lee-Kwang & Lee 1999).

Fuzzy ranking methods can be categorized into three classes (Wang & Kerre, 2001a,b). The first class transforms a number of fuzzy sets into crisp numbers, and ranks the resulting crisp numbers, e.g. (Abbasbandy & Asady, 2006, Chu & Tsao, 2002, Karnik & Mendel, 2001, Wu & Mendel, 2009). The respective methods are easy and straightforward. But, it was warned that "by reducing the whole of our analysis to a single number, we are losing much of the information we have purposely been keeping throughout our calculations" (Freeling, 1980). As such, the first class is suitable when only the sequence of fuzzy sets needs to be determined. The second class ranks fuzzy sets through pairwise comparisons, e.g. (Chen & Lee, 2010a, Yuan, 2001). The respective methods analyze the preference relation of two or more fuzzy sets. As an example, fuzzy set *A* is larger than fuzzy set *B* by 60%. The methods are useful when both preference relations between fuzzy sets and their ranking orders have to be determined. They are suitable for dealing with subjective judgements, especially in areas such as social sciences (Yuan, 1991). However, it is challenging to obtain consistent fuzzy ranking results based on pairwise comparisons (Yuan, 1991). Furthermore, it is difficult to achieve certain reasonable ordering properties, as stated in (Wang and Kerre, 2001a, b). The third class maps a number of fuzzy sets to crisp numbers based on a pre-defined reference set(s) for comparison, e.g. (Chen & Lee, 2010b, Huynh, Nakamori & Lawry, 2008, Lee-Kwang & Lee, 1999). The respective methods are beneficial because a decision maker's viewpoints can be reflected. However, it is challenging when multiple viewpoints do not lead to the same ranking outcome, while a final decision still needs to be made (Yuan, 1991).

Although many fuzzy ranking methods are available in the literature, a general solution across a variety of situations is yet to be known (Huynh, Nakamori & Lawry, 2008). Most of the existing methods emphasize on ranking type-1 fuzzy sets (T1FSs). However, recent advances in fuzzy decision making reveal that the use of higher dimensional fuzzy sets, such as interval type-2 fuzzy sets (IT2FSs), provides better flexibility in preserving and processing linguistic uncertainties. Unfortunately, a generic fuzzy ranking method that can handle both T1FSs and IT2FSs is still new. Therefore, research in fuzzy ranking should focus on undertaking both T1FSs and IT2FSs.

Besides that, a given fuzzy decision making problem may require different fuzzy ranking methods. As an example, consider a peer assessment problem related to cooperative learning of students. It is more meaningful to obtain performance indices that reflect a student's contribution in a group (i.e., weighting) and subsequently rank the student accordingly, instead of sorting the students' achievements (represented in fuzzy sets) sequentially (Cheng & Chen, 2009). In such situation, ranking and weighting of fuzzy sets are two important criteria pertaining to a fuzzy ranking method. Another example is a two-person game involving fuzzy profit and loss, in which the players' viewpoints or attitudes need to be analyzed (Lee-Kwang & Lee, 1999). In this case, a fuzzy ranking method that integrates the decision maker's viewpoints is an important criterion.

To fill the research gaps, fuzzy ranking methods that can flexibly rank both T1FSs and IT2FSs are investigated in details in this thesis. Furthermore, important features, such as ranking and weighting, and integrating decision makers' viewpoints are examined. These are the main novelties of this research, and the main contributions of this thesis.