SEMESTER 2

MATHEMATICS

for Asasi UNIMAS

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PREFACE

Mathematics for Asasi UNIMAS Semester 2 is written for student at Centre for Pre-University Universiti Malaysia Sarawak and all local matriculation centre. This book contain all the required sub-topic for pre-university for Mathematics Semester 2.

The objective of this book is to develop the understanding of mathematical concepts and their application. At the same time, to build problem solving skills and to formulate problems into mathematical terms and try to solve the problems. All mathematical concepts are presented in simple English for easy understanding.

This books contains eight chapter that had been planned and arranged carefully based on the university syllabus. In each chapter, all concepts and skills presented for each subtopic are accompanied by detailed explaination. Question and solution for each chapter provide a wide range of examination type question based on the concepts and theories learnt.

The topics are arranged in a systematic manner to make learning easier and more effective. The mathematical concepts of each topic presented in a comprehensive for easy understanding. Answers are also provided for all question on each chapter. The book is also suitable for first year undergraduate Statistics of a degree or diploma programme.

We hope this book will assist Asasi students in mastering mathematical concepts as well as preparing them for their examination.

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1

DIFFERENTIATIONS

1.1 Derivative of a Function

The derivative of a function $f(x)$ with respect to x from first principles is

$$f'(x) = \frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Example 1.1

Find the derivatives of the following functions from the first principles

(a) $f(x) = 3x + 2$

(b) $f(x) = 5x^2 - 4$

Solution

(a) $f(x) = 3x + 2$, $f(x+h) = 3(x+h) + 2$

$$f'(x) = \lim_{h \to 0} \frac{(3(x+h)+2) - (3x+2)}{h}$$

$$= \lim_{h \to 0} \frac{3x + 3h + 2 - 3x - 2}{h}$$

$$= \lim_{h \to 0} \frac{3h}{h}$$

$$= \lim_{h \to 0} 3$$

$$= 3$$

(b) $f(x) = 5x^2 - 4$, $f(x+h) = 5(x+h)^2 - 4$

$$f'(x) = \lim_{h\to 0} \frac{\left(5(x+h)^2 - 4\right) - \left(5x^2 - 4\right)}{h}$$

$$= \lim_{h\to 0} \frac{5\left(x^2 + 2xh + h^2\right) - 4 - 5x^2 + 4}{h}$$

$$= \lim_{h\to 0} \frac{5x^2 + 10xh + 5h^2 - 5x^2}{h}$$

$$= \lim_{h\to 0} \frac{5h^2 + 10xh}{h}$$

$$= \lim_{h\to 0} \frac{h(5h + 10x)}{h}$$

$$= \lim_{h\to 0} 5h + 10x$$

$$= 5(0) + 10x$$

$$= 10x$$

1.2 Rules of Differentiation

1. The constant rule

$$\frac{d}{dx}(c) = 0$$, where c is real number.

2. The power rule

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

3. The constant multiple rule

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)] = cf'(x)$$

4. The sum and difference rule

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$$

5. The product rule

$$\frac{d}{dx}[f(x)\,g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

6. The quotient rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2} \quad \text{where } g(x) \neq 0$$

7. The chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} \quad \text{where } y = f(u) \text{ and } u = f(x).$$

A simpler version of the chain rule: $\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}g'(x)$

Example 1.2

Find $\frac{dy}{dx}$ of the following functions.

(a) $y = 5$

(b) $y = x^3$

(c) $y = 8x^5$

(d) $y = \frac{10}{\sqrt{x}}$

(e) $y = x^5 + 10x - 8x^3$

(f) $y = (8x^4 + 12)(5x^2 - 10)$

(g) $y = \dfrac{x^3 + 1}{x - 4}$

(h) $y = (x^{10} + 17)^5$

Solution

(a) $y = 5$

$$\frac{dy}{dx} = \frac{d}{dx}(5)$$
$$= 0$$

(b) $y = x^3$

$$\frac{dy}{dx} = \frac{d}{dx}(x^3)$$
$$= 3x^2$$

(c) $y = 8x^5$

$$\frac{dy}{dx} = \frac{d}{dx}(8x^5)$$
$$= 8\frac{d}{dx}(x^5)$$
$$= 8(5)x^4$$
$$= 40x^4$$

(d) $$y = \frac{10}{\sqrt{x}}$$

$$= 10x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(10x^{-\frac{1}{2}}\right)$$

$$= 10\frac{d}{dx}\left(x^{-\frac{1}{2}}\right)$$

$$= 10\left(-\frac{1}{2}\right)x^{-\frac{3}{2}}$$

$$= -5x^{-\frac{3}{2}}$$

$$= -\frac{5}{\sqrt{x^3}}$$

(e) $$y = x^5 + 10x - 8x^3$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^5 + 10x - 8x^3\right)$$

$$= \frac{d}{dx}\left(x^5\right) + \frac{d}{dx}(10x) - \frac{d}{dx}\left(8x^3\right)$$

$$= \frac{d}{dx}\left(x^5\right) + 10\frac{d}{dx}(x) - 8\frac{d}{dx}\left(x^3\right)$$

$$= 5x^4 + 10 - 8(3)x^2$$

$$= 5x^4 + 10 - 24x^2$$

(f) $y=(8x^4+12)(5x^2-10)$

$$\frac{dy}{dx}=\frac{d}{dx}(8x^4+12)(5x^2-10)$$

$$=(8x^4+12)\frac{d}{dx}(5x^2-10)+(5x^2-10)\frac{d}{dx}(8x^4+12)$$

$$=(8x^4+12)(10x)+(5x^2-10)(32x^3)$$

$$=80x^5+120x+160x^5-320x^3$$

$$=240x^5-320x^3+120x$$

(g) $y=\dfrac{x^3+1}{x-4}$

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x^3+1}{x-4}\right)$$

$$=\frac{(x-4)\frac{d}{dx}(x^3+1)-(x^3+1)\frac{d}{dx}(x-4)}{(x-4)^2}$$

$$=\frac{(x-4)(3x^2)-(x^3+1)(1)}{(x-4)^2}$$

$$=\frac{3x^3-12x^2-x^3-1}{(x-4)^2}$$

$$=\frac{2x^3-12x^2-1}{(x-4)^2}$$

(h) $y = (x^{10} + 17)^5$

$$\frac{dy}{dx} = \frac{d}{dx}(x^{10} + 17)^5$$

$$= 5(x^{10} + 17)^4 \frac{d}{dx}(x^{10} + 17)$$

$$= 5(x^{10} + 17)^4 10x^9$$

$$= 50x^9(x^{10} + 17)^4$$

Example 1.3

Find $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$ of the following functions.

(a) $y = 5x^{10}$

(b) $y = 5x^3 - 2x$

Solution

(a) $y = 5x^{10}$

$$\frac{dy}{dx} = \frac{d}{dx}(5x^{10})$$

$$= 5\frac{d}{dx}(x^{10})$$

$$= 5(10x^9)$$

$$= 50x^9$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}\left(50x^9\right)$$

$$= 50\frac{d}{dx}\left(x^9\right)$$

$$= 50\left(9x^8\right)$$

$$= 450x^8$$

$$\frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)$$

$$= \frac{d}{dx}\left(450x^8\right)$$

$$= 450\frac{d}{dx}\left(x^8\right)$$

$$= 450\left(8x^7\right)$$

$$= 3600x^7$$

(b) $y = 5x^3 - 2x$

$$\frac{dy}{dx} = \frac{d}{dx}\left(5x^3 - 2x^2\right)$$

$$= 5\frac{d}{dx}\left(x^3\right) - 2\frac{d}{dx}\left(x^2\right)$$

$$= 5(3)x^2 - 2(2x)$$

$$= 15x^2 - 4x$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}\left(15x^2 - 4x\right)$$

$$= 15\frac{d}{dx}\left(x^2\right) - 4\frac{d}{dx}(x)$$

$$= 15(2)x - 4$$

$$= 30x - 4$$

$$\frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)$$

$$= \frac{d}{dx}(30x - 4)$$

$$= 30\frac{d}{dx}(x) - \frac{d}{dx}(4)$$

$$= 30$$

1.3 Differentiation of Exponential, Logarithmic and Trigonometric Functions

1. $\frac{d}{dx}\left(e^x\right) = e^x$. If $u = g(x)$, then $\frac{d}{dx}\left(e^u\right) = e^u \frac{du}{dx}$.

2. $\frac{d}{dx}\left(a^x\right) = a^x \ln a$, where $a \in R$. If $u = g(x)$, then $\frac{d}{dx}\left(a^u\right) = a^u \frac{du}{dx} \ln a$.

3. $\frac{d}{dx}(\ln x) = \frac{1}{x}$. If $u = g(x)$, then $\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$.

4. $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$, where $a > 0$, $a \neq 1$. If $u = g(x)$, then
   $\frac{d}{dx}(\log_a u) = \frac{1}{u \ln a}\frac{du}{dx}$.

5. Function derivatives

Table 1.1

Function	Derivatives
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$

6. Function derivatives

Table 1.2

Function	Derivatives
$\sin f(x)$	$\cos f(x) \frac{d}{dx} f(x)$
$\cos f(x)$	$-\sin f(x) \frac{d}{dx} f(x)$
$\tan f(x)$	$\sec^2 f(x) \frac{d}{dx} f(x)$
$\sec f(x)$	$\sec f(x) \tan f(x) \frac{d}{dx} f(x)$
$\operatorname{cosec} f(x)$	$-\operatorname{cosec} f(x) \cot f(x) \frac{d}{dx} f(x)$
$\cot f(x)$	$-\operatorname{cosec}^2 f(x) \frac{d}{dx} f(x)$

Example 1.4

Find the derivatives of the following functions.

(a) $y = e^{5x}$

(b) $y = 6^{x}$

(c) $y = e^{4x-2} + \frac{1}{e^{x}}$

(d) $y = 7^{4x} + e^{x}$

(e) $y = x^{2}e^{3x+1}$

Solution

(a)

$$y = e^{5x}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{5x}\right)$$

$$= 5e^{5x}$$

(b)

$$y = 6^{x}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(6^{x}\right)$$

$$= 6^{x} \ln 6$$

(c)

$$y = e^{4x-2} + \frac{1}{e^{x}}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{4x-2} + \frac{1}{e^{x}}\right)$$

$$= \frac{d}{dx}\left(e^{4x-2}\right) + \frac{d}{dx}\left(e^{-x}\right)$$

$$= 4e^{4x-2} - e^{-x}$$

$$= 4e^{4x-2} - \frac{1}{e^{x}}$$

(d) $y = 7^{4x} + e^{x}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(7^{4x} + e^{x}\right)$$

$$= \frac{d}{dx}\left(7^{4x}\right) + \frac{d}{dx}\left(e^{x}\right)$$

$$= 7^{4x}(4)\ln 7 + e^{x}$$

(e) $y = x^2 e^{3x+1}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^2 e^{3x-1}\right)$$

$$= x^2 \frac{d}{dx}\left(e^{3x+1}\right) + \left(e^{3x+1}\right)\frac{d}{dx}\left(x^2\right)$$

$$= \left(x^2\right)\left(3e^{3x}\right) + \left(e^{3x+1}\right)(2x)$$

$$= 3x^2 e^{3x} + 2xe^{3x+1}$$

Example 1.5

Find the derivatives of the following functions.

(a) $y = \ln 2x$

(b) $y = \ln(5x+2)$

(c) $y = \log 5x$

(d) $y = \log_3 \sqrt{5x-3}$

Solution

(a) $y = \ln 2x$

Let $u = 2x$

$$\frac{dy}{dx} = \frac{d}{dx}(\ln 2x)$$
$$= \frac{1}{2x} \cdot \frac{d}{dx}(2x)$$
$$= \frac{1}{2x} \cdot (2)$$
$$= \frac{1}{x}$$

(b) $y = \ln(5x + 2)$

Let $u = 5x + 2$

$$\frac{dy}{dx} = \frac{d}{dx}(\ln(5x+2))$$
$$= \frac{1}{5x+2} \cdot \frac{d}{dx}(5x+2)$$
$$= \frac{1}{5x+2} \cdot (5)$$
$$= \frac{5}{5x+2}$$

(c) $y = \log 5x$

Let $u = 5x$

$$\frac{dy}{dx} = \frac{d}{dx}(\log 5x)$$

$$= \frac{1}{5x \ln 10} \cdot \frac{d}{dx}(5x)$$

$$= \frac{1}{5x \ln 10} \cdot (5)$$

$$= \frac{1}{x \ln 10}$$

(d) $y = \log_3 \sqrt{5x-3}$

Let $u = 5x - 3$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\log_3 (5x-3)^{\frac{1}{2}}\right)$$

$$= \frac{1}{(5x-3)^{\frac{1}{2}} \ln 3} \cdot \frac{d}{dx}(5x-3)^{\frac{1}{2}}$$

$$= \frac{1}{(5x-3)^{\frac{1}{2}} \ln 3} \cdot \left(\frac{1}{2}(5x-3)^{-\frac{1}{2}} \cdot \frac{d}{dx}(5x-3)\right)$$

$$= \frac{1}{(5x-3)^{\frac{1}{2}} \ln 3} \cdot \left(\frac{1}{2}(5x-3)^{-\frac{1}{2}} \cdot (5)\right)$$

$$= \frac{5}{2(5x-3)\ln 3}$$

Example 1.6

Find the derivatives of the following functions.

(a) $y = \sin 3x$

(b) $y = \tan 5x^3$

(c) $y = \sec(2x^3 + 4)$

(d) $y = \cot^4 3x$

Solution

(a)
$$y = \sin 3x$$
$$\frac{dy}{dx} = \frac{d}{dx}(\sin 3x)$$
$$= 3\cos 3x$$

(b)
$$y = \tan 5x^3$$
$$\frac{dy}{dx} = \frac{d}{dx}(\tan 5x^3)$$
$$= \sec^2 5x^3 \frac{d}{dx}(5x^3)$$
$$= \sec^2 5x^3(15x^2)$$
$$= 15x^2 \sec^2 5x^3$$

(c) $y = \sec(2x^3 + 4)$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sec(2x^3 + 4)\right)$$

$$= \sec(2x^3 + 4)\tan(2x^3 + 4)\frac{d}{dx}(2x^3 + 4)$$

$$= \sec(2x^3 + 4)\tan(2x^3 + 4)\cdot(6x^2)$$

$$= 6x^2 \sec(2x^3 + 4)\tan(2x^3 + 4)$$

(d) $y = \cot^4 3x$

$$\frac{dy}{dx} = \frac{d}{dx}(\cot^4 3x)$$

$$= \frac{d}{dx}(\cot 3x)^4$$

$$= -4\cot^3 3x \operatorname{cosec}^2 3x \frac{d}{dx}(3x)$$

$$= -4\cot^3 3x \operatorname{cosec}^2 3x \cdot (3)$$

$$= -12\cot^3 3x \operatorname{cosec}^2 3x$$

1.4 Implicit Differentiation

1. If y is not alone on one side of the equation, the equation defines y implicitly as function of x. For example $y^2 + x^2 = y - 2x$.

2. To find $\frac{dy}{dx}$ of an implicit function, differentiate each terms with respect to x.

Example 1.7

Find $\frac{dy}{dx}$ for the following functions.

(a) $y^2 + 3x = 5x^2 - y$

(b) $5x + xy^2 = 5y$

Solution

(a)
$$y^2 + 3x = 5x^2 - y$$

$$\frac{d}{dx}\left(y^2 + 3x\right) = \frac{d}{dx}\left(5x^2 - y\right)$$

$$\frac{d}{dx}\left(y^2\right) + \frac{d}{dx}(3x) = \frac{d}{dx}\left(5x^2\right) - \frac{d}{dx}(y)$$

$$\frac{d}{dy}\left(y^2\right)\frac{dy}{dx} + 3 = 10x - \frac{d}{dy}(y)\frac{dy}{dx}$$

$$2y\frac{dy}{dx} + 3 = 10x - 1\frac{dy}{dx}$$

$$2y\frac{dy}{dx} + 1\frac{dy}{dx} = 10x - 3$$

$$(2y + 1)\frac{dy}{dx} = 10x - 3$$

$$\frac{dy}{dx} = \frac{10x - 3}{2y + 1}$$