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Research Note

CVBEM for a system of second-order elliptic partial differential equations

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A boundary element method based on the Cauchy's integral formulae and the theory of complex hypersingular integrals is devised for the numerical solution of boundary value problems governed by a system of second-order elliptic partial differential equations. The elliptic system has applications in physical problems involving anisotropic media. © 1998 Elsevier Science Ltd. All rights reserved

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1 INTRODUCTION

Consider the system of second-order elliptic partial differential equations given by

$$\sum_{j=1}^{2}\sum_{p=1}^{2}\sum_{k=1}^{N} a_{ijkp}\frac{\partial^2\phi_k}{\partial x_j\partial x_p}=0 \ (i=1,\ 2,\ \cdots,\ N), \qquad (1)$$

where ϕ_k $(k=1,\ 2,\ \cdots,\ N)$ are functions of x_1 and x_2 and a_{ijkp} $(j, p=1, 2$ and $i, k=1, 2, \cdots,$ $N)$ are real constant coefficients which satisfy the symmetry conditions $a_{ijkp}=a_{kpij}$ and are such that

$$\sum_{j=1}^{2}\sum_{p=1}^{2}\sum_{i=1}^{N}\sum_{k=1}^{N} a_{ijkp}\lambda_{ij}\lambda_{kp}>0$$

$$\text{for every non-zero } N\times 2 \text{ real matrix } [\lambda_{ij}]. \qquad (2)$$

We are interested in solving eqn (1) in a region $\mathcal{R}$ bounded by a simple closed curve C (on the $0x_1x_2$ plane) subject to

$$\left.\begin{array}{lll} \phi_k(x_1,\ x_2)=\mu_k(x_1,\ x_2) & \text{for} & (x_1,\ x_2)\in C_1 \\ P_i(x_1,\ x_2)=Q_i(x_1,\ x_2) & \text{for} & (x_1,\ x_2)\in C_2 \end{array}\right\} \qquad (3)$$

where μ_k and Q_i are suitably prescribed functions of x_1 and x_2, C_1 and C_2 are non-intersecting curves such that $C=C_1\cup C_2$ and

$$P_i=\sum_{j=1}^{2}\sum_{p=1}^{2}\sum_{k=1}^{N} a_{ijkp}\frac{\partial\phi_k}{\partial x_p}n_j \ (i=1,\ 2,\ \cdots,\ N) \qquad (4)$$

with n_j $(j=1, 2)$ being components of the unit outer normal vector to $\mathcal{R}$ on C.

The boundary value problem defined by eqns (1) and (3) has important applications in engineering. As an example, the steady-state temperature distribution in a flat plate which is thermally anisotropic and homogeneous obeys eqn (1) with $N=1$. The temperature and heat flux are given by ϕ_1 and (P_1, P_2) respectively, and a_{1j1p} are the heat conduction coefficients.

The plane static deformation of a homogeneous anisotropic elastic solid is governed by eqn (1) with $N=2$ and x_1 and x_2 as the Cartesian coordinates. The Cartesian displacement and traction are given by (ϕ_1, ϕ_2) and (P_1, P_2) respectively. The coefficients a_{ijkp} are the elastic moduli of the material occupying the solid. For a specific case, the elastostatic behaviour of a transversely isotropic material which has transverse planes perpendicular to the $0x_1x_2$ plane and which undergoes plane deformation is governed by

$$C\frac{\partial^2\phi_1}{\partial x_1^2}+L\frac{\partial^2\phi_1}{\partial x_2^2}+(F+L)\frac{\partial^2\phi_2}{\partial x_1\partial x_2}=0, \qquad (5)$$

$$C\frac{\partial^2\phi_2}{\partial x_2^2}+L\frac{\partial^2\phi_2}{\partial x_1^2}+(F+L)\frac{\partial^2\phi_1}{\partial x_1\partial x_2}=0,$$

which is a special case that can be recovered from eqn (1) if we let $N=2$ and $a_{2222}=A$, $a_{1111}=C$, $a_{1122}=a_{2211}=F$, $a_{1212}=a_{2121}=a_{1221}=a_{2112}=L$ and the remaining a_{ijkl} be zero. [The constants A, F, C and L are independent elastic coefficients.]

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