International Symposium on Mathematical Sciences & Computing Research (iSMSC) 2015 (iSMSC' 15)

Numerical Simulation of U ıtion with Chezy Friction

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***Abstract*—This paper studies and simulates numerically the evolution of undular bore under the effect of damping in the framework of the perturbed extended Korteweg-de Vries equation. Here, we consider Chezy frictional term to be the damping term in the perturbed extended Korteweg-de Vries equation. Numerical simulations show that under the influence of the friction, the undular bore with thick leading wave will transform into KdV-like solitary wave as the leading wave of the undular bore. The amplitude of the "thick" leading wave will remain the same for some time even though there is dissipation effect.**

Keywords-undular bore; damping; Chezy friction

1. INTRODUCTION

The Gardner equation or more commonly known as the extended Korterweg-de Vries (eKdV) equation

$$u_t + \alpha u u_x + \beta u^2 u_x + u_{xxx} = 0 \qquad (1)$$

is an universal model to describe nonlinear wave evolution in stratified fluid flows, for instance large-amplitude internal waves in ocean (see [1–4] and references therein). The sign of the coefficient β depends on the fluid stratification.

The solitary wave solution of the eKdV equation (1) also depends on the signs of the coefficients α and β. In this paper, we only consider the case where the coefficient β is negative. The solitary wave solution of equation (1) is given by

$$u(x,t) = \frac{\alpha}{\beta}\frac{B^2-1}{1+B\cosh[\gamma(x-\gamma^2 t)]} \qquad (2)$$

where

$$\gamma^2 = \frac{\alpha^2}{6\beta}\left(B^2-1\right), a = \frac{\alpha}{\beta}\left(B-1\right).$$

The solitary wave (2) is characterized by a single parameter B [1, 4].

When $\beta < 0$, equation (2) has a single family of solitary waves such that $0 < B < 1$. When the amplitude is small, (2) transform into the KdV-type solitary waves ($B \to 1$). However, when $B \to 0$, (2) describes the so-called "thick" solitary wave (also known as "table-top" soliton) (refer to Fig. 1). As $B \to 0$, the wave amplitude approaches the limiting value [1, 4]

$$a_{lim} = \frac{\alpha}{|\beta|}.$$

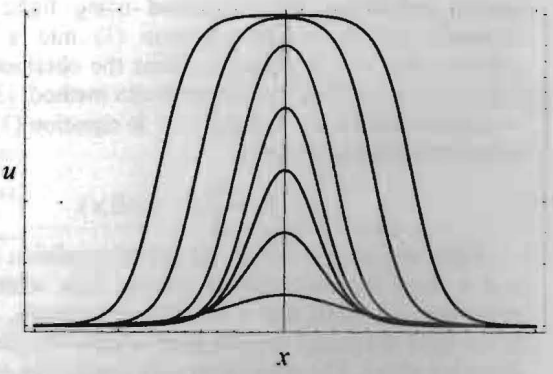

Figure 1. The shape of the solitary wave solution of equation (2)

The evolution of large-amplitude solitary waves with the effect of damping was studied by Grimshaw, Pelinovsky and Talipova [4] in the framework of the perturbed extended KdV (peKdV) equation. The peKdV equation is given by

$$u_t + \alpha u u_x + \beta u^2 u_x + u_{xxx} + \nu T\{u\} = 0 \qquad (3)$$

where the term $T\{u\}$ is the term to describe the effect of damping and ν is a drag coefficient. Here, we consider the fluid flow to have a turbulent boundary layer. Thus, the term $T\{u\}$ can be replaced by a quadratic term or more commonly known as Chezy friction, $T = |u|u$ [4–7].

In this paper, we would like to study and simulate numerically the evolution of undular bores with the effect of