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An Evolutionary-Based Similarity Reasoning Scheme for Monotonic Multi-Input Fuzzy Inference Systems

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***Abstract*—In this paper, an Evolutionary-based Similarity Reasoning (ESR) scheme for preserving the monotonicity property of the multi-input Fuzzy Inference System (FIS) is proposed. Similarity reasoning (SR) is a useful solution for undertaking the incomplete rule base problem in FIS modeling. However, SR may not be a direct solution to designing monotonic multi-input FIS models, owing to the difficulty in getting a set of monotonically-ordered conclusions. The proposed ESR scheme, which is a synthesis of evolutionary computing, *sufficient conditions*, and SR, provides a useful solution to modeling and preserving the monotonicity property of multi-input FIS models. A case study on Failure Mode and Effect Analysis (FMEA) is used to demonstrate the effectiveness of the proposed ESR scheme in undertaking real world problems that require the monotonicity property of FIS models.**

Keywords- Multi-input fuzzy inference system, monotonicity propery, similarity reasoning, fuzzy rule interpolation

I. INTRODUCTION

Recent advances in fuzzy modeling focus on evolutionary fuzzy systems and similarity reasoning (SR)-based techniques. The former attempts to use Evolutionary Computation (EC) techniques to search, learn, or optimize a fuzzy model. Indeed, many problems in fuzzy modeling can be expressed as an optimization problem (either single objective or multi-objectives), and EC have shown to be a promising solution to these problems. Jang *et al*, [1] examined the use of a data-driven fuzzy modeling as a system identification problem which could be expressed as an optimization problem. Ishibuchi *et al.* [2] proposed a Genetic Algorithm (GA)-based technique for selecting a small number of significant fuzzy if-then rules to construct a compact fuzzy classification system with high accuracy. Besides, multi-objective GAs have been used to search for a pareto set of fuzzy models with interpretability and accuracy trade-off [3]. The latter has been proposed as a solution to undertake the incomplete rule base problems. Similarity Reasoning (SR) techniques, e.g. Analogical Reasoning (AR) [4] and Fuzzy Rule Interpolation (FRI) [5], have been exemplified as a type of qualitative reasoning by Zadeh [6]. A conventional Fuzzy Inference System (FIS) assumes that an observation of a rule is mapped to zero, if its conclusion is not defined [7]. However, this may not always be true, and is the so-called "tomato classification problem" [7]. SR is able to deduce a conclusion for an observation, based on a set of incomplete rule base, whereby each rule assumes in the form of antecedent and consequent. In short, a similarity measure (either based on overlapping of membership functions or distance) is obtained between the observation and antecedents of the available rules from the incomplete rule base. A mathematical function is adopted so that the property (or properties) of the conclusion is obtained from the consequent parts of the available rules [8], usually with a weighted addition or average. Similarity measure is then used to indicate the relative weightage of each consequent of the available rules contributing to the conclusion. In this paper, FRI is viewed as a type of SR technique. FRI considers the distance between the observation and the antecedent as a similarity measure, which is further used as an indication to what extend the property (or properties) of the consequent part contributes to the conclusion.

Another recent trend in FIS modeling is the fulfillment of the monotonicity property. Consider an FIS model, $y = f(x_1, x_2, \dots, x_i, \dots, x_n)$, that satisfies the monotonicity condition between its output, y, with respect to its i^{th} input, x_i. Output y monotonically increases or decreases as x_i increases, i.e. $f(x_1, x_2, \dots, x_i^1, \dots, x_n) \le f(x_1, x_2, \dots, x_i^2, \dots, x_n)$ or $f(x_1, x_2, \dots, x_i^1, \dots, x_n) \ge f(x_1, x_2, \dots, x_i^2, \dots, x_n)$, respectively, for $x_i^1 < x_i^2$. The importance of this line of study has been highlighted in a number of recent publications [9-15]. Among the important aspects include: (1) many real-world systems and control problems obey the monotonicity property [10-11, 14-16]; (ii) the validity of the FIS output needs to be ensured for undertaking comparison, selection, and decision making problems [10, 12-13]; (iii) in the case when the number of data samples is small or the fuzzy rule set is incomplete, it is important to fully exploit the available qualitative information/ knowledge [9]; (iv) taking the additional qualitative information/knowledge of the system into consideration makes the model identification process less vulnerable to noise and inconsistencies in data samples, as well as mitigates the over-fitting phenomenon [9]. However, there are only a few articles that address the issues on how to design monotonicity-preserving FIS models [10].

From the literatures, several investigations on the mathematical conditions for a monotonic FIS model have been reported. Generally, theoretical proof of exact monotonicity in FIS is difficult [11]. But, there are some mathematical conditions that are useful for preserving monotonicity in FIS models. In [14], a set of mathematical conditions (i.e., the *sufficient conditions*) have been derived with the assumption that the first derivative of a Sugeno FIS is always *greater than or equal to* zero, or *less than or equal to* zero, for a monotonically increasing or decreasing function, respectively. The *sufficient conditions* suggest that two mathematical conditions (at the antecedent and consequent parts) are essential to obtain a monotonicity-preserving FIS model. For a fuzzy partition (at rule antecedent), maintaining a monotonically-ordered rule base can preserve the monotonicity property. This condition has been used and extended in [10, 12-13, 15]. In [9], it has been verified that for three basic T-norms (minimum, product, and Lukasiewicz), a monotonic input-output behavior is obtained for any monotonic rule bases. Some useful guidelines have also been proposed [9]. The relationships among the monotonicity property, monotonic rule base, and comparable fuzzy sets for single-input-rule-modules-connected FIS model are discussed in [11]. The findings in [9-15] indicate that it is important to maintain a monotonic rule base for FIS models.

The aim of this paper is to investigate on the use of SR for preserving the monotonicity property of multi-input FIS models. The focus is on the zero-order Sugeno FIS model coupled with the *sufficient conditions*. In our previous work [17-18], it has been shown that conventional SR that attempts to deduce each FIS rule consequent separately, with a simple weighted average or addition, is useful; but, it may not be a direct solution to preserving the monotonicity property in multi-input FIS models. This is because SR suffers from the difficulty in getting a set of monotonically-ordered conclusions, with respect to a set of observations which are comparable among themselves as well as comparable with the existing incomplete rule base. Thus, in [18], we have investigated the use of optimization techniques for SR with simulated data.

In this study, an Evolutionary-based SR (ESR) scheme is investigated. Instead of a simple weighted average or weighted addition, SR for monotonicity-preserving multi-input FIS models is formulated as a constrained optimization problem. The proposed ESR scheme adopts the *sufficient conditions* [14] as the hard constraints, and employs certain features of SR as the objective function. An evolutionary computing technique is used to search for a set of conclusions for the associated observations. This is a practical SR model, as the constraints in monotonicity-preserving multi-input FIS models are included. We further demonstrate the use of the ESR scheme with a real case study, i.e., an FIS-based Risk Priority Number (RPN) model in Failure Mode and Effect Analysis (FMEA) methodology.

This paper is organized as follows. In section II, the background of FIS models, the *sufficient conditions*, and SR is presented. A summary of our previous analysis [17-18] is also described. In section III, the proposed ESR scheme is explained in detail. A case study and the results are reported in section IV. Concluding remarks are presented in section V.

II. BACKGROUND

A. Fuzzy Inference Systems

The fuzzy production rules for an n-input FIS model, where $n > 1$ can be represented as follows.

$R^{j_1,j_2,\dots,j_n}$:
$If\ (x_1 is\ A_1^{j_1}) AND\ (x_2 is\ A_2^{j_2}) \dots AND (x_n is\ A_n^{j_n}),$
$THEN (y\ is\ B^{j_1,j_2,\dots,j_n})$

The AND operator in the rule antecedent part is the product function. For the x_i domain, its membership functions are $\mu_i^1(x_i), \mu_i^2(x_i), \dots,$ and $\mu_i^{n_i}(x_i)$. The output is obtained by using the weighted average of a representative value, $b^{j_1,j_2,\dots,j_n}$, with respect to its compatibility grade, as in (1).

$$y = \frac{\sum_{j_n=1}^{j_n=M_n} \dots \sum_{j_2=1}^{j_2=M_2} \sum_{j_1=1}^{j_1=M_1} \left(\mu_1^{j_1}(x_1) \times \mu_2^{j_2}(x_2) \times .. \times \mu_n^{j_n}(x_n) \times b^{j_1,j_2,\dots,j_n} \right)}{\sum_{j_n=1}^{j_n=M_n} \dots \sum_{j_2=1}^{j_2=M_2} \sum_{j_1=1}^{j_1=M_1} \left(\mu_1^{j_1}(x_1) \times \mu_2^{j_2}(x_2) \times .. \times \mu_n^{j_n}(x_n) \right)} \quad (1)$$

where $b^{j_1,j_2,\dots,j_n}$ is a representative value of $B^{j_1,j_2,\dots,j_n}$, i.e., $b^{j_1,j_2,\dots,j_n} = Rep\ (B^{j_1,j_2,e\ j_n})$. The representative value represents the overall location of the membership function (MF). It can be obtained with defuzzification, or with the point that the membership value is 1.

B. Sufficient Conditions

The first derivative of an FIS model, as in (1), returns a weighted addition series. The *sufficient conditions* assume that all the components in the weighted addition series are always *greater than or equal to* zero, or *less than* or *equal to* zero. In general, two conditions can be derived [10, 12-14]:

Condition 1. At the rule antecedent part, $(d\mu_i^p(x)/dx)/\mu_i^p(x) \geq (d\mu_i^q(x)/dx)/\mu_i^q(x)$, where $p > q$. Note that $(d\mu(x)/dx)/\mu(x)$ is the ratio between the rate of change in the membership degree and the membership degree itself. The derivative of a Gaussian MF with respect to x is $G'(x) = -((x-c)/\sigma^2)G(x)$. Note that $(d\mu(x)/dx)/\mu(x)$ for a Gaussian MF , i.e., $(G'(x)/G(x))$, returns a linear function, i.e. $E(x) = G'(x)/G(x) = -(1/\sigma^2)x + (c/\sigma^2)$.

Condition 2. At the rule consequent part, $b^{j_1,j_2,\dots,j_i=p,\dots,j_n} - b^{j_1,j_2,\dots,j_i=q,\dots,j_n} \geq 0$ or $b^{j_1,j_2,\dots,j_i=p,\dots,j_n} - b^{j_1,j_2,\dots,j_i=q,\dots,j_n} \leq 0$ for $dy/dx_i \geq 0$ or $dy/dx_i \leq 0$, respectively. This condition suggests that a monotonically-ordered rule base is required.

C. Similarity Reasoning

In this paper, SR is viewed as a computing paradigm, as in Fig. 1. An example of SR [6] is as follows.

If pressure is *high* Then volume is *small*
If pressure is *low* Then volume is *large*
Thus: If pressure is *medium* Then volume is $(w_1 \wedge small + w_2 \wedge large)$, where $w_1 = sup(high \wedge medium)$, and $w_2 = sup(low \wedge medium)$.

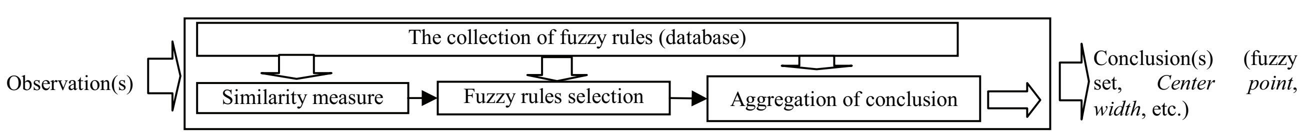

Figure 1 The Similarity reasoning paradigm

The above notion was further extended, and a number of SR schemes have been proposed to allow an unknown conclusion of an observation (in the form of a fuzzy set) to be deduced or predicted, based on a collection of fuzzy rules (database). Note that the "If" part of a fuzzy rule is called the antecedent while the "Then" part is called the consequent. The "If" part of an unknown rule is called the observation, while the "Then" part is called the conclusion.

An observation (in the form of a fuzzy set) acts as an input to the computing paradigm. The observation is compared with the antecedent of each fuzzy rule in the database, and a similarity measure is produced for each fuzzy rule. The similarity measure represents how similar a fuzzy rule is with respect to the observation. There are many ways how this similarity measure can be derived. For example, similarity measure models have been reviewed, compared, and analyzed in [4,8]. In [5, 7, 19], another class of similarity measure that is based on distance has been proposed.

Fuzzy rules from the database are selected for aggregation and for arriving at the conclusions. As an example, in [4], a threshold has been introduced to select fuzzy rules with similarity measure above the threshold. In [19], the n closest fuzzy rules from the observation are selected for use with the Fuzzy Rule Interpolation (FRI) scheme. The importance of the ordering criteria in the selection of fuzzy rules is pointed out in [5,17].

The conclusion is formed by aggregating the selected fuzzy rules. In this stage, a mathematical function is adopted so that the property (or properties) of the conclusion is obtained. The similarity measure is used as an indication of the degree to which the property (or properties) of the consequent of each selected fuzzy rule contributes to the conclusion. In [5], the *center point* and *width* of the conclusion are aggregated using a distance-based similarity measure via a weighted average function. In [6], the conclusion is aggregated using a similarity measure via a weighted addition function. Usually, each conclusion is predicted separately.

D. An Example

In the following section, a summary of our previous work [17-18] is presented. In order to have a better understanding of the work, a numerical example is provided, as follows. A monotonic two-input FIS model, $z = f(x, y)$, is considered. It is known that the relationship between $f(x, y)$ and its inputs, x and y, increases monotonically. Each x and y consists of five Gaussian MFs, i.e., , $mf_{x1}, mf_{x2}, mf_{x3}, mf_{x4}$ and mf_{x5}, and mf_{y1}, mf_{y2}, mf_{y3}, mf_{y4} and mf_{y5}, respectively. These MFs fulfil ***Condition 1***, and can be projected using the ratio as suggested in ***Condition 1*** $(d\mu_i(x)/dx)/\mu_i(x)$, for x and y, respectively.

A complete rule base is expected to have 25 rules. The output domain, z, consists of five MFs , namely mf_{z1}, $mf_{z2}, mf_{z3}, mf_{z4}$ and mf_{z5}, with the representative values of 1, 2, 3, 4, and 5, respectively. In this example, assume that there are eight available rules, as summarized in Fig. 2. Note that the available rules satisfy ***Condition 2***. This problem can also be presented as a rule matrix, as in Fig. 3. As an example, R_1 from Fig. 2 is represented as $R_1: mf_{z1}$, and located in the cell with coordinate (mf_{x1}, mf_{y1}). Observation $\boldsymbol{A}_1^*$, with coordinate (mf_{x1}, mf_{y2}) is mapped to an unknown conclusion, $\boldsymbol{B}_1^*$, which needs to be predicted by using SR.

R_1: If x is mf_{x1} and y is mf_{y1}, Then z is mf_{z1}
R_2: If x is mf_{x1} and y is mf_{y3}, Then z is mf_{z3}
R_3: If x is mf_{x2} and y is mf_{y4}, Then z is mf_{z5}
R_4: If x is mf_{x3} and y is mf_{y1}, Then z is mf_{z2}
R_5: If x is mf_{x3} and y is mf_{y2}, Then z is mf_{z2}
R_6: If x is mf_{x5} and y is mf_{y1}, Then z is mf_{z3}
R_7: If x is mf_{x5} and y is mf_{y3}, Then z is mf_{z4}
R_8: If x is mf_{x5} and y is mf_{y5}, Then z is mf_{z5}

Figure 2. The fuzzy rules in data base

mf_{x5}	$R_6: mf_{z3}$	$\boldsymbol{A}_{16}^*$->$\boldsymbol{B}_{16}^*$	$R_7: mf_{z4}$	$\boldsymbol{A}_{17}^*$->$\boldsymbol{B}_{17}^*$	$R_8: mf_{z5}$
mf_{x4}	$\boldsymbol{A}_{11}^*$->$\boldsymbol{B}_{11}^*$	$\boldsymbol{A}_{12}^*$->$\boldsymbol{B}_{12}^*$	$\boldsymbol{A}_{13}^*$->$\boldsymbol{B}_{13}^*$	$\boldsymbol{A}_{14}^*$->$\boldsymbol{B}_{14}^*$	$\boldsymbol{A}_{15}^*$->$\boldsymbol{B}_{15}^*$
mf_{x3}	$R_4: mf_{z2}$	$R_5: mf_{z2}$	$\boldsymbol{A}_8^*$->$\boldsymbol{B}_8^*$	$\boldsymbol{A}_9^*$->$\boldsymbol{B}_9^*$	$\boldsymbol{A}_{10}^*$->$\boldsymbol{B}_{10}^*$
mf_{x2}	$\boldsymbol{A}_4^*$->$\boldsymbol{B}_4^*$	$\boldsymbol{A}_5^*$->$\boldsymbol{B}_5^*$	$\boldsymbol{A}_6^*$->$\boldsymbol{B}_6^*$	$R_3: mf_{z5}$	$\boldsymbol{A}_7^*$->$\boldsymbol{B}_7^*$
mf_{x1}	$R_1: mf_{z1}$	$\boldsymbol{A}_1^*$->$\boldsymbol{B}_1^*$	$R_2: mf_{z3}$	$\boldsymbol{A}_2^*$->$\boldsymbol{B}_2^*$	$\boldsymbol{A}_3^*$->$\boldsymbol{B}_3^*$
	mf_{y1}	mf_{y2}	mf_{y3}	mf_{y4}	mf_{y5}

Figure 3 Rule matrix for a numerical example

Fig. 4 depicts a surface plot of z versus x and y, using FRI based on a weighted average function. A simple distance, i.e., the Euclidian distance, between the representative values of fuzzy sets, is used. Note that the use of the representative values has been suggested in [19]. In short, a non-monotonic surface is obtained.

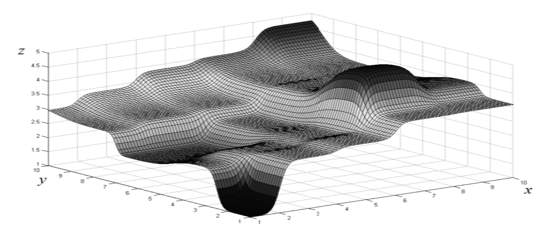

Figure 4 Surface plot of z versus x and y, using an FRI

This phenomenon occurs because a set of non-monotonically-ordered conclusions is deduced by FRI. Two situations as highlighted in [17-18] are:

Situation 1*:* A deduced conclusion of an observation may not be comparable with the collection of fuzzy rules.

Situation 2: It is difficult to deduce a set of conclusions (for a set of observations) that are comparable among one another.

Situation 1 can be solved by adding an ordering criterion [17]. But, ***Situation 2*** suggests that it is necessary to consider the relationship among the conclusions in an SR model. Based on the example, how do we ensure that $B_6^* \geq B_5^*$? Optimization techniques provide a solution to this problem.

III. THE EVOLUTIONARY-BASED SIMILARITY REASONING SCHEME

A. A General Formulation

Consider a multi-input FIS model with $n_{available}$ ($n_{available} > 1$) fuzzy rules $(R_1, R_2, \dots, R_{n_{available}})$, $R_n: A_n \rightarrow B_n$,. There are n_{empty} observations ($A_1^*, A_2^*, \dots, A_{n_{empty}}^*$), where $n_{empty} > 1$. The conclusions for the n_{empty} observations are $B_1^*, B_2^*, \dots, B_{n_{empty}}^*$, respectively. We suggest a SR scheme for monotonicity-preserving multi-input FIS models as a constrained optimization problem. Here, we predict the representative value, instead of the actual membership function, i.e.,

Minimizing

$f\left(b_1^*, b, \dots, b_{n_{empty}}^*\right)$, where $b = rep(B)$

Subject to

Constraint #1: Each conclusion is comparable with the rule collection, i.e, $\overline{b_i} \geq b_i \geq \underline{b_i}$. $\overline{b_i}$ and $\underline{b_i}$ are the highest and lowest possible representative values for the consequent of the observation, respectively. As an example, in Fig. 3, the conclusion of observation $\boldsymbol{A_6^*}$ is $\boldsymbol{B_6^*}$, $\overline{b_6^*} \geq b_6^* \geq \underline{b_6^*}$. The fuzzy rules that are expected to have higher and lower consequents are highlighted in dark-grey and light-grey, respectively. Hence, $\overline{b_6^*}$= **min** (*rep* (mf_{z5}), *rep* (mf_{z4}), *rep* (mf_{z5})), $\underline{b_6^*}$= **max** (*rep* (mf_{z1}), *rep* (mf_{z3})).

Constraint #2: Conclusions are comparable among themselves. In other words, a monotonic ordering of conclusions exist, i.e., $b_i \geq b_j \geq b_k$. As an example, in Fig. 3, $rep(\boldsymbol{B_4^*}) \leq rep(\boldsymbol{B_5^*}) \leq rep(\boldsymbol{B_6^*}) \leq rep(\boldsymbol{B_7^*})$.

In this paper, we investigate the use of a *proximity measure* function that is to be minimized in SR. Each conclusion is further predicted with an SR scheme (as explained in section II(C)), and a set of *reference consequents* is obtained. A *reference conclusion* of a conclusion, $Rep\,(B^*)$, is denoted by $Rep\,(B^*)_R$. For a set of candidates denoted by $(Rep\,(B_1^*)_C, Rep\,(B_2^*)_C, \dots, Rep\,(B_{n_{empty}}^*)_C)$, the *proximity measure* is expressed with a root mean square function, as in (2). It is a measure of "closeness" between the candidates and the *reference conclusions*. The lower the *proximity measure*, the closer the candidate is.

$$Proximity\ measure = \sqrt[2]{\sum_{n=1}^{n_{empty}} (b_{n\ R}^* - b_{n\ C}^*)^2}, \quad \text{where } b = rep(B) \tag{2}$$

B. Genetic Algorithem Search

The problem formulated in Section III (A) can be solved using an evolutionary computation technique, e.g. the Genetic Algorithm (GA) [20]. GA is a population-based stochastic optimization technique. Each potential solution set, in a form of $\left(b_1^*, b, \dots, b_{n_{empty}}^*\right)$, is represented by a chromosome. The GA search procedure is used to find the solution set with the best fitness value. Fig. 5 shows the GA procedure.

%Note: Initialize variable
Determine the **lowest possible representative value** and **highest possible representative value** of each conclusion.
Determine an ***reference Conclusion*** for each conclusion with an SR scheme
Initial **conclusion_candidate_set**(s)
While stopping criteria is not triggered
{
Evaluate **conclusion_candidate_set**(s) with **(3)**
Record the best **conclusion_candidate_set**(s)
Crossover
Mutation
Evaluate the stopping criteria
}

Figure 5. Pseudo-code of the proposed GA based SR scheme

The hard constraints, i.e., **constraints #1** and ***#2***, are considered as penalties of the objective function to be minimized, as in (3).

$$Objective\ function = w \times \begin{Bmatrix} 0\ if\ constraints\ \#1 \text{ and } \#2\ fulfilled \\ 1\ if\ constraints\ \#1 \text{ and } \#2\ not\ fulfilled \end{Bmatrix} + proximity\ measure \tag{3}$$

IV. EXPERIMENTAL RESULTS AND DISCUSSION

A. Failure Mode and Effect Analysis

Failure Mode and Effective Analysis (FMEA) is a popular problem prevention methodology that can be interfaced with many engineering and reliability models [13, 21]. FMEA uses an RPN model to evaluate the risk associated with each failure mode. The RPN model considers three risk factors, i.e., severity (S), occurrence (O), and detect (D), and produces a RPN score ($RPN = g_{RPN}(S, O, D)$). The three input factors are estimated by domain experts in accordance with the scale from "1" to "10" based on a set of commonly agreed evaluation criteria, which are presented in the scale tables.

In this paper, the focus is on designing a monotonicity-preserving FIS-based RPN model. FIS-based RPN models are popular, and they have been widely used [13, 21]. They act as a solution for decision making and selection problems for FMEA methodology. An FIS-based RPN model considers S, O, and D as the inputs and the RPN as the output. MFs of S, O, and D are generated from the designated scale tables. The relationships between S, O, and D and the RPN are expressed by a set of fuzzy If-Then rules. In our previous work [13, 21], it has been shown that an effective FIS-based RPN model should satisfy the monotonicity property. This is because the inputs (i.e., S, O, and D ratings) are defined in such a way that the higher the rating, the more critical the situation is. The output (i.e., the RPN score) is a measure of the failure risk. The monotonicity property is important to allow a valid comparison among all failure modes to be made [12-13]. A prediction is deemed illogical if the RPN model yields a contradictory result [12-13].

To validate the proposed procedure, a series of experiments with data and information collected from a semiconductor manufacturing process is conducted. Specifically, the test handler process of the Flip Chip Ball

Grid Array (FCBGA) product is considered. MFs of S, O, and D of the FIS-based RPN model are constructed using the scale tables in [21]. A fuzzy rules base is formed by collecting information from the domain experts, i.e., engineers in charge of the test handler process. Experiments are conducted according to the following conditions : (1) the fuzzy rule base is incomplete (50% of the fuzzy rules are randomly selected) and without SR; (2) the fuzzy rule base is incomplete (50% of the fuzzy rules are randomly selected) and FRI is used to deduce the missing fuzzy rules; (3) the fuzzy rule base is incomplete (50% of the fuzzy rules are randomly selected) and the ESR scheme (as in section III) is used to deduce the missing fuzzy rules. Note that when SR is not used, the conclusions are mapped to zero.

B. *Experimental Results*

Fig. 6 depicts the objective function versus the number of generations of the GA simulation, with, $w = 20$, for 150 iterations. The objective function of the best individual is subject to GA operations, i.e., crossover and mutation.

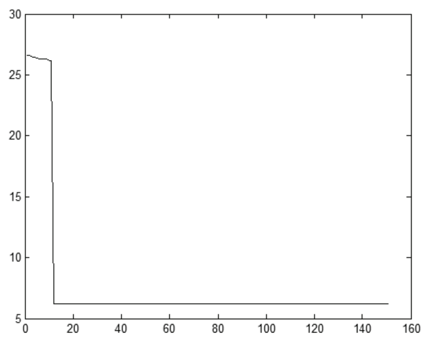

Figure 6 Objective function versus the number of generations of the GA-based procedure for 150 iterations.

The monotonicity property of an FIS model can be observed by using the surface plot. Figures 7, 8, and 9 depict the surface plots of the RPN scores versus O and D when S is fixed at 8, for 50% fuzzy rules.

Without FRI, the surface plot, as shown in Fig. 7, is not monotonic. There are areas where the RPN scores are close to zero because some of the conclusions are undefined, and they are mapped to zero. This is the co-called "tomato classification problem".

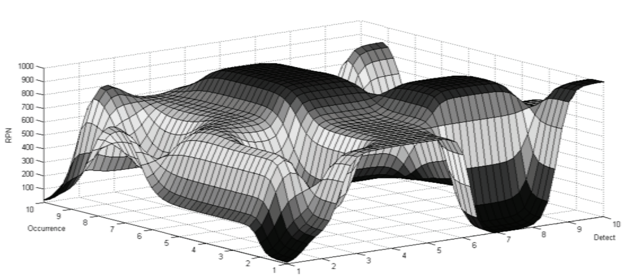

Figure 7 Surface plot for 50% fuzzy rules and without FRI

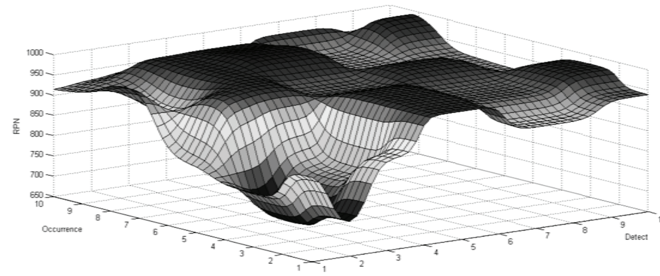

Figure 8 Surface plot for 50% fuzzy rules and with FRI

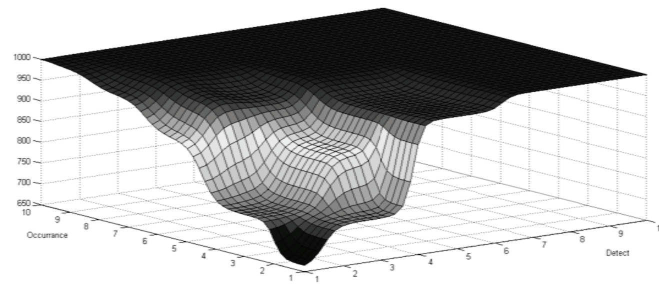

Figure 9 Surface plot for 50% fuzzy rules and with ESR

The use of FRI improves the situation partially. Fig. 8 depicts a "smoother" surface than that of Fig. 7. The effect of the "tomato classification problem" is reduced, as the RPN scores are not mapped to zero. However, the monotonicity property is not satisfied. This is because the rule base obtained is not monotonically-ordered. With the ESR scheme, a monotonic surface plot is obtained, as shown in Fig. 9. In short, the monotonicity property is fulfilled by using the proposed ESR scheme.

C. *Evaluation*

Fulfillment of the monotonicity property of FIS models can also be evaluated by a test for monotonicity [22]. In this paper, the test for monotonicity of the FIS-based RPN model is conducted by comparing the outputs in pairs. The aim of the test is to examine the relationships between the output (the RPN score) and the inputs (S, O, or D). The test allows the monotonic relationship to be represented by a numerical value, from 0 to 1. We attempt to generate all possible comparable pairs between the output and each of the inputs. The number of monotonic pairs is counted, and the degree of monotonicity is obtained as in (4).

$$Degree\ of\ monotonicity = \frac{\#\ Monotone\ pairs(D)}{\#Comparable\ pairs\ (D)} \quad (4)$$

The $Degree\ of\ monotonicity$ is 1 for an ideal monotonic FIS model. The $Degree\ of\ monotonicity$ is 0 if the FIS model violates totally the condition of monotonicity. A higher $Degree\ of\ monotonicity$ implies a better fulfillment of the monotonicity property by the FIS model under test.

As an example, comparable pairs in the form of [S,O,D] are generated, i.e., [1,1,1] and [2,1,1], [2,1,1] and [3,1,1],…, [9,1,1] and [10, 1,1], …, and finally [9, 10, 10] and [10, 10, 10]. The RPN scores for [1,1,1] and [2,1,1] are compared, and its fulfillment towards monotonicity is checked. The same goes to other comparable pairs. In this case, 900 comparable pairs are generated between the RPN and S. The same applies to the RPN and O as well as the RPN and D. Thus, a total of 2700 comparable pairs are available. Table 1 shows the results.

Table1 The measure of monotonicity

Experiment	#Comparable pairs	#Monotonic pairs	Degree of monotonicity
Without SR	2700	1376	0.51
With FRI	2700	1981	0.73
With ESR	2700	2700	1.00

As can be seen in Table 1, without SR, there are 1376 monotonic pairs, and the *degree of monotonicity* is 0.51. An improvement is achieved with FRI, i.e., 1981 monotonic pairs, with the *degree of monotonicity* of 0.73. The ESR scheme is able to generate 2700 monotonic pairs, with the *Degree of monotonicity* of 1. This indicates that a full fulfilment of monotonicy among the RPN and S, O, and D ratings. In other words, the monotonicity property is fully perserved with the use of the ESR scheme.

V. SUMMARY

In this paper, it is argued that SR that predicts each rule consequent separately, using a simple weighted average function, may not be a direct solution for designing a monotonic multi-input FIS model. An alternative SR scheme, i.e., ESR, is thus proposed. It is formulated as a constrained optimization problem. An optimization procedure, i.e., GA, is adopted to search for a set of conclusions that obey the *sufficient conditions*, with the minimum proximity measure. An experiment has been conducted with real data collected from industry, i.e., an FIS-based RPN model for FMEA. The results show that without SR, the output does not fulfill the monotonicity property, and some regions are mapped to zero, hence leading to the "tomato classification problem". With SR, the situation is improved, but the output is still not able to satisfy the monotonicity property. With the proposed ESR scheme, the monotonicity property can be preserved. As a result, ESR provides a solution to designing multi-input FIS models that preserve the monotonicity property.

For further work, investigations into other evolutionary models, e.g. particle swarm optimization, harmonic search etc., can be conducted. Alternative methods of handling constraints will be studied. In addition to FMEA, experiments in different domains are needed to fully evaluate the proposed ESR scheme in designing useful monotonic multi-input FIS models for various applications.

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