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# Time Series Econometrics

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Major lines of the development of time series econometrics are discussed. The presentation is primarily aimed to system theorists. Emphasis is both on history and on recent developments. A discussion of basic ideas and driving forces and of important model classes (more than of identification procedures), together with a critical view on the relevance for application is given. [1]

## 1. Introduction

The use of statistical methods in order to extract information from economic data has a long history dating back to the last century. In this contribution the focus is on data-based model building (rather than e.g. on seasonal adjustment or the construction of stylized facts) from time series (rather than e.g. from cross sectional) data.

There is a wide range of aims for time series econometrics. The main aims are: forecasting, policy simulation, estimation of deep i.e. economically meaningful parameters and empirical evaluation of conflicting theories.

The main areas of application for time series econometrics are macroeconomics and finance. For a long time macroeconometrics was the most important part of econometrics. The idea was to provide a tool for forecasting the gross national product and its components and for quantitative economic policy on the one hand, and for problems of economic theory on the other hand.

At present, applications in finance have attracted great attention. The increasing importance of financial markets together with new products on these markets such as options, has created a demand for new tools, for instance, to

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name one example, for option pricing. The particular features of finance data, such as varying volatilities, together, partially at least, with large sample sizes, have led to special models and methods. Recently, in addition applications of time series econometrics to microdata, e.g. for forecasting of sales or inventories in firms are increasing in number.

For a long time econometric research and applications were concentrated on a few countries such as the English speaking countries and the Netherlands. In the late fifties and in the sixties of our century, econometrics spread over to many countries and in the sixties and early seventies many econometric models were built. However e.g. in German speaking countries, at this time still a substantial number of ecomomists were against econometrics, partly from principal reasons, such as that econometrics is inappropriate for a market economy or that mathematics is of no use for economics. On the other hand, at the same time, many econometricians were much too optimistic about the practical value of econometric models. The first oil crisis showed the fragility of model-based forecasts and caused an 'oil crisis of econometrics'. Today the range in evaluating the contribution of econometrics has become narrower from both sides. It has become clear that econometrics is auxiliary in character: In order to make reasonable use of econometric tools, economic expertise and good judgement are required. This makes econometrics still to a certain degree to an art, where results very much depend on assumptions, which are hard to verify. The idea, that econometric analysis can be performed in an 'automated' way, in the sense that from data alone, without economic reasoning, only by using mathematics and computers, valid models may be obtained, has turned out to be too optimistic.

Due to the nature of the subject, econometrics faces specific difficulties: In many cases a qualitative understanding of the main basic forces and economic mechanisms still is the primary aim of analysis. The targets of economic policy are shifting according to new situations and needs, and so does the angle of looking at the economy. In particular in macroeconomics, data are subject to substantial measurement errors and contain only a limited amount of information, and also theories are very imprecise. Usually the data are nonexperimental. The economy is changing in time and there are no 'real constants' in economic theory. New phenomena may evolve in the economy, which are not reflected in past observed data or the corresponding past data are too short for a reasonable analysis. All this creates an intrinsic tension between precision and actuality in econometric analysis.

Both, (model based) time series econometrics and system identification are concerned with finding a good model from data. Thus time series econometrics, time series statistics and system identification show common features. One thing, time series econometrics and system identification have in common, is the analysis of 'structural' properties such as identifiability, which are rather neglected or 'assumed away' in many cases in statistics. Nevertheless the interactions and exchanges of ideas between econometrics and systems theory are surprisingly limited. The Kalman filter had an impact in econometrics too, and conversely so did the results on maximum likelihood estimation and on causality testing obtained in econometrics for system identification. There has been an increasing intensity of interaction in the sixties and seventies of our century. In the authors opinion the work of Hannan and coworkers on ARMA and

ARMAX model identification is the best example for this interaction. During the last fifteen years however, the two areas drifted apart again. Certainly, fields like neural nets, wavelets or chaos did attract attention in both areas, however major developments in each area turn out to be widely unknown among workers in the respective other area. As far as econometrics is concerned, one reason for this is the emphasis on models and methods which are genuine for the specific features of economic data and theories.

## 2. A Short History of Econometrics

The history of econometrics may be divided into three parts: The 'Pre-Cowles-Commission Time', the 'Cowles Commission Time' and 'Modern Times' beginning shortly after 1970. It should be noted, that recently two books on the history of econometric ideas (Epstein [7], Morgan [20]) have been published.

We will not deal with the 'Pre-Cowles Commission Time' here, despite of the fact that a number of important developments, such as periodogram analyses of the business cycle or the development of MA and AR models by Yule, fall into this period. The term 'Cowles-Commission Time' here will be used in the broad sense. The formation of econometrics as a field of its own can be dated to the thirties of our century. The Econometric Society was founded in 1930, the Cowles-Commission in 1932 and the first volume of 'Econometrica' appeared in 1933. The birth of econometrics is closely related to Keynesian macroeconomics and the development of national accounting schemes. In the thirties mathematical economists like Frisch, Kalecki and Samuelson used (linear) difference- or differential equation systems for explaining 'macrodynamics' in particular business cycles. For economic analysis and in particular for quantitative economic policy (such as deficit spending) the numerical values of the coefficients of these equation systems were of interest. For this reason in particular Frisch and Tinbergen tried to estimate these coefficients using correlation analysis or ordinary least squares. Tinbergen's models for the Netherlands and the United States mark the first peak in this development. The linear (difference) equation systems used were in 'structural form', i.e. in a form where the equations came from economic theory, containing considerable a priori information, e.g. in form of zero restrictions on some coefficients. As a simple example consider the Keynesian system

$$\begin{aligned} C_t &= \alpha + \beta Y_t + u_t \\ Y_t &= C_t + I_t \end{aligned} \tag{1}$$

where consumption $C_t$ and income $Y_t$ are endogenous, investment $I_t$ is exogenous and $u_t$ is noise. $\alpha$ and $\beta$ are unknown parameters. The system above shows instantanious feedback (called simultaneity in econometrics) between the output variables $C_t$ and $Y_t$. As a consequence, due to nonorthogonality of $Y_t$ and $u_t$, ordinary least squares gives inconsistent estimates.

The subsequent work of the Cowles Commission has triggered an intellectual revolution. The motivation for this work was the idea that Tinbergen's macromodels were basically correct and that only 'technological innovations' for estimation were needed for the final breakthrough. The main innovations were the use of stochastic models (obtained by stochastic assumptions on the noise term, HAAVELMO [12]), which made estimation and testing part of mathematical statistics, and a theory of identification for (in general MIMO) linear static or ARX systems (KOOPMANS ET AL. [17], MANN & WALD [19]). In the latter context the careful analysis of the problem of identifiability and the quasi maximum likelihood estimation (MLE) have to be emphasized. The MLE's are consistent and asymptotically efficient. A major problem at this time was the computational burden associated with MLE. For this reason, in the years following numerically simpler estimation procedures such as two stage least squares have been developed, which still are consistent, however in general lack asymptotic efficiency. The first model, which has been estimated with Cowles Commission methods, was the Klein I model for the USA. It should be said however, that despite of the fact that the inconsistency of ordinary least squares in simultaneous equations was one of the main reasons for the development of the Cowles Commission methods, many models are still estimated by ordinary least squares.

The emphasis of the methodological work of the Cowles Commission was on parameter estimation for models specified by economic theory. Problems of data driven specification (selection of variables, determination of maximum lags or of the correlation structure of the noise from data) received much less attention. This corresponded to the prevailing idea, that macroeconomic theory would be able to formulate the specification of the 'true model' a priori, and that only the unknown parameters have to be determined from data.

The methodological contributions of the Cowles Commission have turned out to be pioneering, despite of the fact that today in econometrics there is much less emphasis on simultaneous estimation methods. On the other hand, the Cowles Commission was much too optimistic as far as the practical relevance of the models was concerned. In this context it is interesting to read the famous Keynes-Tinbergen debate (see MORGAN [20]).

During the sixties and early seventies of our century, econometrics became a well established discipline, at least in western countries. In many countries large 'structural' macro models were built and used for forecasting and policy simulation (e.g. for the fine tuning of the business cycle). Some of these models had several hundreds of equations. Most of the models were 'overidentified' in the sense that they contained a great number of (e.g. zero) restrictions, which significantly reduced the dimension of the parameter space.

In the early seventies however there was an increasing criticism of conventional structural macroeconomic model building. A comparison of the forecasting performance with small SISO models, identified by the Box-Jenkins methodology, showed that the latter models outperformed the large models, at least in the short run. The first oil crisis (1974–1975) with its significant nonstationarities in the data, led to serious mispredictions with large models and caused a marked change of minds. This was what the author calls the end of the Cowles Commission time and the beginning of Modern Times in econometrics.

In general econometricians now became much more aware of the limitations of their tools. To a good part the econometric community responded to the new challenges by technological innovations. Thereby the following main lines can be identified:

1. The first line was to develop tools for data-driven specification. It was understood that one problem with the building of big macro models was a rather careless use of a priori specifications. Consequently tests and diagnostic checks e.g. for determining the noise structure, the functional form of the relations or for structural breaks have been developed. Now typically a whole battery of such specification tests is used. In doing so, clearly a reuse of sample problem arises. At about the same time information criteria, e.g. for order estimation in AR and ARMA models or for determining the number of regressors in linear regressions were developed in system theory and statistics e.g. by Akaike and Rissanen and subsequently used in econometrics. Whereas the basic intention with information criteria was to further automize identification procedures, also the ideas of exploratory and interactive data analysis, advanced in particular by Tukey found attention in econometrics. A third area related to the specification problem was the analysis of sensitivity (Leamer [18]) and robustness with respect to a priori assumptions.
2. The second main line was the further development of time series econometrics. The development of tests for causality (Granger [10]) and vector autoregressive (VAR) modeling (Sims [25]) have to be mentioned in this context. VAR modeling in a certain sense was a counterrevolution against structural model building. No a priori classification of the observed variables into inputs and outputs was required and no a priori zero restrictions on the parameters were imposed; the idea was to obtain such information from the data (e.g. by testing for zeros on coefficients). Clearly in this context the curse of dimensionality is a major problem, since in macroeconometrics a system with say 8 or 10 variables (inputs plus outputs) is rather small. The idea of Sims and coworkers was to partially overcome this difficulty by Bayesian modeling with a special prior (favouring unit root models), whose hyperparameters have to be estimated. Nevertheless VAR models are comparably small. In macroeconometrics now both, the conventional structural approach (complemented with a number of specification tests), where a lot of a priori restrictions are imposed, and more data driven approaches, such as the VAR approach, are used. In the first

approach, clearly more weight is given to economic theory, with the argument that macrodata are very imprecise (which is correct). In the second approach, more weight is given to information coming from data, with the argument that theory is very imprecise (which is correct too).

Up to now, there is no clearcut opinion forming, which approach to prefer; however there is a tendency to use structural model building, where the main economic mechanisms are reflected in the model, if the main purpose is economic analysis, policy simulation or medium term forecasting. On the other hand, for short term forecasting VAR or black box type models are favored.

A rather complete theory of identification of stable MIMO ARMA and ARMAX systems, consisting of the modules structure theory (realization and parametrization), maximum likelihood type estimation and order estimation, has been worked out in the seventies and eighties (Dunsmuir & Hannan [4], Hannan & Kavalieris [13], Hannan & Deistler [14]), which however had not much resonance in econometrics.

Nonlinear models of various kinds have been considered. Nonlinear black box models as well as highly structured model classes are considered. In the latter case the specific nonlinear structure may come from modeling of specific data features or from economic theory. A, in a certain sense, rather complete estimation theory for parametric nonlinear models is given e.g. in Gallant [8] or Pötscher & Prucha [22]. The (unsolved) difficulty thereby are the rather complicated assumptions, which reflect the fact, that in these cases no structure theory is available.

Nonparametric methods are used increasingly, because of their flexibility and since in fields like finance large sample sizes are not uncommon (see e.g. Robinson [23]). This area has many different facets such as nonparametric cointegration analysis or nonparametric ARCH models, two give two examples. Neural nets have been further developed and extensively used in econometrics in particular for finance data.

Problems related to chaos, e.g. test procedures for discriminating chaos from stochastic behaviour have been investigated; however it is fair to say that chaos modeling up to now has not become popular in the field of econometrics.

Given the limitations of space we will present two very important areas, namely a particular form of nonstationarity and long memory on the one hand, and ARCH models, as a special class of nonlinear models on the other hand, in more detail. However also this presentation will be rather short and, since these models are relatively unknown to a systems engineering audience, introductory. Emphasis is put on the description of the models rather than on identification procedures. As in the whole paper also here the number of references has been reduced to a minimum by even omitting very important ones, because of space limitations.

3. Finally, the third main line in responding to the criticism of classical econometrics was the rapid development of microeconometrics with special models such as qualitative response models or models for censured data. We will not deal with these really important developments here, since the time series aspect in these cases is not dominant.

## 3. Linear Nonstationarities and Long Memory: Integrated Processes, Cointegration and Fractional Integration

Many economic time series show apparent nonstationarities such as trends in means and variances. In the classical approach often, using transformations such as differencing, the data were transformed to stationarity. The disadvantage of differencing is that information at frequency zero is lost. As will be pointed out below this information is essential for economic analysis, since it contains the information about steady state equilibria. From this point of view, the idea to model, rather than to remove, nonstationarities was suggesting itself.

A very important class of nonstationary models are linear unit roots models, which generate integrated processes:

A stochastic process $(y_t \mid t \in \mathbb{N})$ is called *integrated* (of order one) if its first differences $(1-z)y_t$ (where $z$ denotes the backward shift) are stationary, whereas $(y_t)$ is not stationary. The definition can be extended in an obvious way to orders two etc., however we do not consider this case here. For simplicity, here we will assume that $(1-z)y_t$ is stationary ARMA. In this case an integrated process can be generated by an ARMA model, which has stable roots and roots equal to one (called a unit root model). An example for an integrated process is a random walk with drift

$$y_t = y_{t-1} + c + \varepsilon_t$$

which shows linear trends in means and variances. Here $\varepsilon_t$ is white noise and $c$ is a constant.

The statistical analysis of integrated models shows that the convergence of estimates may be faster than in the stationary case and that the limiting laws are no longer Gaussian. As a simple example consider the scalar $AR(1)$ system

$$y_t = \rho y_{t-1} + \varepsilon_t \tag{2}$$

where $\varepsilon_t$ is white noise.

As is well known, in the stationary case ($|\rho| < 1$), the ordinary least squares estimator $\hat{\rho_T}$ of $\rho$ (T denotes sample size) is consistent and has the property

$$\sqrt{T}(\hat{\rho_T} - \rho) \xrightarrow{L} N\left(0, 1-\rho^2\right)$$

where $\xrightarrow{L}$ denotes convergence in law and $N\left(\mu, \sigma^2\right)$ denotes a Gaussian distribution with mean $\mu$ and variance $\sigma^2$.

For $\rho = 1$, the situation is different. Let us assume that the initial value is zero; then

$$y_t = \varepsilon_1 + \cdots + \varepsilon_t$$

Let $\sigma^2 = \mathbb{E}\varepsilon_t^2$ and

$$X_T(r) = \begin{cases} 0 & \text{for} \quad 0 \le r < \frac{1}{T} \\ \frac{y_1}{T} & \text{for} \quad \frac{1}{T} \le r < \frac{2}{T} \\ \vdots & \vdots \quad\quad \vdots \\ \frac{y_T}{T} & \text{for} \quad r = 1 \end{cases}$$

Then we have the functional central limit theorem

$$\frac{\sqrt{T} X_T(.)}{\sigma^2} \xrightarrow{L} W(.)$$

where $W(.)$ is the standard Brownian motion.

Now

$$\int_0^1 X_T(r)dr = \frac{y_1}{T^2} + \cdots + \frac{y_{T-1}}{T^2}$$

Using the continuous mapping theorem we obtain

$$T(\hat{\rho_T} - 1) = \frac{\frac{1}{T}\sum y_{t-1}\varepsilon_t}{\frac{1}{T^2}\sum y_{t-1}^2} \xrightarrow{L} \frac{\frac{1}{2}\{W(1)^2 - 1\}}{\int_0^1 W(r)^2 dr}$$

This in particular shows that the ordinary least squares estimator in this case converges faster than in the stationary case ('superconsistency' i.e. $T$- rather than $\sqrt{T}$-consistency). The limiting distribution for $\hat{\rho_T}$ can be used for testing the null hypothesis $\rho = 1$ against the stationary alternative.

It should be emphasized, that the nonstationarities of integrated processes are very special ones. Even in a general ARMA context, when one also allows for poles of the transfer function on other places at the unit circle and inside the unit circle (the latter case is the explosive one) unit roots are highly nongeneric, not to speak of nonlinear nonstationary models. Clearly in an economic context both stationary and integrated processes can only serve as rough approximations for a certain time period; but even with this understanding, in the author's opinion, in a number of applications, modeling of nonstationarities with integrated processes is used too uncritical.

Perhaps the most important development in modern time series econometrics is cointegration analysis (Granger [11], Engle & Granger [6]). Cointegration analysis is concerned with special integrated processes.

Formally, an integrated vector process $(y_t)$ is called *cointegrated*, if there exists an $\alpha \in \mathbb{R}^n, \alpha \neq 0$ such that $(\alpha y_t)$ is stationary.

The main idea is to model an equilibrium relation between the nonstationary components of a vector process $(y_t)$ in the sense that $\alpha$ describes a long run (static equilibrium) relation, because 'stationarity' is interpreted as 'relatively small'. The number of linearly independent vectors $\alpha$, such that $\alpha y_t$ is stationary, is called *cointegrating rank*. The idea of cointegration is closely related to the idea of error correction (Engle & Granger [6]).

Cointegrated processes may be represented as follows: Let

$$(1-z)y_t = u_t = a^{-1}(z)b(z)\varepsilon_t$$

be a unit root model, where $a(z) = \sum A_j z^j, A_j \in \mathbb{R}^{n\times n}, A_0 = I$, and $b(z)$ is defined analogously, where $(\varepsilon_t)$ is white noise and $(u_t)$ is stationary ARMA, with spectral density unequal to zero at frequency zero. Writing

$$a^{-1}(z)b(z) = k(z) = k(1) + \tilde{k}(z)$$

where $\tilde{k}(1) = 0$, we obtain

$$y_t = (1-z)^{-1}k(1)\varepsilon_t + (1-z)^{-1}\tilde{k}(z)\varepsilon_t \tag{3}$$

where $(1-z)^{-1}\tilde{k}(z)\varepsilon_t$ is stationary. In the cointegrated case $k(1)$ must be singular and unequal to zero (in the case $k(1) = 0$ the process $(y_t)$ would be stationary). $(1-z)^{-1}k(1)\varepsilon_t$ can be interpreted as common trends generated by a lower dimensional, integrated, factor process. Thus (3) gives a factor (or errors-in-variables) model interpretation of cointegration, whith stationary noise $(1-z)^{-1}\tilde{k}(z)\varepsilon_t$. Note however that in general the two terms on the r.h.s. of (3) are not uncorrelated.

Let $A$ denote a matrix, whose rows form a basis for the left kernel of $k(1)$. Clearly these rows are cointegrating vectors. Now $A$ can be interpreted as a static long run equilibrium relation, which is exact for the first term on the r.h.s. of (3), i.e. $A(1-z)^{-1}k(1)\varepsilon_t = 0$, and the stationary part $A(1-z)^{-1}\tilde{k}(z)\varepsilon_t$ describes the (in the long run relatively small) deviations from equilibrium.

A very elegant setting for cointegration can be developed in an autoregressive framework as follows: Assume that

$$a(z)y_t = \varepsilon_t \tag{4}$$

where $\det a(z) \neq 0, |z| < 1$ and $|z| = 1$ except for $z = 1$. Then we may write

$$(1-z)y_t = \Gamma_1(1-z)y_{t-1} + \cdots + \Gamma_{p-1}(1-z)y_{t-p-1} + \Pi y_{t-p} + \varepsilon_t \tag{5}$$

where $\Pi = -a(1)$. If $\Pi$ has full rank $n$, then $(y_t)$ is stationary; if $\Pi = 0$, then $(y_t)$ is integrated but not cointegrated. Let $r$ denote the rank of $\Pi$ (which is the cointegrating rank) and assume $0 < r < n$. Then $\Pi = BA'$, where $A, B \in \mathbb{R}^{n\times r}$ and $A$ is a matrix, whose rows are the cointegrating vectors.

Based on these properties a full information (Gaussian) ML procedure has been developed (Johansen [15, 16]) consisting of tests for the cointegrating rank and estimation of $A$ and $B$. Johansen's approach commences from a Gaussian likelihood in the parameters $\Gamma_1, \cdots, \Gamma_{p-1}, \Omega = \mathbb{E}\varepsilon_t\varepsilon_t', B$ and $A$, depending in addition on the integer $r$. By stepwise concentrating parameters out, a concentrated likelihood, depending on $A$ and $r$, $L(A, r)$ say, is obtained. Johansen proposes two likelihood ratio tests for the nullhypothesis of (at most) $r(< n)$ (linearily independent) cointegrating vectors; the first test is against the alternative of $r+1$ cointegrating vectors and the second test is against the alternative of no cointegration (i.e. stationarity). The nonstandard limiting distributions corresponding to the null hypothesis have been derived. Johansen also derives the asymptotic properties for the MLE's $\hat{A}_T$ and $\hat{B}_T$ of $A$ and $B$ respectively under the assumption that $A$ and $B$ are identifiable by imposing suitable restrictions. In particular the limiting distribution of $T(\hat{A}_T - A)$ is shown to be a mixture of Gaussian distributions.

For an important alternative approach see PHILLIPS [21].

To repeat, cointegration is one of the most important developments in recent time series econometrics, with a great number of applications in macroeconomics and finance. This is partly because integrated processes are reasonably good models for a number of evidently nonstationary economic variables and partly because relations 'at frequency zero' (which previously often have been removed from data) are important for economic analysis. Despite of its undoubted success and importance, in the author's opinion this area is overemphasized at present. In many applications integration is not the only reasonable alternative to stationarity and the linear static relations expressed by the cointegrating matrix $A$ are a very simple form of economic relations. In addition the whole concept of cointegration is very much oriented towards a priori ideas from economics; this might be one reason, why it never spread out to systems engineering.

In many cases, in testing for integration versus stationary ARMA, the results are not clearcut. This was one reason for introducing *long memory* (long range dependence) models in econometrics. These models serve as a bridge between integrated and stationary ARMA processes. Other major reasons for the interest in long memory were phenomena of persistence of shocks in data and rates of decay in sample autocorrelations, which were neither consistent with integrated nor with stationary ARMA processes. Long memory models first have been used in physical sciences. As far as empirical evidence in econometrics is concerned, finance data are most important. A particulary interesting aspect is long memory in volatilities in certain finance data.

*Fractionally integrated, in particular ARFIMA models*

$$(1-z)^d y_t = u_t \tag{6}$$

are the most popular class of long memory models, see e.g. BAILLIE & KING [2]. Here $d \in (0,1)$ is called the order and $(u_t)$ is stationary ARMA, with a spectral density having no zeros. For noninteger $d$, from the Taylor series expansion, we obtain:

$$(1-z)^d = 1 - dz + d(d-1)\frac{z^2}{2!}\cdots$$

and by inverting this expression the solution for (6) is obtained. The parameters, which have to be identified, are the ARMA parameters of $(u_t)$ and the order $d$. The system (6) is infinite dimensional; it is stable (in the sense that

$y_t$ is stationary) for $d \in (0, 0.5)$ and unstable otherwise. In the stable case, $(y_t)$ is linearly regular with a Wold decomposition

$$y_t = \sum_{j=0}^{\infty} k_j \varepsilon_{t-j}$$

where the coefficients clearly satisfy $\sum \|k_j\|^2 < \infty$, where however $\sum \|k_j\| = \infty$. Therefore for $d \in (0, 0.5)$ fractionally integrated processes have a spectral density, which has a pole at frequency zero (which gives a nice interpretation of long memory). For $d = 0.5$, and $(u_t)$ white noise, $(y_t)$ is already nonstationary and a discrete-time version of $1/f$ noise. From a more abstract point of view, for the stationary case, ARFIMA processes provide a particular parametrization for spectral densities of the form (for simplicity of notation the scalar case is considered):

$$\begin{aligned} f_y(\lambda) &= \frac{\sigma^2}{2\pi} \left| \frac{b(e^{-i\lambda})}{a(e^{-i\lambda})} \right|^2 |1 - e^{-i\lambda}|^{-2d} \\ &= f_u(\lambda) |1 - e^{-i\lambda}|^{-2d} \end{aligned} \tag{7}$$

where the ARMA process $(u_t)$ (with spectral density $f_u$) is of the form $u_t = a^{-1}(z) b(z) \varepsilon_t, (\varepsilon_t)$ white noise and $\sigma^2 = \mathbb{E}\varepsilon_t^2$.

Several approaches for estimation and testing for ARFIMA models have been used. From (7) we obtain

$$\log(f_y(\lambda)) = \log(f_u(0)) - d \log(4 \sin^2(\lambda/2)) + \log(f_u(\lambda)/f_u(0))$$

In Geweke & Porter-Hudak [9] a semiparametric estimation procedure for $d$ has been suggested inspired by the formula above. They estimate $d$ from a regression, using in a log-log scale the periodogram of $y_t$ and the frequencies $\lambda$ in a neighborhood of zero. The asymptotic properties of this estimator have been investigated e.g. in Robinson [24].

An alternative is (Gaussian) MLE of $d$ and the ARMA parameters (simultaneously).

ARFIMA models are standard in econometrics now (also in combination with other models), however they have not attained a popularity comparable to cointegrated processes.

## 4. ARCH and Related Models for Volatility Clustering

Finance data often have a number of special features such as volatility clustering (episodes of high variation and episodes of low variation), leverage effects (i.e. movements in means are negatively correlated with volatility) or unequally spaced observations e.g. in case of tick by tick data. This led to the development of models which are able to reproduce these features.

On the other hand, commencing with Bachelier [1] a number of 'theoretical' stochastic models have been developed to explain features of finance data or to justify and extend algorithms for option-pricing, such as the famous Black-Scholes formula. There is now an increasing tendency to empirically determine unknown parameters in such models. Even in more empirical models, in general in finance, there is much more justification for the use of stochastic models compared to macroeconomics, where genuine stochastic theories are rare.

Here we will consider models for time-varying (conditional) second moments only. As has been mentioned already, time varying variations are a significant feature in many financial time series; in addition they are of particular interest for option pricing. A particularly important class of models are the ARCH (Autoregressive Conditional Heteroskedasticity) models (ENGLE [5]) and their generalizations (BOLLERSLEV ET AL. [3]).

For simplicity, consider the scalar case only. Consider a stationary Gaussian $AR(1)$ process (2); then clearly its expected value $\mathbb{E}y_t$ is equal to zero, whereas its conditional expectation $\mathbb{E}(y_t \mid y_{t-1}, y_{t-2}, \cdots)$ is time-varying. An ARCH process is the analogon for second moments. We have:

$$\begin{array}{rcl} \varepsilon_t & = & \sigma_t z_t \\ \sigma_t^2 & = & c + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 \end{array} \tag{8}$$

where $z_t$ is i.i.d with $\mathbb{E}z_t = 0, \mathbb{E}z_t^2 = 1$ and $c, \alpha_i$ are parameters satisfying $c > 0$ and $\alpha_i \geq 0$. For stationarity of $(\varepsilon_t)$ the condition $\sum \alpha_i < 1$ has to be imposed. Note that in the stationary case $(\varepsilon_t)$ is white noise in the sense of being uncorrelated in time, however

$$\mathbb{E}(\varepsilon_t^2 \mid \varepsilon_{t-1}, \varepsilon_{t-2}, \cdots) = c + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_p \varepsilon_{t-p}^2$$

which gives a nontrivial forecast for the conditional variances.

In ENGLE [5], a ML procedure (under a Gaussian assumption for $z_t$) for parameter estimation and for testing for $\alpha_1 = \cdots = \alpha_p = 0$ is described. A number of other procedures e.g. based on the Generalized Method of Moments are available now.

A number of linear and nonlinear generalizations for the ARCH model (8) are available now, such as GARCH models of the form

$$\sigma_t^2 = c + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2$$

which allow for more flexible lag structures. In order to assure a well defined process it is assumed that $\beta(z) = 1 - \sum \beta_i z^i \neq 0, |z| \leq 1$ and that $c > 0, \alpha_i \geq 0, \beta_i \geq 0$ holds.

In addition $(\varepsilon_t)$ is stationary if

$$(\alpha_1 + \beta_1) + \cdots + (\alpha_p + \beta_p) < 1$$

holds.

ARCH, GARCH and also other models for time varying (conditional) variances are very common now.

## 5. Conclusions

As far as the practical relevance of time series econometrics is concerned, there are still differences in judgement. In the author's opinion modesty, but not resignation is appropriate. The area shows, like other fields too, a certain tendency to be self-referential, trendy and partly also redundant. There is still a tendency of being led astray by doing mathematics for its own sake and of doing empirical analysis without critically evaluating assumptions, tools and the relevance of results. The danger of narrowing the point of view by using mathematical instruments and even more of unrealistic claims, of an 'econometric fundamentalism' and of 'mathematical omnipotence fancies' is still present. In the author's opinion econometrics has to be understood as auxiliary, in the sense that its developments, in the long run at least, are only justified if they really contribute to empirical economic analysis. In applications common sense and intellectual honesty are required. Econometricians have to learn from the needs and experiences of applications.

On the other hand there is no reason for resignation. As has been pointed out, recently a great number of new models and methods as reactions to failures in the past have been developed. Compared to the seventies, the progress in this respect is enormous. In the eighties, econometrics, or its vanguard at least, has become mathematically 'high tech', e.g. by applying and further developing nonstandard asymptotics. An interesting feature during the last ten years was the emphasis on models taking into account genuine features of economic data and theories. This is a reason why, after a period of convergence, time series econometrics and systems identification in engineering drift apart now.

Data banks, software for methods and computer capacities have increased both, the comfort and the number of applications, drastically; in addition economists with less formal training now have easy access to econometric tools for empirical research. Econometrics of finance has become a particularly promising and vibrant area, where now models and methods have been developed in order to take into account special features of finance data. Both, because of its attractivity and because of demand, e.g. from banks, a relatively large number of econometricians moved into this area recently; even system theorists are becoming interested or work in the field. There is a substantial number of real applications; the final judgement concerning success stories however here should be given in ten years. Macroeconometrics definitely at present is not an indispensable tool for economic policy. Nevertheless macromodels are used to a limited extent for forecasting and policy simulations, in most cases as a supplement to or in combination with judgement of experts. As far as competing macroeconomic theories are concerned, econometric analyses are often used to corroborate a specific theory; Summers [26] claims that econometric results almost never finally resolved such a discussion.

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