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Signific consultation
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Signific Consultation

David van Dantzig's Dream of a Practical Significs

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David van Dantzig was one of the founders of our Centre. He inspired a tradition of statistical consultation within the Statistical Department. He also was an active participant in the dutch signific movement; his was the idea of signific consultation. A brief survey is given of Van Dantzig's mathematical career, highlighting his turn to mathematical statistics, and an overview of his contributions to significs. An analysis of the idea of signific consultation will show that significs was in fact a program of judicious mathematization.

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1. INTRODUCTION

The term 'signific consultation' was coined by David van Dantzig (1900-1959). Like all participants in the Dutch signific movement Van Dantzig had his particular understanding of their common aim: the fostering of proper understanding between human beings. He is quite representative of the movement for this reason, and because he is a mathematician. In ascribing a major role to mathematical thought and to logic the significists show a kinship to their contemporaries of the Vienna Circle; they differ from the latter in denying formalist and logicist viewpoints. Signific thinking and Vienna Circle philosophy share a strongly speculative character combined with a commitment to improve the world by intellectual means. Comparing the two, significs stands for the more moderate view on improving communication in society: through understanding and clarification, rather than through forbidding or prescribing certain language uses. Furthermore Dutch signific thought stresses not only the importance of the intended content of a communication but also of its desired effect; in doing so it foreruns and parallels some of the early developments in modern semiotics and pragmatics. We will see these global traits of significs reflected in Van Dantzig's work. In the following, however, the emphasis will be on his particular position: practical significs.

It is fair to say that the existence of significs as an intellectual movement in the Netherlands coincides with Gerrit Mannoury's (1867-1957) acting as its central figure, i.e. from 1917 or 1922 until 1957.

In matters of significs Van Dantzig always refers to Mannoury and rightly so. Virtually every notion he brings up, every method he propagates can be traced back to his master in significs. Van Dantzig's own position is best illustrated where he radically pushes through the practical orientation of significs. In 1948 he expressed his dream in public that one day signific consultation agencies (signifische adviesbureaux) [Dantzig 1949a: p.22] might come into existence, in analogy to the already existing practice of statistical consultation.

The extremity of Van Dantzig's position reveals to us an essential characteristic of significs, namely that it was a program of action. To Van Dantzig and to Mannoury it was a practicable doctrine rather than a speculative theory.

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Mannoury takes up Lady Welby's writings as a slogan, a battle cry, as he notes in his *Handboek der Analytische Signifika* [Mannoury 1947: I p.134] (on Welby and the origins of significs see [Schmitz 1985, 1989]). And why write a handbook, if not for practical purposes?

The signific movement set out to improve our world. Its goal was to enrich and round out language, our apparatus of understanding, in order to prevent misunderstanding and extinguish empty controversy. For example in the realm of statistical consultation Van Dantzig would teach his staff to reach clarity about the customer's intentions before anything else, in order to preclude misunderstanding. Van Dantzig himself undertook several signific analyses of notions in political controversy.

At first glance such activism seems to be consonant with a low inclination towards formalism. Beyond doubt is the rejection of the formalist philosophy: Mannoury opposes the Vienna Circle's 'formalizing tendencies'; Van Dantzig criticizes Carnap and Morris for not considering the very act of speech [Dantzig 1948c]. A closer look shows that Van Dantzig's pursuit is to reach a better world through the application of (signific) theory. Both Mannoury and Van Dantzig are mathematicians and they do in some respect aim at formalizing. Their restriction on formalization, however, is that they view it in relation to human emotion and volition¹. In more precise terms their pursuit should rather be named mathematization, at which they aim judiciously in as far as the so-called emotional and volitional are not completely discarded. In fact we will see that Van Dantzig himself uses the term mathematization. The notion of 'mathematical form of thought' (wiskundige denkvorm) pervades all of significs. *We claim that significs was a program of action and more specifically a program of prudent mathematization.*



Gerrit Mannoury in 1940, picture from the collection of David van Dantzig

Content

The author believes that the historiography of significs has brought forward a good picture of the movement as an intellectual endeavour. Then again little attention has been paid so far to the speculative character of significs and cognate theories and likewise to the practice-orientedness of the movement. David van Dantzig offers us a delightful example of a significist whose career and views clearly show these aspects of significs. Section 2 will deal with his career and views as a mathematician, highlighting his turn to statistics that is so closely connected with signific ideas. Section 3 gives an overview of Van Dantzig's contributions to significs. Section 4 confronts the two in order to analyse the notion of signific consultation. The concluding section 5 seeks to evaluate what the Van Dantzig-story contributes to our understanding of significs, focusing on the relation of mathematics and significs.

1. 'Emotional' or 'emotive' and 'volitional' or 'volitive' are technical terms from the signific idiom. At one point Van Dantzig offers as a translation: 'valuative' and 'incitive (i.e. inspiring to action)' respectively [Dantzig 1949 b: p.54].

2. DAVID VAN DANTZIG

Van Dantzig is wellknown for his founding activities in postwar mathematical statistics in the Netherlands. He acted as a cofounder of the Mathematical Centre² in Amsterdam in 1946, an institute which combines pure mathematics with a dedication to applications. Van Dantzig's branch in the Centre, the Statistical Department, developed a lively practice of statistical consultation.

David van Dantzig lived from 1900 to 1959. His first contact with Gerrit Mannoury dates from 1917, while he was still a student of chemistry. Mannoury's lectures inspired his transfer to mathematics, to pure mathematics, in 1921³. That is to say, he took Mannoury's relativist philosophy and Mannoury's significs, but only later on. First of all Mannoury opened up for him the beauty of pure mathematics.

Van Dantzig qualified in topology and took his degree in 1931 with his former fellow student B.L. van der Waerden on a 31-page thesis *Studiën over Topologische Algebra* [Dantzig 1931]. From 1927 he worked in Delft with J.A. Schouten in differential geometry (calculus of tensors), first as Schouten's assistant and from 1932 as lector. In his 1938 inaugural adress [Dantzig 1938a,b] as an extra-ordinary professor of mathematics he unfolded his "hypothesis of flashes", a view of theoretical physics considering physical phenomena as built up from elementary "events" - rather than from elementary motion -, a view of physics based on a "geometry without points".



David van Dantzig in 1934.



2. Now CWI, Centre for Mathematics and Computer Science. Van Dantzig may be regarded as the originator of the twofold ambition, theory and practice, of the institute. Cf. [Alberts e.a. 1987].

3. Cf. [Dantzig 1931: preface]; Van Dantzig had to give up his studies in 1917 due to family circumstances. When he reentered the University of Amsterdam in 1921, his choice was mathematics.

Then, in 1940, seemingly without any reason Van Dantzig expounds his views on the calculus of probability, 'on mathematical and empiristic foundations of probability calculus' [Dantzig 1941]. This event clearly marks a turning point in his career. In the talk he claims that no axiom system can offer sufficient founding of probability calculus; any such formal system is in need of further empirical foundation. It is the first occasion where Van Dantzig takes the relation of a formal system and reality into explicit consideration. It also indicates the starting point of Van Dantzig's concern with mathematical statistics, entering the field from the side of its foundations.

Reasons for turning to statistics

There are several good reasons for turning to statistics.

Research in theoretical physics may lead towards probabilistic considerations. Such has been the case in thermodynamics (kinetic gas theory) since about 1860. In the period discussed here statistical quantum mechanics was a true novelty. Through his work in calculus of tensors with J.A. Schouten during the nineteen-thirties Van Dantzig had touched upon both these fields of physics. His hypothesis of flashes is the concept for a theory of elementary physical events, a theory of the kind that will evidently call for probabilistic treatment. Moreover, since probability theory had only recently gained the status of a proper mathematical theory through the successful axiomatizations by Kolmogoroff and by Reichenbach, the strive for a physical theory based on it needed no longer to be considered improper. These connections may suffice to show that Van Dantzig ran into matters of probability calculus. For anyone active in application-oriented mathematics it would have been hard to overlook these new developments of probability theory during the interbellum. The 1941 publication of the talk proves that David van Dantzig was well acquainted with the relevant literature.

An even more convincing reason for turning to statistics derives from significs, where its program proposes both empirical psychological and mass psychological investigations. The first, psychological (psycholinguistic) theory, needs testing: by statistical technique. The second, mass psychology, was a hot topic in the signific society from the late thirties through the forties, just as it was in many intellectual circles of the time.

Otto Neurath lived in The Hague from 1933 to 1940, promoting his image-statistics (Bildstatistik) as a means of non-deceiving mass communication. From 1937 on Neurath, carrying with him the Vienna Circle's attainments in probability theory, conducted a lively debate with a group of significists, one of whom was David van Dantzig. Neurath became a friend of the Van Dantzig family while both men lived in The Hague⁴.

In a way, Van Dantzig was forced into mathematical statistics. He had come to an ordinary professorship at the Delft Technical University in 1940. The war meant a hazardous interlude: Jewish, he was expelled from the university and forced to move to Amsterdam.

"Through helpful contacts with some insurance institutes and with the National Aeronautical Laboratory, which fortunately charged him with research problems, he became more and more interested in the practical role mathematical statistics and mathematical physics might play in life. It may almost be seen as a prophecy that on the illegal identity card which was to serve in cases of emergency, his profession was indicated by the word "Statistician" ' [Koksma 1959: p.332]

This observation is made by Van Dantzig's friend and colleague J.F. Koksma, who undertook much to protect him. After the war Koksma also acted as a cofounder of the Mathematical Centre, which realized Van Dantzig's view to ally pure mathematics with an orientation towards applications. In that context a new chair at the University of Amsterdam was created for 'the study of collective phenomena' and occupied by Van Dantzig. In his inaugural lecture, finally held in 1948, he revealed his crucial motive for turning to statistics [Dantzig 1949a: p.22 ff].

4. The author thanks Aart van Bochove for calling my attention to this debate, cf. [Bochove 1986]. Van Dantzig's son recalls Neurath being a friend of the family from the time both men lived in The Hague.

To Van Dantzig's own understanding the crucial motive lies in his relativist philosophy, which, again, he shares with Mannoury. Van Dantzig opposes relativism to all absolutism and dogmatism. In the formation of concepts, so he states, absolute yes-or-no oppositions should be overcome by gradation. Gradual distinctions between one notion and another ought to replace such contradistinctions. On a societal level Van Dantzig champions the roll back of dogmatic attitudes: egotistic behaviour (ik-gedrag), i.e. the capitalist mentality, ought to be replaced by socially conscious we-behaviour (wij-gedrag); ideological behaviour by scientific-relativist behaviour. Factual realisations of such scientific and relativist comportment were offered for one thing by the calculus of probability, for another thing by signfics [Dantzig 1949a: p.23].

At this point we find the first parallel of mathematical statistics⁵ and signfics with Van Dantzig: they stand in equal rights as realisations of scientific relativism. Although gradation and relativization are themselves major signfic themes, e.g. in [Mannoury 1947: II], [Dantzig 1949b], Van Dantzig in so many words claims the dominance of the principle of relativism. We take three consequences from these statements of 1948. First, the reflection present in so many of his publications must primarily be considered as relativist reflection, which does in turn make use of signfic technique.

Second, statistics, even when regarded as applied probability, was motivated by the search for a better world. Moreover, third, the turn to mathematical statistics was closely connected with signfic thought and strongly motivated by the signfic theme of relativization.

View of statistical consultation.

Van Dantzig performed a turn to statistics as a commitment of putting mathematics at the service of society. In doing so he developed his own particular view of statistical consultation.

Van Dantzig devoted the rest of his life and career primarily to the Mathematical Centre. Although he did work and publish on a diversity of subjects like logic, intuitionism, didactics of mathematics, information theory and signfics, his main efforts went into mathematical statistics, statistical consultation and their foundations. During and immediately after the war he conducted some statistical consultations by himself; under his supervision at the Centre a tradition grew of taking these mathematics to practice. Between 1949 and 1959 the consultation division of the Statistical Department treated well over 200 orders of the size that lead to a written report.

Van Dantzig's views on consultation are reflected in characteristic statements he is remembered for by his pupils in statistics, cf. [Alberts e.a. 1987: Ch.8]. 'Find out what the customer really wants' used to be the first adagium in statistical consultation. He would attribute great importance to the analyzing dialogue in consulting and would urge the statistician 'to be involved in the research project before the client starts gathering data'. Rather than a reductionist, Van Dantzig proposed a dualist approach: peeling off and treating with mathematical statistics the "indicative elements" in order to clarify and safeguard the "valuative" content.

A clear example of such dualism is in the report to the largest and most famous order acquired by Van Dantzig for the Mathematical Centre in those years: a set of problems concerning the Delta-works for flood prevention. 'Econometric decision problems for flood prevention' [Dantzig 1956] is the answer to one of these questions; it offers a calculation scheme to evaluate the risks of flooding against the investments for prevention. In this study Van Dantzig separates out the value of human life (i.e. the "cost" of casualties) to be the subject of an "emotional" (or ethical) decision by the responsible politicians. The outcome of such a decision would then serve as input to the hypothetical scheme of calculation, and make it realistic.

This consultation was one of the last things Van Dantzig worked on. He died in 1959, little before the publication of the final report.

5. Van Dantzig does regard mathematical statistics as applied probability, which is, of course, but one way of viewing it. His view differs from straitforward positivism in conveying a subtle picture of what it means to 'apply theory' at all. See fig.I, next section.

3. CONTRIBUTIONS TO SIGNIFICS

The fact that David van Dantzig is less well known as a significist, is mainly due to the comparative isolation of significs. We face the paradoxical situation of a movement which claims the pursuit of better understanding in the world, but which proclaims to that end its very own understanding of this world, formulated in an almost hermetic idiom⁶. Within the movement Van Dantzig may have taken a somewhat extreme position, 'empiristically oriented' according to Mannoury, and may have belonged to the 'younger generation' [Mannoury 1947: I p.157], he was quite dedicated to its cause and active. He seems to have played a major role in stimulating the eighty year old Mannoury to prepare and publish the *Handboek* [Mannoury 1947]. Van Dantzig deserves special recognition for disseminating signific ideas in his lectures and writings. Moreover, if a practice-oriented turn is discerned in later significs, then his name and his influence should be mentioned.

Van Dantzig would always work on many subjects simultaneously. We find wholly and partially signific publications all through his career. The first of these is an early article on the social value of mathematical education [Dantzig 1927], which pleads for significs in education and simultaneously offers a more or less signific analysis of the term "logical thinking". In the paper Van Dantzig announces a truly signific study 'on the psycho-genesis of mathesis' - it never appeared, probably as he was doing so many things at the same time.

His personal character seems to have dictated the ever reoccurring jump from one question to the other, to the underlying and more fundamental question. We will find him working in the fundamental margins of the subjects he is dealing with. Thus, entering the field of mathematical statistics from its foundations, as we saw above, was no accident but rather characteristic of Van Dantzig. His subsequent teachers, G. Mannoury, L.E.J. Brouwer and J.A. Schouten, were studying the foundations of mathematics and of physics; within the spectre they offer Van Dantzig would choose once more the deepest questions. There is no clue to decide, whether it was his character or his relativist philosophy that inspired the passionate inspection of founding propositions.

David van Dantzig was somewhat feared as a debater, always keen on unscrewing, relativizing, the fixed assumptions in someone's position, his own assumptions included.

Two publications in 1938 present a good example of this passion. They raise the question whether physics would not be better off when defined independent of the mathematical notions of space and time. Their content is a radical continuation, leading to the 'hypothesis of flashes', of Van Dantzig's work with Schouten on differential geometry and of his own interest in the theory of relativity. However, they may just as well be regarded as exercises in significs.

The first is an article in *Erkenntnis*: 'Some possibilities of the future development of the notions of space and time' [Dantzig 1938a]; the second is his Delft inaugural lecture *Vragen en schijnvragen over ruimte en tijd* [Dantzig 1938b]. Both publications show the influence of the Vienna Circle. Conforming to the desire of Neurath, editing *Erkenntnis*, Van Dantzig changed the title of the article from 'Some speculations about...' into 'Some possibilities of...'. The title of his lecture *Vragen en schijnvragen...* (Questions and pseudo-questions) directly derives from Carnapian idiom. These influences, however, remain superficial. The true signific tenor is revealed already in the subtitle of the lecture: *Een toepassing van den wiskundigen denkvorm* (An application of the mathematical form of thought). And to Van Dantzig applying the mathematical form of thought was the cornerstone of significs: it meant concept analysis, clarification of notions.

Concept analysis: a general key

Concept analysis is a general key to Van Dantzig's contributions to significs: it is called 'signific work' on the one hand, 'use of mathematical thought' on the other hand. He regards significs as a way par excellence of bringing mathematics to usefulness [Dantzig 1948a].

6. The author would like to propose this paradox as a daring theme for further research.

Most all of Van Dantzig's reflections on mathematics, on statistics and on empirical science contain significant passages. Allusions to significant and admiring notes on Mannoury will regularly appear at the end of an article. One couldn't possibly count all this work under significant. But in these 1938 publications the element of concept analysis is so much dominating, that Van Dantzig may be considered to be practicing significant in them⁷. Notable in these significant is the double role of mathematical thought, both background of significant and major theme subject to significant work; we will return to this point in the final section.

Van Dantzig gave a number of straightforward contributions to significant. Firstly, he developed some applications of significant, the main body of which is a series of analyses of political terms, e.g. on the notion of "freedom", or 'significant considerations on the notions "guilt", "penalty" etc. in relation to the question of annexation'⁸. Secondly, he undertook a series of disseminating activities, namely his lecture courses in Delft (1945/46) and in Amsterdam (1947/48); the latter under the very title *Significa* (Introduction to general significant) [Dantzig 1946b; 1949a]. The never fainting allusion to Mannoury and to significant in writings of varying purport may also be counted under this heading. The third group of Van Dantzig's contributions to significant is given by a number of articles on significant, on significant and semiotics, on Mannoury, on information theory, on "general procedures of empirical science" [Dantzig 1947a,b, 1948b,c, 1958], etcetera. These constitute, more than significant exercises, systematic extensions of the theory of significant.

To the very core of significant belongs 'Significant and its relation to semiotics', in which Van Dantzig distinguishes significant from and defends it against the work of Morris and Carnap ([Dantzig 1948c], this article is more extensively dealt with in A. Eschbach's contribution to these proceedings). The article is symbolic of his work in that it follows the thread of expositions against the views of the Vienna Circle and related authors - against all those who he pleases to gather under the nominators of logicism or formalism. In the discussions, however, Van Dantzig is like most significantists remaining within the realm of, broadly speaking, positivism; so we witness a brotherly dispute.

Leitmotiv

The content of the dispute reveals the leitmotiv of Van Dantzig's work: to develop a particular perspective on methods and foundations of empirical science - and to realize that perspective. Science is viewed as an appearance of the relation of mathematics to reality. The most explicit elaboration of this view is in 'General procedures of empirical science' [Dantzig 1947a]; other examples of the same issue are in [Dantzig 1941, 1946b, 1949b, 1953a]. In it Van Dantzig offers a twelve-state analysis of the process of building and using mathematical models. In discerning these states he in so many words leans upon the traditional significant distinction of linguistic levels. He furthermore invokes Mannoury's notions of "switching on" and "switching off" formalism [Dantzig 1947a: p.10;p.5;p.9].

7. Drawing a sideways conclusion on David van Dantzig: the reflection guiding, and sometimes interfering with, the mathematical work reveals an innate tendency towards speculation, that is being channeled into significant.

8. [Dantzig 1945, 1946a] The issue in the question of annexation, to which Van Dantzig strongly opposed, were certain revisionist proposals in 1945 to "correct" the German-Netherlands' borderline. Further applications to political discourse are in [Dantzig 1947c, 1951, 1953] and in a lecture course on the subject in 1949.

1. Experience	Forgetting
2. Recollection	Simplification
3. Observation	Ellipsis
4. Description	Regularisation
5. Model	Switching on
6. Formalisation	Absolutising
7. Induction	Arranging
8. Axiomatising	Deduction
9. Extension	Switching off
10. Interpretation	Inductive behaviour
11. Expectation	Volition
12. Action	

Figure 1: The general scheme of empirical science as seen by Van Dantzig. Note the states preceding formalisation and following extension [Dantzig 1947a: p. 10].

The scheme is notable for the effort to stay away from logical empiricism. More than one transition (forgetting, simplification, ellipsis, regularisation) separate experienced reality from the formalism that is "switched on" and "switched off". These transitions show Van Dantzig's conviction that one does (should) not discard the incitive and valuative aspects. The incitive and valuative aspects, or rather 'elements', are put aside in order to be reintegrated with the result of formal treatment:

'By undoing, at least partially, ("switching off", according to Mannoury) the formalisation and returning to ordinary (or ordinary scientific) terminology, an *interpretation* of the deduced statements [...] is given. On this interpretation the *expectation* is based, that the fictitious observations occurring in the conclusion of such a deduced statement *will* really occur, *if* the fictitious observations occurring in the hypotheses *do* really occur. This expectation, combined with definite emotions and volitions (wishes) with respect to the realisation of certain imagined events of situations leads to definite *actions*, realising Brouwer's "replacement of aims by means", and which are the (ultimate or intermediate) *purpose* of the scientific activity under consideration' [Dantzig 1947a: p.9; *italics* in the original].

Apparently formalizing is not presented as an end in itself. Actions are held to be the purpose, and they are directed by the formal consideration only after volitions and emotions have been re-attached to it.

Van Dantzig's dualism shows in the fact that according to his view the emotions and volitions are separated out, temporarily set aside, but not discarded. More importantly, whereas logico-positivist literature will hardly distinguish formalization from mathematization, Van Dantzig is precisely concerned with mathematization: formalization now appears as a special procedure within the mathematical form of thought. For these two reasons his view is non-reductionist: he proposes a judicious way of mathematizing. His lecture notes on significs in fact do contain such a plea for prudent deployment of formal procedures: according to what is described as "the signific method"⁹.

Mathematical Modelling

These reflections on general procedures of empirical science constitute the kernel of Van Dantzig's contribution to signific theory (the third group of his contributions to significs). They are presented at several occasions [Dantzig 1946b, 1947a, 1949b]; they elaborate the signific themes of 'language gradation' and 'elements of significance' and thus extend the theory of significs; they are central to a larger class of publications, that add to or disseminate signific thought, e.g. [Dantzig 1948c, 1953a].

Van Dantzig elaborates general signific insights for the special case of mathematical modelling, the 'general procedure' alluded to. Interestingly, in doing so he presents a novelty. He is the first, in 1946,

9. [Dantzig 1949b: pp.36 ff, pp. 50-54]; the signific method is presented in opposition to the formalist method and meant to solve the deadlocks in the latter.

in Dutch literature to come up with the notion of "mathematical model". Simultaneously he reflects on its foundations, like we would indeed expect him to, and sets precautions for its use¹⁰. Typical of his significant view on mathematical modelling is that he analyses the process of modelling starting and ending at the level of experience. This allows the inclusion of statistical consultation in the scheme.

4. SIGNIFIC CONSULTATION AS MATHEMATIZATION

"It is desirable and feasible to distinguish the indicative element in a valuative judgement from the emotive element, to investigate its referential basis and to mathematize it"¹¹.

Quoted is thesis XV appended by Van Dantzig to his dissertation in 1931. It was clearly meant as a significant credo at the time; in 1947 it was still appreciated as such by Mannoury and repeated because of its particular value for significs. The doctrine of distinguishing elements of significance is evoked and linked with mathematizing. One excerpt from thesis XV reads: "It is desirable to mathematize the indicative element in a valuative judgement". In view of the programmatic tenor of the thesis we may state that significs, as a program, was essentially a program of mathematization.

Explaining thesis XV Van Dantzig sketches how significant analysis of a valuative judgement leads to discerning a sequence of finalities, a "finality sequence" (finaalreeks) which halts at a last unexplained wish or passion - where the subject refuses to answer further questioning for his motives. In general someone will want A in order to reach B, so B is desired and the relation of A and B is indicated. B in turn is wanted for reasons of C etcetera. Thus a chain A, B, C, ...K is constructed, whose subsequent relations are the indicative elements: a finality sequence halting at one last desire K¹². Such finality sequences are no different, Van Dantzig proceeds, from Brouwer's "jump from goal to means", which results in the subject perceiving causal sequences and which is thus at the basis of mathematical thinking, cf. [Brouwer 1907: II]. In fact the only difference between causal and finality sequences is believed to be in the concurrent emotions.

Though sketchy, the explanation makes sufficiently clear what is meant here by mathematization and where it comes in. Separating out emotions will leave us with material apt for the mathematical form of thought. The dualism in Van Dantzig's views is now tangible.

Statistical techniques would seem particularly appropriate to assess the efficiency of the finality relations in a sequence of goals and means. Without mention of statistics, Van Dantzig as early as 1931 defines the "efficiency coefficient" of a finality sequence, in order to avoid the recurrence to strict causality. This again makes it easier to imagine that mathematical statistics would strike him as extremely useful in view of his significant pursuit.

Concept analysis was alluded to in the preceding section as a general key to Van Dantzig's contributions to significs. We are now able to pinpoint the doctrine of the so-called elements of significance as the main pattern underlying such analysis: volitive (incitive), emotive and indicative elements are discerned in an expression. Furthermore, mathematizing appears as the central tool. Here is the reason why Van Dantzig could call concept analysis at once 'significant work' and 'use of mathematical thought'. He regarded significs as a science, and as a way par excellence of bringing mathematics to usefulness. Significs was seen as an empirical science: like mathematical statistics as a science of the kind that allows the very practical use in the setting of consultation. Therefore the twelve state scheme of 'General procedures' may be held to apply here to his significant consultation as well. The thought is near, but necessarily remains a suggestion, that Van Dantzig developed the scheme with empirical significs in mind in the first place.

10. [Dantzig 1946b: p.5] The lecture in which he actually introduces the notion of mathematical model, was given in 1945. Like so many authors in the thirties we find him in 1940 plodding his way with such terminology as "formalist systems" and "regularized picture". [Dantzig 1941: p.78 ff]; cf. also [Alberts e.a. 1987: p.84]. In 1945 the relief of the new notion, mathematical model, was there.

11. [Dantzig 1931: separate leaf of theses]. Reprinted with an explanation in [Mannoury 1947: pp.157-159] as an appendix to the notes to section 40. In Dutch: "Het is wenschelijk en mogelijk, het indicatieve element in een waardeeringsoordeel van het emotioneele element te onderscheiden, de betrekingsbasis ervoor te onderzoeken en het vervolgens te mathematiseeren."

12. Note that the possible infinite regress is halted only by the frame of human subjectivity.

Dream

Whereas Van Dantzig founded a certain tradition in statistical consultation, in particular at the Statistical Department of the Mathematical Centre, his dream of signific consultation was to remain an ideal. Still, one can gather a fair impression of what he had in mind from his earlier experience and from his own elucidation in 1948. The applications of significs to the conceptual analysis of political terms may certainly be regarded as an experience in signific consultation [Dantzig 1945, 1946a, 1947c, 1951, 1953]. Other examples could be mentioned, e.g. [Dantzig 1947c], basically concerned with concept analysis and with discerning "indicative" from "emotive" elements.

On introducing the idea of signific consultation agencies in his inaugural address of 1948 Van Dantzig supposed the potential customers to be philosophers and scientists with questions like: "Please, read my manuscript to check if the intentions I have wanted to express will in fact be understood in the same sense by the reader" or "Check if I used these notion properly, significantly responsible, without stretching their significance too far". His final hypothetical example was a remarkable one: that of a priest coming in, having to address a workers meeting: "I know I am a somewhat oldfashioned man, clinging to catholic terminology. Please, be so good to translate this speech into marxian language, maintaining its indicative content and as far as possible its volitive and emotive value" [Dantzig 1949a: p.22].

This last example is curious, because in very the same lecture Van Dantzig denounced both contemporary communism and catholicism as the worst examples of dogmatism and absolutism in the field of politics. It was not like Van Dantzig to be pulling jokes, but we can sample his excitement with the idea of showing to such customer the relativity of terminology, while performing the "translation".

The crucial idea of signific consultation was, of course, to help people to reach clarity in expressing their intentions, e.g. avoid ambiguity, and to that end help clarify these intentions - by discerning the "indicative" from the "emotive". The peeling off or repeated dichotomizing of terms and notions is again the basic pattern. In a different article Van Dantzig discussed the contribution of the mathematical form of thought to the growth of the social sciences. The most important way to contribute, more important than through statistics, were through concept analysis. And significs is the form in which mathematics stands ready for such contribution [Dantzig 1948a]. Significs appears well disposed for consultation.

Again we find signific and statistical consultation compared, this time with a preference for significs. The comparison is based on the fact that both are ways of bringing the mathematical form of thought to service. As Van Dantzig first introduced the idea of signific consultation in relation to the example of statistical consultation, we feel free to continue the comparison.

A rathermore superficial comparison yields that both significs and statistics deal with dispersion phenomena: statistics deals with dispersion among data, e.g. data from measurements; significs deals with dispersion of significance of a concept (signifische spreiding). The underlying link is that in both cases dispersion is treated in a mathematical way.

The more enlightening common element in signific and statistical consultation is the emphasis on the customer's true intention: on finding out what the consultee really wants. This element is even stronger here than in Van Dantzig's style of statistical consultation. One perceives indeed the pretentious overtone of understanding the customer better than he would understand himself. The "better understanding" is sought after by distinguishing the elements of significance and by mathematizing. Still the programmatic pursuit of mathematization is reticent in two interconnected ways. Firstly the very idea of consulting, instead of taking over, reveals a lasting respect for personal intentions. Secondly, the dualist, or dichotomizing, approach aims at retaining a last, in a way "purified", residual of human purpose or intention. Because of the double reticence, we call it a judicious mathematization. The non-reductionist character of signific viewpoints has been mentioned above. Moreover, whereas statistical consultation concentrates on the so-called indicative element, signific consultation cherishes the remainder, the emotional element, as well.

The emphasis on the customer's last intention is common to statistical and signific consultation. In statistics this emphasis is only instrumental, in significs it is the heart of the matter.

5. SIGNIFICS AND MATHEMATICS

Developing a perspective on empirical science, in particular on the role of mathematical thought in it, was found to be the leitmotiv in Van Dantzig's work. Like so many scientists of his generation, Van Dantzig is finding his position in relation to the neopositivism of the Vienna Circle. He could just be considered one of these, had it not been for the signific movement.

Significs lends independent coherence to his views and makes them root in a different tradition. Nevertheless, this does mean that significs appears as a kind of foundational study of mathematics and of empirical science. With Van Dantzig, significs offers here primarily the doctrine of the elements of significance and the 'general procedures of empirical science'. The doctrine is a dualist, dichotomizing, approach of language, distinguishing the indicative elements from the valuative and the incitive. Mathematical thought is connected with the elements of significance in that the indicative element, as it is separated out, is apt for mathematization. The general procedure of empirical science offers a pattern, that as a whole stands for mathematization. Van Dantzig builds upon signific theory to describe the methodic locus and the operation of mathematization. His view of the role of mathematics is a relativization of the formalist standpoint in that it emphasizes the role of experience as a necessary counterpart of formal considerations. Thus the extension of general procedures of empirical science to consultation is only natural.

Because of the non-reductionist purport, that emphasizes the role of experience and the importance of the consultee's intention, we regard Van Dantzig's view of mathematization as a view of judicious mathematization.



David van Dantzig in his office at the Mathematical Centre, about 1953.

However, significs will not merely offer a detached view of foundations of science - be it a philosophical or a psychologicistic view. The dedication marking the signific movement is the pursuit of better understanding. Its style is adhortative, calling for the utmost clarity in expression and in reception of a message. Significs was not just meant to be a reflection of communication, of science in particular, but a program of action. This dedication is directly reflected in the work of Van Dantzig, precisely at the issue of mathematization. In 1931 he argued that mathematizing the indicative element (of a valuative judgement) is feasible and also desirable. The style and wordings of his further writings leave no doubt about his expectation of a salutary effect from mathematical modelling, from signific concept analysis and from mathematization of science in general. Stretching the application of probability theory into statistical consultation shows not only a view of judicious mathematization, but also an extended willingness to bring the mathematical form of thought to service.

Significs proposes mathematization

Thus far in this concluding section, we have discussed significs as a body of meta-level considerations on mathematization, and on communication in general. This discussion suffices to support the claim that significs proposes, according to Van Dantzig and Mannoury, a program of judicious mathematization. Mannoury in 1947 quoted with consent Van Dantzig's thesis XV of 1931 and the explanation to it, because of its importance for significs. Still, we found more and closer links between mathematics and significs.

Signific consultation seems to appear at first as an extension by analogy of statistical consultation. In fact a number of parallel traits have come to sight. It can not be stressed enough, that consultation implies practice statistics and *practice significs*. Both significs and the calculus of probability are theories of the kind that allows such practice use. Specifically in consultation we found the common emphasis on the consultee's intention. Further parallels concern significs and statistics in general: the concern with dispersion phenomena; the dichotomizing or dualist approach, discerning 'emotive' from 'indicative' elements; statistics and significs stand in equal rights, according to Van Dantzig, as adequate scientific expressions of relativist philosophy; both were seen by him as realizations of the service-ability of mathematical thought, with a preference for signific concept analysis.

Significs is mathematization

Paralleling mathematical statistics and significs opens a different level of discussion, exactly because significs is then viewed as an empirical science, moreover as a realization of the service-ability of the mathematical form of thought. Van Dantzig does treat significs as an empirical science, although in no way as traditional applied mathematics. His scheme of 'General procedures' is precisely an effort to escape from the narrow traditional idea of applying mathematics. He sees significs as one way of bringing mathematical thought to usefulness. Therefore his twelve state scheme of general procedures must hold for the empirical science of significs, and a fortiori for signific concept analysis and signific consultation. The scheme, as we saw, sketches the procedure of mathematical modelling. It describes a pattern of mathematization.

Concluding, we discover significs in a double role vis-a-vis mathematical thought. Reflecting upon it, significs proposes a program of mathematization. Realizing the service-ability of mathematical thought, significs is itself mathematization. Summing up: following Van Dantzig's idea of signific consultation leads to the recognition that significs is a program of judicious mathematization.

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