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DEFICIENCY IN INFINITE-DIMENSIONAL MANIFOLDS

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Contents

1. Introduction. In [17] it was shown that if E is a metric topological vector space (MTVS) which is homeomorphic ($\cong$) to its own countable infinite product E^{ω}, and M is any E-manifold (i.e. M is a paracompact manifold modeled on E), then $M \cong M \times E$. Accordingly we define a subset K of M to have E-deficiency (or to be E-deficient) provided that K is closed and there exists a homeomorphism $h: M \to M \times E$ such that $h(K) \subset M \times \{0\}$.

Such sets have proved to be important to the point-set topology of infinite dimensional manifolds because of results of the following two types.

(1) Negligibility theorem. If ℓ^2 is separable infinite-dimensional Hilbert space, M is an ℓ^2-manifold, and $K \subset M$ is a countable union of ℓ^2-deficient sets, then it was shown in [1] that $M \cong M \setminus K$. More general results have been established in [8].

(2) Homeomorphism extension theorems. If M is as in (1), K_1 and K_2 are ℓ^2-deficient sets in M, and $h: K_1 \to K_2$ is a homeomorphism which is homotopic to id_{K_1} (the identity on K_1), then h can be extended to a manifold homeomorphism [6]. For K_1 and K_2 additionally assumed to be ANR's, a similar result has been established for more general linear spaces than ℓ^2 [11].

In applications it is not easy to recognize that some sets have E-deficiency, thus it becomes desirable to have a coordinate-free topological characterization of E-deficiency in E-manifolds M. Such a characterization was obtained in [3] for $M = \ell^2$ and in [7] it was generalized to M any ℓ^2-manifold. It states (using a notion introduced by Anderson in [3]) that $K \subset M$ (where M is an ℓ^2-manifold) is ℓ^2-definient iff K has Property Z (or is a Z-set), where a set F in a space X has Property Z iff F is closed and for each non-null, homotopically trivial open set U in X, $U \setminus F$ is non-null and homotopically trivial. Among other things this enables us to recognize collared, closed sub-manifolds of M (i.e. bounderies of M) as being ℓ^2-deficient and any closed subset of M which is a countable union of ℓ^2-deficient sets is itself ℓ^2-deficient.

The main result of this paper is the following, which generalizes this characterization of E-deficiency to Fréchet manifolds.

Theorem 1. *Let $E \cong E^\omega$ be a Fréchet space, M be an E-manifold, and let $K \subset M$. Then K has E-deficiency iff K has Property Z.*

We remark that there are no known examples of infinite-dimensional Fréchet spaces E which do not satisfy the condition $E \cong E^\omega$. Concerning techniques it should be remarked that the proof of the corresponding result for $M = \ell^2$ [3] used the topology of the Hilbert cube I^∞ and the fact that ℓ^2 can be compactified by I^∞ (since ℓ^2 is homeomorphic to the countable product of lines [4]). The proof we give for Theorem 1 also uses the fact that ℓ^2 can be compactified by I^∞.

Using Theorem 1 above and Theorem 1 of [8] we easily obtain the following result.

Corollary. *Let M be as in Theorem 1 and let $K \subset M$ be a countable union of Z-sets. Then K is strongly negligible in M, i.e. there exists a homeomorphism $h: M \to M \setminus K$ which may be chosen arbitrarily close to id_M.*

(The notion of "arbitrarily close" will be made precise in the next section). We remark that by using different techniques David W. Henderson has recently shown (unpublished) that single Z-sets are strongly negligible in E-manifolds, where $E \cong E^\omega$ is a locally convex (LC) MTVS.

In Theorem 2 we establish a homeomorphism extension theorem which generalizes the extension theorem of [6] (which was proved for ℓ^2-manifolds). In Theorem 2' below we give a simplified version of Theorem 2. The more general statement appears in Section 5.

Theorem 2'. *Let $E \cong E^\omega$ be a LCMTVS and let M be an E-manifold. If K_1 and K_2 are E-deficient subsets of M and $h: K_1 \to K_2$ is a homeomorphism which is homotopic to id_{K_1}, then h can be extended to a manifold homeomorphism.*

2. Preliminaries. In this paper all spaces will be assumed to be metric and all homeomorphisms will be assumed to be onto.

Let X and Y be spaces and let $\mathcal{U}$ be an open cover of Y. Then functions $f, g: X \to Y$ are said to be $\mathcal{U}$-close provided that for each $x \in X$ there exists a $U \in \mathcal{U}$ such that $f(x), g(x) \in U$. A function $F: X \times I \to Y$ (where $I = [0,1]$) is said to be limited by $\mathcal{U}$ provided that for each $x \in X$ there exists a $U \in \mathcal{U}$ such that $F(\{x\} \times I) \subset U$.

If X is a space and $F \subset X$ is closed, then by Lemma 3 of [5] there exists an open cover $\mathcal{U}$ of $X \setminus F$ such that if $h: X \setminus F \to X \setminus F$ is any homeomorphism which is $\mathcal{U}$-close to $id_{X \setminus F}$, then h can be extended to a homeomorphism $\tilde{h}: X \to X$ which satisfies $\tilde{h} \mid F = id_F$. Such a cover of $X \setminus F$ will be called normal (with respect to F).

Let X be a space and let $\{f_i\}_{i=1}^{\infty}$ be a collection of homeomorphisms of X onto itself. Then for each $x \in X$ we let $f(x) = \lim_{i \to \infty} f_i \circ \dots \circ f_1(x)$, if this limit exists. If $f(x)$ exists, for all $x \in X$, then we write $f = L\Pi_{i=1}^{\infty} f_i$, and call it the infinite left product of $\{f_i\}_{i=1}^{\infty}$. We now state a convergence criterion for infinite left products. This is essentially a reformulation of West's version [18] of Theorem 4.2 of [4].

Convergence Procedure. Let X be a (topologically) complete space and let $\mathcal{U}$ be an open cover of X. Then to each homeomorphism $f: X \to X$ and each integer $i > 0$ we can assign an open cover $\mathcal{U}_i(f)$ of X such that if $\{f_i\}_{i=1}^{\infty}$ is any collection of homeomorphisms of X onto itself for which f_{i+1} is $\mathcal{U}_i(f_i \circ \dots \circ f_1)$ - close to id_X, for all $i > 0$, then $f = L\Pi_{i=1}^{\infty} f_i$ gives a homeomorphism of X onto itself which is $\mathcal{U}$-close to id_X.

There is one other notion of deficiency which will be useful in the sequel. Let X be a space which is homeomorphic to $X \times \ell^2$ and let $K \subset X$. Then K has ℓ^2-deficiency provided that K is closed and there exists a homeomorphism $h: X \to X \times \ell^2$ which satisfies $h(K) \subset X \times \{0\}$.

We will represent the Hilbert cube I^{∞} as $\Pi_{i=1}^{\infty} I_i$, where each I_i is the closed interval $[-1,1]$. The set $\Pi_{i=1}^{\infty} I_i^{\circ}$, where $I_i^{\circ} = (-1,1)$, will be denoted by s. In [2] it is shown that $s \times I^{\infty} \cong s$ and we have already remarked that $s \cong \ell^2$.

3. The equivalance of E- and ℓ^2-deficiency. The main result of this section is Theorem 3.1, where we show that in certain spaces E-deficiency and ℓ^2-deficiency are equivalent concepts. A similar proposition was established in [9], where E was additionally assumed to be a Banach space. We remark that the proof we give of Theorem 3.1 below follows in broad outline the proof of the corresponding result of [9], with appropriate modifications being made to overcome the lack of a norm. We will first need a technical result concerning open cones. (By the open cone over a space X (denoted by C(X)) we mean the space $\{v\} \cup (X \times (0,1))$, which is topologized by choosing as a basis the usual topology on $X \times (0,1)$ together with all sets of the form $\{v\} \cup (X \times (0,t))$, for all $t \in (0,1)$. We call v the vertex of the cone). We omit the proof of the lemma, since it is similar to Theorem 5.3 of [17].

Lemma 3.1. Let $E \cong E^\omega$ be a MTVS. Then there exists a homeomorphism $h: E \times [0,1) \times E \to C(E) \times E$ which satisfies the following properties.
(1) $h(E \times \{t\} \times E) = E \times \{t\} \times E$, for all $t \in (0,1)$,
(2) $h(E \times \{0\} \times E) = \{v\} \times E$, where v is the vertex of C(E).

Theorem 3.1. Let $E \cong E^\omega$ be a MTVS, M be an E-manifold, and let $K \subset M$. Then K has E-deficiency iff K has ℓ^2-deficiency.
Proof. Assume K has E-deficiency and let $f: M \to M \times E \times E$ be a homeomorphism such that $f(K) \subset M \times E \times \{0\}$. By the Bartle-Graves-Michael Theorem [15] we have $E \cong (-1,1) \times G$, for some G. Thus

$$E \cong E^\omega \cong (-1,1)^\omega \times G^\omega \cong (-1,1)^\omega \times (-1,1)^\omega \times G^\omega \cong (-1,1)^\omega \times E.$$

Since $(-1,1)^\omega = s \cong \ell^2$ we have $E \cong E \times \ell^2$. Let $g: E \to E \times \ell^2$ be a homeomorphism which satisfies $g(0) = (0,0)$ and let $\tilde{f}: M \to M \times E \times E \times \ell^2$ be defined by $\tilde{f}(x) = (id \times g) \circ f(x)$, where $id \times g: M \times E \times E \to M \times E \times E \times \ell^2$ is defined by $(id \times g)(x,y,z) = (x,y,g(z))$. Then $\tilde{f}$ is a homeomorphism and $\tilde{f}(K) \subset M \times E \times E \times \{0\}$. This implies that K has ℓ^2-deficiency.

On the other hand assume that K has ℓ^2-deficiency. Thus we have a homeomorphism $f: M \to M \times \ell^2$ such that $f(K) \subset M \times \{0\}$. It is easy to modify f to get a homeomorphism $\tilde{f}: M \to M \times E \times [0,1) \times E$ which satisfies

$\tilde{f}(K) \subset M \times E \times \{0\} \times E$. (Use the fact that $\ell^2 \cong \ell^2 \times [0,1)$ [13]). Let h: $E \times [0,1) \times E \to C(E) \times E$ be the homeomorphism described in Lemma 3.1. Then $id_M \times h$: $M \times E \times [0,1) \times E \to M \times C(E) \times E$ is a homeomorphism and $(id_M \times h)\circ\tilde{f}(K) \subset M \times \{v\} \times E$, where v is the vertex of $C(E)$.

In the proof of Lemma 2 of [12] there is a proof that $E \times \ell^2 \cong C(E \times S_1)$, where $S_1 = \{x \in \ell^2 \mid \|x\| = 1\}$. Since $S_1 \cong \ell^2$ [13] we have $E \cong C(E)$. Thus we can modify $(id_M \times h)\circ\tilde{f}$ to get a homeomorphism g: $M \to M \times E$ which satisfies $g(K) \subset M \times \{0\}$. □

4. Deforming a manifold to an ℓ^2- deficient subset. The main result of this section is Theorem 4.2, which shows how to deform certain manifolds onto subsets which have ℓ^2-deficiency. The following lemma is needed for its proof.

Lemma 4.1. *Let X be a space and let K be a closed subset of $X \times I^\infty$ such that $K \subset X \times s$. Then there exists a homeomorphism f: $X \times I^\infty \to X \times I^\infty$ which satisfies* $f(K) \subset X \times \Pi_{i=1}^{\infty} [-\frac{1}{2},\frac{1}{2}]$.

Proof. For each integer $i > 0$ and each $x \in X$ let

$$f_i(x) = \begin{cases} \text{glb } \{t_i \mid (x,t) \in K\}, & \text{if } K \cap (\{x\} \times I^\infty) \neq \phi \\ 1, & \text{if } K \cap (\{x\} \times I^\infty) = \phi \end{cases}$$

where we adopt the convention that if $t \in I^\infty$, then t_i is the i^{th} coordinate of t. It follows routinely that each f_i: $X \to (-1,1]$ is lower semi-continuous. Thus by Dowker's theorem ([10], page 170) there is a continuous function g_i: $X \to (-1,1)$ which satisfies $-1 < g_i(x) < f_i(x)$, for all $x \in X$. Similarly there is a continuous function g_i^1: $X \to (-1,1)$ which satisfies $-1 < g_i(x) < g_i^1(x) < 1$ and $\tau_i \circ \pi_{I^\infty}(K \cap (\{x\} \times I^\infty)) \subset (g_i(x), g_i^1(x))$, for all $x \in X$, where $\tau_i: I^\infty \to I_i$ and π_{I^∞}: $X \times I^\infty \to I^\infty$ are projections.

For each pair a,b, of real numbers satisfying $-1<a<b<1$, there exists a unique piecewise linear homeomorphism $h_{a,b}$: $[-1,1] \to [-1,1]$ which satisfies $h_{a,b}(a) = -\frac{1}{2}$, $h_{a,b}(b) = \frac{1}{2}$, and $h_{a,b}$ is linear an each of the intervals $[-1,a]$, $[a,b]$, and $[b,1]$.

Then define f: $X \times I^\infty \to X \times I^\infty$ by $f(x,(t_i)) = (x,(h_{g_i(x),g_i^1(x)}(t_i)))$, for all $(x,(t_i)) \in X \times I^\infty$. Clearly f fulfills our requirements. □

Theorem 4.1. *Let* $E \cong E^\omega$ *be a* MTVS, M *be an E-manifold*, $K \subset M$ *be* ℓ^2-*deficient, and let* $\mathcal{U}$ *be an open cover of* M. *Then there exists a homotopy* $H: M \times I \to M$ *such that* $H_0 = id$, $H_t|K = id$, *for all* $t \in I$, $H_1 : M \to M$ *is an embedding such that* $H_1(M)$ *is* ℓ^2-*deficient, and* H *is limited by* $\mathcal{U}$.

Proof. Since $I^\infty \times s \cong s$ we can use an argument like that used in Lemma 6 of [7] to prove that a closed set $F \subset M$ has ℓ^2-deficiency iff there exists a homeomorphism of M onto $M \times I^\infty$ taking F into $M \times \{0\}$. Thus let $f: M \to M \times I^\infty$ be a homeomorphism such that $f(K) \subset M \times \{0\}$.

Using techniques like those used in Lemma 4.1 we can clearly construct a homotopy $G : M \times I^\infty \times I \to M \times I^\infty$ such that $G_0 = id$, $G_1 : M \times I^\infty \to M \times I^\infty$ is a closed embedding such that $G_1(M \times I^\infty) \subset M \times s$, $G_t|M \times \{0\} = id$, for all t, and G is limited by $f(\mathcal{U})$, the cover of $M \times I^\infty$ induced by f and $\mathcal{U}$. Then define $H : M \times I \to M$ by $H_t(x) = f^{-1} \circ G_t \circ f(x)$. All we have to do is show that $H_1(M)$ has ℓ^2-deficiency.

Note that $f \circ H_1(M)$ is a closed subset of $M \times I^\infty$ which satisfies $f \circ H_1(M) \subset M \times s$. Using Lemma 4.1 there exists a homeomorphism $g: M \times I^\infty \to M \times I^\infty$ such that $g \circ f \circ H_1(M) \subset M \times \Pi_{i=1}^\infty [-\frac{1}{2},\frac{1}{2}]$. From [2] it follows that there exists a homeomorphism $h: I^\infty \to I^\infty \times I^\infty$ such that $h(\Pi_{i=1}^\infty [-\frac{1}{2},\frac{1}{2}]) \subset I^\infty \times \{0\}$. Let $\tilde{h}: M \times I^\infty \to M \times I^\infty \times I^\infty$ be defined by $\tilde{h}(x,y) = (x,h(y))$. Then $\tilde{h} \circ g \circ f \circ H_1(M) \subset M \times I^\infty \times \{0\}$. By our comments above this proves that $H_1(M)$ has ℓ^2-deficiency. □

5. <u>A proof of Theorem 2</u>. The main result of this section is Theorem 2, where we generalize the homeomorphism extension theorem of [6]. The proof we give follows in broad outline the proof given in [6], but there are a few technicalities which have to be overcome in order to make the proof work. We will need the following mapping replacement theorem which resembles Theorem 3.1 of [6].

Lemma 5.1. *Let $E \cong E^{\omega}$ be a LCMTVS, M be an E-manifold, X be a space which can be embedded as a closed subset of E, $A \subset X$ be closed, and let $f: X \to M$ be a continuous function such that $f|A$ is a homeomorphism of A onto an E-deficient subset of M. If $\mathfrak{U}$ is any open cover of M, then there exists an embedding $g: X \to M$ such that $g(X)$ is E-deficient, $g|A = f|A$, and g is $\mathfrak{U}$-close to f.*

Proof. Using Theorem 4.1 and the fact that E is an AR [16], a proof can be given which is similar to Theorem 3.1 of [6]. □

We will also need the following generalization of Theorem 2 of [7].

Lemma 5.2. *Let $E \cong E^{\omega}$ be a MTVS, M be a connected E-manifold, and let $K \subset M$ be an E-deficient set. Then M can be embedded as an open subset of E so that K is taken onto an E-deficient (and therefore closed) subset of E.*

Proof. The proof proceeds routinely as in Theorem 2 of [7] provided we note that (1) M can be embedded as an open subset of E, and (2) there exists a homotopy $H: E \times I \to E$ such that $H_0 = id_E$, H_t is a homeomorphism (onto), for $0 \leq t < 1$, and $H_1: E \to E \setminus \{0\}$ is a homeomorphism. The first assertion is just Theorem 4 of [12] and the second assertion follows since a corresponding property is true for ℓ^2 [4] and also since $E \cong E \times \ell^2$ (as was noted in Theorem 3.1). □

Theorem 2. *Let $E \cong E^{\omega}$ be a LCMTVS, M be an E-manifold, and let K_1, K_2 be E-deficient subsets of M for which there exists a homotopy $H: K_1 \times I \to M$ such that $H_0 = id_{K_1}$ and $H_1: K_1 \to K_2$ is a homeomorphism. If $\mathfrak{U}$ is an open cover of M such that H is limited by $\mathfrak{U}$, then there exists an ambient*

<u>invertible isotopy</u> $G : M \times I \to M$ <u>which satisfies</u> $G_o = id_M$, $G_1|K_1 = H_1$, <u>and</u> G <u>is limited by</u> $St^3(\mathcal{U})$ (<u>the</u> 3^{rd} <u>star of the cover</u> $\mathcal{U}$).
(An isotopy $G : X \times I \to X$ is said to be an <u>ambient invertible isotopy</u> provided that each level is an onto homeomorphism and $G^* : X \times I \to X$, defined by $G^*_t(x) = G^{-1}_t(x)$, is continuous).
Proof. First note that a homeomorphism extension theorem for E (without the limitation by covers) is easy to establish for E-deficient subsets of E. One merely uses the technique of Klee [14], as used in [2]. Thus in the case that $K_1 \cap K_2 = \phi$ we can use Lemma 5.1 and the techniques of [6] to obtain our desired ambient invertible isotopy.

On the other hand assume that $K_1 \cap K_2 \neq \phi$. It follows routinely that K_1 and K_2 are Z-sets, and therefore $K_1 \cup K_2$ is a Z-set. Using an unpublished result of David W. Henderson there exists, for each open cover $\mathcal{U}'$ of M, a homotopy $F : M \times I \to M$ such that $F_o = id_M$, $Cl(F_1(M)) \cap (K_1 \cup K_2) = \phi$(where Cl denotes closure), and F is limited by $\mathcal{U}'$. Thus by Lemma 5.1 and an appropriate choice of $\mathcal{U}'$, there exists an embedding $F^* : K_2 \times I \to M$ such that $F^*_o = id_{K_2}$, $F^*(K_2 \times I)$ is an E-deficient set in M for which $F^*_1(K_2) \cap (K_1 \cup K_2) = \phi$, and F^* is limited by $\mathcal{U}$.

Using the above remarks there exists an ambient invertible isotopy $G^* : M \times I \to M$ such that $G^*_o = id_M$, $G^*_1|K_2 = F^*_1$, and G^* is limited by $\mathcal{U}$. Note that K_1 and $F^*_1(K_2)$ are disjoint E-deficient subsets of M and $F^*_1 \circ H_1 : K_1 \to F^*_1(K_2)$ is a homeomorphism which is homotopic to id_{K_1}, with a homotopy that is limited by $St(\mathcal{U})$. We can once more use the above techniques to find an ambient invertible isotopy $H^* : M \times I \to M$ such that $H^*_o = id_M$, $H^*_1|K_1 = F^*_1 \circ H_1$, and H^* is limited by $St^2(\mathcal{U})$. Then the obvious composition $G = (G^*)^{-1}H^*$ fulfills our requirements. □

6. Proof of Theorem 1. The step from E-deficiency to Property Z is straightfoward and resembles Theorem 9.1 of [3]. For the other implication let $K \subset M$ have Property Z and let $h : M \to M \times I^\infty$ be a homeomorphism. Using the representation for I^∞ and s given in Section 2 let $B(I^\infty) = I^\infty \setminus s$. It is shown in [3] that there is a homeomorphism of I^∞ onto itself which sends $B(I^\infty)$ into s. We can obviously write $B(I^\infty) = \bigcup_{n=1}^{\infty} C_n$, where each C_n is compact. Thus using the above comment and the techniques of Theorem 4.1 it follows that each $M \times C_n$ is E-deficient in $M \times I^\infty$.

We will describe a sequence $\{g_i\}_{i=1}^{\infty}$ of homeomorphisms of $M \times I^\infty$ onto itself whose left product $g = L\Pi_{i=1}^{\infty} g_i$ gives a homeomorphism of $M \times I^\infty$ onto itself which satisfies $g \circ h(K) \subset M \times s$. Then we can apply the techniques of Theorem 4.1 to conclude that $g \circ h(K)$ is E-deficient.

Since $h(K)$ is a Z-set in $M \times I^\infty$ we can use the technique of the proof of Theorem 2 to get a homeomorphism $g_1 : M \times I^\infty \to M \times I^\infty$ such that $g_1 \circ h(K) \cap (M \times C_1) = \phi$. Now invoking the Convergence Procedure of Section 2 we need to produce a homeomorphism $g_2 : M \times I^\infty \to M \times I^\infty$ which is $\mathcal{U}$-close to $id_{M \times I^\infty}$, for any prechosen open cover $\mathcal{U}$ of $M \times I^\infty$, and $g_2 \circ g_1 \circ h(K) \cap (M \times (C_1 \cup C_2)) = \phi$. Once more using the fact that $M \times (C_1 \cup C_2)$ is E-deficient and $g_1 \circ h(K)$ is a Z-set, we can use the techniques of Theorem 2 to obtain the desired g_2.

Thus using an inductive procedure we can choose homeomorphisms $g_i : M \times I^\infty \to M \times I^\infty$ so that $g_i \circ \cdots \circ g_1\ h(K) \cap (M \times (\bigcup_{n=1}^{i} C_n)) = \phi$ and $g = L\Pi_{i=1}^{\infty} g_i$ gives a homeomorphism of $M \times I^\infty$ onto itself. Since we are able to select each g_i arbitrarily close to $id_{M \times I^\infty}$ we can choose $\{g_i\}_{i=1}^{\infty}$ so that $g \circ h(K) \cap (M \times B(I^\infty)) = \phi$, thus $g \circ h(K) \subset M \times s$ and we are done. □

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