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ABSTRACT
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SPACE-MAPPING TECHNIQUES APPLIED TO THE OPTIMIZATION OF A SAFETY ISOLATING TRANSFORMER

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Abstract – **Space-mapping optimization techniques allow to align low-fidelity and high-fidelity models in order to reduce the computational time and increase the accuracy of the solution. The main idea is to build an approximate model from the difference of responses between both models. Therefore the optimization process is computed on the surrogate model. In this paper, some recent approaches of space-mapping techniques such as aggressive-space-mapping, output-mapping and manifold-mapping algorithms are applied to optimize a safety isolating transformer. The electric, magnetic and thermal phenomena of the device are modelled by an analytical model and a 3D finite elements model. It is considered as a benchmark for multi-level optimization to test different algorithms.**

Introduction

The design of complex electric devices requires the development of multi-level, multi-physics and multi-disciplinary models. It is thus a question to increase the complexity and the accuracy of the models, physics or geometrical, by taking account of the multi-physics aspect, the strong non-linearity phenomena and the geometries in all their complexity. This requires controlling the fields of validity of the models and of the couplings. These researches require obviously sophisticated mathematical techniques. But the optimization of performances of the device cannot be applied directly to such complicated and computationally expensive models.

In practice, the space-mapping (SM) techniques [1] aligning low-fidelity and high-fidelity models speed up the optimization procedures by exploiting auxiliary coarse models that are faster to compute. In this paper, different strategies of multi-level optimization with SM techniques and with sequential quadratic programming (SQP) method [2] will be presented. Therefore, the aggressive-space-mapping (ASM) with trust region [3]-[4], an output-mapping (OM) and the manifold-mapping (MM) [5]-[6] algorithms are applied for the optimization of a safety isolating transformer [7]. The benchmark is established by one analytical model and one 3D finite element (FEA) model that simulate the electric, magnetic and thermal phenomena in the device. Comparison is finally made with a single level optimization method that is the sequential quadratic programming (SQP), taken as reference.

Optimization problem

The multi-optimization problem is benchmark of safety isolating transformer that is presented at Compumag 2007 conference [7]. The optimization problem contains 7 design variables and 7 non-linear inequality constraints (Fig. 1). The design variables are lying between prescribed lower and upper limits. The purpose is to minimize the total mass of the transformer. It is shown in the Fig. 1 and in [7]. The electric device is thus designed by a combination of an analytical model and a 3D FEA model.

The Kapp assumption where the voltage drop due to the magnetizing current is neglected and some assumptions for the thermal model are taken into account for the analytical coarse model. Equations modeling the physical phenomena of the device are ranked using specific algorithms [8]. This model leads to an implicit system of 8 equations and other equations solved sequentially. The non-linear implicit system does not depend on the initial values. It is solved using the Jacobian and Levenberg-Marquart algorithm. A complete description of the analytical model is available in [7] such as Matlab, MathCad, pdf, and Pro@Design files.

Since the transformer is symmetric, thermal and magnetic phenomena are both modelled on the eighth of transformer by using 3D FEA. For the electromagnetic modelling, all magnetic and electric quantities are assumed sinusoidal. Full load and no-load simulations are used to compute all the characteristics. A magneto-thermal weak coupling is also considered. The copper and iron losses are computed with the magnetic sinusoidal steady-state solver and introduced as heat sources in the thermal static solver. The copper temperature is used to compute the coils resistors introduced in the magnetic solver and this loop is performed until convergence of temperatures. Matlab and Opera3D files can be found in [7].

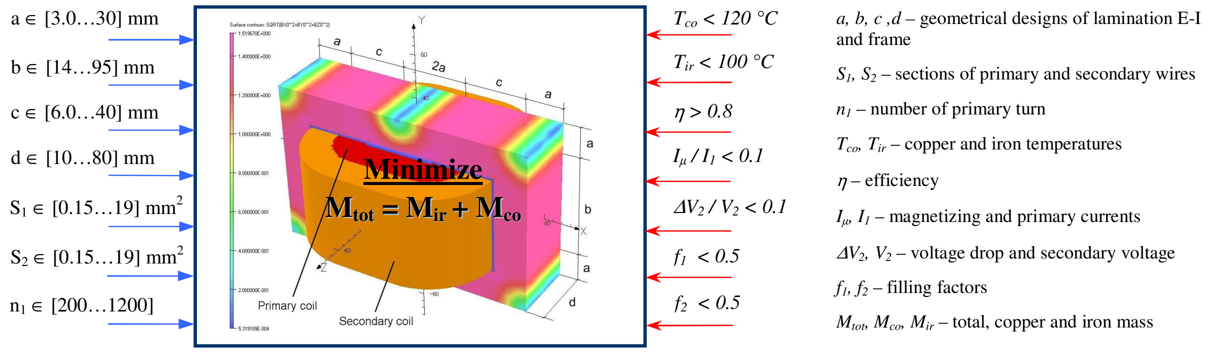

Fig. 1. Optimization problem of a safety isolating transformer

Space-mapping optimization techniques

Space Mapping (SM)

Let us consider an optimization problem with design variables $\mathbf{x}$ in the design space $\mathbf{x} \in X \subset R^n$ and specification $\mathbf{y} \in R^m$. The accurate behavior of electromagnetic devices is often studied using models that have large computational costs, e.g., finite element models. In space-mapping terminology [1], these models are called *fine* models. The fine model response is denoted by $\mathbf{f}(\mathbf{x}) \in R^m$. The problem we set out to solve in this paper can be stated as:

$$\text{find } \mathbf{x}^*_f \in X \text{ such that } \mathbf{x}^*_f = \underset{x \in X}{arg\,min} \left\| f(x) - y \right\| \tag{1}$$

where *argmin* denotes the argument of the minimum and || || indicates a suitable norm. Space-mapping needs a second, possibly less accurate but computationally cheaper model, called *coarse* model. The coarse models are assumed to be defined over the same design space $\mathbf{z} \in X$. Their response is denoted by $\mathbf{c}(\mathbf{z}) \in R^m$. The auxiliary optimization problem can be formulated as:

$$\text{find } \mathbf{z}^* \in X \text{ such that } \mathbf{z}^* = \underset{z \in X}{arg\,min} \left\| c(z) - y \right\| \tag{2}$$

The space-mapping function (parameter extraction) $\mathbf{p}$: $X \rightarrow X$ maps the design variable $\mathbf{x}$ with corresponding fine model response $\mathbf{f}(\mathbf{x})$ to the design variable $\mathbf{p}(\mathbf{x})$ that minimizes the fine and coarse model discrepancy, i.e.:

$$\mathbf{x} \rightarrow \mathbf{p}(\mathbf{x}) = \underset{z \in X}{arg\,min} \left\| c(z) - f(x) \right\| \tag{3}$$

This function allows the surrogate $\mathbf{c}(\mathbf{p}(\mathbf{x}))$ for $\mathbf{f}(\mathbf{x})$ to be defined. The space-mapping solution is defined through the use of this surrogate as follows:

$$\text{find } \mathbf{x}^*_{sm} \in X \text{ such that } \mathbf{x}^*_{sm} = \underset{z \in X}{arg\,min} \left\| c(p(z)) - y \right\| \tag{4}$$

In practice (4) is solved iteratively by replacing $\mathbf{p}(\mathbf{x})$ by an sequence of approximations $\{\mathbf{p}_k(\mathbf{x})\}_{k>1}$ converging to $\mathbf{p}(\mathbf{x})$ for $k \rightarrow \propto$ yielding a sequence of iterands $\{\mathbf{x}_{k,\,sm}\}_{k>1}$ converging to $\mathbf{x}^*_{sm}$. Typically $\mathbf{p}(\mathbf{x})$ is expanded in a first order Taylor series and Broyden's method is employed to update the gradient $\nabla\mathbf{p}(\mathbf{x})$ from iteration k to k+1. The space-mapping technique thus replaces the fine model optimization problem (1) by the surrogate problem (4) and solves the latter by the following sequence of coarse model optimization problems:

$$\text{find } \mathbf{x}_{k,\,sm} \in X \text{ such that } \mathbf{x}_{k,\,sm} = \underset{z \in X}{arg\,min} \left\| c(p_k(z)) - y \right\| \tag{5}$$

each requiring a fine model evaluation to compute $\mathbf{p}_k(\mathbf{x})$. However, we have that:

$$\mathbf{x}^*_{sm} \neq \mathbf{x}^*_f \tag{6}$$

unless additional assumptions on $\mathbf{p}(\mathbf{x})$ (and thus indirectly on the coarse model) are imposed.

Aggressive Space Mapping (ASM)

The basic idea behind space-mapping optimization is the following: if the fine model allows for an almost reachable design (i.e., $\mathbf{f}(\mathbf{x}^*) \approx \mathbf{y}$), from (2) and (3) we expect:

$$\mathbf{p}(\mathbf{x}^*) = \underset{z \in X}{arg\,min} \left\| c(z) - f(x^*) \right\| \approx \underset{z \in X}{arg\,min} \left\| c(z) - y \right\| = \mathbf{z}^* \tag{7}$$

Based on this relation, the original space-mapping approach assumes $\mathbf{p}(\mathbf{x}^*) \approx \mathbf{z}^*$. Therefore, the primal space-mapping approach seeks for a solution of the minimization problem:

$$\mathbf{x}^*_p = \underset{x \in X}{arg\,min} \left\| p(x) - z^* \right\| \tag{8}$$

For Aggressive Space Mapping (ASM) algorithm [3], the space-mapping function is approximated by linearization to obtain:

$$\mathbf{p}_k(\mathbf{x}) = \mathbf{p}(\mathbf{x}_k) + \mathbf{B}_k(\mathbf{x}\text{-}\mathbf{x}_k) \tag{9}$$

In each space-mapping iteration step the matrix $\mathbf{B}_k$ is adapted by a rank-one update. For that purpose a Boyden-type approximation for the Jacobian of the space-mapping function $\mathbf{p}(\mathbf{x})$ is used:

$$B_{k+1} = B_k + \frac{p(x_{k+1}) - p(x_k) - B_k h}{h^T.h} h^T \tag{10}$$

where $\mathbf{h} = \mathbf{x}_{k+1} - \mathbf{x}_k$. This is combined with original space mapping, so that $\mathbf{x}_{k+1} = \mathbf{x}_k - \mathbf{B}_k^{-1}(\mathbf{p}(\mathbf{x}_k) - \mathbf{z}^*)$ and the ASM is obtained. Note that Broyden's method cannot assure that every $\mathbf{B}_k$ is non singular, so, the inverse in the original space-mapping schema should be read as a pseudo-inverse. A trust region strategy [5] is introduced in ASM to improve the robustness of the algorithm. The trust region is updated each iteration by comparing the difference of coarse and fine objective functions between two consecutive iterations. ASM just solves the equation (8) by a quasi-Newton iteration with an approximate Jacobian. We notice that only one evaluation of the fine model is needed per iteration.

Manifold-Mapping (MM)

The manifold-mapping (MM) technique [5]-[6] exploits coarse model information and defines a surrogate optimization problem whose solution, unlike in (6), does coincide with $\mathbf{x}^*_f$. The key ingredient is the manifold-mapping function between the coarse and fine model image spaces $\mathbf{c}(X) \subset \mathrm{R}^m$ and $\mathbf{f}(X) \subset \mathrm{R}^m$. This function $\mathbf{S}$: $\mathbf{c}(X) \rightarrow \mathbf{f}(X)$ maps the point $\mathbf{c}(\mathbf{x}^*_f)$ to $\mathbf{f}(\mathbf{x}^*_f)$ and the tangent space of $\mathbf{c}(X)$ at $\mathbf{x}^*_f$ to the tangent space of $\mathbf{f}(X)$ at $\mathbf{x}^*_f$. It allows the surrogate model $\mathbf{S}(\mathbf{c}(\mathbf{x}))$ and the manifold-mapping solution to be defined as follows:

$$\text{find } \mathbf{x}^*_{mm} \in X \text{ such that } \mathbf{x}^*_{mm} = \underset{z \in X}{arg\,min} \left\| S(c(z)) - y \right\| \tag{11}$$

The manifold-mapping function S(x) is approximated by a sequence $\{\mathbf{S}_k(\mathbf{x})\}_{k>1}$ yielding a sequence of iterands $\{\mathbf{x}_{k,\, mm}\}_{k>1}$ converging to $\mathbf{x}^*_{mm}$. The individual iterands are defined by coarse model optimization:

$$\text{find } \mathbf{x}^*_{k,\, mm} \in X \text{ such that } \mathbf{x}^*_{k,\, mm} = \underset{z \in X}{arg\,min} \left\| S_k(c(z)) - y \right\| \tag{12}$$

At each iteration k, the construction of $\mathbf{S}_k$ requires the singular value decomposition of the matrices ΔC_k and ΔF_k of size $m \times \min(k, n)$ whose columns span the coarse and fine model tangent space in the current iterand, respectively. Denoting these singular value decompositions by:

$$\Delta C_k = U_{k,c} \Sigma_{k,c} V_{k,c}^T \text{ and } \Delta F_k = U_{k,f} \Sigma_{k,f} V_{k,f}^T \tag{13}$$

We introduce the updated objective $\mathbf{y}_k$ as:

$$\mathbf{y}_k = \mathbf{c}(\mathbf{x}_k) - [\Delta C_k \Delta F_k^{\psi} + (I - U_{k,c} U_{k,c}^T)] (\mathbf{f}(\mathbf{x}_k) - \mathbf{y}) \tag{14}$$

where superscript ψ denotes the pseudo-inverse. With this notation, the problem (12) is asymptotically equivalent to:

$$\text{find } \mathbf{x}^*_{k,\,mm} \in X \text{ such that } \mathbf{x}^*_{k,\,mm} = \underset{z\in X}{arg\,min}\left\| c(z) - y_k \right\| \tag{15}$$

The equation (15) is solved by sequential quadratic programming (SQP) method [2]. By construction:

$$\mathbf{x}^*_{mm} = \mathbf{x}^*_{f} \tag{16}$$

Output-Mapping (OM)

The main idea of the output-mapping (OM) technique is to modify the coarse model by adding some corrective coefficients $\theta \in \mathrm{R}^m$ in order to align the results of the coarse model with those of the fine model. The coefficients are updated at each iteration to minimize the discrepancy between both models. Then, they are introduced in the coarse model to compute $\mathbf{x}^*_{om}$ for the next iteration. The mapping between both models is in fact due to these corrective coefficients. A surrogate model is built and the optimization is computed on it. The original problem (3) is transformed as follows:

$$\text{find } \mathbf{x}^*_{om} \in X \text{ such that } \mathbf{x}^*_{om} = \underset{z\in X}{arg\,min}\left\| (c(z, \theta^i_k)) - y \right\| \tag{17}$$

where θ^i_k denotes the corrective coefficient for the i^{th} result of the coarse model at iteration k. In the first iteration, each component of the vector θ is set to 1. Each resolution (17) is made with SQP.

The corrective coefficients are updated by the Nelder-Mead's Simplex method [9] at each iteration using least-squares function to minimize the differences between both models:

$$\text{find } \theta_k \in \mathrm{R}^m \text{ such that } \theta_k = \underset{z\in X}{arg\,min}\left\| \sum_{i=1}^{m} (c(z^*_k, \theta^i_k) - f(z^*_k))^2 \right\| \tag{18}$$

where m is the number of results of the coarse model. $\mathbf{f}(\mathbf{z}^*_k)$ denotes the responses of the fine model with the design variables found previously on the surrogate model $\mathbf{c}(\mathbf{z}, \theta^i_k)$. In fact, this simplex method is a direct search method that does not use numerical gradient. For this problem with 7 non-linear inequality constraints, in which two mechanical constraints (f_1, f_2 in Fig.1.) do not depend on the 3D FEA, so that, there are only five corrective coefficients introduced. Therefore, a simplex (hexa-5-tope) in 5 dimensional spaces is characterized by the 6 distinct vectors that are its vertices. The method is a pattern search that compares function values at the 6 vertices of the hexa-5-tope. The worst vertex where the objective function is largest, is rejected and replaced with a new vertex. The step is repeated until the diameter of the simplex is less than the specified tolerance. We have thus:

$$\mathbf{x}^*_{om} = \mathbf{x}^*_{f} \tag{19}$$

The OM and the MM algorithms do not use parameters extraction (PE). PE is a delicate issue in SM algorithms [1] and may leads to poor results. Therefore OM and MM are more robust for multi-level optimizations.

Results and discussion

The three SM algorithms presented in this paper are applied for the optimization of a safety isolating transformer where a 3D FEA is the fine model that is expensive with the magneto-thermal coupling and a set of analytical equations belonging to the electric, magnetic and thermal lumped circuits is used as the coarse model which is fast but less accurate.

The results of ASM, MM, and OM algorithms are shown in Table 1 and compared with those of the single level optimization algorithm that is SQP. All algorithms stop when the maximal difference between the coarse and fine models is less than 0.1%. The solutions found by MM and OM are very close. The MM an OM techniques have accurate solutions with only three 3D FEA. The solution found by ASM after two 3D FE simulations is not good, because the total mass is higher than the one found by OM and MM.

In the case of ASM, the optimum depends extremely on the trust region. Therefore, an analysis of convergences for different trust regions (t.r.) is done in Fig. 2a. In Fig. 2, the maximum constraint is plotted with lines and values can be read from the right axis. The total mass (objective function) is plotted using histograms and the values can be read from the left axis. Among trust regions chosen based on the normalization of design variables (Fig. 2a), the t.r. = 0.015 with two 3D FEA gives the best solution with a minimal objective function (total mass) and all constraints are fulfilled (maximal constraint is 99.54%). The ASM algorithm with t.r. = 0.02 is not convergent. The one with t.r. = 0.0067 provides a solution with one more 3D FEA and a mass greater than the one with t.r. = 0.015. Therefore, the best solution of ASM algorithm (t.r. = 0.015) is kept for comparison with MM and OM in Table 1.

TABLE 1. RESULTS OF THE MULTI-LEVEL AND SINGLE-LEVEL OPTIMIZATIONS

Parameters		Single-level optimization	Multi-level optimizations		
		SQP	ASM	MM	OM
a	mm	14.94	12.99	12.98	12.98
b	mm	48.43	51.32	50.35	50.35
c	mm	15.64	16.71	16.57	16.56
d	mm	38.27	44.26	42.81	42.77
S_1	mm^2	0.310	0.3302	0.3302	0.3304
S_2	mm^2	2.787	2.962	2.969	2.972
n_1	-	611.78	645.67	631.48	630.78
M_{tot}	kg	2.294	2.410	2.306	2.304
Number of 3D FEA	-	44	2	3	3
T_{co}	°C	120.65	118.17	119.26	119.19
T_{ir}	°C	100.44	98.97	100.09	100.06
η	%	89.24	89.30	89.42	89.43
I_μ/I_1	%	10.09	7.23	9.75	9.85
$\Delta V_2/V_2$	%	7.60	7.79	7.58	7.55
f_1	%	50.06	49.73	50.00	50.00
f_2	%	50.09	49.77	50.00	50.00
Maximal constraint	%	100.90	99.54	100.09	100.06

Fig. 2b shows that the convergences of MM and OM are fast and the total masses decrease slightly. ASM with t.r. = 0.015 with only two 3D FEA converges faster than MM and OM. However, the total mass is much greater than the ones of MM and OM.

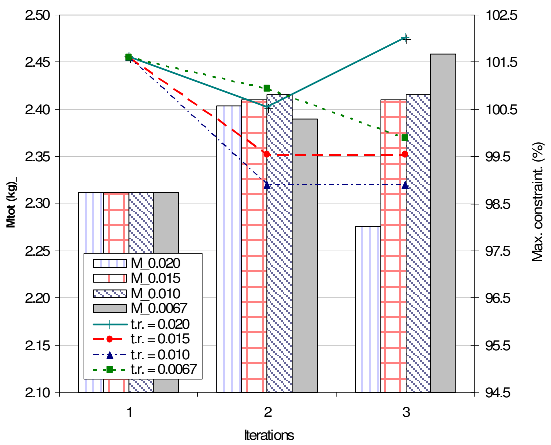

Fig. 2a. Convergences of ASM algorithm with different trust region

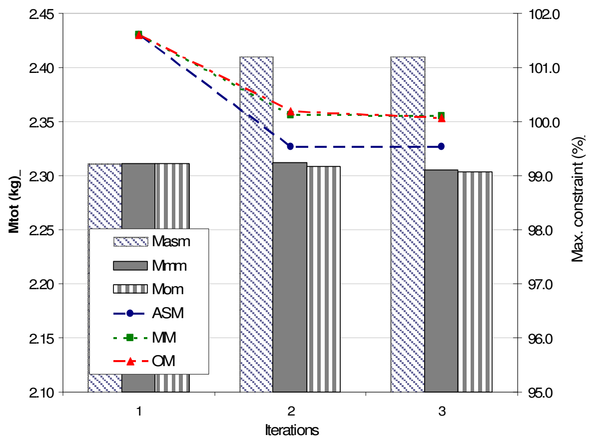

Fig. 2b. Convergences of ASM, MM and OM algorithms

The solution found with the single-level optimization is poor with a very expensive computational time. Indeed, 44 FEA are required to find a solution whose maximal constraint is 0.9% higher than permitted.

In the optimization of the safety transformer, it appears than multi-level optimization techniques using jointly a fine and a coarse model is very superior to single-level algorithms with a computation time 10 times smaller. Amongst the SM techniques, OM and MM give very good results.

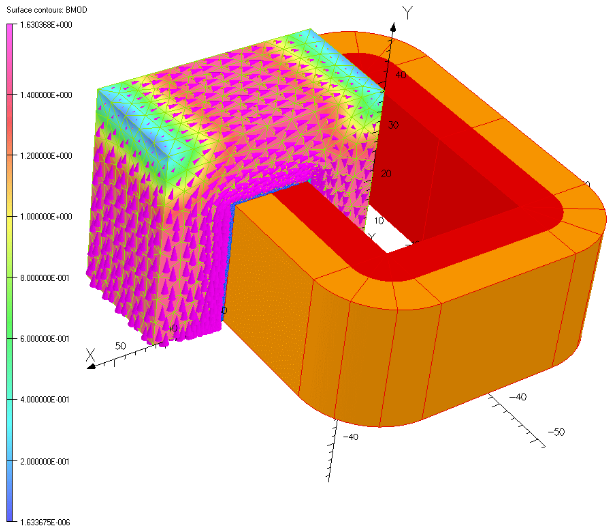

Fig. 3a. Flux density in the optimal transformer found by OM

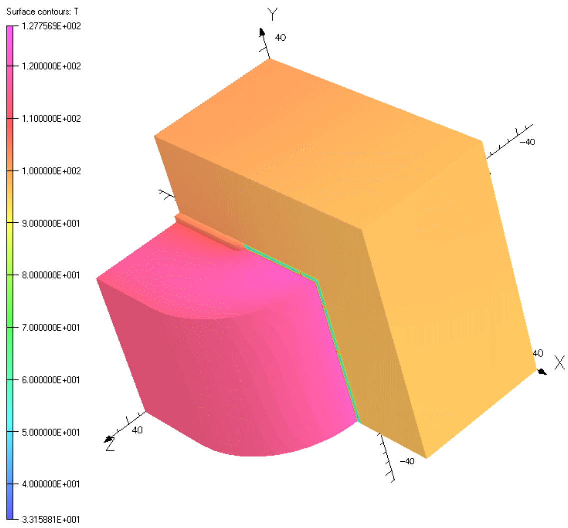

Fig. 3b. Temperatures in the optimal transformer found by OM

Conclusion

In order to speed-up the optimization of electrical devices, more than one model can be used. In this paper, two models are presented. The first one consists in a set of analytical equations belonging to the lumped circuits for the modelling of electric, magnetic and thermal phenomena. This model is fast but coarse. The second model is a 3D FEA with a magneto-thermal weak coupling. It is accurate (fine) but requires a big computation time.

Different space-mapping techniques are applied for the optimization of a safety isolating transformer. The first one is the aggressive space-mapping algorithm which leads to poor results. Output-mapping and manifold-mapping algorithms are more robust and gave good results using only three FEA.

The solution found with the single-level optimization is poor and required 44 FEA. In the optimization of the safety transformer, it appears than multi-level optimization techniques using jointly a fine and a coarse model are very superior to single-level algorithm with a computation time 10 times smaller.

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