Centrum voor Wiskunde en Informatica
Centre for Mathematics and Computer Science

R.D. Gill

Statistical investigation of weather conditions in the German Bight near Sylt, June-July 1984

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Statistical investigation of weather conditions in the German Bight near Sylt, June-July 1984

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Amsterdam

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1. Summary

The company Van Oord B.V. was engaged in beach-replenishment on the island of Sylt, Summer 1984. Their material suffered some damage during heavy storms in June 1984 which also delayed the work. Continuing bad weather in July 1984 prevented essential repair work and let to further damage and delay. The questions we address here are: could weather conditions of such severity reasonably have been foreseen? Were the weather conditions indeed of exceptional severity? We especially wish to quantify the answers to these questions.

We answer the questions by a statistical analysis of data on summer weather conditions in this area, confronting the data of 1984 with the data for a series of past years. Data are available from two weather stations: from List on the island of Sylt itself, but only for 15 years; and from the more distant station Elbe 1, but for a considerably longer series of 48 years (both numbers excluding 1984). The analysis is based on an overall or aggregate measurement of "severity of weather conditions" during the months June and July respectively. We quantify this concept as the amount of time there was wind at (Beaufort scale) force v or more ($v = 4$ to 9 incl.) from the sector West and North-West. This quantification is partly motivated by data-availability, but mainly in order to focus on the most relevant measurement of weather severity with respect to the operation of beach-replenishment. Concentration on a few simple measurements also allows a relatively simple statistical analysis.

We first show that, as one would expect, there is a very strong correlation between the List and Elbe 1 series of data. The further analysis is therefore mainly based on the longer Elbe 1 series. We show that the June 1984 weather conditions were indeed highly exceptional: weather of this severity has not been experienced for the past fifty years. Extrapolation techniques suggest a frequency of such severe conditions of less than once in a hundred years. The July 1984 conditions were also severe, and comparable to the worst in the last fifty years.

2. Available data

In the preliminary report [1] it is argued that beach-replenishment work on Sylt could not be continued at Westerly or North-Westerly wind of force 6 or more. The material being used could be expected to withstand a short period of force 8 wind from this sector, but not a longer continued period. On the other hand, repair work and routine maintenance cannot be carried out at force 4 wind or more. The weather conditions in June 1984 seemed to be of exceptional severity in all these respects. Although there were no heavy storms in July 1984 the weather conditions continued to be bad and the need to carry out essential repairs continually delayed work.

On the basis of the above considerations we decided that the severity of weather conditions in a certain period can be meaningfully measured as the total length of time in that period during which there is wind blowing at force v or more, $v = 4,6$ or 8, from the relevant sector. We added the values $v = 5,7$ and 9 to our analysis for completeness, but do not report on them here.

The available data [2] gives the distribution of wind strengths and directions over each month as

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follows:

for **List**: per Beaufort scale number and per 30° sector (12 sectors), for the years 1966-1980 (15 years) and 1984, in hours;

for **Elbe 1**: per Beaufort scale number and per 90° sector (4 sectors), for the years 1930-1939 and 1946-1983 (48 years) and 1984, in parts per thousand.

According to [1] the relevant sector is 225° to 345° ; this consists of four 30° sectors as registerd at List. For Elbe 1 we took the smaller "Sector West", 225° to 315°. The discrepancy of 30° is annoying. However we preferred to work with these differing sectors for the two series rather than constructing a 225° to 345° measurement for Elbe 1 by some interpolation technique, or taking the smaller "Sector West" for both series.

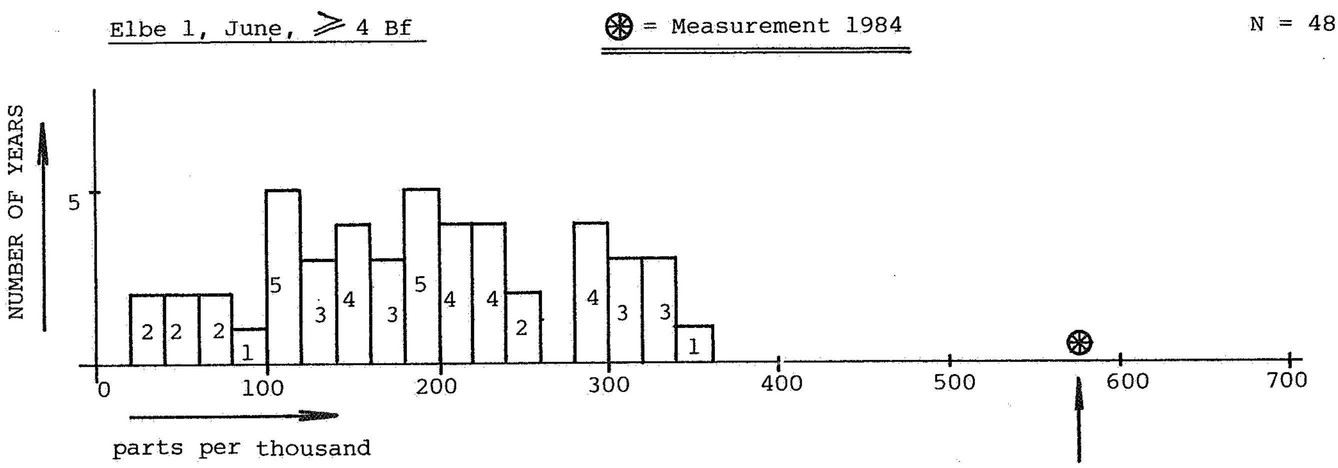
The List series was converted from "hours" to "parts per thousand" (p.p.th.). The data actually analysed is therefore, per month, and for the available years, the total amount of time (in p.p.th.) there was force v or more wind, for $v = 4, 5, 6, 7, 8$ and 9, from the sector 225° - 315° (Elbe 1) or 225° - 345° (List) (See "basic data set"). We concentrate below on $v = 4, 6$ and 8.

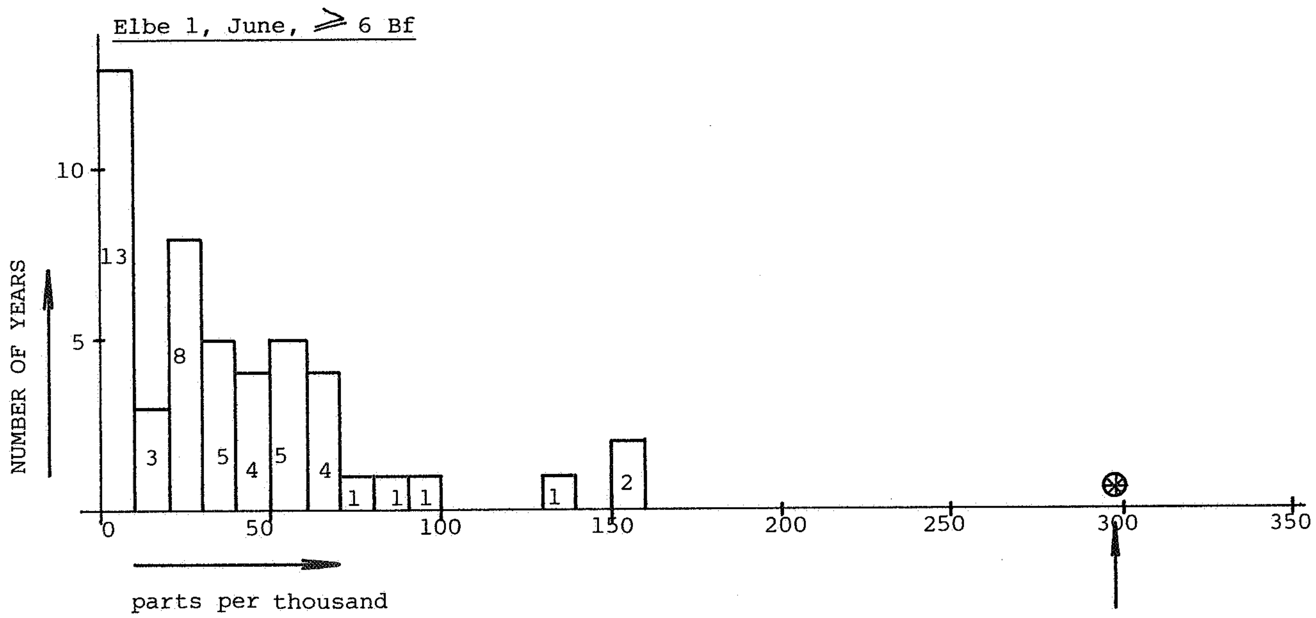
3. **Statistical analysis**

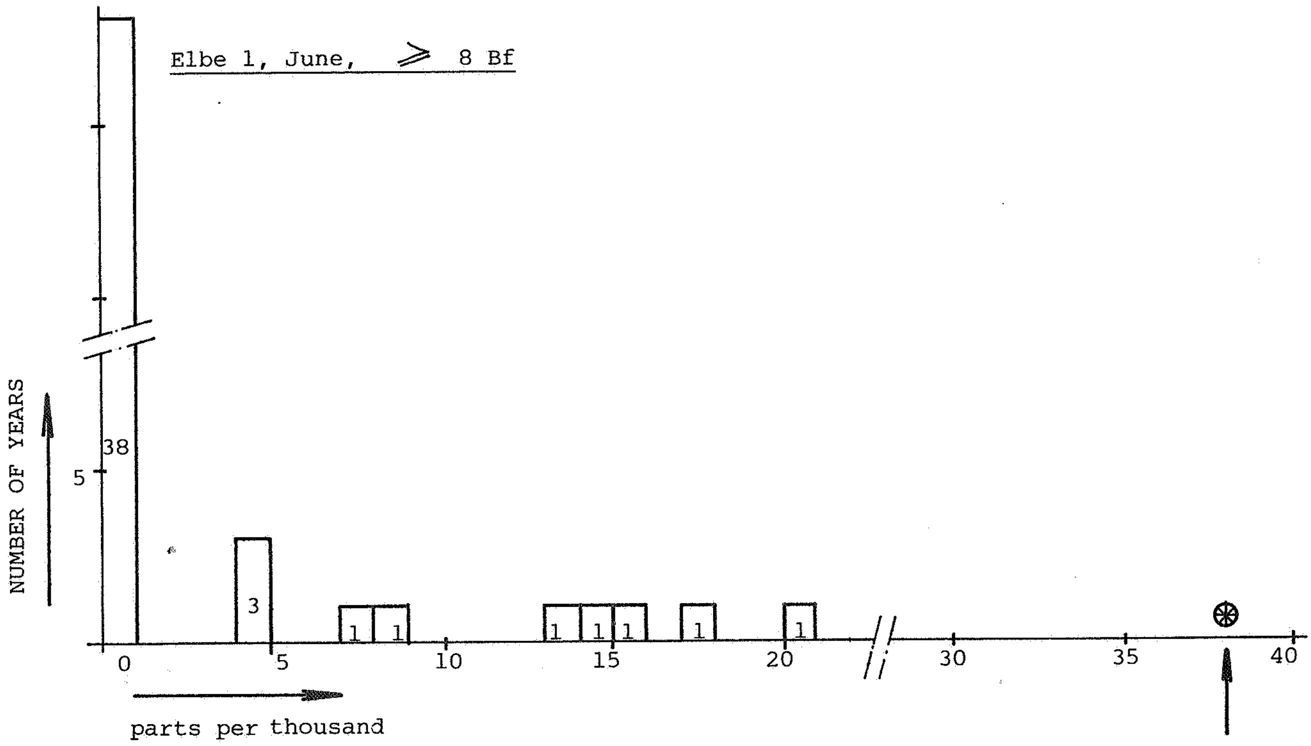
The first analysis consisted of a number of graphical and numerical summaries of the data. We present below, for the months June and July, and for $v = 4, 6, 8$:

- histograms of the series of observations (figures 1 to 12);
- scattergrams of Elbe 1 vs. List data based on the overlapping 15 years, plus 1984 (figures 13 to 18).

Also we give means and standard deviations (excluding 1984), the 1984 observations, and (Pearson) correlation between the corresponding Elbe 1 and List observations (excluding 1984). (Table 1)

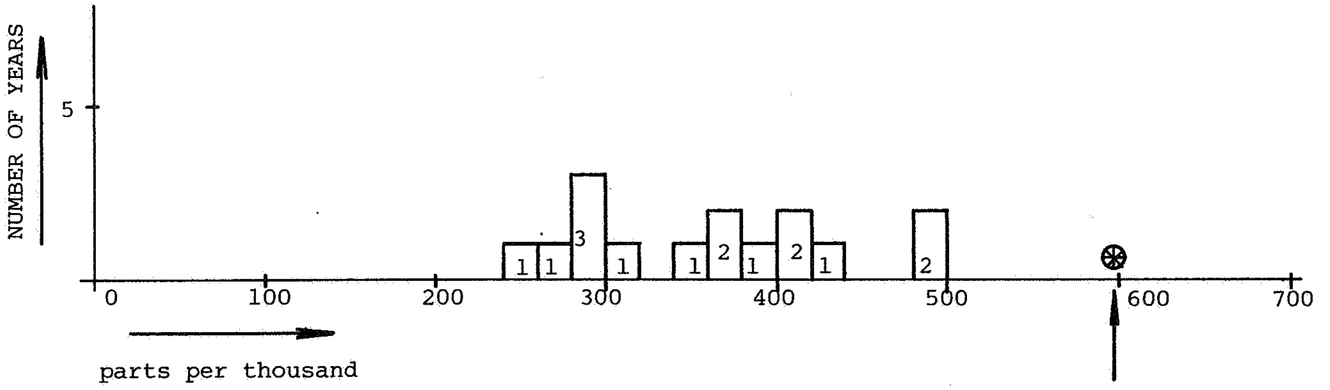




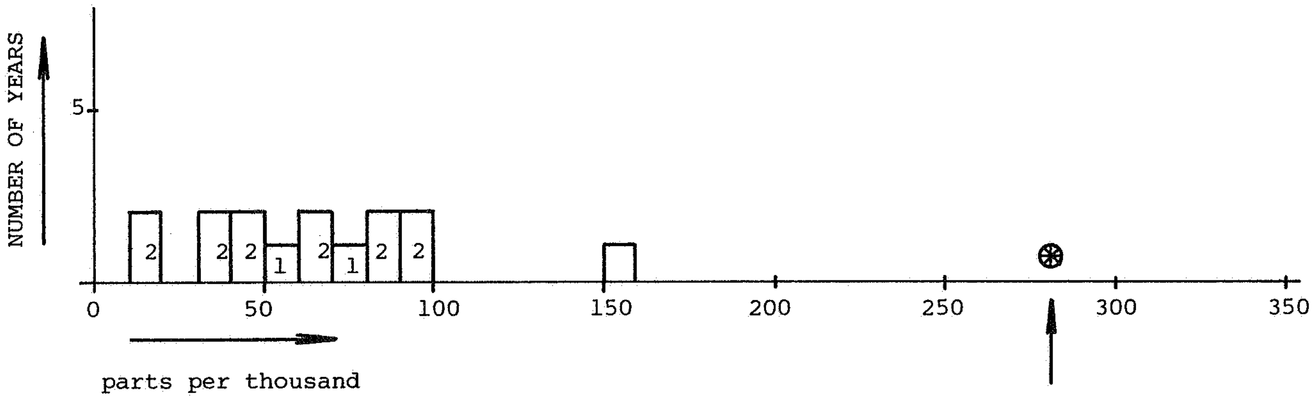


Figures 1 to 3 - Histograms

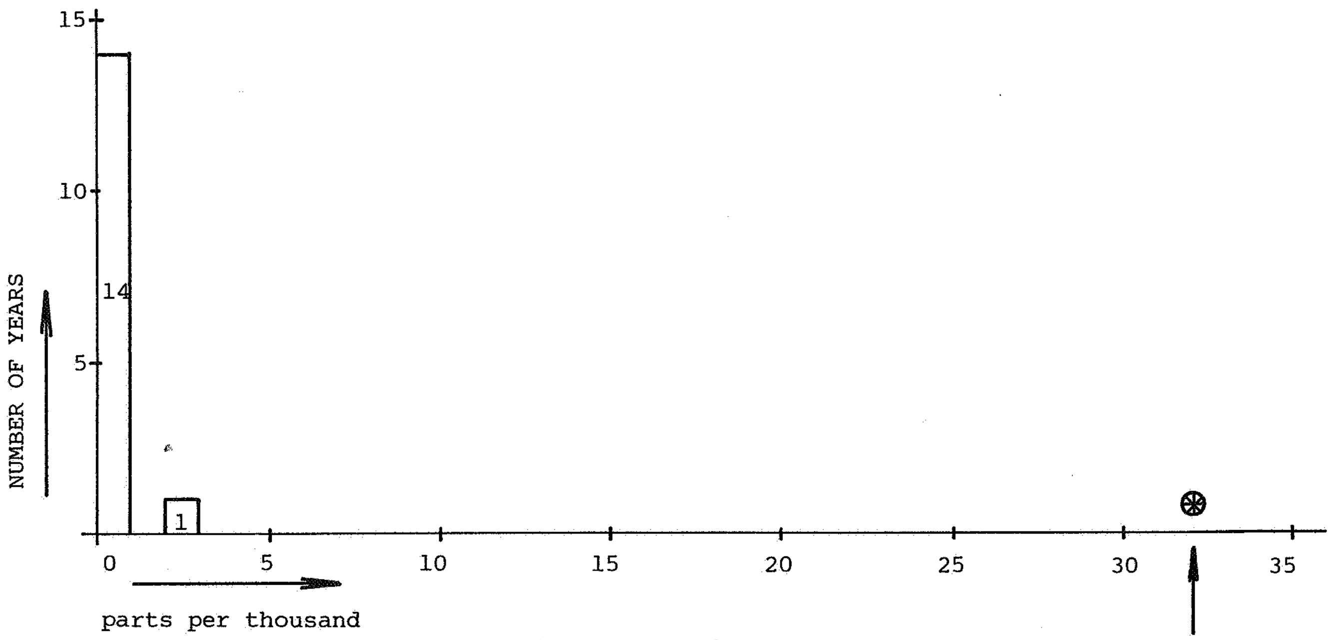
List, June, ≥ 4 Bf ⊛ = Measurement 1984 N = 15



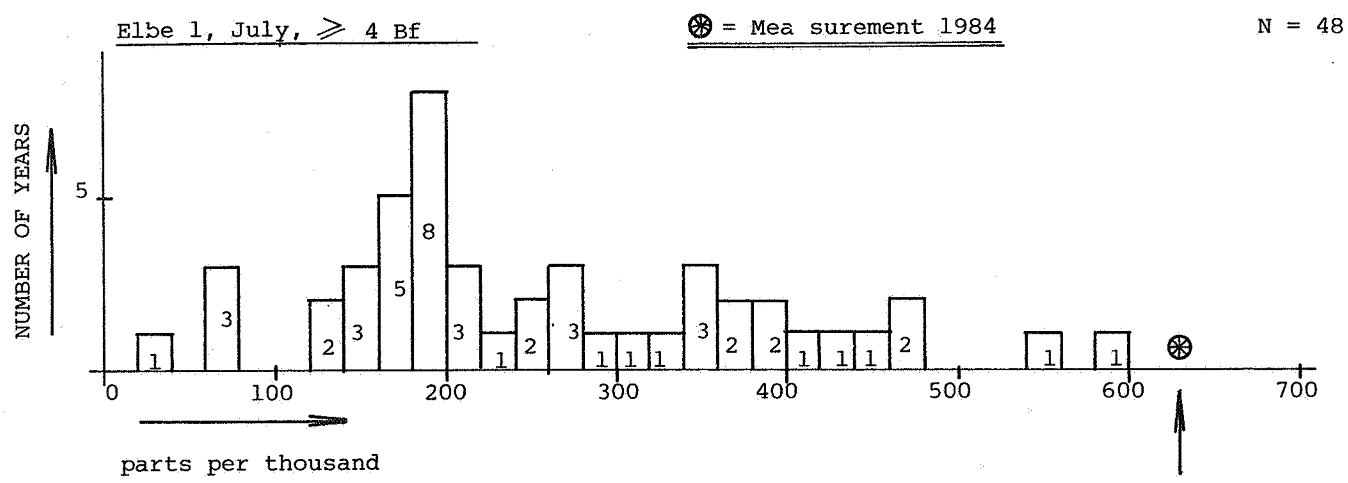
List, June, ≥ 6 Bf

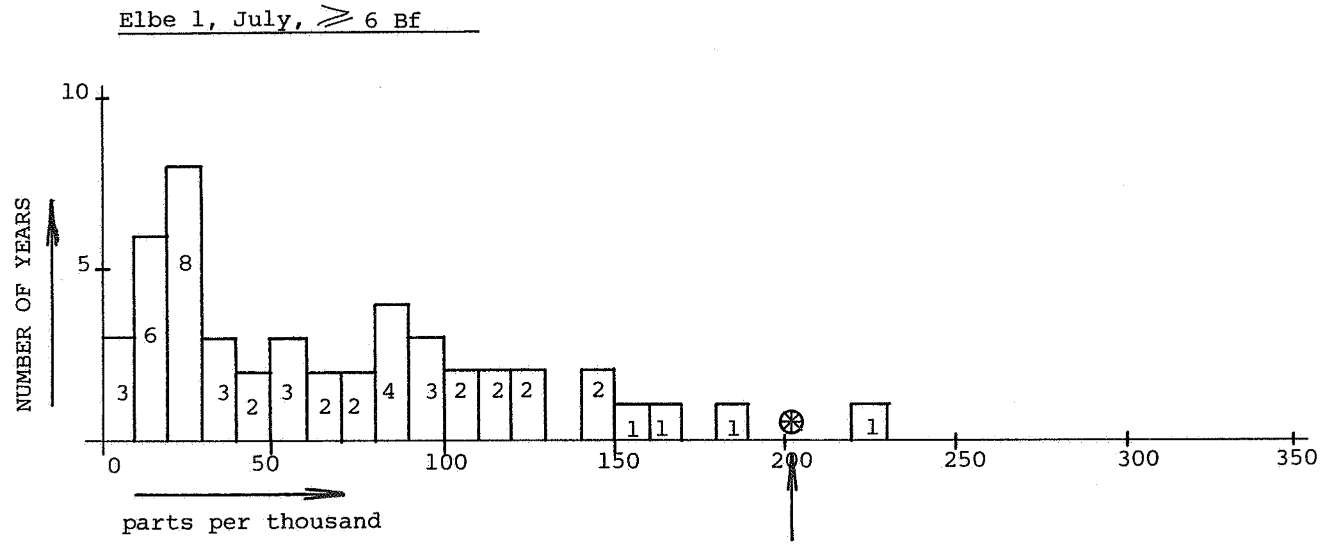


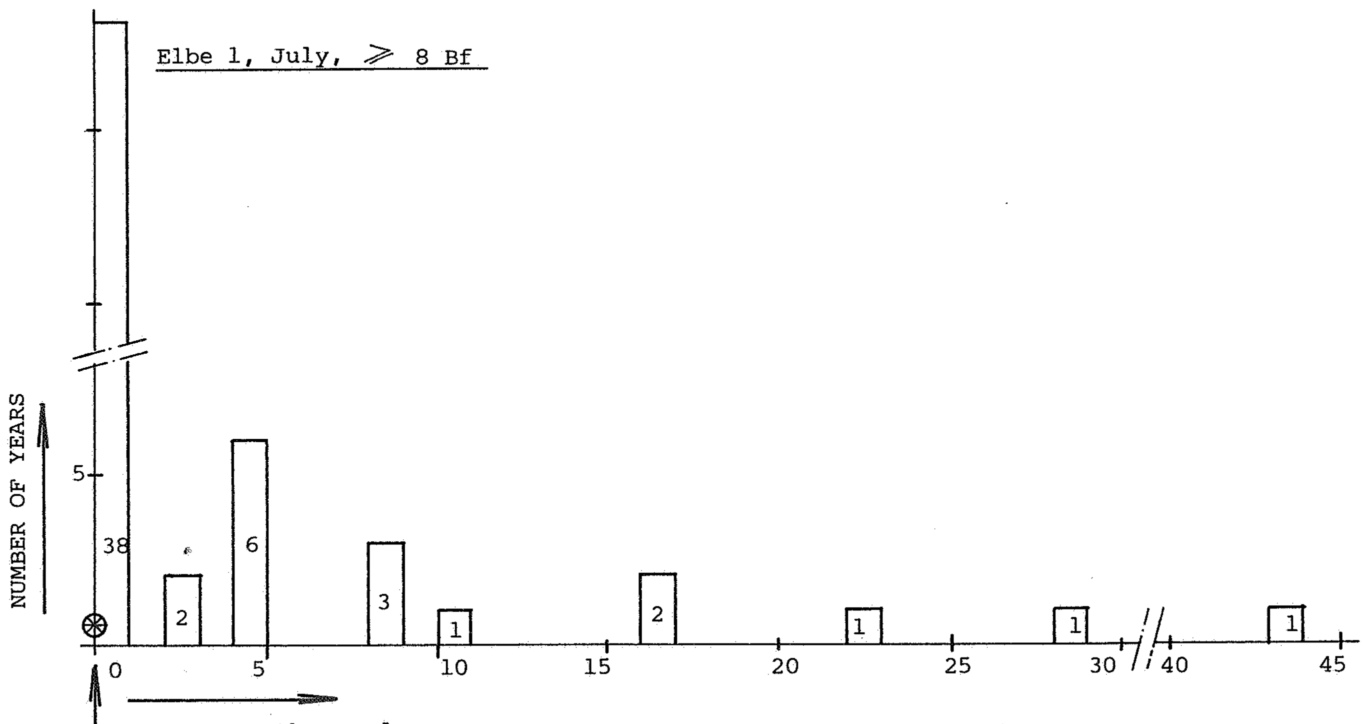
List, June, ≥ 8 Bf



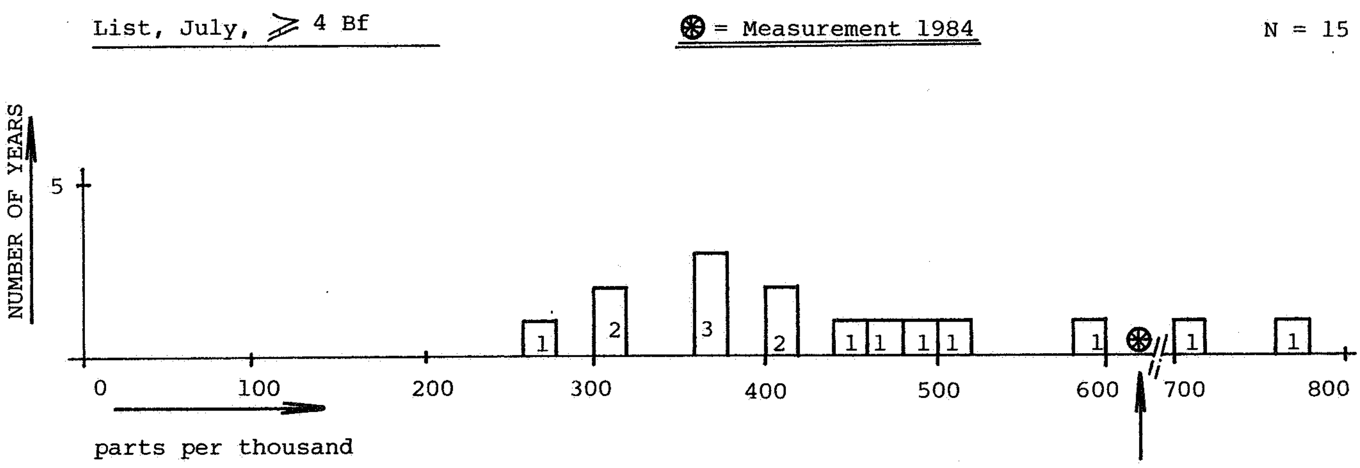
Figures 4 to 6 - Histograms

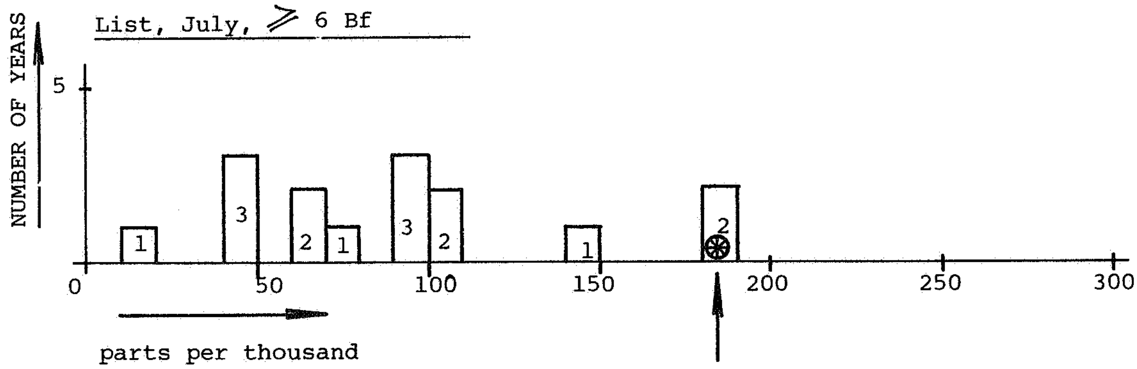


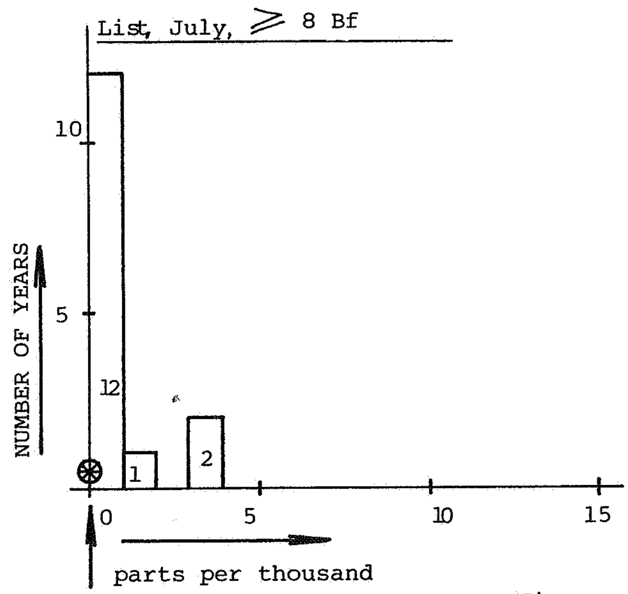




Figures 7 to 9 - Histograms







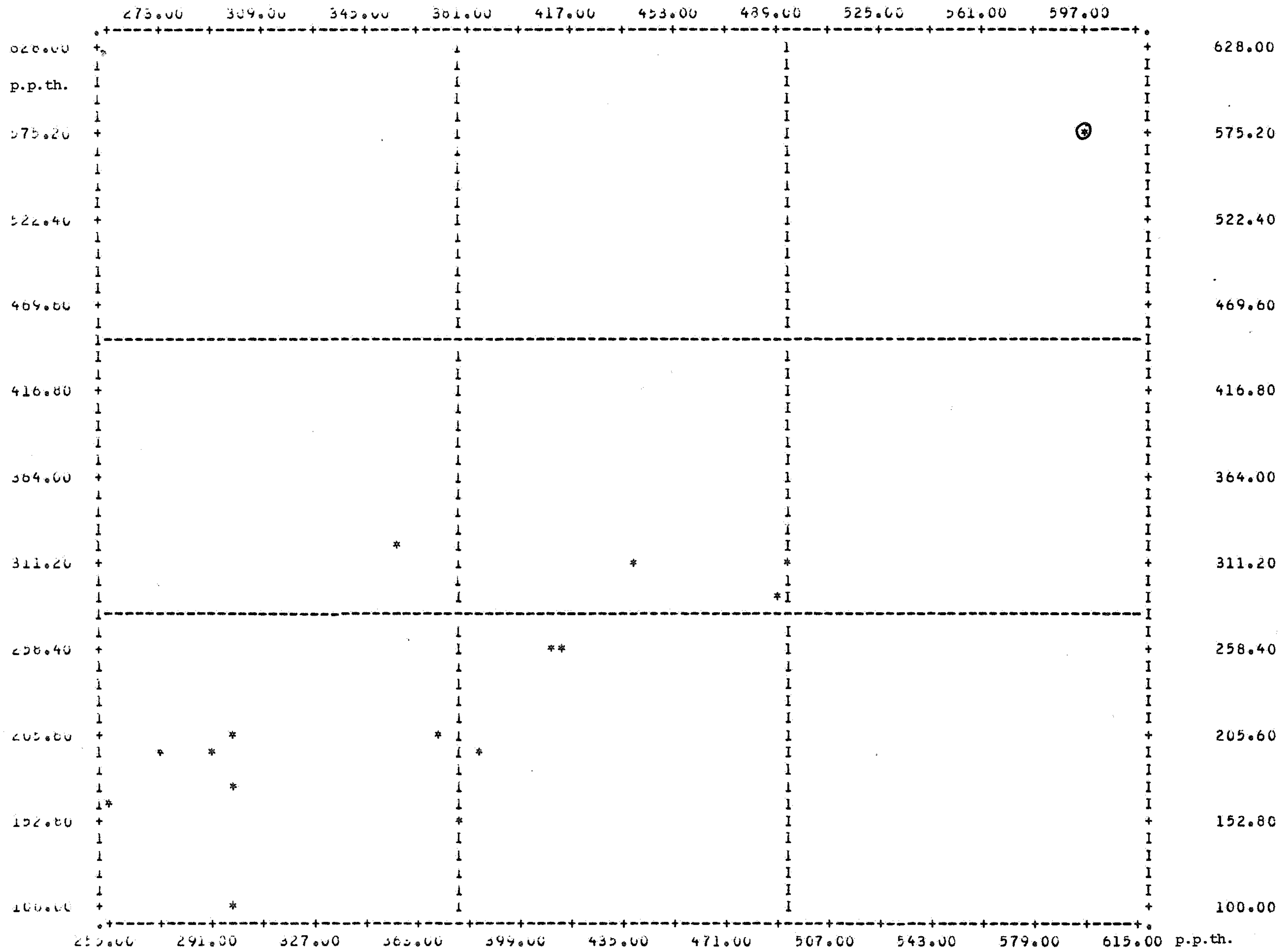
Figures 10 to 12 - Histograms

SCATTERGRAM JUNI (ELBE VS LIST)

Elbe vs. List, windforce ≥ 4 Bf, n = 15
June
1984 = ⊛

Figure 13

SCATTERGRAM OF (DOWN) ELBE4
(ACROSS) LIST4



SCATTERGRAM JUNI (ELBE VS LIST)

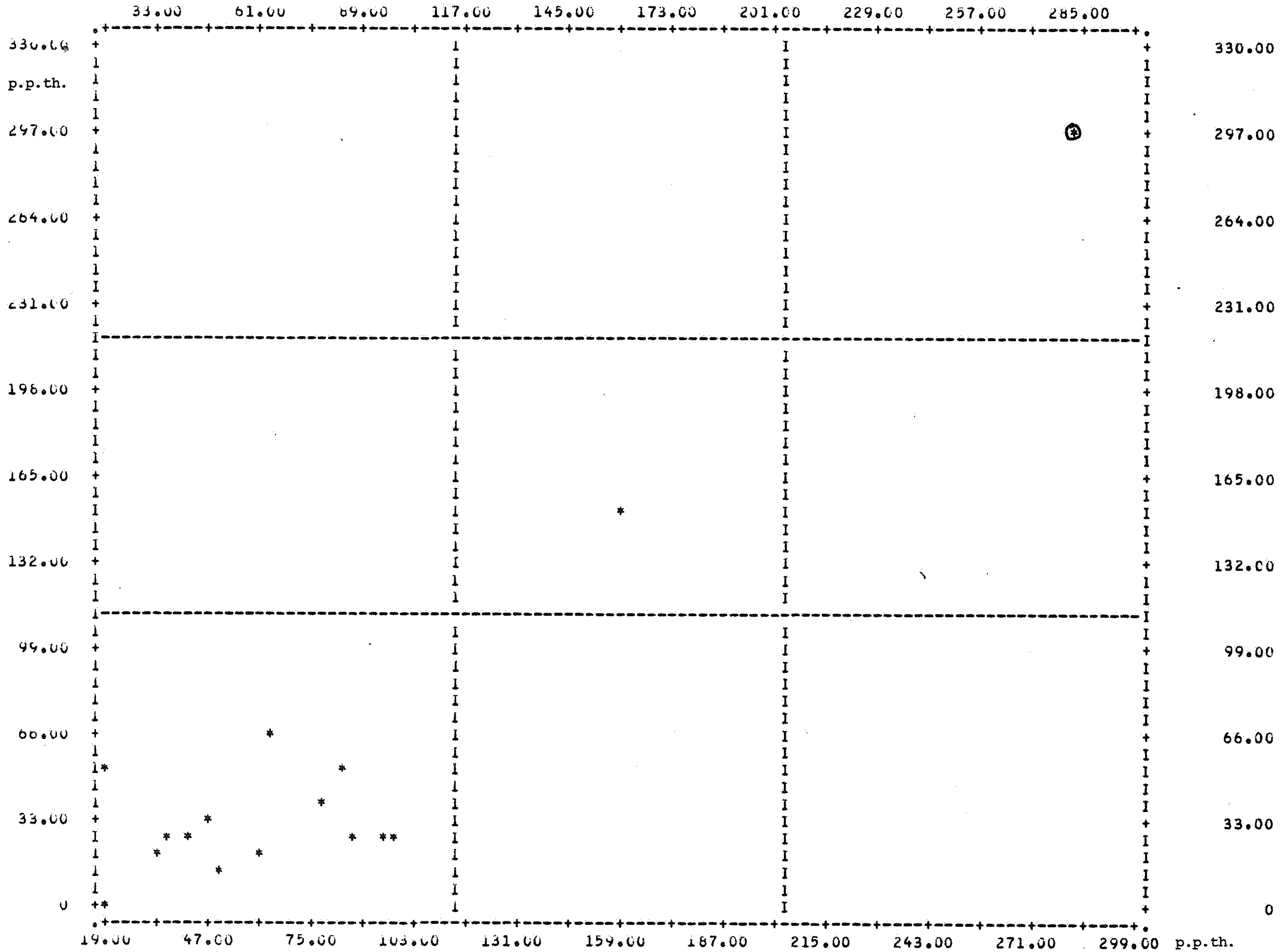
SCATTERGRAM OF (DOWN) ELBE6
(ACROSS) LIST6

Elbe vs. List, windforce ≥ 6 Bf, n = 15

June

1984 = ⊛

Figure 14

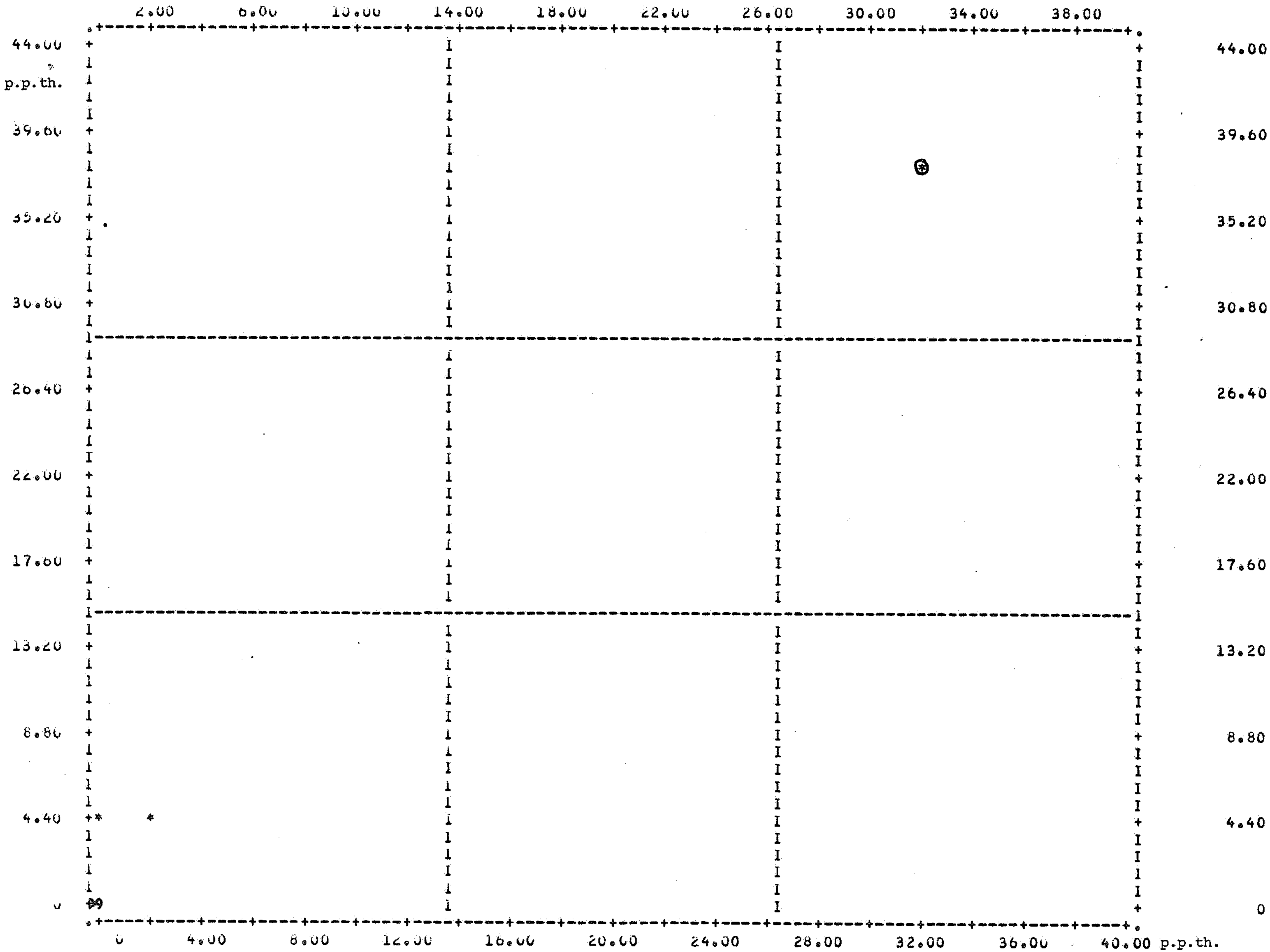


SCATTERGRAM JUNI (ELBE VS LIST)

SCATTERGRAM OF (DOWN) ELBE8
(ACROSS) LIST8

Elbe vs. List, windforce ≥ 8 Bf, n = 15
June
1984 = ⊛

Figure 15

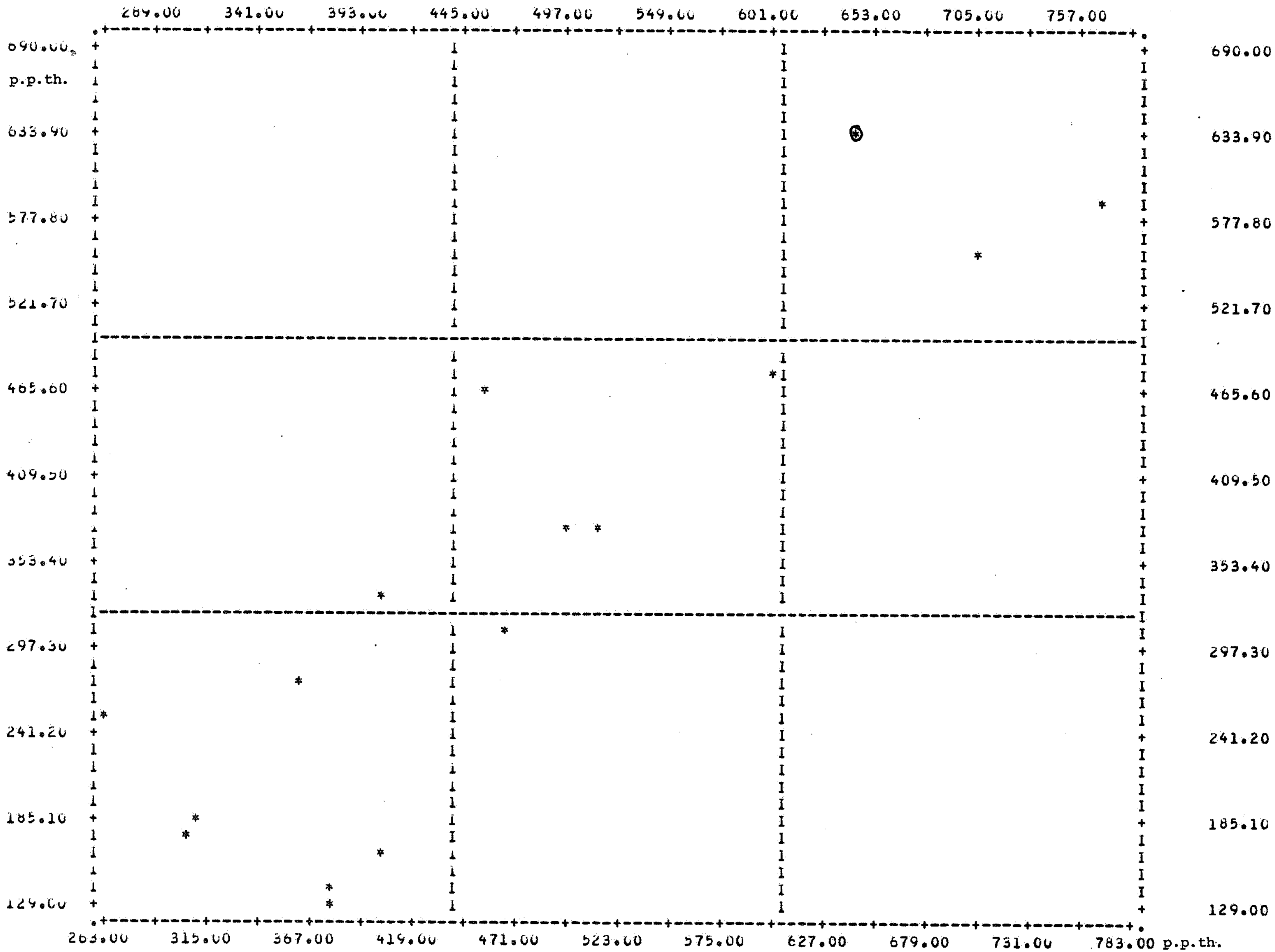


SCATTERGRAM JULI (ELBE VS LIST)

SCATTERGRAM OF (DOWN) ELBE4
(ACROSS) LIST4

Elbe vs. List, windforce ≥ 4 Bf, n = 15
July
1984 = ⊛

Figure 16

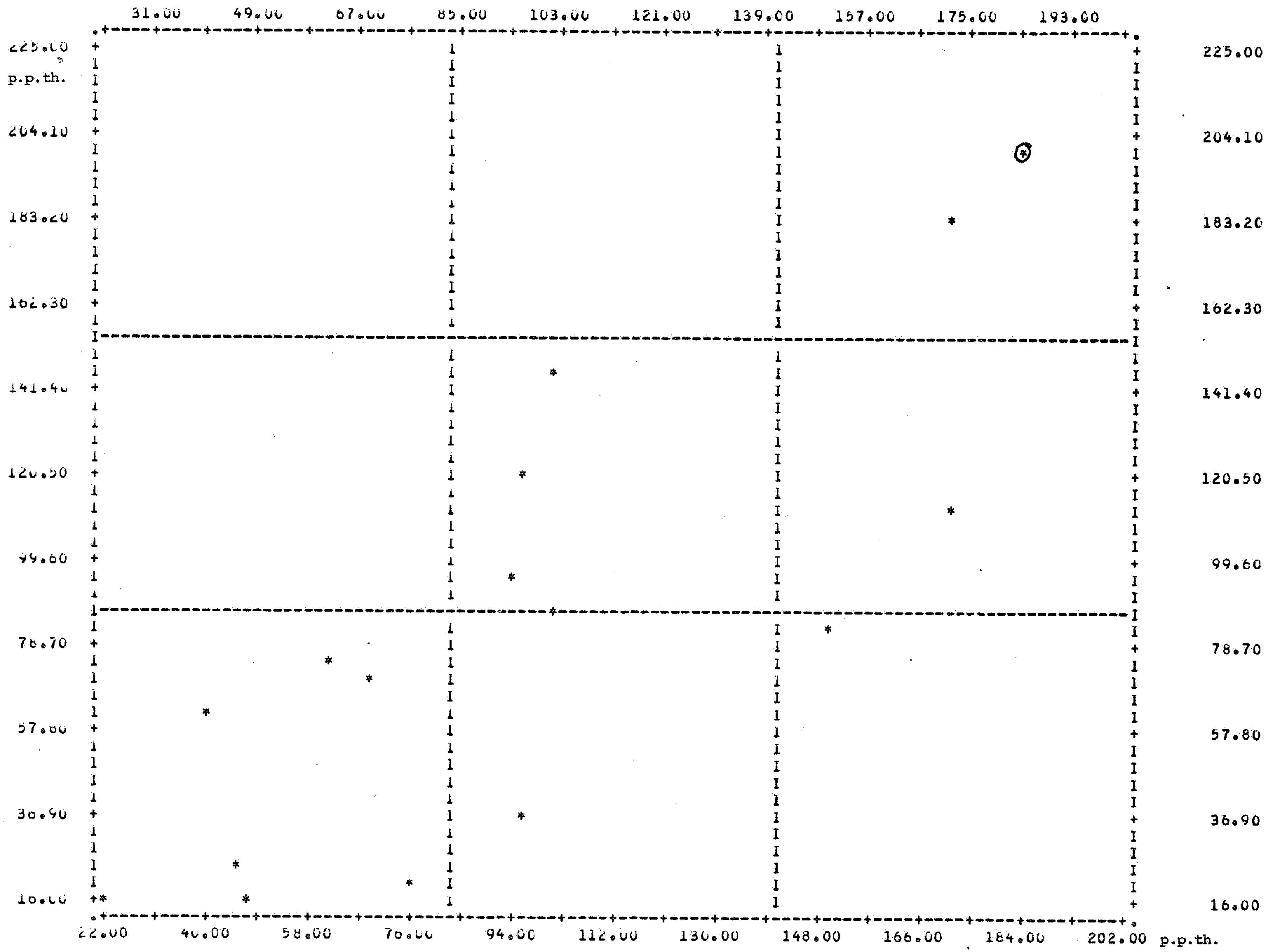


SCATTERGRAM JULI (ELBE VS LIST)

SCATTERGRAM OF (DOWN) ELBE6
(ACROSS) LIST6

Elbe vs. List, windforce ≥ 6 Bf, n = 15
July
1984 = ⊛

Figure 17

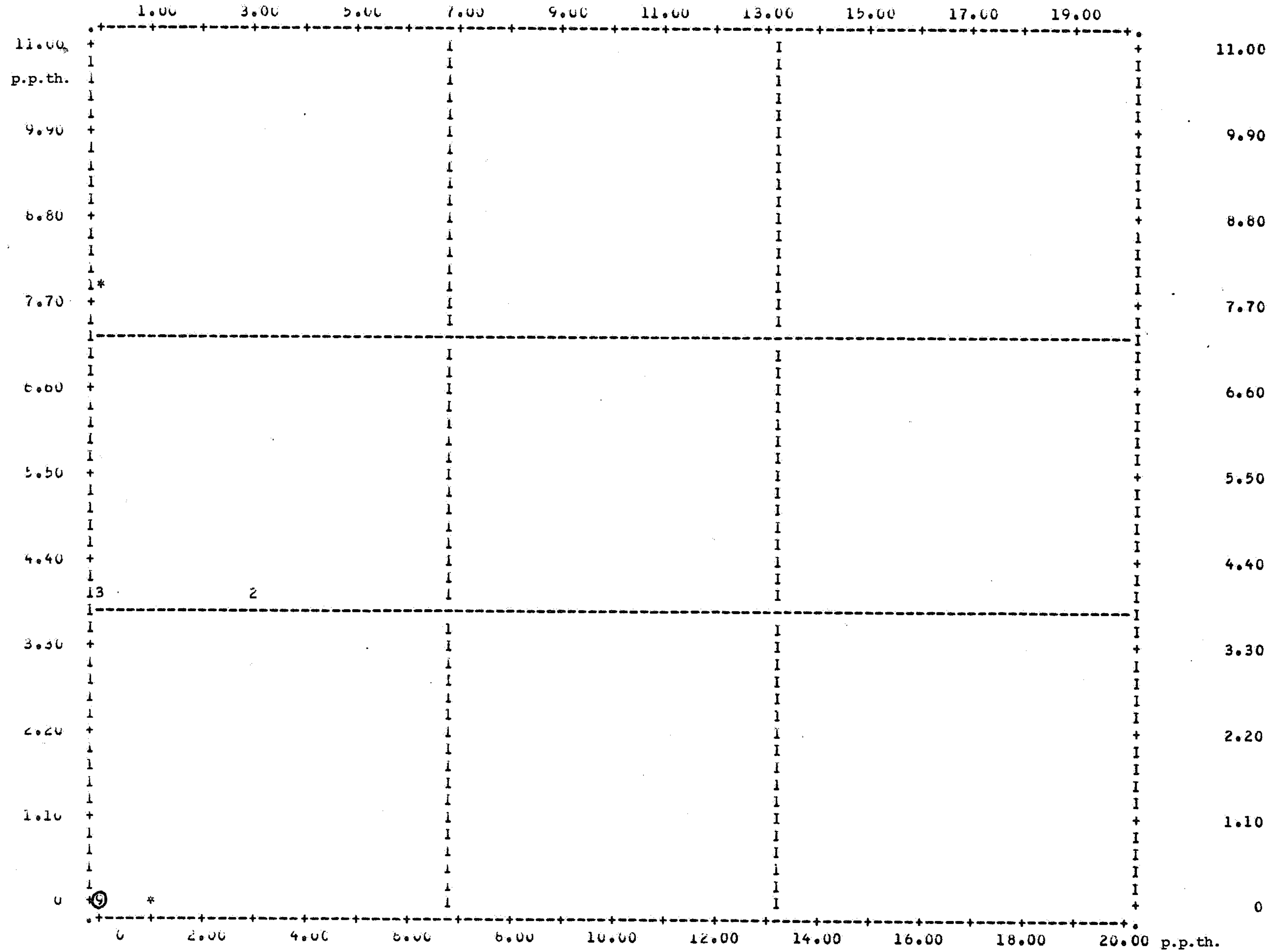


SCATTERGRAM JULI (ELBE VS LIST)

SCATTERGRAM OF (DOWN) ELBE8
(ACROSS) LIST8

Elbe vs. List, windforce ≥ 8 Bf, n = 15
July
1984 = (*)

Figure 18



ELBE 1, JUNE (n = 48)

	≥ 4	≥ 6	≥ 8
Mean	188.96	38.29	2.21
Standard deviation	88.16	38.11	5.07
Observation 1984	576	297	38

LIST, JUNE (n = 15)

	≥ 4	≥ 6	≥ 8
Mean	363.07	64.53	0.13
Standard deviation	75.88	36.98	0.52
Observation 1984	598	281	32

ELBE 1, JULY (n = 48)

	≥ 4	≥ 6	≥ 8
Mean	257.00	67.29	4.35
Standard deviation	128.20	53.41	8.86
Observation 1984	632	201	0

LIST, JULY (n = 15)

	≥ 4	≥ 6	≥ 8
Mean	454.13	89.33	0.47
Standard deviation	145.23	45.94	1.06
Observation 1984	642	184	0

CORRELATIONS (n = 15)

	≥ 4	≥ 6	≥ 8
June	.71	.69	.68
July	.87	.74	.29

Table 1: Summary statistics

From these results we draw the following conclusions:
1) the correlation between corresponding Elbe 1 and List measurements is very high;
2) the June 1984 measurements are strikingly larger than any observation in the corresponding June series;
3) the July 1984 measurements are about as large as the largest observations in the corresponding July series.

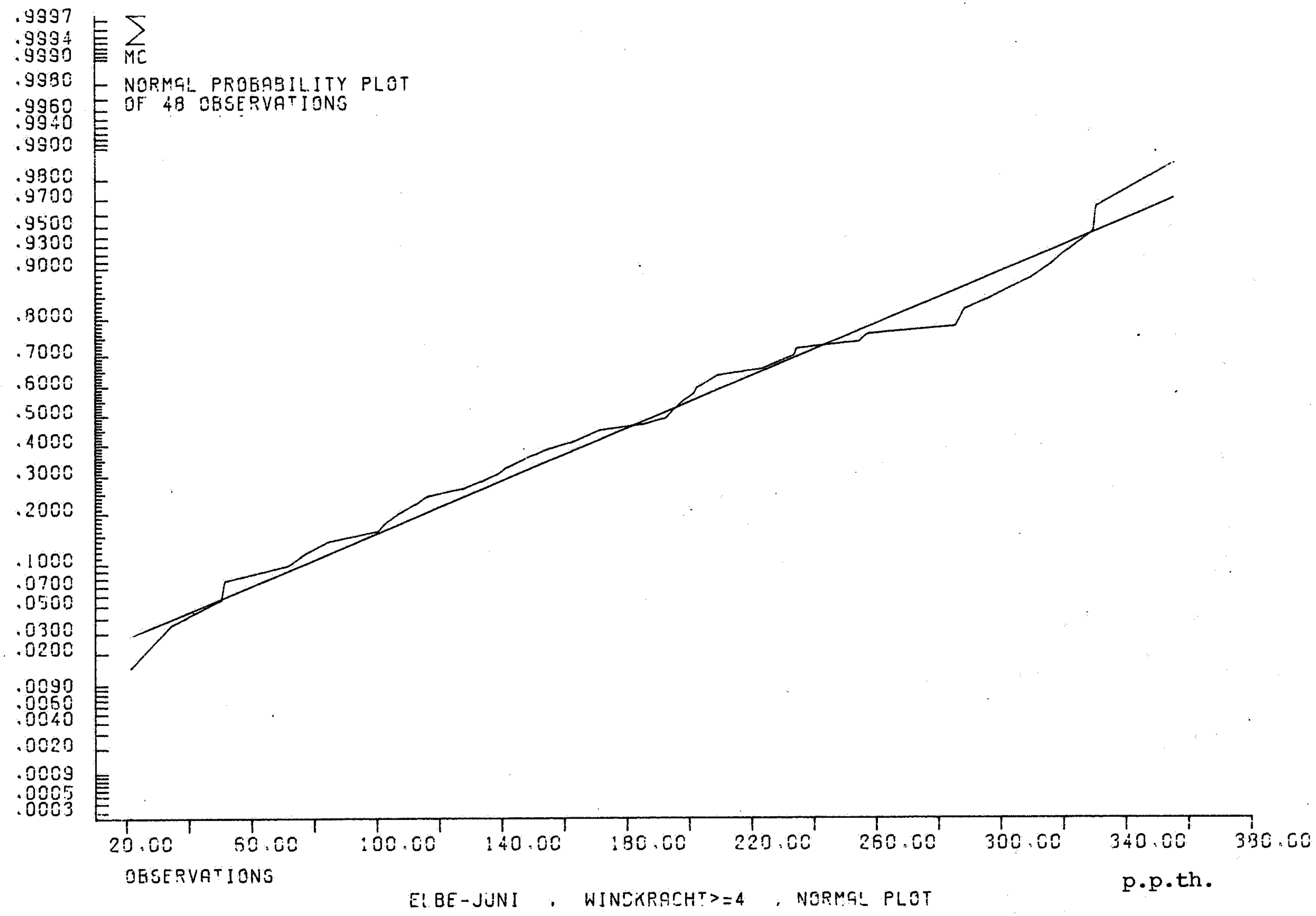
The only exception to this is the July $\geqslant 8$ measurement (and also the low July $\geqslant 8$ correlation).

In order to quantify the exteremeness of the June measurement we have to resort to extrapolation. A priori there is no special reason why the amount of time in the month there is $\geqslant$ force v wind from a certain sector should be distributed over the years according to any special probability distribution (e.g. normal, exponential, log-normal,...). We have therefore for simplicity and for an exploratory investigation chosen two simple and familiar distributions (normal and exponential) and constructed "probability plots" (see e.g. [3]) of the empirical distribution over the years of our measurement versus these theoretical distributions. From the plots one can read off, for each value of t p.p.th. on the horizontal scale, the relative frequency of observations less than or equal to t on the vertical scale. This is an estimate of the probability that a future measurement is less than or equal to t . More importantly, one can use the plots to check the assumption of an exponential or a normal distribution respectively: if the plot approximates a straight line (in the exponential case, passing through the origin), then the observations do appear to come from the corresponding distribution. A clear departure from a straight line is strong evidence against this assumption. Of course, the smaller the number of observations, the more sample variability obscures the picture.

A complicating factor in the present application is that at larger values of v , in an increasing number of years our measurement takes exactly the value zero. So we have made probability plots of the non-zero measurements only (a decreasing number of observations as v increases). The statistical model we are investigating is therefore: with a certain probability the measurement of wind occurrence is non-zero; conditional on being non-zero it is normally or exponentially distributed.

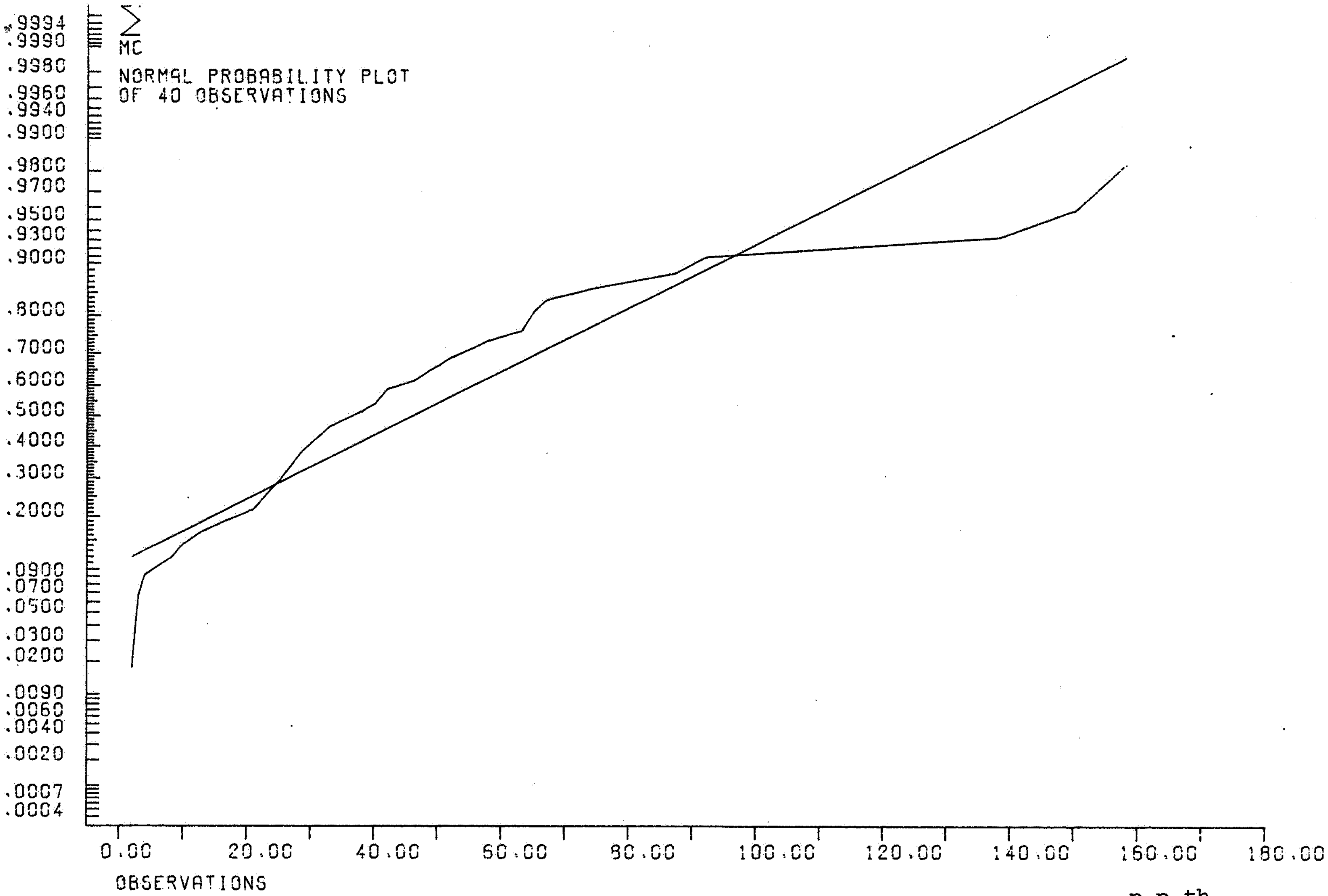
We present below the normal and exponential plots for $v \geqslant 4$, $v \geqslant 6$ and $v \geqslant 8$, the months June and July, but for the Elbe 1 data only (figures 19 to 30). The plots for the intermediate wind strengths, and for the List data, fit into the pattern we describe in a moment (except that due to smaller sample series, the List data is less conclusive).

FIGURE 19



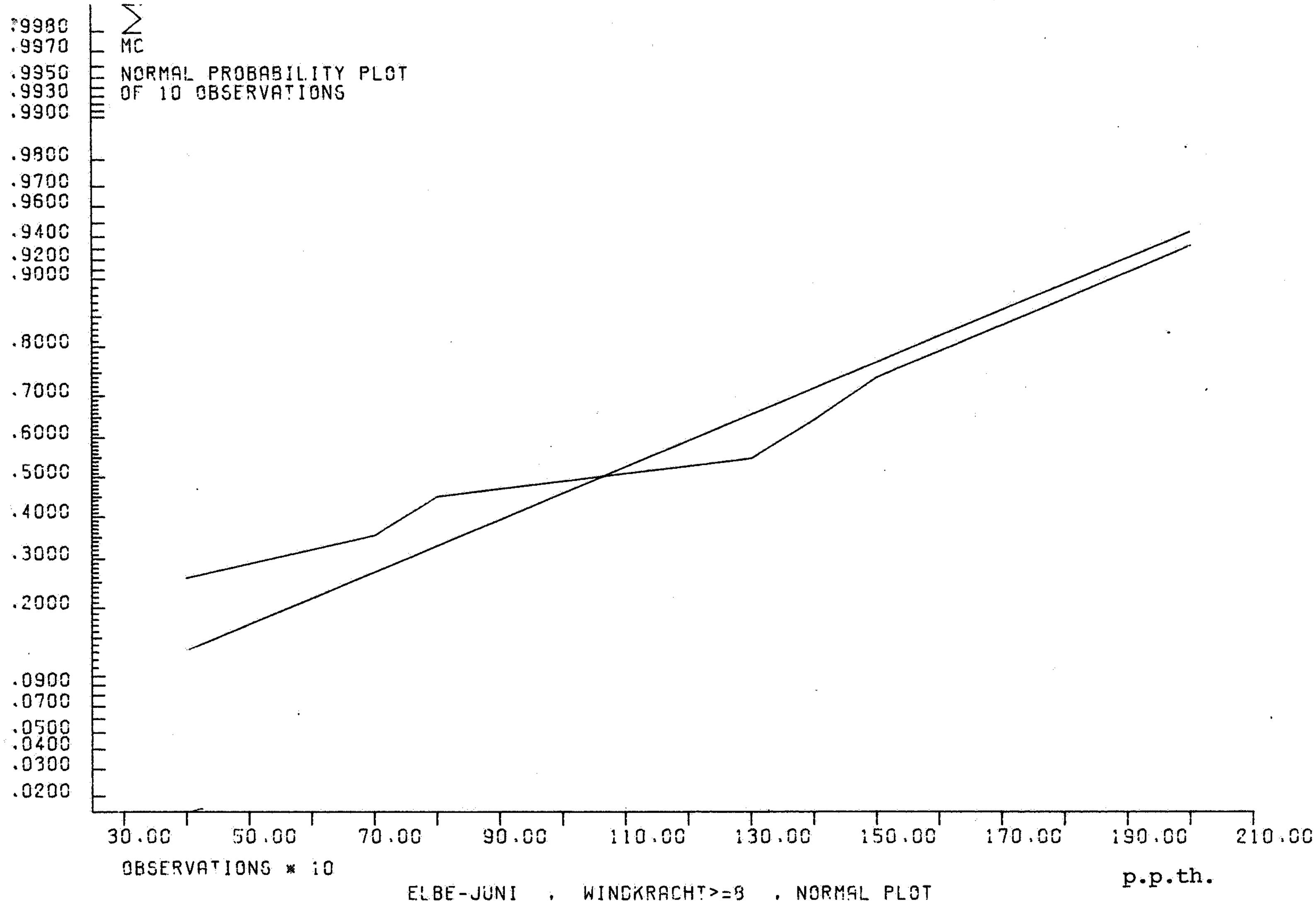
MC
NORMAL PROBABILITY PLOT
OF 48 OBSERVATIONS
.9997
.9994
.9990
.9980
.9960
.9940
.9900
.9800
.9700
.9500
.9300
.9000
.8000
.7000
.6000
.5000
.4000
.3000
.2000
.1000
.0700
.0500
.0300
.0200
.0090
.0060
.0040
.0020
.0009
.0005
.0003
20.00
60.00
100.00
140.00
180.00
220.00
260.00
300.00
340.00
380.00
OBSERVATIONS
p.p.th.
ELBE-JUNI , WINDKRACHT>=4 , NORMAL PLOT

FIGURE 20



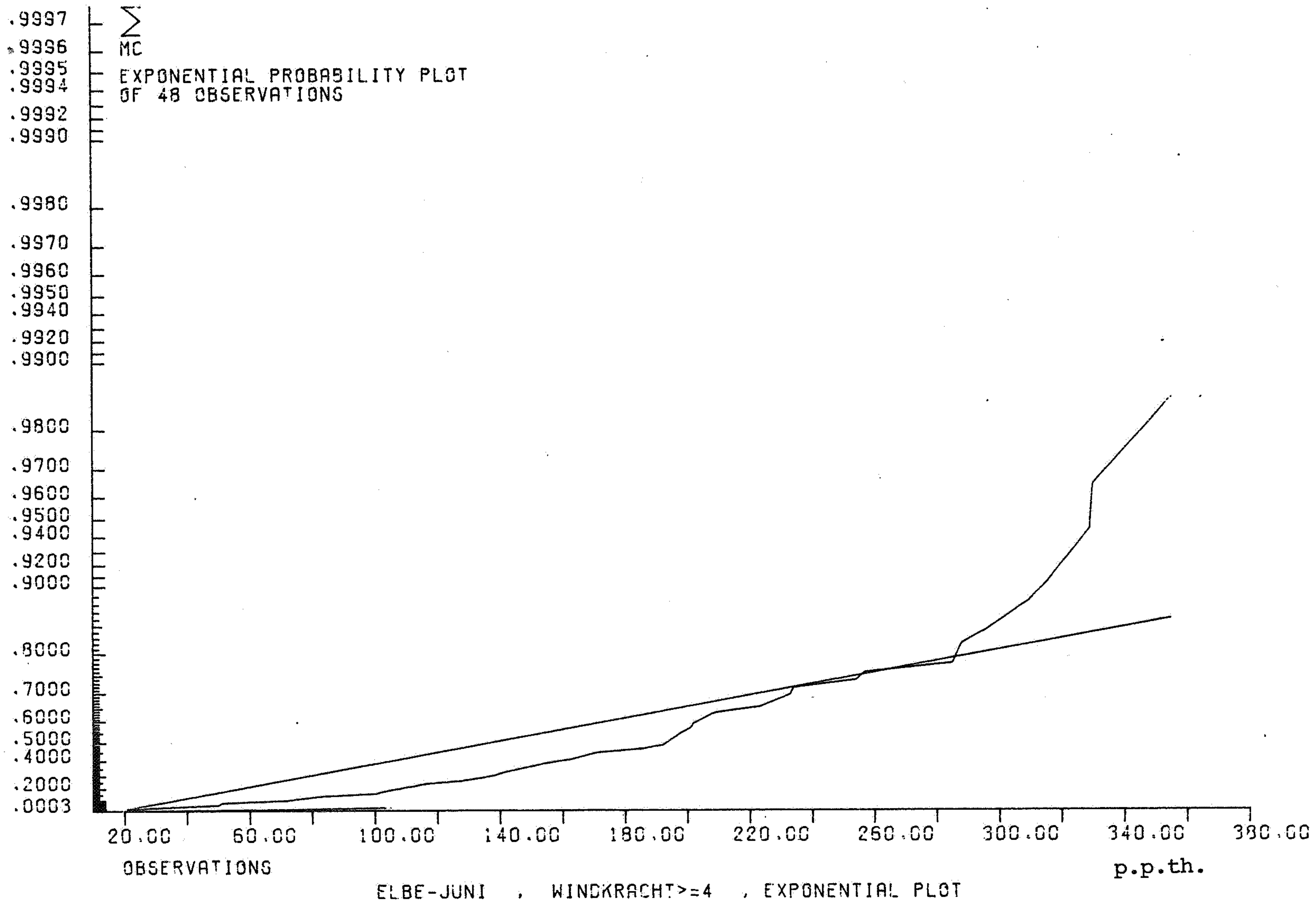
ELBE-JUNI , WINDKRACHT>=6 , NORMAL PLOT

FIGURE 21



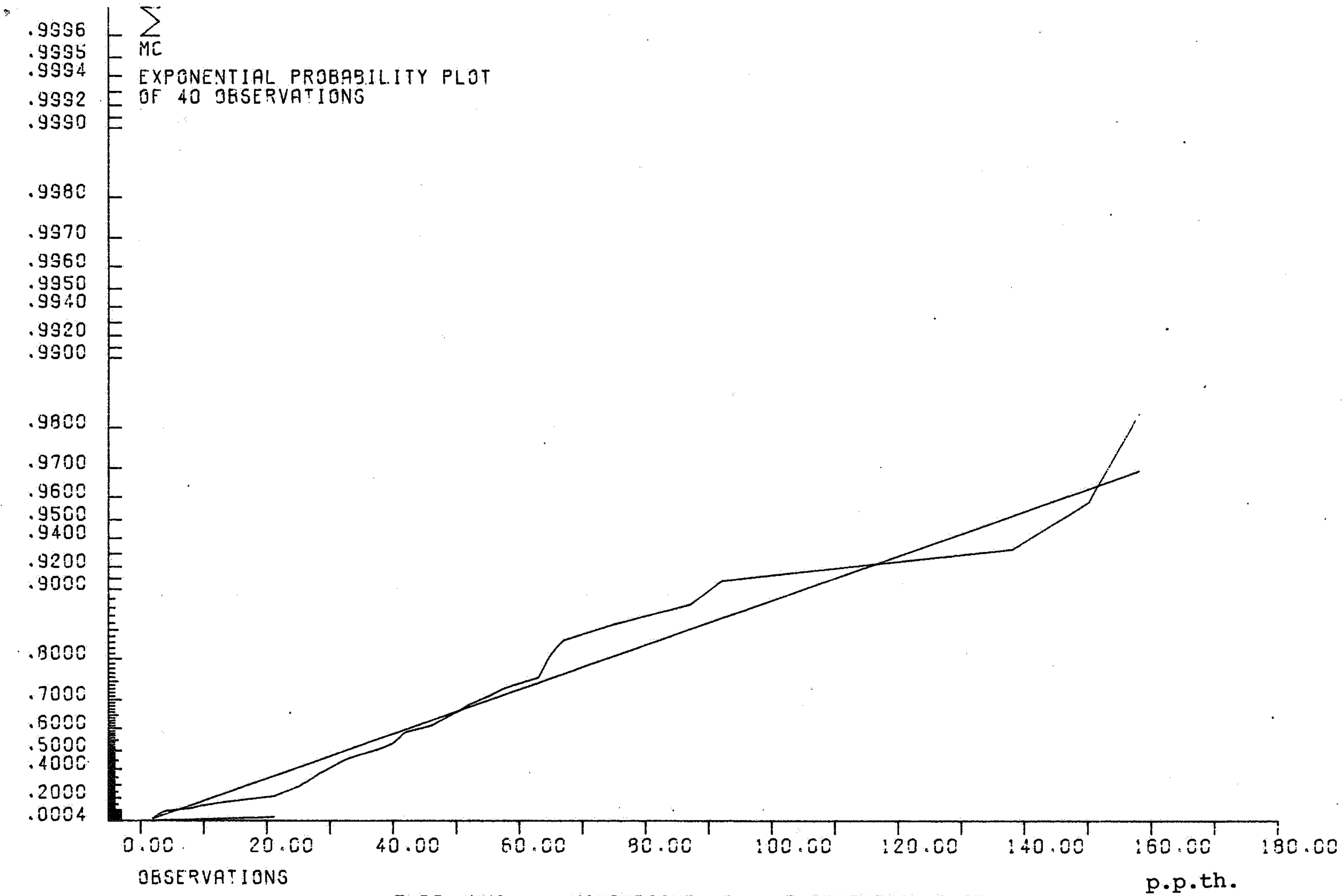
ELBE-JUNI , WINDKRACHT>=8 , NORMAL PLOT

FIGURE 22



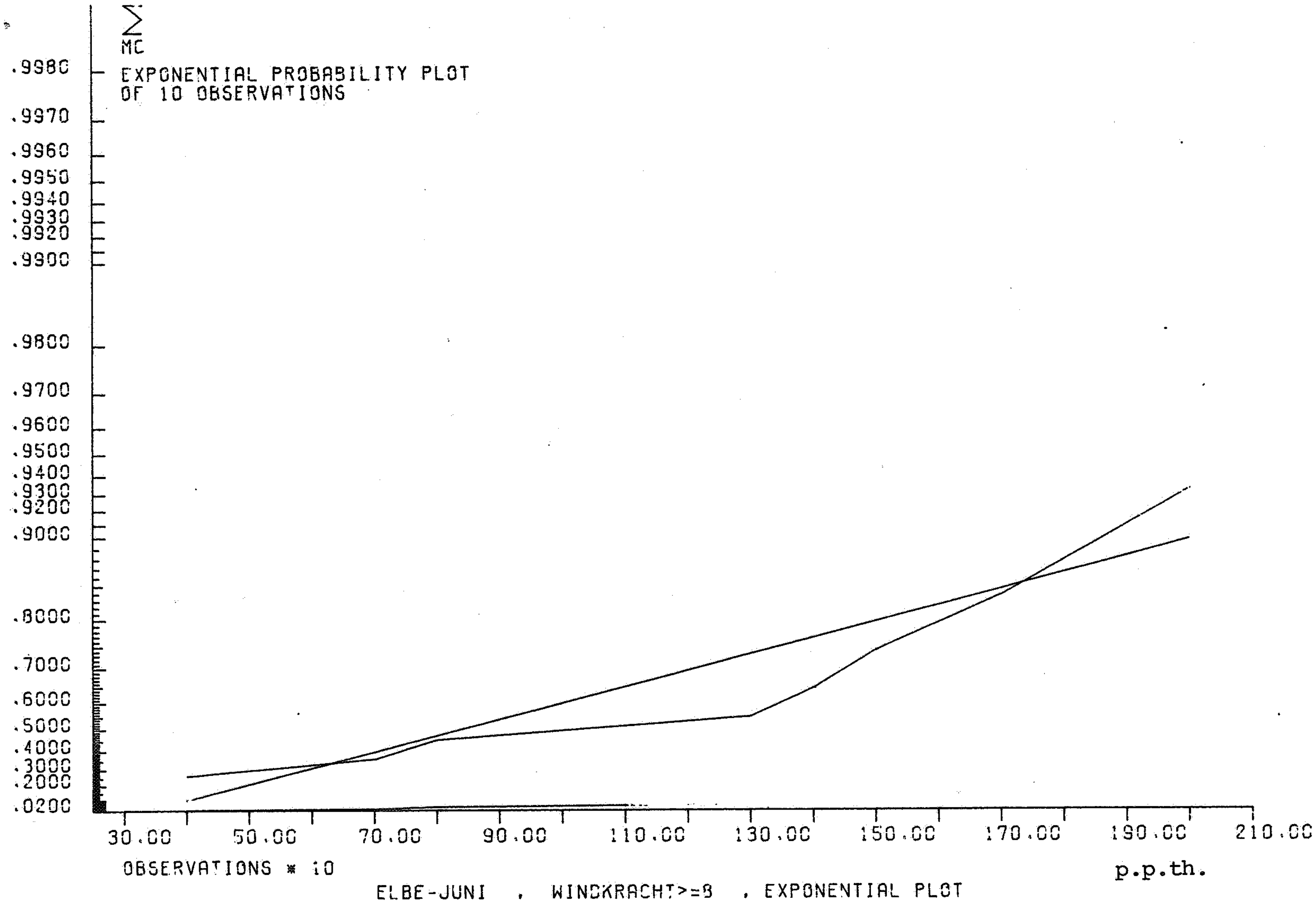
∑
MC
EXPONENTIAL PROBABILITY PLOT
OF 48 OBSERVATIONS
.9997
.9996
.9995
.9994
.9992
.9990
.9980
.9970
.9960
.9950
.9940
.9920
.9900
.9800
.9700
.9600
.9500
.9400
.9200
.9000
.8000
.7000
.6000
.5000
.4000
.2000
.0003
20.00
60.00
100.00
140.00
180.00
220.00
260.00
300.00
340.00
380.00
OBSERVATIONS
p.p.th.
ELBE-JUNI , WINDKRACHT>=4 , EXPONENTIAL PLOT

FIGURE 23



ELBE-JUNI , WINDKRACHT>=6 , EXPONENTIAL PLOT

FIGURE 24



∑
MC
EXPONENTIAL PROBABILITY PLOT
OF 10 OBSERVATIONS
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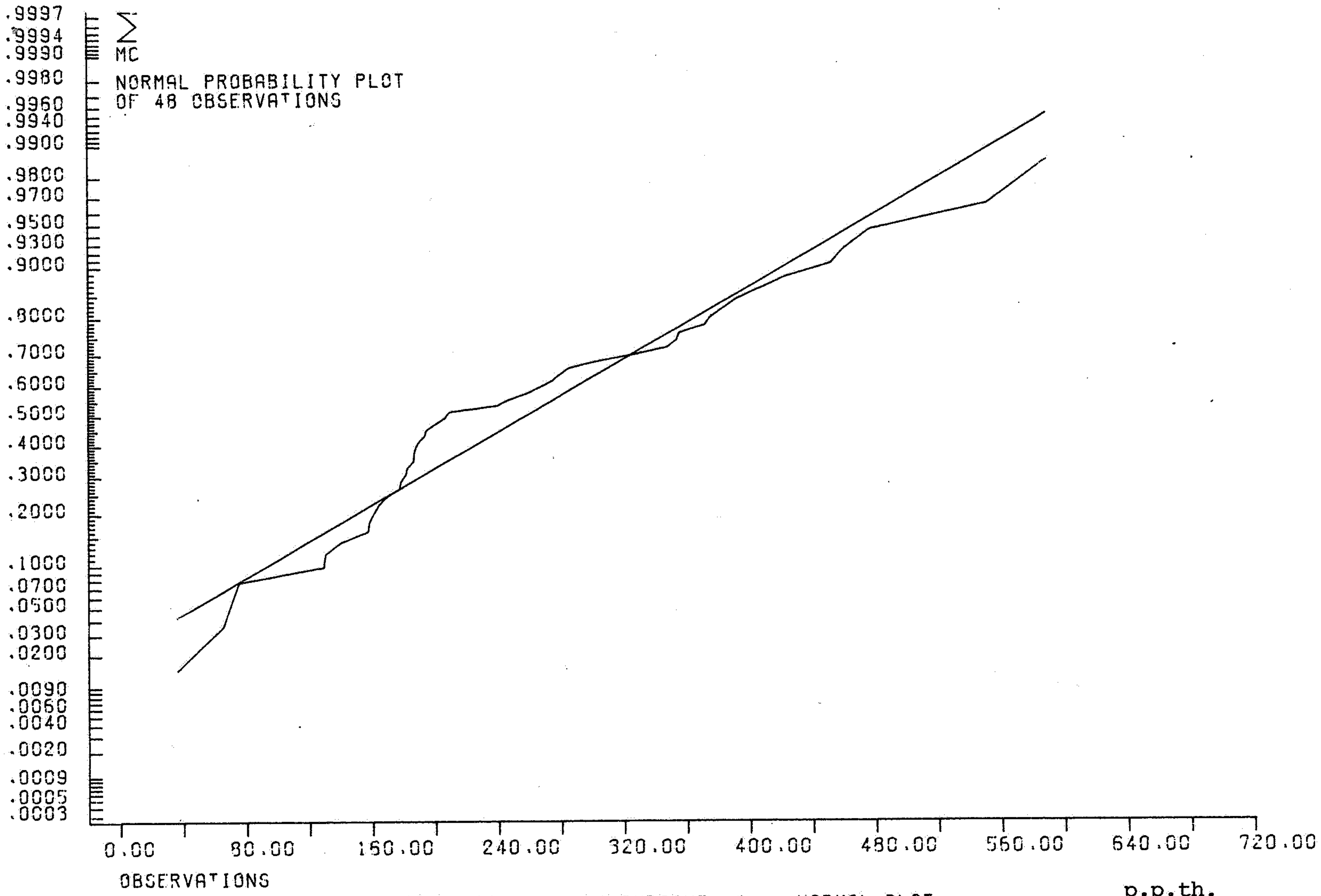
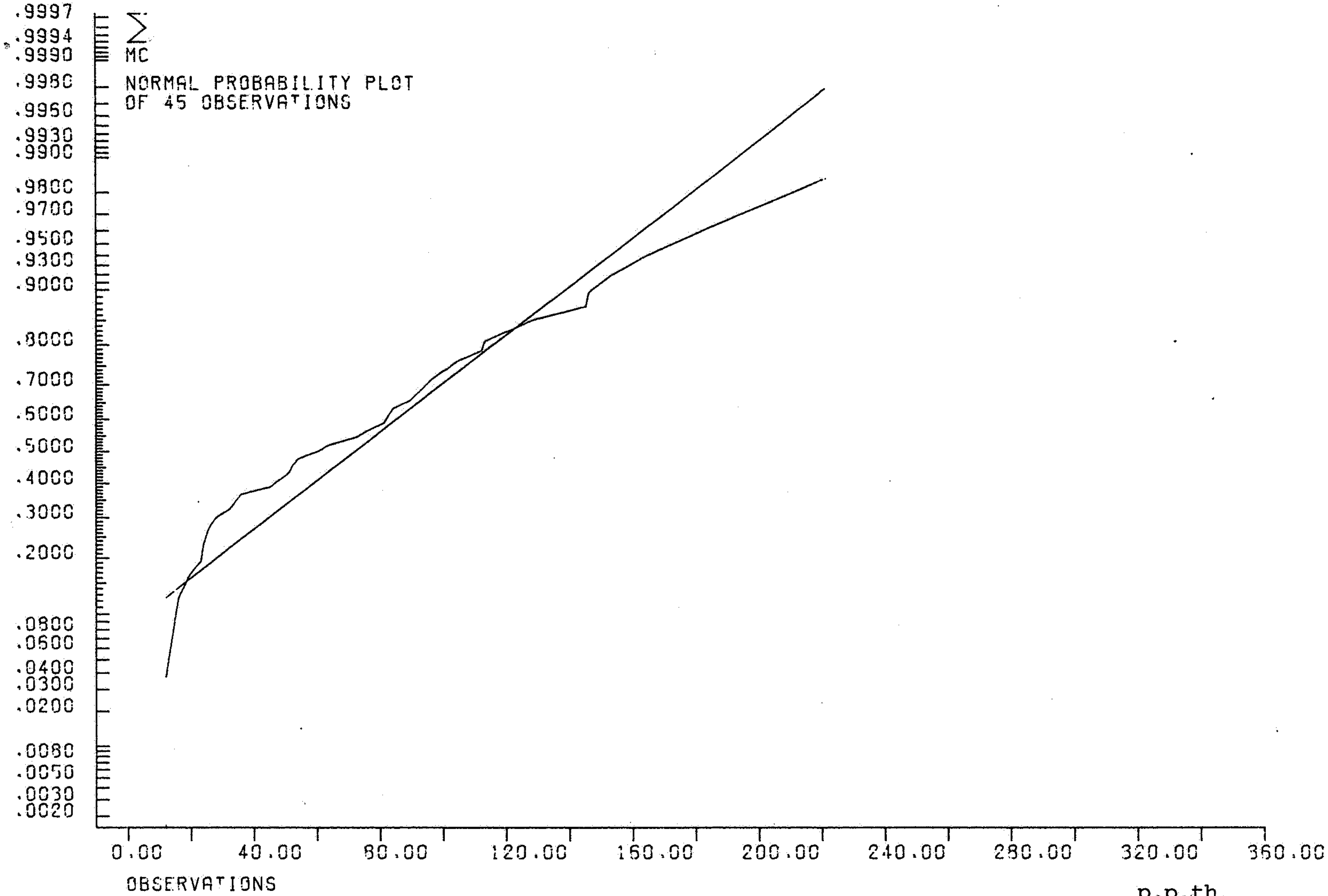
.9970
.9960
.9950
.9940
.9930
.9920
.9900
.9800
.9700
.9600
.9500
.9400
.9300
.9200
.9000
.8000
.7000
.6000
.5000
.4000
.3000
.2000
.0200
30.00
50.00
70.00
90.00
110.00
130.00
150.00
170.00
190.00
210.00
OBSERVATIONS * 10
p.p.th.
ELBE-JUNI , WINDKRACHT>=8 , EXPONENTIAL PLOT

FIGURE 25



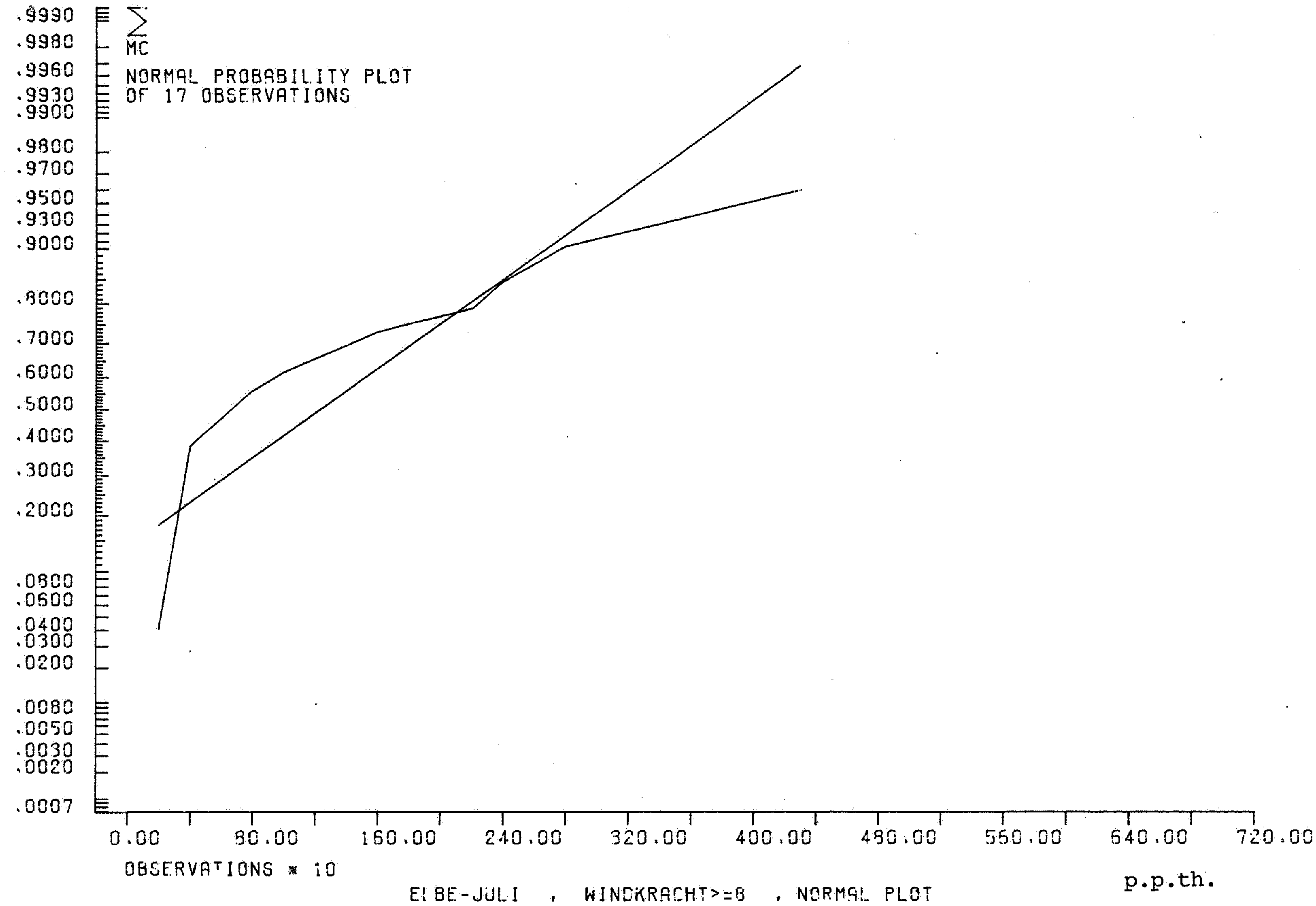
ELBE-JULI , WINDKRACHT>=4 , NORMAL PLOT

FIGURE 26



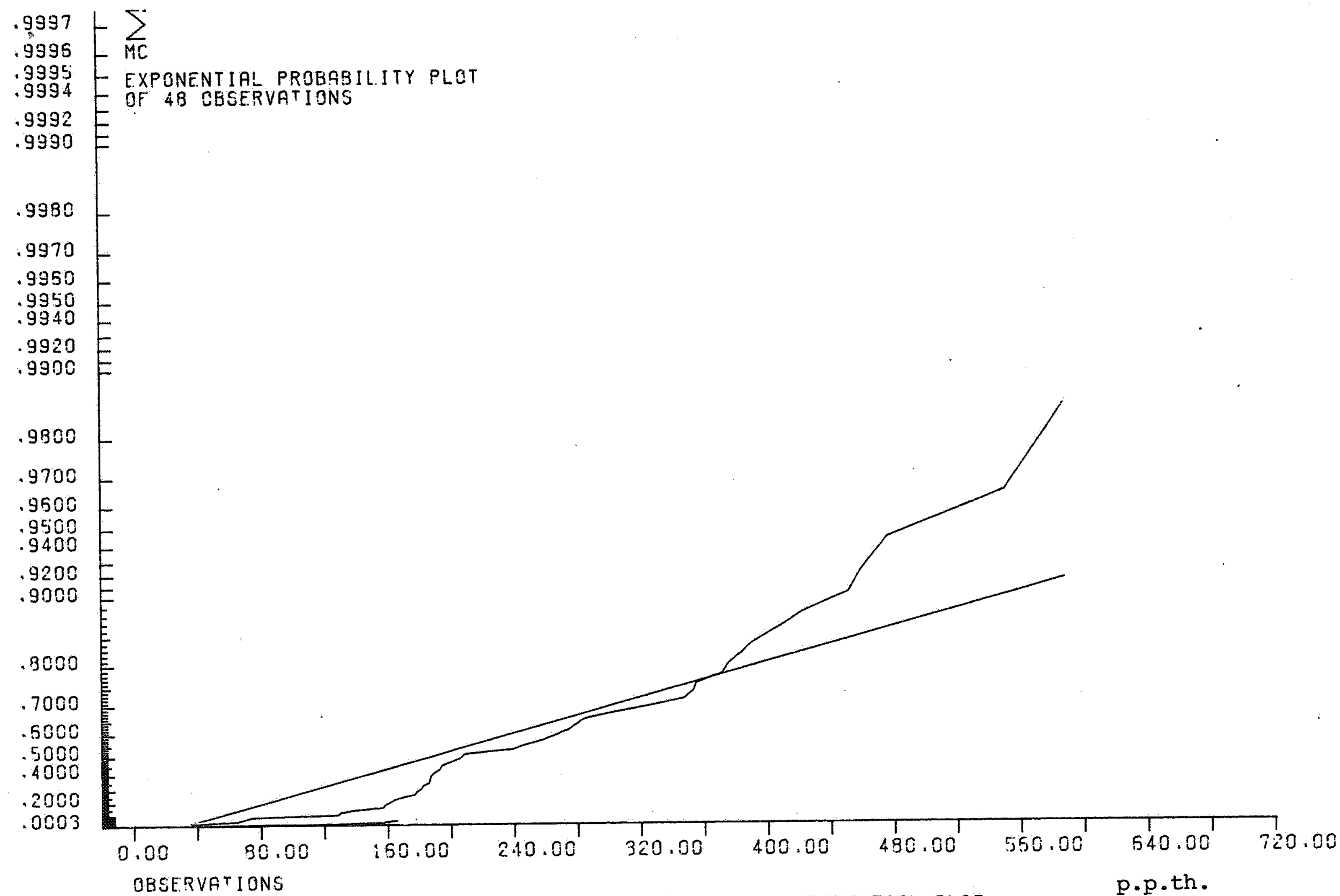
ELBE-JULI , WINDKRACHT>=6 , NORMAL PLOT

FIGURE 27



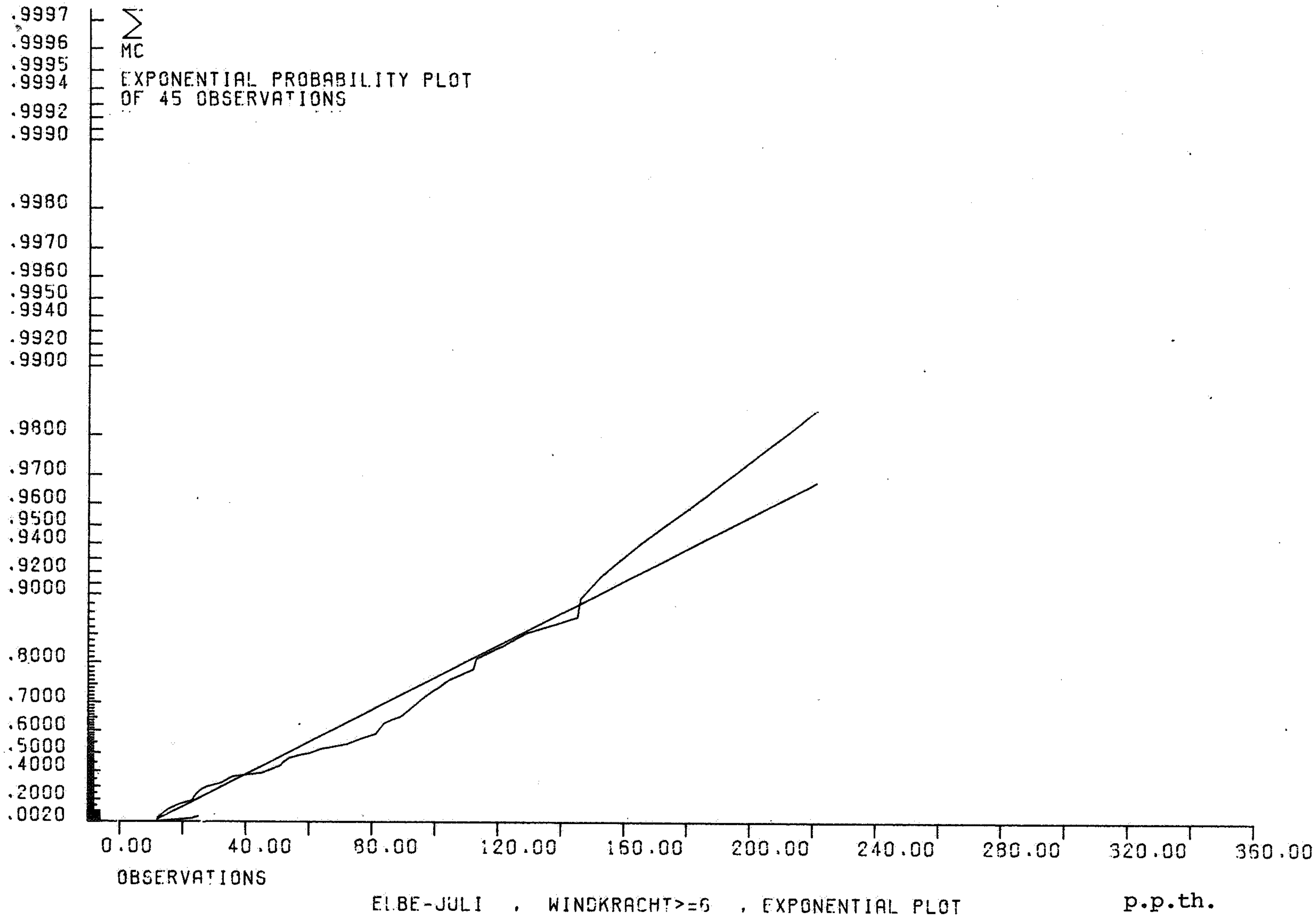
p.p.th.

FIGURE 28



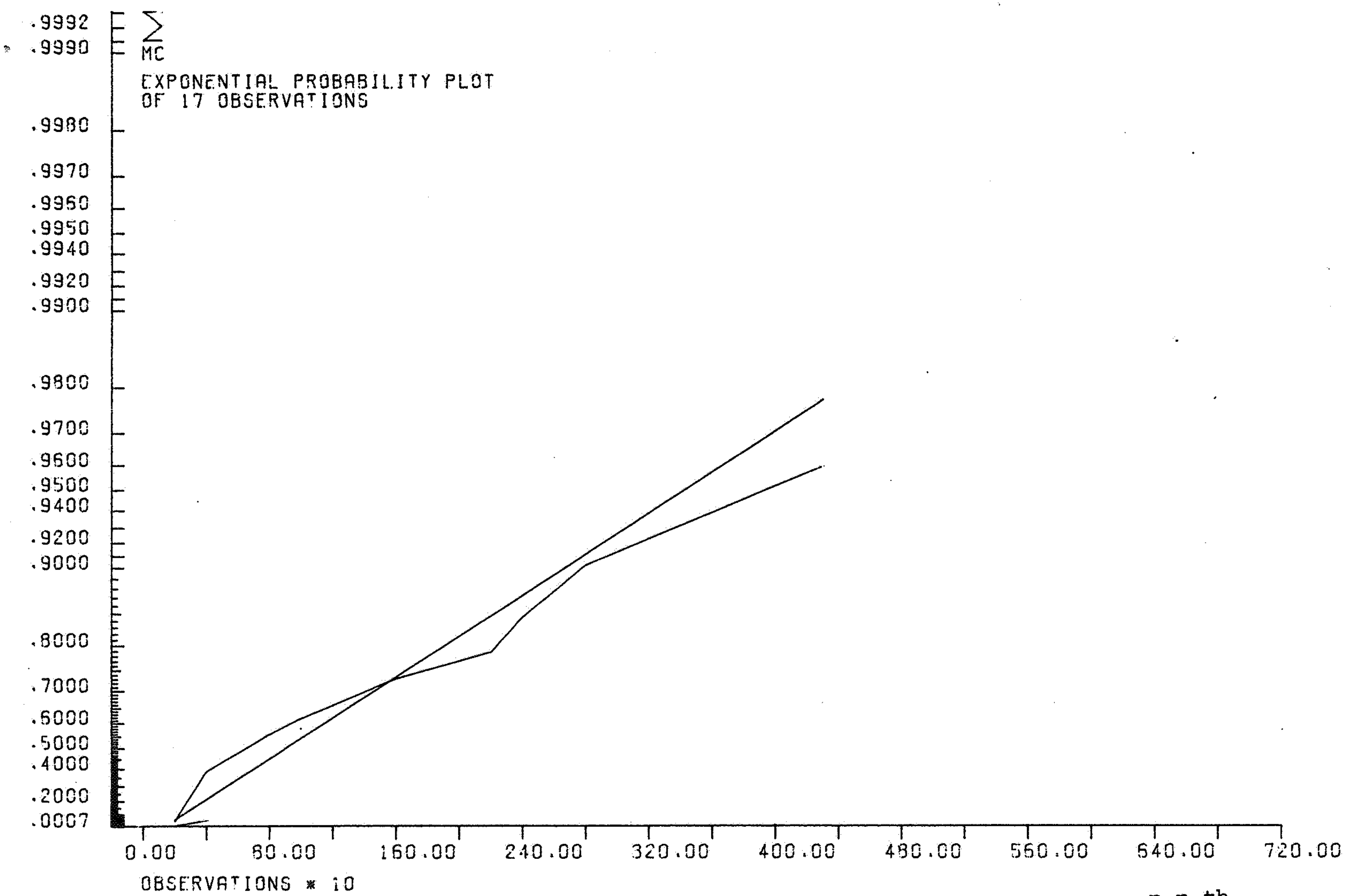
ELBE-JULI , WINDKRACHT>=4 , EXPONENTIAL PLOT

FIGURE 29



.9997
.9996
.9995
.9994
.9992
.9990
.9980
.9970
.9960
.9950
.9940
.9920
.9900
.9800
.9700
.9600
.9500
.9400
.9200
.9000
.8000
.7000
.6000
.5000
.4000
.2000
.0020
∑
MC
EXPONENTIAL PROBABILITY PLOT
OF 45 OBSERVATIONS
0.00
40.00
80.00
120.00
160.00
200.00
240.00
280.00
320.00
360.00
OBSERVATIONS
ELBE-JULI , WINDKRACHT>=6 , EXPONENTIAL PLOT
p.p.th.

FIGURE 30



ELBE-JULI , WINDKRACHT>=8 , EXPONENTIAL PLOT

The plots show very clearly the following behaviour. For $v \geqslant 4$ the non-zero measurements (all measurements in fact) seem close to normally distributed. As the threshhold wind velocity increases they depart more and more from a normal distribution and become more closely exponentially distributed. At $v \geqslant 6$ we have most clearly an exponential distribution. At $v \geqslant 8$ the number of non-zero observations has become too small to discriminate clearly between the two distributions. However because of the earlier systematic shift from normal to exponential with increasing wind velocity, the exponential distribution seems better supported here.

We therefore chose the following parametric statistical analysis in order to estimate the probability that the 1984 Elbe 1 measurement will be equalled or exceeded in any future year. This estimate is based on the available data up to but not including 1984. It is also based on the unverifiable assumption that the frequency of very large measurements continues to follow the normal or exponential distribution. Such an assumption (common in civil engineering) is unavoidable if we are to draw any conclusions at all. The resulting figures should be used to draw qualitative rather than quantitative conclusions. The $v \geqslant 4$ estimate is based on the assumption that this observation is drawn each year independently from a fixed normal distribution (whose mean μ and variance σ^2 we estimate by the sample mean and variance). The $v \geqslant 6$ and $v \geqslant 8$ estimates are based on the assumption that the corresponding observations are non-zero with a certain probability p (which we estimate with the sample fraction of non-zero observations); conditional on being non-zero they are exponentially distributed (with a mean λ^{-1} which we estimate by the sample mean of non-zero observations). The parameters p and λ are of course different in the $v \geqslant 6$ and $v \geqslant 8$ cases. The results are presented in Table 2 below. In addition to this "best estimate" of the probability of exceedance of the 1984 observations, we also present a 95% confidence upper limit to the probability. This is relevant because fifty random values do not precisely determine the probability distribution from which they are drawn. Again assuming a normal or exponential distribution as appropriate, we are 95% certain that the unknown probability we want to estimate is smaller than this upper limit. The methods used (a combination of some standard techniques) are described in the mathematical appendix. The two models are illustrated by figures 31 and 32. The extra statistics needed for these computations are given in Table 3.

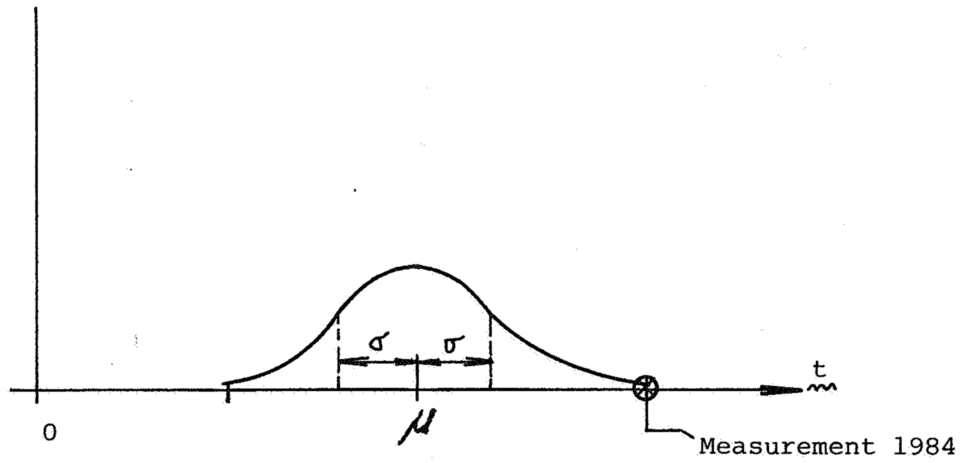
	ELBE 1, JUNE, ≥ 4 (normal model)	ELBE 1, JULY, ≥ 4 (normal model)
Estimated probability of exceedance of 1984 observation	$.5 \times 10^{-5}$	.0018
95% confidence upper limit to probability of exceedance of 1984 observation	$.15 \times 10^{-3}$	.0087

	ELBE 1, JUNE, ≥ 6 (exponential model)	ELBE 1, JULY, ≥ 6 (exponential model)
Estimated probability of exceedance of 1984 observation	.0013	.0570
95% confidence upper limit to probability of exceedance of 1984 observation	.0070	.1136

	ELBE 1, JUNE, ≥ 8 (exponential model)	ELBE 1, JULY, ≥ 8 (exponential model)
Estimated probability of exceedance of 1984 observation	.0058	* --
95% confidence upper limit to probability of exceedance of 1984 observation	.0395	--

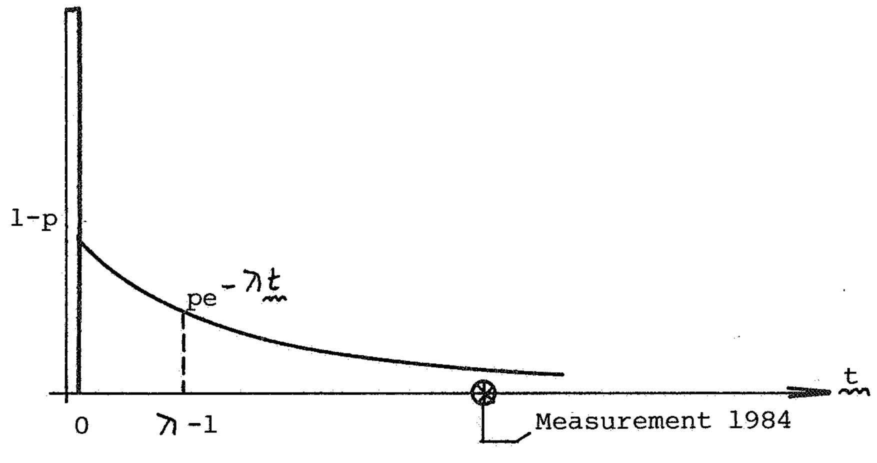
* (The 1984 July ≥ 8 observation was zero.)

Table 2: Estimated probabilities of exceedance of 1984



The distribution of amount of time $\underset{\sim}{v} \geqslant 4$ Bf (normal model)

FIGURE 31



The distribution of amount of time $\underset{\sim}{v} \geqslant 6$ Bf or $\underset{\sim}{v} \geqslant 8$ Bf (exponential model)

FIGURE 32

ELBE 1 JUNE (n = 48)

	≥ 4	≥ 6	≥ 8
No. of +ve observations	48	40	10
Mean of +ve observations	188.96	45.95	10.60
No. of observations exceeding 1984 observations	0	0	0

ELBE 1 JULY (n = 48)

	≥ 4	≥ 6	≥ 8
No. of +ve observations	48	45	17
Mean of +ve observations	257.00	71.78	12.294
No. of observations exceeding 1984 observations	0	1	17

Table 3: Extra statistics for exponential model

The estimates of the probability of an observation as extreme as that of 1984 confirm the general impression from the histograms that the weather conditions of June 1984 were exceptionally severe in all respects. Even though there was no force 8 or more wind in July 1984, the amount of wind at force 4 or more (and to a lesser extent, force 6 or more) was also exceptionally large. These conclusions are not altered when we take account of the fact that the estimates are subject to random variation, being based on a sample of less than fifty years: the 95% confidence upper limits to the probability of an observation as extreme as that of 1984 tell the same story. For June, the estimated probability of an observation as large of that of June 1984 is $.5\times10^{-5}$ for the amount of time the wind velocity v is force 4 or more; $.1\times10^{-2}$ for $v\geqslant6$; and $.6\times10^{-2}$ for $v\geqslant8$. The 95% confidence upper limits are $.2\times10^{-3}$, $.7\times10^{-2}$ and $.4\times10^{-1}$ respectively. In July 1984, there was no force 8 or more wind (as is common). However the estimated probabilities for $v\geqslant4$ and $v\geqslant6$ are $.2\times10^{-2}$ and $.6\times10^{-1}$ respectively with 95% confidence upper limits $.9\times10^{-2}$ and .1.

4. Conclusions

We have shown that the June 1984 weather conditions were indeed highly exceptional: weather of this severity has not been experienced for the past fifty years. Extrapolation techniques suggest a frequency of such severe conditions of less than once in a hundred years. The July 1984 conditions were also severe, and comparable to the worst in the last fifty years.

References

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Appendix: Mathematical background

1. Normal model
2. Exponential model

1. Normal model
Let $\overline{X}$ and S be the sample mean and standard deviation based on a sample of size n from the $N(\mu,\sigma^2)$ distribution. The probability of exceedance of a specified level x^* by a future observation from this distribution is $\gamma = 1-\Phi((x^* - \mu)/\sigma)$ where Φ is the standard cumulative normal distribution. One can estimate γ by

$$\hat{\gamma} = 1-\Phi\left[\frac{x^* - \overline{X}}{S}\right]$$

We construct an approximate $100 \times (1 - \alpha)\%$ confidence upper limit for γ as follows. First construct a confidence lower limit for $(x^* - \mu)/\sigma$. Now $\overline{X}$ and S are stochastically independent, $\overline{X} \sim N(\mu,\sigma^2/n)$ and for large n, $S \dot{\sim} N(\sigma, \sigma^2/2n)$. Here "$\sim$" means "is distributed as" and "$\dot{\sim}$" means "is approximately distributed as". This result follows from using the δ-method ([4], p. 385,386) and the fact that $(n-1)S^2/\sigma^2 \sim \chi^2_{n-1} \dot{\sim} N(n-1, 2(n-1))$. Using the δ-method again we find

$$\frac{x^* - \overline{X}}{S} \dot{\sim} N\left[\frac{x^* - \mu}{\sigma}, \frac{\sigma^2/n}{\sigma^2} + \frac{1}{\sigma^4}(x^* - \mu)^2 \cdot \frac{\sigma^2}{2n}\right]$$

$$= N\left[\frac{x^* - \mu}{\sigma}, \frac{1}{n}\left[1 + ½\left[\frac{x^* - \mu}{\sigma}\right]^2\right]\right].$$

An approximate $100 \times (1 - \alpha)\%$ confidence lower limit for $(x^* - \mu)/\sigma$ is therefore

$$\frac{x^* - \overline{X}}{S} - u_\alpha\left\{\frac{1}{n}\left[1 + ½\left[\frac{x^* - \overline{X}}{S}\right]^2\right]\right\}^{½}$$

where u_α is the upper $1-\alpha$ fractile of the standard normal distribution; i.e. $\Phi(u_\alpha) = 1-\alpha$, in particular $u_{.05} = 1.645$. The approximation becomes more accurate as n becomes larger. A $100 \times (1\text{-}\alpha)\%$ confidence upper limit for $\gamma = 1 - \Phi^{-1}((x^* - \mu)/\sigma)$ is therefore

$$1 - \Phi^{-1}\left[\frac{x^* - \overline{X}}{S} - u_\alpha\left\{\frac{1}{n}\left[1 + ½\left[\frac{x^* - \overline{X}}{S}\right]^2\right]\right\}^{½}\right]$$

2. Exponential model
Suppose $X_1, \cdots, X_n$ are independent and identically distributed with

$$Pr(X_i > 0) = 1 - Pr(X_i = 0) = p$$

and

$$Pr(X_i > t \mid X_i > 0) = e^{-\lambda t}.$$

Thus the X_i 's are positive with probability p ; and conditional on being positive they are exponentially distributed with parameter λ (and mean λ^{-1}). We wish to estimate

$$\gamma = Pr(X_i > x^*) = pe^{-\lambda x^*}$$

for a given value x^* .
Let R be the number of positive observations, let $\hat{p} = R/n$ and let $\overline{X}^+$ be their mean

$(\bar{X}^+ = \Sigma_{i:X_i>0} X_i \,/\, R)$. Then $R \sim Bin(n,p)$ so $\hat{p} \mathrel{\dot\sim} N(p, p(1-p)/n)$.
Conditional on $R = r$, $r\bar{X}^+ \sim Gamma(r,\lambda) \mathrel{\dot\sim} N(r/\lambda, r/\lambda^2)$ if r is large. So $\bar{X}^+ \mathrel{\dot\sim} N(\lambda^{-1}, r^{-1}\lambda^{-2}) \mathrel{\dot\sim} N(\lambda^{-1}, (pn)^{-1}\lambda^{-2})$ if n is large so that $r/n \approx p$. Thus for large n, $\hat{p}$ and $\bar{X}^+$ are approximately independent and normally distributed.

A natural estimator of γ is

$$\hat{\gamma} = \hat{p}e^{-x^* / \bar{X}^+}.$$

We compute an approximate $100 \times (1-\alpha)\%$ confidence upper limit for γ via a confidence upper limit for

$$\log\gamma = \log p - x^* / \bar{X}^+.$$

We have by the δ-method again that

$$\begin{aligned}\log\hat{p} - x^* / \bar{X}^+ &\mathrel{\dot\sim} N(\log p - \lambda x^*, \\ &\quad \frac{1}{p^2}\frac{p(1-p)}{n} + \frac{x^{*2}}{\lambda^{-4}}(pn)^{-1}\lambda^{-2}) \\ &= N(\log p - \lambda x^*, \frac{1}{pn}(1-p + (\lambda x^*)^2)).\end{aligned}$$

This approximation is good if n is large and p is not too close to zero or one.

An approximate $100 \times (1-\alpha)\%$ confidence upper limit for $\log\gamma$ is therefore

$$\log\hat{p} - \frac{x^*}{\bar{X}^+} + u_\alpha\left\{\frac{1}{R}(1-\hat{p} + (x^* / \bar{X}^+)^2)\right\}^{½}$$

and for γ is

$$\hat{p}e^{-x^* / \bar{X}^+}\exp\left[u_\alpha\left\{\frac{1}{R}(1-\hat{p} + (x^* / \bar{X}^+)^2)\right\}^{½}\right].$$

BASIC DATA SET

ELBE 1 - JUNE

YEAR	WINDFORCE >=4	>=5	>=6	>=7	>=8	>=9
1930	34	7	0	0	0	0
1931	185	96	41	14	7	0
1932	77	23	3	0	0	0
1933	107	30	0	0	0	0
1934	133	92	65	21	14	14
1935	155	55	3	3	0	0
1936	51	3	0	0	0	0
1937	141	85	52	22	15	0
1938	315	209	158	62	17	7
1939	21	4	2	0	0	0
1946	202	82	40	26	20	6
1947	103	36	0	0	0	0
1948	127	56	10	0	0	0
1949	112	62	29	0	0	0
1950	138	67	38	21	13	0
1951	71	25	0	0	0	0
1952	223	85	26	13	0	0
1953	50	8	0	0	0	0
1954	171	71	33	4	0	0
1955	84	21	0	0	0	0
1956	233	116	8	0	0	0
1957	233	108	33	8	0	0
1958	116	33	4	0	0	0
1959	285	159	63	4	0	0
1960	196	92	38	25	0	0
1961	329	191	58	8	0	0
1962	355	263	138	75	4	0
1963	209	138	46	25	8	0
1964	330	167	50	0	0	0
1965	234	188	92	29	0	0
1966	171	104	50	12	4	0
1967	309	171	29	4	0	0
1968	196	117	42	13	0	0

1969	196	75	0	0	0	0
1970	100	50	25	8	0	0
1971	209	113	21	4	0	0
1972	201	105	55	13	0	0
1973	192	117	17	4	0	0
1974	309	192	29	0	0	0
1975	162	87	33	4	0	0
1976	288	138	25	8	0	0
1977	254	154	25	4	0	0
1978	321	246	150	37	4	0
1979	150	67	13	0	0	0
1980	257	190	65	8	0	0
1981	288	217	75	25	0	0
1982	150	100	67	0	0	0
1983	295	187	87	4	0	0
1984	576	455	297	109	38	13

ELBE 1 - JULY

YEAR	WINDFORCE					
	>=4	>=5	>=6	>=7	>=8	>=9
1930	209	111	52	10	0	0
1931	266	172	97	45	22	6
1932	130	52	23	0	0	0
1933	187	71	26	0	0	0
1934	239	142	84	58	10	0
1935	285	189	146	93	43	23
1936	156	82	26	0	0	0
1937	247	165	105	4	0	0
1938	75	14	0	0	0	0
1939	189	98	51	23	2	0
1946	194	108	54	32	16	0
1947	65	38	0	0	0	0
1948	187	88	32	2	0	0
1949	157	113	28	0	0	0
1950	161	96	36	24	16	4
1951	75	40	0	0	0	0
1952	193	76	24	0	0	0
1953	383	242	81	0	0	0
1954	355	109	20	0	0	0
1955	36	20	16	0	0	0
1956	200	172	112	56	28	0
1957	178	97	12	0	0	0
1958	392	234	129	64	24	0
1959	177	141	60	0	0	0
1960	279	178	101	8	0	0
1961	451	338	165	44	0	0
1962	347	230	97	48	8	0
1963	206	97	16	4	0	0
1964	407	290	153	36	8	0
1965	423	350	221	60	4	0
1966	375	258	145	24	4	0
1967	169	48	16	0	0	0
1968	164	91	36	16	4	0

ELBE 1 JULY (CONTINUED)

1969	303	129	24	8	4	0
1970	476	311	121	12	4	0
1971	140	96	84	48	4	0
1972	182	89	16	12	0	0
1973	326	197	64	12	8	0
1974	588	491	185	20	0	0
1975	274	165	72	12	0	0
1976	129	89	20	0	0	0
1977	371	246	97	28	0	0
1978	460	291	89	0	0	0
1979	550	364	113	4	0	0
1980	256	153	76	24	0	0
1981	353	187	45	0	0	0
1982	181	96	48	12	0	0
1983	186	89	12	0	0	0
1984	632	455	201	40	0	0

LIST - JUNE

YEAR	WINDFORCE >=4	>=5	>=6	>=7	>=8	>=9
1966	297	191	83	29	2	0
1967	491	338	98	0	0	0
1968	274	169	78	5	0	0
1969	292	179	19	0	0	0
1970	297	168	86	40	0	0
1971	371	194	32	3	0	0
1972	300	118	19	0	0	0
1973	385	200	60	0	0	0
1974	437	198	35	0	0	0
1975	255	163	47	1	0	0
1976	489	238	42	17	0	0
1977	409	253	96	8	0	0
1978	356	259	159	44	0	0
1979	379	194	50	6	0	0
1980	414	242	64	0	0	0
1984	596	409	281	62	32	6

LIST - JULY

YEAR	WINDFORCE					
	>=4	>=5	>=6	>=7	>=8	>=9
1966	499	278	101	4	0	0
1967	305	114	47	15	0	0
1968	404	249	96	32	3	0
1969	465	252	45	8	3	0
1970	599	383	96	1	0	0
1971	379	248	149	36	0	0
1972	308	139	22	0	0	0
1973	406	202	40	3	0	0
1974	770	561	172	9	0	0
1975	361	164	68	3	0	0
1976	376	192	76	33	1	0
1977	514	337	94	16	0	0
1978	457	277	101	7	0	0
1979	706	484	171	16	0	0
1980	263	145	62	22	0	0
1984	642	430	184	17	0	0