SA

stichting
mathematisch
centrum

MC

AFDELING MATHEMATISCHE STATISTIEK SW 18/72 JULY

DANA QUADE

ANALYZING RANDOMIZED BLOCKS BY WEIGHTED RANKINGS

SA

2e boerhaavestraat 49 amsterdam

Printed at the Mathematical Centre, 49, 2e Boerhaavestraat, Amsterdam.

The Mathematical Centre, founded the 11-th of February 1946, is a non-profit institution aiming at the promotion of pure mathematics and its applications. It is sponsored by the Netherlands Government through the Netherlands Organization for the Advancement of Pure Research (Z.W.O), by the Municipality of Amsterdam, by the University of Amsterdam, by the Free University at Amsterdam, and by industries.

ANALYZING RANDOMIZED BLOCKS BY WEIGHTED RANKINGS

by

Dana Quade *)

University of North Carolina and Mathematical Center (Amsterdam)

SUMMARY

In a randomized-blocks design, let X_{ij} be the yield of the j-th treatment in the i-th block, for $i = 1,\dots,n$ and $j = 1,\dots,m$, and let R_{ij} be the within-block rank of X_{ij}. Let V_i be a measure of apparent variability in the i-th block, with Q_i the corresponding rank. Let $t_1, t_2, \dots, t_m$ be constants such that $\Sigma t_j = 0$, and let $s_1, \dots, s_n$ be constants such that $0 \leq s_1 \leq \dots \leq s_n$. Then a test statistic of the form

$$C_n = (m-1) \sum_{j=1}^{m} \{ \sum_{i=1}^{n} s_{Q_i} t_{R_{ij}} \}^2 / \sum_{i=1}^{n} s_i^2 \sum_{j=1}^{m} t_j^2$$

is proposed for the hypothesis H_0 of no treatment effects, given that block effects are additive. Such a statistic is strictly distribution-free under H_0, and under reasonable conditions it is asymptotically distributed for large n as a χ^2 with (m-1) degrees of freedom. The special case where $t_j = j - (m+1)/2$ and $s_i = i$, which generalizes Wilcoxon's matched-pairs signed-rank test, is studied in more detail; a table is provided for m = 3 with n = 3(1)7, m = 4 with n = 3 or 4, and m = 5 with n = 3.

---

*) Supported by a Research Career Development Award (No. GM-38906) from the National Institute of General Medical Sciences.

ANALYZING RANDOMIZED BLOCKS BY WEIGHTED RANKINGS

Suppose $m \geq 2$ treatments are applied once each to $n \geq 2$ different blocks. Let X_{ij} be the yield of the j-th treatment in the i-th block, for $i = 1,\ldots,n$ and $j = 1,\ldots,m$. We assume that the blocks are independent: i.e., that

$$\text{(I)} \qquad \begin{cases} \text{The random vectors } \underline{X}_i = (X_{i1},\ldots,X_{im}), \text{ for} \\ i = 1,\ldots,n, \text{ are mutually independent.} \end{cases}$$

Let F_i be the joint distribution function of $X_{i1},\ldots,X_{im}$, where for simplicity we assume also that F_i is such that

$$\text{(II)} \qquad P\{X_{ij} = X_{ij'}\} = 0 \quad \text{for } j \neq j',$$

and hence with probability 1 there will be no ties within blocks.

The hypothesis of interest is that of "no treatment effects"; specifically

$$H_0\text{: } X_{i1},\ldots,X_{im} \text{ are interchangeable for each } i,$$

which means that each F_i is symmetric in its m arguments. This hypothesis can be treated by the well-known "method of n rankings". Let R_{ij} be the within-block rank of X_{ij}; by the assumption of no ties, $(R_{i1},\ldots,R_{im})$ is for each $i = 1,\ldots,n$ a permutation of the integers $(1,\ldots,m)$. And let $t_1,\ldots,t_m$ be any constants such that $\Sigma t_j = 0$, $\Sigma t_j^2 > 0$. Then we may use as test statistic

$$A_n = \frac{(m-1) \sum_{j=1}^{m} \{ \sum_{i=1}^{n} t_{R_{ij}} \}^2}{n \sum_{j=1}^{n} t_j^2} .$$

Such a statistic is strictly distribution free under H_0, and thus can easily be tabulated for small values of m and n. As n tends to infinity -

see Puri & Sen [1, Theorem 7.2.1] - the statistic A_n is asymptotically distributed as χ^2 with (m-1) degrees of freedom. Two familiar special cases are the test of Friedman, which can be obtained by taking $t_j = j - (m+1)/2$, and that of Brown & Mood, obtained by taking $t_j = \text{sgn}(j-(m+1)/2))$.

In the usual "parametric" treatment of this problem, there is made an additional assumption (besides normality) of "block additivity", of which one version is that

$$\text{(III)} \quad \begin{cases} \text{There exist quantities } \mu_1,\ldots,\mu_n \text{ (block effects)} \\ \text{such that for } i = 1,\ldots,n \\ F_i(X_{i1},\ldots,X_{im}) = F(X_{i1}-\mu_i,\ldots,X_{im}-\mu_i). \end{cases}$$

In such a situation the treatments may properly be compared not only within blocks but also between them; and the method of n rankings, which uses only the within-block comparisons, is inefficient. Since this assumption is widely accepted, there is need for a nonparametric method which takes it into account.

For the special case where m = 2, a standard nonparametric procedure does exist for using the information in between-block comparisons: the Wilcoxon matched-pairs signed-rank test. Let $D_i = |X_{i1}-X_{i2}|$ be the absolute difference between the yields of the two treatments in the i-th block, and let Q_i be the corresponding rank, for $i = 1,\ldots,n$. Let also $Z_i = 0$ or 1 according as $X_{i1} < X_{i2}$ or $X_{i1} > X_{i2}$. Then the signed-rank statistic is $W = \Sigma Z_i Q_i$. Under H_0 this statistic has mean $n(n+1)/4$ and variance $n(n+1)(2n+1)/24$; for small n the exact distribution has been tabulated, and for large n it is asymptotically normal. The signed-rank test has an asymptotic relative efficiency of $3/\pi$ with respect to the analysis of variance test for the classic parametric model. This value compares with $2/\pi$ for the method of n rankings (which, with m = 2, reduces to the simple sign test).

A more general class of tests, for the case $m \geq 2$, is provided by the following "method of ranking after alignment". From the observations in the i-th block calculate some measure of location $M_i = \phi(X_{i1},\ldots,X_{im})$, where the function ϕ is symmetric in its m arguments and is such that

$$\phi(x_1+c,\ldots,x_m+c) = \phi(x_1,\ldots,x_m) + c$$

for any c - for example, the block mean or median.
Then for $i = 1,\ldots,n$ and $j = 1,\ldots,m$ define aligned observations $Y_{ij} = X_{ij} - M_i$, from which the block effects have been removed. Let R_{ij} in this method be the rank of Y_{ij} within the set of all $N = mn$ aligned observations. Now, given constants $s_1,\ldots,s_N$ such that $\Sigma s_k = 0$, $\Sigma s_k^2 > 0$, let $Z_{ij} = s_{R_{ij}}$ be the score corresponding to the observation X_{ij}. Then the test statistic is

$$B_n = \frac{(m-1) \sum_{j=1}^{m} \{\sum_{i=1}^{n} Z_{ij}\}^2}{\sum_{i=1}^{n} \sum_{j=1}^{m} Z_{ij}^2 - \sum_{i=1}^{n} \{\sum_{j=1}^{m} Z_{ij}\}^2/m} .$$

If $m > 2$ the statistic B_n is distribution-free under H_0 only conditionally. Given the sets of scores which occur in the various blocks, the $(m!)^n$ samples obtainable by within-block permutations of the scores are equally likely. Thus the (conditional) significance level can be calculated as the proportion of such samples which yield values of B_n equal to or greater than the value actually observed. Given that the blocks are additive, as n tends to infinity the statistic B_n is asymptotically distributed as χ^2 with (m-1) degrees of freedom, provided that the sequence of sets of constants $\{s_i(N)\colon i = 1,\ldots,N\}$ for each $N = mn$ satisfies conditions of Chernoff-Savage type - see Puri & Sen [1, section 7.3]. Nevertheless, it is difficult to see that the method of ranking after alignment has any great advantage over the simple randomization test based on the analysis of variance statistic using the original observations. This latter procedure requires rather less computational effort to produce exact (conditional) significance levels for small n, and under reasonable assumptions it has similar asymptotic properties for large n. Thus a simpler test is needed.

By the way, it may not be obvious that the two-sided signed-rank test is equivalent to a special case of the test based on B_n. With $m = 2$, so that $N = 2n$, let

$$s_k = \begin{cases} [\frac{k+1}{2}] & \text{for } k \text{ even} \\ -[\frac{k+1}{2}] & \text{for } k \text{ odd} \end{cases}, \quad k = 1,\dots,N,$$

where [x] denotes the greatest integer not greater than x. Note that $\Sigma s_k = 0$, $\Sigma s_k^2 = n(n+1)(2n+1)/3 > 0$. Then the score Z_{ij} corresponding to the observation X_{ij} will equal Q_i or $-Q_i$ according as X_{ij} is the larger or the smaller of X_{i1} and X_{i2}. Furthermore,

$$\sum_{i=1}^{n} Z_{i1} = -\sum_{i=1}^{n} Z_{i2} = 2\{W - \frac{n(n+1)}{4}\}$$

and thus

$$B_n = \frac{24\{W - n(n+1)/4\}^2}{n(n+1)(2n+1)} .$$

Let us now generalize the signed-rank test in a different way, to obtain what may be called a "method of n weighted rankings". Under Assumption III all blocks are equally variable; and if some appear more variable than others, they are perhaps better referred to as more discriminating. It seems intuitively reasonable that these blocks should receive greater weight in the analysis. Thus let us calculate from the observations in the i-th block some measure of (apparent) variability $V_i = \psi(X_{i1},\dots,X_{im})$, where the function ψ is symmetric in its m arguments and is such that

$$\psi(x_1+c,\dots,x_m+c) = \psi(x_1,\dots,x_m)$$

for all c - for example, the within-block variance or range. And let Q_i be the rank of V_i for $i = 1,\dots,n$; for simplicity we assume that

(IV) $\qquad P\{V_i = V_{i'}\} = 0 \quad \text{for } i \neq i',$

so that with probability 1 there will be no ties among the V's. Then the relative weight given to the i-th block will be s_{Q_i}, for $i = 1,\dots,n$, where the s's are constants such that $0 \leq s_1 \leq \ldots \leq s_n$. Finally, let R_{ij} be the within-block rank of X_{ij}, and let $t_1, \ldots, t_m$ be constants, as in the method of n (unweighted) rankings; then the proposed test statistic is

$$C_n = \frac{(m-1) \sum_{j=1}^{m} \{ \sum_{i=1}^{n} s_{Q_i} t_{R_{ij}} \}^2}{\sum_{i=1}^{n} s_i^2 \sum_{j=1}^{m} t_j^2} .$$

The quantities $s_{Q_i} t_{R_{ij}}$ may be called scores for this procedure.

To see that the signed-rank statistic is indeed equivalent to a special case of C_n, let $m = 2$, $t_1 = -1$, $t_2 = 1$; and let $V_i = D_i$ and $s_i = i$ for $i = 1, \ldots, n$. Then

$$\sum_{i=1}^{n} s_{Q_i} t_{R_{i1}} = - \sum_{i=1}^{n} s_{Q_i} t_{R_{i2}} = 2\{W - \frac{n(n+1)}{4}\}$$

and

$$C_n = \frac{24\{W - n(n+1)/4\}^2}{n(n+1)(2n+1)} .$$

The method of n (unweighted) rankings is also clearly a special case, obtained by taking $s_i = 1$ for all i.

It is clear that any statistic of the form C_n is strictly distribution-free under H_0, and could easily be tabulated for small values of m and n. For large n we may use the following

Theorem. Suppose Assumptions I through IV hold, and in addition the sequence of weights $s_i(n)$ for $i = 1, \ldots, n$ and $n = 1, 2, \ldots$ satisfies the Wald-Wolfowitz condition, namely

$$\text{(V)} \qquad \frac{\sum_{i=1}^{n} [s_i(n) - \bar{s}(n)]^r}{\{ \sum_{i=1}^{n} [s_i(n) - \bar{s}(n)]^2 \}^{r/2}} = O(n^{1-r/2}) \quad \text{for } r = 3, 4, \ldots,$$

where $\bar{s}(n) = \Sigma s_i(n)/n$. Then, under H_0, as n tends to infinity the statistic C_n is asymptotically distributed as χ^2 with (m-1) degrees of freedom.

(Note. Assumptions II and IV simplify the exposition by prohibiting ties, but they are not really essential.)

Proof. (The theorems referred to below are as numbered by Puri & Sen [1]). Define the treatment totals

$$H_j = \sum_{i=1}^{n} s_{Q_i} t_{R_{ij}} \qquad \text{for } j = 1,\dots,m,$$

and consider an arbitrary contrast in them, say $L_n = \Sigma\lambda_j H_j$ where $\Sigma\lambda_j = 0$ and $\Sigma\lambda_j^2 > 0$. We have

$$L_n = \sum_{i=1}^{n} s_{Q_i} W_{in}, \qquad \text{where } W_{in} = \sum_{j=1}^{m} \lambda_j t_{R_{ij}}.$$

By Assumptions I and III (and II) the W's are independent and identically distributed random variables, and they are clearly bounded. Under H_0 they have mean 0 and variance $(\Sigma t_j^2)(\Sigma\lambda_j^2)/(m-1) > 0$, so they satisfy the conditions of Theorem 3.4.5. Furthermore, by Assumptions I and III (and IV) the Q's are a random permutation of the integers 1,...,n; and under H_0 they are independent of the W's. Thence it is easy to verify that L_n has mean 0 and variance

$$\sigma_n^2 = (\Sigma s_i^2)(\Sigma t_j^2)(\Sigma\lambda_j^2)/(m-1).$$

Thus, using Assumption V, by Theorem 3.4.1 L_n/σ_n is asymptotically a standard normal variable. The present Theorem follows using the same argument as for Theorem 7.2.1.

For any pair of blocks, say the i-th and i'-th, let

$$T_{ii'} = \sum_{j=1}^{m} t_{R_{ij}} t_{R_{i'j}} \Big/ \sum_{j=1}^{m} t_j^2 ;$$

the statistic T may be interpreted as a measure of rank correlation between blocks. Now consider a weighted average of these rank correlations over all pairs of blocks, namely

$$U_n = \sum_{i<i'} S_{ii'} T_{ii'} ,$$

where the weights, which sum to unity, are the random variables

$$S_{ii'} = \frac{2\, s_{Q_i}\, s_{Q_{i'}}}{(\Sigma s_i)^2 - \Sigma s_i^2} .$$

Then it is easily verified that

$$C_n = (m-1)\ \{1 + \frac{(\Sigma s_i)^2 - \Sigma s_i^2}{\Sigma s_i^2}\ U_n\}.$$

Under Assumptions I through IV we have

$$E[T] = \theta = \sum_{j=1}^{m} \theta_j^2 \ / \sum_{j=1}^{m} t_j^2 \qquad \text{where } \theta_j = E[t_{R_{ij}}]$$

and

$$E[S] = \frac{2}{n(n-1)} ,$$

so

$$E[U_n] = \frac{n(n-1)}{2} \operatorname{cov}(S,T) + \theta$$

and

$$E[C_n] = (m-1)\ \{1 + \frac{(\Sigma s_i)^2 - \Sigma s_i^2}{\Sigma s_i^2}\ [\frac{n(n-1)}{2} \operatorname{cov}(S,T) + \theta]\}.$$

The test based on C_n will be consistent against any sequence of alternatives for which $E[C_n]$ tends to infinity as n increases. In particular, the following conditions are jointly sufficient for consistency: (i) $\operatorname{cov}(S,T) \geq 0$; (ii) $\Sigma s_i^2/(\Sigma s_i)^2 \to 0$; and (iii) $\Sigma\theta_j^2 > 0$. In the special case where the rankings are unweighted, i.e. where $s_i = 1$ for all i, the covariance of (i) is exactly 0, the ratio of (ii) is 1/n, and (iii) is both necessary and sufficient. With weighted rankings any reasonable choice of the measure of variability and of the constants $s_1,\ldots,s_n$ should produce a positive correlation between S and T and thus increase the efficiency of the test.

Returning to the null-hypothesis distribution of C_n, let us set down formulas for its lower moments. We have $E[C_n] = m-1$, and a little algebra yields

$$\mathrm{Var}[C_n] = 2(m-1)v_n \quad \text{where } v_n = \frac{(\Sigma s_i^2)^2 - \Sigma s_i^4}{(\Sigma s_i^2)^2} .$$

More extensive algebra yields

$$E[C_n - (m-1)]^3 = \frac{4(m-1)^2}{(\Sigma s_i^2)^3} \{\frac{2}{m-1} [(\Sigma s_i^2)^3 - 3\Sigma s_i^2 \Sigma s_i^4 + 2\Sigma s_i^6] + \frac{m(\Sigma t_j^3)^2}{(m-2)(\Sigma t_j^2)^3} [(\Sigma s_i^3)^2 - \Sigma s_i^6]\}.$$

In what follows we shall suppose that $\Sigma t_j^3 = 0$. The skewness of C_n is then

$$\beta_1 = \frac{\{E[C_n - (m-1)]^3\}^2}{\{\mathrm{Var}[C_n]\}^3} = \frac{8}{m-1} w_n,$$

where

$$w_n = \frac{[(\Sigma s_i^2)^3 - 3\Sigma s_i^2 \Sigma s_i^4 + 2\Sigma s_i^6]^2}{[(\Sigma s_i^2)^2 - \Sigma s_i^4]^3} .$$

(Under Assumption V, v_n and w_n tend to unity as n increases.) These results suggest the following asymptotic chi-square approximations to the null-hypothesis distribution of C_n:

(one moment exact)

$$C_n \sim \chi^2 \quad \text{with } (m-1) \text{ d.f.}$$

(two moments exact)

$$\frac{C_n}{v_n} \sim \chi^2 \qquad \text{with } \frac{m-1}{v_n} \qquad \text{d.f.}$$

(three moments exact)

$$\frac{C_n - (m-1)}{\sqrt{v_n w_n}} + \frac{m-1}{w_n} \sim \chi^2 \qquad \text{with } \frac{m-1}{w_n} \qquad \text{d.f.}$$

Before the method of weighted rankings can be applied in practice, we must choose: (1) the measure of variability V_i; (2) the set of constants $t_1,\ldots,t_m$; and (3) the set of weights $s_1,\ldots,s_n$. In all three cases further study is needed to determine the best choice, but some tentative recommendations can be made.

For the measure of variability, the within-block variance is probably as good a statistic as any; or the within-block range may be used if an easy calculation is wanted.

For the constants $t_1,\ldots,t_m$ a reasonable choice seems to be

$$t_j = j - \frac{m+1}{2}, \quad j = 1,\ldots,m.$$

Then the measure T of rank correlation is the well-known Spearman rho. If the rankings are unweighted, the test statistic C_n reduces to Friedman's chi-square. The proper choice of $t_1,\ldots,t_m$ in the unweighted case is discussed in Puri & Sen [1, section 7.2.3].

Finally, for the weights $s_1,\ldots,s_n$ the choice

$$s_i = i, \quad i = 1,\ldots,n$$

suggests itself immediately, on two grounds: first, because these weights are simple; and second, because they provide a generalization of the signed-rank statistic. Another possibility is to choose $s_1 = \ldots = s_k = 0$ and $s_{k+1} = \ldots = s_n = 1$, where $k = k(n)$ is (say) the nearest integer to γn for some γ, $0 < \gamma < 1$. Such a choice is equivalent to discarding the k least discriminating blocks and performing an unweighted analysis of the remainder; it has the advantage of not requiring any new table of weighted rankings. Note that both ways of choosing the s's satisfy the Wald-Wolfowitz condition V.

But let us consider in more detail the procedure which results when $t_j = j - (m+1)/2$ for $j = 1,\ldots,m$ and $s_i = i$ for $i = 1,\ldots,n$. We have

$$\Sigma t_j = 0, \quad \Sigma t_j^2 = \frac{m^3-m}{12}, \quad \Sigma t_j^3 = 0$$

and

$$\Sigma s_i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \Sigma s_i^4 = \frac{3n^2+3n-1}{5}\,\Sigma s_i^2$$

$$\Sigma s_i^6 = \frac{3n^4+6n^3-3n+1}{7}\,\Sigma s_i^2,$$

from which we calculate

$$v_n = \frac{(n-1)(10n^2+7n-6)}{5n(n+1)(2n+1)}$$

and

$$w_n = \frac{5(n-2)^2(2n-1)^2(70n^3+77n^2-153n-180)^2}{49n(n-1)(n+1)(2n+1)(10n^2+7n-6)^3}.$$

The treatment totals are

$$H_j = \sum_{i=1}^{n} Q_i\left(R_{ij} - \frac{m+1}{2}\right), \quad j = 1,\ldots,m.$$

Then

$$C_n = \frac{72H}{m(m+1)n(n+1)(2n+1)},$$

where $H = \Sigma H_j^2$ is a convenient index: H is always an integer unless $m = 2 \pmod 4$ and $n = 1$ or $2 \pmod 4$, and even then 2H is always an integer. In the Appendix to this paper is given a table of the exact null-hypothesis distribution of C_n for m = 3 treatments with n = 3, 4, 5, 6, or 7 blocks; for m = 4 treatments with n = 3 or 4 blocks; and for m = 5 treatments with n = 3 blocks. These results were obtained by the unsophisticated method of generating all possible permutations, using the computational facilities of the Mathematical Center at Amsterdam. (I am grateful to Jack Alanen for assistance in programming, and to the Center for a grant of computer time.)

A simple example will illustrate the procedure. Suppose we have m = 3 treatments and n = 6 blocks, with raw data (X_{ij}'s) as follows:

		Blocks					
		1	2	3	4	5	6
	1	31	40	39	50	40	40
Treatments	2	23	36	38	43	28	35
	3	30	32	43	54	31	33

The corresponding within-block ranks (R_{ij}'s) are then

		Blocks					
		1	2	3	4	5	6
	1	3	3	2	2	3	3
Treatments	2	1	2	1	1	1	2
	3	2	1	3	3	2	1

Subtracting (m+1)/2 = 2 gives the values of $t_{R_{ij}}$:

		Blocks					
		1	2	3	4	5	6
	1	1	1	0	0	1	1
Treatments	2	-1	0	-1	-1	-1	0
	3	0	-1	1	1	0	-1

The within-block variances are:

Block 1	2	3	4	5	6
19	16	7	31	39	13

The corresponding ranks Q_i, which are also the values of s_{Q_i}, are:

Block 1	2	3	4	5	6
4	3	1	5	6	2

Hence the scores $s_{Q_i} t_{R_{ij}}$ are:

		Block 1	2	3	4	5	6
Treatments	1	-4	3	0	0	6	2
	2	-4	0	-1	-5	-6	0
	3	0	-3	1	5	0	-2

Finally, the treatment totals are: $H_1 = 15$, $H_2 = -16$, and $H_3 = 1$, so $H = H_1^2 + H_2^2 + H_3^2 = 482$, and

$$C_n = \frac{(72)(482)}{(12)(546)} = 5.297.$$

The table of the Appendix gives P = .05877 as the corresponding exact significance level.

REFERENCE

[1] M.L. Puri & P.K. Sen (1971). Nonparametric Methods in Multivariate Analysis. New York, Wiley.

APPENDIX

EXACT DISTRIBUTION OF THE STATISTIC C_n

m = 3, n = 3

H	C_n	f	Σf	P
0	.000	1	36	1.0000000000
2	.143	2	35	.9722222222
6	.429	4	33	.9166666667
8	.571	3	29	.8055555556
14	1.000	2	26	.7222222222
18	1.286	2	24	.6666666667
24	1.714	4	22	.6111111111
26	1.857	4	18	.5000000000
32	2.286	1	14	.3888888889
38	2.714	2	13	.3611111111
42	3.000	4	11	.3055555556
54	3.857	2	7	.1944444444
56	4.000	2	5	.1388888889
62	4.429	2	3	.0833333333
72	5.143	1	1	.0277777778

m = 3, n = 4

H	C_n	f	Σf	P
0	.000	1	216	1.0000000000
2	.067	10	215	.9953703704
6	.200	6	205	.9490740741
8	.267	14	199	.9212962963
14	.467	16	185	.8564814815
18	.600	6	169	.7824074074
24	.800	8	163	.7546296296
26	.867	12	155	.7175925926
32	1.067	8	143	.6620370370
38	1.267	12	135	.6250000000
42	1.400	8	123	.5694444444
50	1.667	6	115	.5324074074
54	1.800	2	109	.5046296296
56	1.867	16	107	.4953703704
62	2.067	12	91	.4212962963
72	2.400	5	79	.3657407407
74	2.467	8	74	.3425925926
78	2.600	4	66	.3055555556
86	2.867	6	62	.2870370370
96	3.200	2	56	.2592592593
98	3.267	10	54	.2500000000
104	3.467	10	44	.2037037037
114	3.800	4	34	.1574074074
122	4.067	4	30	.1388888889
126	4.200	4	26	.1203703704
128	4.267	1	22	.1018518519
134	4.467	2	21	.0972222222
146	4.867	4	19	.0879629630
150	5.000	2	15	.0694444444
152	5.067	4	13	.0601851852
158	5.267	4	9	.0416666667
168	5.600	2	5	.0231481481
182	6.067	2	3	.0138888889
200	6.667	1	1	.0046296296

m = 3, n = 5

H	C_n	f	Σf	P
0	.000	6	1296	1.0000000000
2	.036	33	1290	.9953703704
6	.109	34	1257	.9699074074
8	.145	24	1223	.9436728395
14	.255	58	1199	.9251543210
18	.327	35	1141	.8804012346
24	.436	26	1106	.8533950617
26	.473	52	1080	.8333333333
32	.582	22	1028	.7932098765
38	.691	58	1006	.7762345679
42	.764	58	948	.7314814815
50	.909	23	890	.6867283951
54	.982	26	867	.6689814815
56	1.018	34	841	.6489197531
62	1.127	42	807	.6226851852
72	1.309	22	765	.5902777778
74	1.345	38	743	.5733024691
78	1.418	48	705	.5439814815
86	1.564	34	657	.5069444444
96	1.745	16	623	.4807098765
98	1.782	46	607	.4683641975
104	1.891	24	561	.4328703704
114	2.073	34	537	.4143518519
122	2.218	26	503	.3881172840
126	2.291	32	477	.3680555556
128	2.327	14	445	.3433641975
134	2.436	26	431	.3325617284
146	2.655	20	405	.3125000000
150	2.727	14	385	.2970679012
152	2.764	22	371	.2862654321
158	2.873	18	349	.2692901235
162	2.945	12	331	.2554012346
168	3.055	24	319	.2461419753
182	3.309	38	295	.2276234568
186	3.382	22	257	.1983024691
194	3.527	14	235	.1813271605
200	3.636	10	221	.1705246914
206	3.745	20	211	.1628086420
216	3.927	6	191	.1473765432
218	3.964	8	185	.1427469136
222	4.036	16	177	.1365740741
224	4.073	12	161	.1242283951
234	4.255	10	149	.1149691358
242	4.400	5	139	.1072530864
248	4.509	8	134	.1033950617
254	4.618	14	126	.0972222222

m = 3, n = 5 (continued)

H	C_n	f	Σf	P
258	4.691	10	112	.0864197531
266	4.836	14	102	.0787037037
278	5.055	10	88	.0679012346
288	5.236	2	78	.0601851852
294	5.345	12	76	.0586419753
296	5.382	8	64	.0493827160
302	5.491	4	56	.0432098765
312	5.673	6	52	.0401234568
314	5.709	4	46	.0354938272
326	5.927	4	42	.0324074074
338	6.145	7	38	.0293209877
342	6.218	6	31	.0239197531
344	6.255	2	25	.0192901235
350	6.364	6	23	.0177469136
362	6.582	4	17	.0131172840
366	6.655	4	13	.0100308642
378	6.873	4	9	.0069444444
398	7.236	2	5	.0038580247
422	7.673	2	3	.0023148148
450	8.182	1	1	.0007716049

m = 3, n = 6

H	C_n	f	Σf	P
0	.000	22	7776	1.0000000000
2	.022	117	7754	.9971707819
6	.066	140	7637	.9821244856
8	.088	94	7497	.9641203704
14	.154	222	7403	.9520318930
18	.198	131	7181	.9234825103
24	.264	108	7050	.9066358025
26	.286	214	6942	.8927469136
32	.352	90	6728	.8652263374
38	.418	198	6638	.8536522634
42	.462	240	6440	.8281893004
50	.549	102	6200	.7973251029
54	.593	112	6098	.7842078189
56	.615	158	5986	.7698045267
62	.681	182	5828	.7494855967
72	.791	94	5646	.7260802469
74	.813	172	5552	.7139917695
78	.857	202	5380	.6918724280
86	.945	170	5178	.6658950617
96	1.055	82	5008	.6440329218
98	1.077	240	4926	.6334876543
104	1.143	132	4686	.6026234568
114	1.253	168	4554	.5856481481
122	1.341	156	4386	.5640432099
126	1.385	166	4230	.5439814815
128	1.407	56	4064	.5226337449
134	1.473	132	4008	.5154320988
146	1.604	124	3876	.4984567901
150	1.648	74	3752	.4825102881
152	1.670	106	3678	.4729938272
158	1.736	116	3572	.4593621399
162	1.780	74	3456	.4444444444
168	1.846	114	3382	.4349279835
182	2.000	206	3268	.4202674897
186	2.044	132	3062	.3937757202
194	2.132	110	2930	.3768004115
200	2.198	44	2820	.3626543210
206	2.264	102	2776	.3569958848
216	2.374	48	2674	.3438786008
218	2.396	86	2626	.3377057613
222	2.440	108	2540	.3266460905
224	2.462	68	2432	.3127572016
234	2.571	98	2364	.3040123457
242	2.659	45	2266	.2914094650
248	2.725	68	2221	.2856224280
254	2.791	76	2153	.2768775720
258	2.835	96	2077	.2671039095

m = 3, n = 6 (continued)

H	C_n	f	Σf	P
266	2.923	148	1981	.2547582305
278	3.055	64	1833	.2357253086
288	3.165	34	1769	.2274948560
294	3.231	118	1735	.2231224280
296	3.253	54	1617	.2079475309
302	3.319	54	1563	.2010030864
312	3.429	62	1509	.1940586420
314	3.451	56	1447	.1860853909
326	3.582	46	1391	.1788837449
338	3.714	82	1345	.1729681070
342	3.758	62	1263	.1624228395
344	3.780	44	1201	.1544495885
350	3.846	46	1157	.1487911523
362	3.978	52	1111	.1428755144
366	4.022	48	1059	.1361882716
378	4.154	48	1011	.1300154321
384	4.220	24	963	.1238425926
386	4.242	34	939	.1207561728
392	4.308	64	905	.1163837449
398	4.374	40	841	.1081532922
402	4.418	44	801	.1030092593
416	4.571	30	757	.0973508230
422	4.637	30	727	.0934927984
434	4.769	62	697	.0896347737
438	4.813	38	635	.0816615226
446	4.901	32	597	.0767746914
450	4.945	18	565	.0726594650
456	5.011	30	547	.0703446502
458	5.033	28	517	.0664866255
474	5.209	32	489	.0628858025
482	5.297	22	457	.0587705761
486	5.341	10	435	.0559413580
488	5.363	20	425	.0546553498
494	5.429	32	405	.0520833333
504	5.538	16	373	.0479681070
512	5.626	10	357	.0459104938
518	5.692	36	347	.0446244856
536	5.890	14	311	.0399948560
542	5.956	18	297	.0381944444
546	6.000	36	279	.0358796296
554	6.088	18	243	.0312500000
558	6.132	14	225	.0289351852
566	6.220	16	211	.0271347737
578	6.352	5	195	.0250771605
582	6.396	14	190	.0244341564
584	6.418	8	176	.0226337449

m = 3, n = 6 (continued)

H	C_n	f	Σf	P
600	6.593	6	168	.0216049383
602	6.615	18	162	.0208333333
608	6.681	10	144	.0185185185
614	6.747	6	134	.0172325103
618	6.791	8	128	.0164609053
626	6.879	10	120	.0154320988
632	6.945	8	110	.0141460905
648	7.121	2	102	.0131172840
650	7.143	6	100	.0128600823
654	7.187	10	94	.0120884774
662	7.275	10	84	.0108024691
666	7.319	10	74	.0095164609
672	7.385	6	64	.0082304527
674	7.407	8	58	.0074588477
686	7.538	12	50	.0064300412
698	7.670	4	38	.0048868313
702	7.714	8	34	.0043724280
722	7.934	7	26	.0033436214
728	8.000	2	19	.0024434156
746	8.198	4	17	.0021862140
762	8.374	4	13	.0016718107
774	8.505	4	9	.0011574074
806	8.857	2	5	.0006430041
842	9.253	2	3	.0003858025
882	9.692	1	1	.0001286008

$m = 3$, $n = 7$

H	C_n	f	Σf	P
0	.000	82	46656	1.0000000000
2	.014	488	46574	.9982424554
6	.043	438	46086	.9877829218
8	.057	510	45648	.9783950617
14	.100	938	45138	.9674639918
18	.129	422	44200	.9473593964
24	.171	456	43778	.9383144719
26	.186	926	43322	.9285408093
32	.229	480	42396	.9086934156
38	.271	884	41916	.8984053498
42	.300	782	41032	.8794581619
50	.357	416	40250	.8626971879
54	.386	384	39834	.8537808642
56	.400	888	39450	.8455504115
62	.443	842	38562	.8265174897
72	.514	395	37720	.8084705075
74	.529	798	37325	.8000042867
78	.557	708	36527	.7829003772
86	.614	768	35819	.7677254801
96	.686	376	35051	.7512645748
98	.700	1132	34675	.7432055898
104	.743	784	33543	.7189429012
114	.814	650	32759	.7021390604
122	.871	688	32109	.6882073045
126	.900	628	31421	.6734610768
128	.914	353	30793	.6600008573
134	.957	656	30440	.6524348422
146	1.043	668	29784	.6383744856
150	1.071	294	29116	.6240569273
152	1.086	674	28822	.6177554870
158	1.129	640	28148	.6033093278
162	1.157	270	27508	.5895919067
168	1.200	586	27238	.5838048697
182	1.300	1172	26652	.5712448560
186	1.329	516	25480	.5461248285
194	1.386	572	24964	.5350651578
200	1.429	290	24392	.5228052126
206	1.471	546	24102	.5165895062
216	1.543	254	23556	.5048868313
218	1.557	550	23302	.4994427298
222	1.586	488	22752	.4876543210
224	1.600	548	22264	.4771947874
234	1.671	480	21716	.4654492455
242	1.729	238	21236	.4551611797
248	1.771	500	20998	.4500600137
254	1.814	486	20498	.4393432785
258	1.843	426	20012	.4289266118

m = 3, n = 7 (continued)

H	C_n	f	Σf	P
266	1.900	898	19586	.4197959534
278	1.986	442	18688	.4005486968
288	2.057	207	18246	.3910751029
294	2.100	576	18039	.3866383745
296	2.114	432	17463	.3742926955
302	2.157	424	17031	.3650334362
312	2.229	366	16607	.3559456447
314	2.243	404	16241	.3481009945
326	2.329	394	15837	.3394418724
338	2.414	524	15443	.3309970850
342	2.443	318	14919	.3197659465
344	2.457	368	14601	.3129501029
350	2.500	344	14233	.3050625857
362	2.586	340	13889	.2976894719
366	2.614	318	13549	.2904020919
378	2.700	282	13231	.2835862483
384	2.743	146	12949	.2775420096
386	2.757	330	12803	.2744127229
392	2.800	479	12473	.2673396776
398	2.843	296	11994	.2570730453
402	2.871	296	11698	.2507287380
416	2.971	308	11402	.2443844307
422	3.014	280	11094	.2377829218
434	3.100	546	10814	.2317815501
438	3.129	232	10268	.2200788752
446	3.186	252	10036	.2151063100
450	3.214	106	9784	.2097050754
456	3.257	226	9678	.2074331276
458	3.271	254	9452	.2025891632
474	3.386	206	9198	.1971450617
482	3.443	234	8992	.1927297668
486	3.471	110	8758	.1877143347
488	3.486	238	8648	.1853566529
494	3.529	460	8410	.1802554870
504	3.600	192	7950	.1703960905
512	3.657	108	7758	.1662808642
518	3.700	396	7650	.1639660494
536	3.829	196	7254	.1554783951
542	3.871	186	7058	.1512774348
546	3.900	332	6872	.1472908093
554	3.957	170	6540	.1401748971
558	3.986	182	6370	.1365312071
566	4.043	172	6188	.1326303155
578	4.129	82	6016	.1289437586
582	4.157	150	5934	.1271862140
584	4.171	166	5784	.1239711934
600	4.286	68	5618	.1204132373

m = 3, n = 7 (continued)

H	C_n	f	Σf	P
602	4.300	300	5550	.1189557613
608	4.343	162	5250	.1125257202
614	4.386	160	5088	.1090534979
618	4.414	148	4928	.1056241427
626	4.471	146	4780	.1024519890
632	4.514	134	4634	.0993227023
648	4.629	59	4500	.0964506173
650	4.643	146	4441	.0951860425
654	4.671	114	4295	.0920567558
662	4.729	116	4181	.0896133402
666	4.757	106	4065	.0871270576
672	4.800	112	3959	.0848551097
674	4.814	112	3847	.0824545610
686	4.900	238	3735	.0800540123
698	4.986	108	3497	.0749528464
702	5.014	92	3389	.0726380316
722	5.157	146	3297	.0706661523
726	5.186	52	3151	.0675368656
728	5.200	194	3099	.0664223251
734	5.243	102	2905	.0622642318
744	5.314	80	2803	.0600780178
746	5.329	90	2723	.0583633402
758	5.414	86	2633	.0564343278
762	5.443	82	2547	.0545910494
774	5.529	70	2465	.0528335048
776	5.543	84	2395	.0513331619
794	5.671	74	2311	.0495327503
798	5.700	140	2237	.0479466735
800	5.714	40	2097	.0449459877
806	5.757	132	2057	.0440886488
818	5.843	70	1925	.0412594307
824	5.886	66	1855	.0397590878
834	5.957	54	1789	.0383444787
842	6.014	64	1735	.0371870713
854	6.100	116	1671	.0358153292
864	6.171	28	1555	.0333290466
866	6.186	64	1527	.0327289095
872	6.229	56	1463	.0313571674
878	6.271	52	1407	.0301568930
882	6.300	88	1355	.0290423525
888	6.343	46	1267	.0271562071
896	6.400	50	1221	.0261702675
906	6.471	40	1171	.0250985940
914	6.529	48	1131	.0242412551
926	6.614	38	1083	.0232124486
936	6.686	38	1045	.0223979767
938	6.700	62	1007	.0215835048

m = 3, n = 7 (continued)

H	C_n	f	Σf	P
942	6.729	28	945	.0202546296
950	6.786	34	917	.0196544925
962	6.871	66	883	.0189257545
968	6.914	18	817	.0175111454
974	6.957	36	799	.0171253429
978	6.986	26	763	.0163537380
992	7.086	36	737	.0157964678
998	7.129	18	701	.0150248628
1014	7.243	38	683	.0146390604
1016	7.257	32	645	.0138245885
1022	7.300	50	613	.0131387174
1026	7.329	22	563	.0120670439
1032	7.371	28	541	.0115955075
1046	7.471	28	513	.0109953704
1050	7.500	14	485	.0103952332
1058	7.557	10	471	.0100951646
1064	7.600	48	461	.0098808299
1082	7.729	18	413	.0088520233
1086	7.757	20	395	.0084662209
1094	7.814	16	375	.0080375514
1098	7.843	16	359	.0076946159
1106	7.900	30	343	.0073516804
1112	7.943	20	313	.0067086763
1118	7.986	30	293	.0062800069
1134	8.100	10	263	.0056370027
1142	8.157	16	253	.0054226680
1152	8.229	5	237	.0050797325
1154	8.243	14	232	.0049725652
1158	8.271	8	218	.0046724966
1176	8.400	18	210	.0045010288
1178	8.414	24	192	.0041152263
1184	8.457	16	168	.0036008230
1194	8.529	14	152	.0032578875
1202	8.586	6	138	.0029578189
1206	8.614	10	132	.0028292181
1208	8.629	14	122	.0026148834
1214	8.671	10	108	.0023148148
1226	8.757	12	98	.0021004801
1238	8.843	6	86	.0018432785
1248	8.914	10	80	.0017146776
1250	8.929	2	70	.0015003429
1256	8.971	10	68	.0014574760
1274	9.100	16	58	.0012431413
1302	9.300	4	42	.0009002058
1304	9.314	8	38	.0008144719
1314	9.386	4	30	.0006430041

m = 3, n = 7 (continued)

H	C_n	f	Σf	P
1338	9.557	6	26	.0005572702
1352	9.657	1	20	.0004286694
1358	9.700	2	19	.0004072359
1376	9.829	4	17	.0003643690
1406	10.043	4	13	.0002786351
1418	10.129	4	9	.0001929012
1464	10.457	2	5	.0001071674
1514	10.814	2	3	.0000643004
1568	11.200	1	1	.0000214335

m = 4, n = 3

H	C_n	f	Σf	P
0	.000	1	576	1.0000000000
2	.086	3	575	.9982638889
4	.171	2	572	.9930555556
6	.257	10	570	.9895833333
8	.343	7	560	.9722222222
10	.429	8	553	.9600694444
12	.514	2	545	.9461805556
14	.600	12	543	.9427083333
16	.686	2	531	.9218750000
18	.771	13	529	.9184027778
20	.857	9	516	.8958333333
22	.943	4	507	.8802083333
24	1.029	14	503	.8732638889
26	1.114	19	489	.8489583333
30	1.286	12	470	.8159722222
32	1.371	3	458	.7951388889
34	1.457	6	455	.7899305556
36	1.543	15	449	.7795138889
38	1.629	18	434	.7534722222
40	1.714	7	416	.7222222222
42	1.800	14	409	.7100694444
44	1.886	4	395	.6857638889
46	1.971	6	391	.6788194444
48	2.057	2	385	.6684027778
50	2.143	15	383	.6649305556
52	2.229	3	368	.6388888889
54	2.314	26	365	.6336805556
56	2.400	14	339	.5885416667
58	2.486	2	325	.5642361111
62	2.657	12	323	.5607638889
64	2.743	1	311	.5399305556
66	2.829	20	310	.5381944444
68	2.914	14	290	.5034722222
70	3.000	12	276	.4791666667
72	3.086	14	264	.4583333333
74	3.171	19	250	.4340277778
76	3.257	4	231	.4010416667
78	3.343	6	227	.3940972222
80	3.429	3	221	.3836805556
82	3.514	4	218	.3784722222
84	3.600	10	214	.3715277778
86	3.686	14	204	.3541666667
88	3.771	4	190	.3298611111
90	3.857	20	186	.3229166667
94	4.029	6	166	.2881944444
96	4.114	2	160	.2777777778
98	4.200	10	158	.2743055556

m = 4, n = 3 (continued)

H	C_n	f	Σf	P
100	4.286	6	148	.2569444444
102	4.371	8	142	.2465277778
104	4.457	11	134	.2326388889
106	4.543	4	123	.2135416667
108	4.629	4	119	.2065972222
110	4.714	16	115	.1996527778
114	4.886	10	99	.1718750000
116	4.971	8	89	.1545138889
118	5.057	4	81	.1406250000
120	5.143	6	77	.1336805556
122	5.229	5	71	.1232638889
126	5.400	10	66	.1145833333
130	5.571	6	56	.0972222222
132	5.657	8	50	.0868055556
134	5.743	8	42	.0729166667
136	5.829	1	34	.0590277778
140	6.000	2	33	.0572916667
144	6.171	1	31	.0538194444
146	6.257	6	30	.0520833333
148	6.343	1	24	.0416666667
150	6.429	8	23	.0399305556
152	6.514	2	15	.0260416667
154	6.600	2	13	.0225694444
160	6.857	1	11	.0190972222
162	6.943	3	10	.0173611111
164	7.029	3	7	.0121527778
170	7.286	3	4	.0069444444
180	7.714	1	1	.0017361111

m = 4, n = 4

H	C_n	f	Σf	P
0	.000	3	13824	1.0000000000
2	.040	43	13821	.9997829861
4	.080	26	13778	.9966724537
6	.120	64	13752	.9947916667
8	.160	67	13688	.9901620370
10	.200	64	13621	.9853153935
12	.240	22	13557	.9806857639
14	.280	164	13535	.9790943287
16	.320	15	13371	.9672309028
18	.360	98	13356	.9661458333
20	.400	126	13258	.9590567130
22	.440	62	13132	.9499421296
24	.480	104	13070	.9454571759
26	.520	202	12966	.9379340278
30	.600	120	12764	.9233217593
32	.640	33	12644	.9146412037
34	.680	114	12611	.9122540509
36	.720	106	12497	.9040075231
38	.760	172	12391	.8963396991
40	.800	92	12219	.8838975694
42	.840	108	12127	.8772424769
44	.880	56	12019	.8694299769
46	.920	100	11963	.8653790509
48	.960	18	11863	.8581452546
50	1.000	206	11845	.8568431713
52	1.040	82	11639	.8419415509
54	1.080	182	11557	.8360098380
56	1.120	192	11375	.8228443287
58	1.160	42	11183	.8089554398
62	1.240	232	11141	.8059172454
64	1.280	11	10909	.7891348380
66	1.320	182	10898	.7883391204
68	1.360	148	10716	.7751736111
70	1.400	72	10568	.7644675926
72	1.440	107	10496	.7592592593
74	1.480	240	10389	.7515190972
76	1.520	42	10149	.7341579861
78	1.560	88	10107	.7311197917
80	1.600	56	10019	.7247540509
82	1.640	78	9963	.7207031250
84	1.680	128	9885	.7150607639
86	1.720	216	9757	.7058015046
88	1.760	68	9541	.6901765046
90	1.800	162	9473	.6852575231
94	1.880	160	9311	.6735387731
96	1.920	40	9151	.6619646991
98	1.960	195	9111	.6590711806

m = 4, n = 4 (continued)

H	C_n	f	Σf	P
100	2.000	66	8916	.6449652778
102	2.040	68	8850	.6401909722
104	2.080	212	8782	.6352719907
106	2.120	112	8570	.6199363426
108	2.160	42	8458	.6118344907
110	2.200	228	8416	.6087962963
114	2.280	136	8188	.5923032407
116	2.320	194	8052	.5824652778
118	2.360	104	7858	.5684317130
120	2.400	116	7754	.5609085648
122	2.440	202	7638	.5525173611
126	2.520	192	7436	.5379050926
128	2.560	18	7244	.5240162037
130	2.600	50	7226	.5227141204
132	2.640	96	7176	.5190972222
134	2.680	218	7080	.5121527778
136	2.720	92	6862	.4963831019
138	2.760	120	6770	.4897280093
140	2.800	60	6650	.4810474537
142	2.840	56	6590	.4767071759
144	2.880	32	6534	.4726562500
146	2.920	236	6502	.4703414352
148	2.960	46	6266	.4532696759
150	3.000	132	6220	.4499421296
152	3.040	140	6088	.4403935185
154	3.080	100	5948	.4302662037
158	3.160	120	5848	.4230324074
160	3.200	26	5728	.4143518519
162	3.240	103	5702	.4124710648
164	3.280	160	5599	.4050202546
166	3.320	100	5439	.3934461806
168	3.360	80	5339	.3862123843
170	3.400	152	5259	.3804253472
172	3.440	22	5107	.3694299769
174	3.480	132	5085	.3678385417
176	3.520	22	4953	.3582899306
178	3.560	94	4931	.3566984954
180	3.600	107	4837	.3498987269
182	3.640	132	4730	.3421585648
184	3.680	76	4598	.3326099537
186	3.720	120	4522	.3271122685
190	3.800	44	4402	.3184317130
192	3.840	6	4358	.3152488426
194	3.880	201	4352	.3148148148
196	3.920	64	4151	.3002748843
198	3.960	84	4087	.2956452546

m = 4, n = 4 (continued)

H	C_n	f	Σf	P
200	4.000	123	4003	.2895688657
202	4.040	48	3880	.2806712963
204	4.080	32	3832	.2771990741
206	4.120	208	3800	.2748842593
208	4.160	17	3592	.2598379630
210	4.200	64	3575	.2586082176
212	4.240	97	3511	.2539785880
214	4.280	42	3414	.2469618056
216	4.320	96	3372	.2439236111
218	4.360	78	3276	.2369791667
222	4.440	96	3198	.2313368056
224	4.480	32	3102	.2243923611
226	4.520	64	3070	.2220775463
228	4.560	44	3006	.2174479167
230	4.600	140	2962	.2142650463
232	4.640	21	2822	.2041377315
234	4.680	122	2801	.2026186343
236	4.720	48	2679	.1937934028
238	4.760	60	2631	.1903211806
242	4.840	66	2571	.1859809028
244	4.880	67	2505	.1812065972
246	4.920	70	2438	.1763599537
248	4.960	84	2368	.1712962963
250	5.000	74	2284	.1652199074
254	5.080	104	2210	.1598668981
256	5.120	3	2106	.1523437500
258	5.160	56	2103	.1521267361
260	5.200	68	2047	.1480758102
262	5.240	36	1979	.1431568287
264	5.280	66	1943	.1405526620
266	5.320	96	1877	.1357783565
268	5.360	10	1781	.1288339120
270	5.400	64	1771	.1281105324
272	5.440	15	1707	.1234809028
274	5.480	52	1692	.1223958333
276	5.520	68	1640	.1186342593
278	5.560	56	1572	.1137152778
280	5.600	24	1516	.1096643519
282	5.640	44	1492	.1079282407
286	5.720	48	1448	.1047453704
288	5.760	9	1400	.1012731481
290	5.800	81	1391	.1006221065
292	5.840	26	1310	.0947627315
294	5.880	48	1284	.0928819444
296	5.920	59	1236	.0894097222
298	5.960	22	1177	.0851417824
300	6.000	16	1155	.0835503472

m = 4, n = 4 (continued)

H	C_n	f	Σf	P
302	6.040	44	1139	.0823929398
304	6.080	6	1095	.0792100694
306	6.120	58	1089	.0787760417
308	6.160	50	1031	.0745804398
310	6.200	22	981	.0709635417
312	6.240	16	959	.0693721065
314	6.280	82	943	.0682146991
318	6.360	36	861	.0622829861
320	6.400	7	825	.0596788194
322	6.440	18	818	.0591724537
324	6.480	38	800	.0578703704
326	6.520	60	762	.0551215278
328	6.560	18	702	.0507812500
330	6.600	16	684	.0494791667
332	6.640	22	668	.0483217593
334	6.680	36	646	.0467303241
336	6.720	6	610	.0441261574
338	6.760	44	604	.0436921296
340	6.800	20	560	.0405092593
342	6.840	34	540	.0390625000
344	6.880	28	506	.0366030093
346	6.920	16	478	.0345775463
350	7.000	40	462	.0334201389
354	7.080	14	422	.0305266204
356	7.120	42	408	.0295138889
358	7.160	8	366	.0264756944
360	7.200	15	358	.0258969907
362	7.240	32	343	.0248119213
364	7.280	6	311	.0224971065
366	7.320	12	305	.0220630787
370	7.400	8	293	.0211950231
372	7.440	10	285	.0206163194
374	7.480	34	275	.0198929398
376	7.520	8	241	.0174334491
378	7.560	16	233	.0168547454
382	7.640	8	217	.0156973380
386	7.720	21	209	.0151186343
388	7.760	2	188	.0135995370
390	7.800	10	186	.0134548611
392	7.840	7	176	.0127314815
394	7.880	2	169	.0122251157
396	7.920	4	167	.0120804398
398	7.960	24	163	.0117910880
400	8.000	2	139	.0100549769
402	8.040	4	137	.0099103009
404	8.080	24	133	.0096209491
406	8.120	2	109	.0078848380
408	8.160	4	107	.0077401620

m = 4, n = 4 (continued)

H	C_n	f	Σf	P
410	8.200	14	103	.0074508102
414	8.280	8	89	.0064380787
416	8.320	4	81	.0058593750
418	8.360	2	77	.0055700231
420	8.400	4	75	.0054253472
422	8.440	8	71	.0051359954
424	8.480	1	63	.0045572917
426	8.520	12	62	.0044849537
428	8.560	2	50	.0036168981
432	8.640	2	48	.0034722222
434	8.680	8	46	.0033275463
436	8.720	1	38	.0027488426
440	8.800	4	37	.0026765046
446	8.920	8	33	.0023871528
450	9.000	5	25	.0018084491
452	9.040	6	20	.0014467593
458	9.160	6	14	.0010127315
464	9.280	1	8	.0005787037
468	9.360	3	7	.0005063657
482	9.640	3	4	.0002893519
500	10.000	1	1	.0000723380

m = 5, n = 3

H	C_n	f	Σf	P
0	.000	1	14400	1.0000000000
2	.057	4	14399	.9999305556
4	.114	7	14395	.9996527778
6	.171	18	14388	.9991666667
8	.229	26	14370	.9979166667
10	.286	32	14344	.9961111111
12	.343	6	14312	.9938888889
14	.400	50	14306	.9934722222
16	.457	37	14256	.9900000000
18	.514	50	14219	.9874305556
20	.571	36	14169	.9839583333
22	.629	44	14133	.9814583333
24	.686	74	14089	.9784027778
26	.743	106	14015	.9732638889
28	.800	36	13909	.9659027778
30	.857	92	13873	.9634027778
32	.914	60	13781	.9570138889
34	.971	90	13721	.9528472222
36	1.029	63	13631	.9465972222
38	1.086	98	13568	.9422222222
40	1.143	97	13470	.9354166667
42	1.200	56	13373	.9286805556
44	1.257	98	13317	.9247916667
46	1.314	132	13219	.9179861111
48	1.371	50	13087	.9088194444
50	1.429	178	13037	.9053472222
52	1.486	33	12859	.8929861111
54	1.543	148	12826	.8906944444
56	1.600	152	12678	.8804166667
58	1.657	92	12526	.8698611111
60	1.714	74	12434	.8634722222
62	1.771	142	12360	.8583333333
64	1.829	109	12218	.8484722222
66	1.886	138	12109	.8409027778
68	1.943	64	11971	.8313194444
70	2.000	150	11907	.8268750000
72	2.057	121	11757	.8164583333
74	2.114	208	11636	.8080555556
76	2.171	98	11428	.7936111111
78	2.229	62	11330	.7868055556
80	2.286	151	11268	.7825000000
82	2.343	110	11117	.7720138889
84	2.400	68	11007	.7643750000
86	2.457	236	10939	.7596527778
88	2.514	114	10703	.7432638889
90	2.571	244	10589	.7353472222
92	2.629	90	10345	.7184027778
94	2.686	172	10255	.7121527778

m = 5, n = 3 (continued)

H	C_n	f	Σf	P
96	2.743	112	10083	.7002083333
98	2.800	112	9971	.6924305556
100	2.857	99	9859	.6846527778
102	2.914	108	9760	.6777777778
104	2.971	227	9652	.6702777778
106	3.029	188	9425	.6545138889
108	3.086	64	9237	.6414583333
110	3.143	258	9173	.6370138889
112	3.200	64	8915	.6190972222
114	3.257	172	8851	.6146527778
116	3.314	124	8679	.6027083333
118	3.371	128	8555	.5940972222
120	3.429	148	8427	.5852083333
122	3.486	172	8279	.5749305556
124	3.543	96	8107	.5629861111
126	3.600	174	8011	.5563194444
128	3.657	135	7837	.5442361111
130	3.714	210	7702	.5348611111
132	3.771	58	7492	.5202777778
134	3.829	258	7434	.5162500000
136	3.886	170	7176	.4983333333
138	3.943	92	7006	.4865277778
140	4.000	90	6914	.4801388889
142	4.057	114	6824	.4738888889
144	4.114	170	6710	.4659722222
146	4.171	248	6540	.4541666667
148	4.229	71	6292	.4369444444
150	4.286	192	6221	.4320138889
152	4.343	150	6029	.4186805556
154	4.400	156	5879	.4082638889
156	4.457	74	5723	.3974305556
158	4.514	150	5649	.3922916667
160	4.571	123	5499	.3818750000
162	4.629	118	5376	.3733333333
164	4.686	118	5258	.3651388889
166	4.743	178	5140	.3569444444
168	4.800	82	4962	.3445833333
170	4.857	214	4880	.3388888889
172	4.914	48	4666	.3240277778
174	4.971	158	4618	.3206944444
176	5.029	184	4460	.3097222222
178	5.086	106	4276	.2969444444
180	5.143	102	4170	.2895833333
182	5.200	92	4068	.2825000000
184	5.257	130	3976	.2761111111
186	5.314	126	3846	.2670833333
188	5.371	56	3720	.2583333333

m = 5, n = 3 (continued)

H	C_n	f	Σf	P
190	5.429	160	3664	.2544444444
192	5.486	64	3504	.2433333333
194	5.543	198	3440	.2388888889
196	5.600	70	3242	.2251388889
198	5.657	120	3172	.2202777778
200	5.714	134	3052	.2119444444
202	5.771	78	2918	.2026388889
204	5.829	62	2840	.1972222222
206	5.886	170	2778	.1929166667
208	5.943	57	2608	.1811111111
210	6.000	94	2551	.1771527778
212	6.057	52	2457	.1706250000
214	6.114	100	2405	.1670138889
216	6.171	116	2305	.1600694444
218	6.229	106	2189	.1520138889
220	6.286	54	2083	.1446527778
222	6.343	64	2029	.1409027778
224	6.400	86	1965	.1364583333
226	6.457	102	1879	.1304861111
228	6.514	28	1777	.1234027778
230	6.571	116	1749	.1214583333
232	6.629	56	1633	.1134027778
234	6.686	130	1577	.1095138889
236	6.743	62	1447	.1004861111
238	6.800	38	1385	.0961805556
240	6.857	40	1347	.0935416667
242	6.914	66	1307	.0907638889
244	6.971	29	1241	.0861805556
246	7.029	74	1212	.0841666667
248	7.086	70	1138	.0790277778
250	7.143	86	1068	.0741666667
252	7.200	36	982	.0681944444
254	7.257	80	946	.0656944444
256	7.314	48	866	.0601388889
258	7.371	26	818	.0568055556
260	7.429	26	792	.0550000000
262	7.486	30	766	.0531944444
264	7.543	46	736	.0511111111
266	7.600	54	690	.0479166667
268	7.657	14	636	.0441666667
270	7.714	58	622	.0431944444
272	7.771	13	564	.0391666667
274	7.829	30	551	.0382638889
276	7.886	22	521	.0361805556
278	7.943	44	499	.0346527778
280	8.000	38	455	.0315972222
282	8.057	20	417	.0289583333
284	8.114	38	397	.0275694444

m = 5, n = 3 (continued)

H	C_n	f	Σf	P
286	8.171	26	359	.0249305556
288	8.229	11	333	.0231250000
290	8.286	42	322	.0223611111
292	8.343	8	280	.0194444444
294	8.400	24	272	.0188888889
296	8.457	37	248	.0172222222
298	8.514	16	211	.0146527778
300	8.571	12	195	.0135416667
302	8.629	18	183	.0127083333
304	8.686	10	165	.0114583333
306	8.743	18	155	.0107638889
308	8.800	4	137	.0095138889
310	8.857	18	133	.0092361111
312	8.914	20	115	.0079861111
314	8.971	20	95	.0065972222
316	9.029	6	75	.0052083333
320	9.143	11	69	.0047916667
324	9.257	3	58	.0040277778
326	9.314	12	55	.0038194444
328	9.371	3	43	.0029861111
330	9.429	12	40	.0027777778
332	9.486	6	28	.0019444444
334	9.543	6	22	.0015277778
340	9.714	3	16	.0011111111
342	9.771	4	13	.0009027778
344	9.829	4	9	.0006250000
350	10.000	4	5	.0003472222
360	10.286	1	1	.0000694444