

7482

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Theory of Areal Spaces

An areal space is defined as a space endowed a priori with a measure of an m-dimensional plane element:

$$dS = F(x^i_\alpha, p^i_\alpha) du^1 \dots du^m \quad \alpha = 1, 2, \dots, m (< n)$$

which should be regarded as the area of an infinitesimal domain on a subspace of m dimensions; $x^i = x^i(u^\alpha)$, putting

$$p^i_\alpha = \frac{\partial x^i}{\partial u^\alpha}.$$

This space is a Finsler space for $m=1$ and a Cartan space for $m=n-1$.

If we can derive a tensor of second order g_{ij} (rank n) from F and its first and second derivatives by an algebraic operation the space is called of the submetric class.

On making use of the Legendre's form of F:

$$L^{\alpha\beta}_{ij} = \frac{1}{F} \frac{\partial^2 F}{\partial p^i_\alpha \partial p^j_\beta} - \frac{\partial \log F}{\partial p^i_\alpha} \frac{\partial \log F}{\partial p^j_\beta} + \frac{\partial \log F}{\partial p^i_\beta} \frac{\partial \log F}{\partial p^j_\alpha}$$

we can obtain the m-tensor

$$(1) \quad g_{i_1 i_2 \dots i_m}; \quad j_1 j_2 \dots j_m$$

which splits to

$$(2) \quad m! g [i_1 [j_1 g_{i_2 j_2} \dots g_{i_m j_m}] j_m]$$

if the space is one of Riemannian, Finsler and Cartan spaces. When (1) is equal to (2) in a space of the submetric class, then the space is named of the metric class.

The main results are as follows:

(1) The space of the metric class must be one of Riemannian, Finsler and Cartan spaces, that is, it must be

$$m=1, \quad m=n-1 \quad \text{or} \quad \frac{\partial}{\partial p^k_\alpha} g_{ij} = 0.$$

(2) In general, an areal space should be of the submetric class, that is, the tensor g_{ij} can be derived from

$$F, \quad \frac{\partial F}{\partial p^i_\alpha}, \quad \frac{\partial^2 F}{\partial p^i_\alpha \partial p^j_\beta}$$

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by an algebraic operation, exceptⁱⁿ some special cases.

Therefore, for the general theory of areal spaces it is sufficient to consider an areal space of the submetric class. By this reason, we shall determine the connection parameters

$\Gamma_{jk}^i, C_{j,k}^{i\alpha}$ in the space of submetric class, that is the introduction of the theory of areal spaces. There are many unsolved but very interesting problems in this theory.