HERMITE NUMERICAL METHOD TO ESTIMATE THE RADON AND RADIUM EFFECTS OF THE SOIL IN BARTELLA REGION

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Abstract

The Hermite numerical interpolation method used to estimate and predict the radon and radium concentrations, from some of the experimental measurements made on the soil samples of the Bartella region in Iraq. The results obtained from deductible mathematical method were close to real experimental results. Predicted results are in the range of 80.04–4051.59 Bq/m³ radon concentrations and 0.322–17.276 Bq/Kg radium concentrations corresponding to 200–10000 tracks of alpha particles. The maximum errors were less than 0.289 and 1.172 percent for radon and radium concentrations, respectively. The effects of radium and radon radiations estimated of soil in the studied area depend on its concentrations, a higher number of alpha particle tracks mean high radiation concentration permissible values, by the Organization alltk Economic Cooperation and Development (OECD), the studied area is safe from hazards on health of radium and radon exposure from the soil.

Keywords: concentrations of radon, concentrations of radium, Hermite method, soil samples, applied mathematics, numerical interpolation.

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1. Introduction

Applied mathematics is one of the important mathematics branches, it solves applied sciences different problems, it deals with all the stages for solving the problems, namely, given a problem, formulate a mathematical model that describes it, through analytical or numerical methods, solve the model, compare the model results with experimental results, in the case that they disagree qualitatively, go back and reformulate the problem [1]. The numerical Hermite interpolation method used in applied mathematics, which gives values with small amount error from the real values. The method can be used to estimate and predict results close to the experiments from a few sample values when measurements completely are not available for any reason [2]. The Hermite interpolation numerical method is an interpolating method of data points as a function of polynomial. The Hermite interpolating generated polynomial is related closely to the polynomial of Newton, both from the calculation of divided are derived [3]. Hermite approximations method can be generated from the basis of Newton interpolatory formula of divided difference at x_0, x_1, \dots, x_n , that is:

$$P_n(x) = f(x_0) + \sum_{k=1}^n f[x_0, x_1, \dots, x_k](x - x_0)(x - x_1) \dots (x - x_{k-1}).$$
(1)

Tabla 1

The method uses connection between the n^{th} derivative of function and the n^{th} divided difference [4]. The entries used for the first three columns of divided-difference in determining the Hermite polynomial $H_{11}(x)$ for x_0, x_1, \dots, x_5 , are shown in the following **Table 1**.

| | Table I | | |
|-----------------------|--------------------|---|--|
| , | The Newton's divid | led difference | |
| x | $f(x_0)$ | First divided differently | Second, divided differently |
| <i>x</i> ₀ | $f[x_0]$ | $f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{x_1 - x_0}$ | |
| x_1 | $f[x_1]$ | $f[x_1, x_2] = \frac{f[x_2] - f[x_1]}{x_2 - x_1}$ | $f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$ |
| <i>x</i> ₂ | $f[x_2]$ | $f[x_2, x_3] = \frac{f[x_3] - f[x_2]}{f[x_2, x_3]}$ | $f[x_1, x_2, x_3] = \frac{f[x_2, x_3] - f[x_1, x_2]}{x_3 - x_1}$ |
| <i>x</i> ₃ | $f[x_3]$ | $f[x_{3}, x_{4}] = \frac{f[x_{4}] - f[x_{3}]}{f[x_{3}, x_{4}]}$ | $f[x_2, x_3, x_4] = \frac{f[x_3, x_4] - f[x_2, x_3]}{x_4 - x_2}$ |
| <i>x</i> ₄ | $f[x_4]$ | $f[x_{1}, x_{5}] = \frac{f[x_{5}] - f[x_{4}]}{f[x_{5}] - f[x_{4}]}$ | $f[x_3, x_4, x_5] = \frac{f[x_4, x_5] - f[x_3, x_4]}{x_5 - x_3}$ |
| <i>x</i> ₅ | $f[x_5]$ | $x_{5} - x_{4}$ | |

In the same manner the remaining entries generated as that of Newton's divided difference **Table 1**, the Hermite polynomial is given by [5]:

$$H_{2n+1}(x) = f[x_0] + \sum_{k=1}^{2n+1} f[x_0, x_1, \dots, x_k](x - x_0)(x - x_1) \dots (x - x_{k-1}).$$
(2)

Recently, many researcher have been studied the estimation subject as, the finite element method (FEM) which is an efficient method for solving the engineering and mathematical physics most diverse problems [6]. The absorbed dose estimation of surface skin for the standard radio-logical examinations of the patient [7]. Estimation solar radiation diffuse over Iraq [8]. Estimation the concentration of the atmospheric CO_2 in Iraq [9]. Estimation soil erosion by the neural network models [10]. Method for time series prediction by small sets of experimental samples [11]. Estimation the air Kerma of entrance surface and dose area product for the examined patient by the long term X-ray examination [12]. Mathematical estimation of the inserted sand column bearing capacity in soft clay soil [13]. Estimation of the soil samples radiological hazardous effects in Nineveh province [14], and others applications [15–17].

In this research, Hermite numerical interpolation method used to obtain a mathematical model for estimatting the radon and radium radiations concentrations amounts, and the effects of the soil of Bartella region, the estimations and measurements were very close together since the very small deviation between them. Radon gas and radium are radioactive elements produced from the decay of well known unstable uranium material element, the decay of radioactive elements occurs with emitting alpha particles, beta particles, or electromagnetic radiations. The effects of these radioactive elements when entries to the human body cause many diseases, an important one is cancers especially lung cancer by the radon. So measurement and control of their concentrations are very important.

2. Materials and Methods

The Hermite method used to estimate the radiation concentrations in the soil samples of Bartella region located in the Nineveh plain region of Iraq, which its an open plain area, mostly has red soils, where N denotes the radiation emitted alpha particle track counts recorded by a CR-39 nuclear track detector used in measurements the soil sample in the studied area, R_k is the radiation concentration, experimental results taken from [18]. The method can be explained as follows:

For the radon element if there are some values of the sample's track count with their corresponding concentration listed in **Table 2**.

Table 2

Initial experimental track count with their radon concentrations

| Tracks | count N | Concentra | tions $f(N)$ |
|--------|---------|-----------|--------------|
| N_0 | 800 | $f(N_0)$ | 323.2 |
| N_1 | 933 | $f(N_1)$ | 377.1 |
| N_2 | 1040 | $f(N_2)$ | 420.2 |
| N_3 | 1200 | $f(N_3)$ | 484.8 |

As in (2) and Table 1, Hermite polynomial of this situation will be

$$R_{k} = f(N_{0}) + f[N_{0}, N_{1}](N - N_{0}) + f[N_{0}, N_{1}, N_{2}](N - N_{0})(N - N_{1}) + f[N_{0}, N_{1}, N_{2}, N_{3}](N - N_{0})(N - N_{1})(N - N_{2}).$$
(3)

Substituting the given experimental values for radon from Table 2 into (3) let's obtain

$$R_k = 0.40526N - 1.008. \tag{4}$$

This equation by substituting N estimated the results of radon concentrations.

For the radium element, some values of the sample's track count with their corresponding concentration listed in **Table 3**.

Table 3

Initial experimental track count with their radium concentrations

| Tracks | count N | Concentrat | tions f(N) |
|--------|---------|------------|------------|
| N_0 | 800 | $f(N_0)$ | 1.3 |
| N_1 | 933 | $f(N_1)$ | 1.59 |
| N_2 | 1040 | $f(N_2)$ | 1.77 |
| N_3 | 1200 | $f(N_3)$ | 2.04 |

As in (3) with Substituting the given experimental values for radium from Table 3, let's obtain

$$R_k = 0.00173N - 0.024. \tag{5}$$

By substituting N estimated the results of radium concentrations.

3. Results

The estimated (predicted) results listed in the **Table 4**, and the comparison between estimated and experimental results listed in **Table 5**.

The estimated results of radon and radium concentrations from the mathematical method depend on N, which represent the number of alpha particle tracks, the range of N is between 200–10000 track as in **Table 4**. Comparison between estimated results and experimental available range of N between 800–6000 were made, and the percentage deviation form experimental results calculated, the obtained results showed that the calculated results for radon and radium concentrations are accurate and close to experiments since the deviation values are very small, as in **Table 5**.

Table 4

Estimated results of radon and radium obtained from mathematical method

| N | Radon R _k Est. | Radium <i>R_k</i> Est. |
|-------|---------------------------|----------------------------------|
| 200 | 80.04 | 0.322 |
| 400 | 161.09 | 0.668 |
| 600 | 242.14 | 1.014 |
| 800 | 323.2 | 1.36 |
| 1200 | 485.3 | 2.05 |
| 1600 | 647.41 | 2.74 |
| 2400 | 971.62 | 4.1 |
| 3600 | 1458 | 6.2 |
| 3906 | 1582 | 6.73 |
| 5066 | 2052 | 8.74 |
| 6000 | 2431 | 10.36 |
| 7000 | 2835.81 | 12.086 |
| 8000 | 3241.07 | 13.816 |
| 9000 | 3646.33 | 15.546 |
| 10000 | 4051.59 | 17.276 |

Table 5

Comparison between the obtained estimated and available experimental results for radon and radium

| | | | Radon | | | Radium |
|------|----------------------------------|---------------------------------|--------------------------------------|-----------------------------------|----------------------------------|--|
| N | Radon <i>R</i> _k Est. | Radon <i>R_k</i> Exp. | $R_{k Est.} - R_{k Exp} / R_{k Exp}$ | Radium <i>R</i> _k Est. | Radium <i>R_k</i> Exp. | $\left R_{k\ Est.}-R_{k\ Exp}\right /R_{k\ Exp}$ |
| | | | ×100 % | | | ×100 % |
| 800 | 323.2 | 323.2 | 0 | 1.36 | 1.36 | 0 |
| 933 | 377.1 | 377.1 | 0 | 1.59 | 1.59 | 0 |
| 1040 | 420.46 | 420.2 | 0.062 | 1.77 | 1.77 | 0 |
| 1200 | 485.3 | 484.8 | 0.103 | 2.05 | 2.04 | 0.49 |
| 1280 | 517.7 | 517.1 | 0.116 | 2.19 | 2.18 | 0.459 |
| 1600 | 647.41 | 646.4 | 0.156 | 2.74 | 2.73 | 0.366 |
| 1733 | 701.31 | 700.3 | 0.144 | 2.97 | 2.96 | 0.338 |
| 2093 | 847.2 | 845.7 | 0.177 | 3.6 | 3.57 | 0.84 |
| 2266 | 917.3 | 915.8 | 0.164 | 3.9 | 3.87 | 0.775 |
| 2400 | 971.62 | 969.7 | 0.198 | 4.1 | 4.1 | 0 |
| 2933 | 1187.6 | 1185 | 0.221 | 5 | 5 | 0 |
| 3240 | 1312 | 1309 | 0.229 | 5.58 | 5.53 | 0.904 |
| 3600 | 1458 | 1454 | 0.275 | 6.2 | 6.14 | 0.977 |
| 3906 | 1582 | 1578 | 0.253 | 6.73 | 6.67 | 0.9 |
| 4133 | 1674 | 1670 | 0.24 | 7.13 | 7.05 | 1.135 |
| 4266 | 1728 | 1723 | 0.29 | 7.36 | 7.28 | 1.099 |
| 4520 | 1831 | 1826 | 0.274 | 7.8 | 7.68 | 1.563 |
| 4573 | 1852 | 1847 | 0.271 | 7.89 | 7.81 | 1.024 |
| 5066 | 2052 | 2047 | 0.244 | 8.74 | 8.65 | 1.04 |
| 5466 | 2214 | 2208 | 0.272 | 9.43 | 9.33 | 1.072 |
| 6000 | 2431 | 2424 | 0.289 | 10.36 | 10.24 | 1.172 |

4. Discussion

When the proposed mathematical method applied, perfect and conformed results were obtained, the maximum errors were less than 0.289 and 1.172 percent for radon and radium concentrations, respectively, which means that the estimated results are very close to the results of the experiment. The estimated of radon concentration values are in the range of 80.04–4051.59 Bq/m³ radon concentrations and 0.322–17.276 Bq/Kg radium concentrations corresponding to 200–10000 tracks of alpha particles, The values are below 370 Bq/Kg the recommended radium concentration permissible values of (OECD) [19]. The estimated results limitation is its dependence on the accuracy of available experimental measurements that the calculation built upon. The modification and development can be made by taking the average value of several measurements on the each soil sample to increase the accuracy of estimated results, and using different detectors in measurement and comparing between them.

Fig. 1 represents the estimated and experimental radium concentrations which shown that there are predicted values before and after experimental values in the same line means that the radium increasing with increasing of the number of alpha particle tracks, this figure can be used as the calibration curve between tracks and radium concentration.

Fig. 2 shows the radium concentration values with the rearrangement experimental samples, it's from one sample to another varied due to radium content in the sample location. The Bartella region is an open area around houses, it has a large amount of wind and air causes raised radon gas from the soil to diffusion to air, so the results of the measurements show very low values of radon and radium concentrations. The upper limit results are still within the allowed limit, therefore the area under study is safe for life from natural radiation.



Fig. 1. Radium estimated and experimental concentration variation with alpha track numbers



Sample

Fig. 2. Radium estimated and experimental concentration variation in the samples

Fig. 3 shows the obtained good positive correlation between estimated and experimental radium concentrations, it is evident they close together with small amount difference of 1.172 percent between them. This means that using the Hermite interpolation method gives good predicted results.



Fig. 3. Radium estimated with experimental concentrations variations.

5. Conclusions

In this research, the concentrations of radon and radium in soil samples estimated and predict from some of the experimental measurements, made on the soil samples of the Bartella region.

The higher number of alpha particle tracks means high radiation concentration, the high radiation will cause human health damage.

The values are below 370 Bq/Kg the recommended radium concentration permissible values by the Organization for Economic Cooperation and Development (OECD).

A good positive correlation (1.00) has been obtained between the estimated and experimental radium concentration, it is evident they close together with a small amount difference of 1.172 percent between them. This means that using the Hermite interpolation method gives good predicted results.

The Fig. 1 can be used as the calibration curve between number of the tracks and radium concentration.

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