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STUDY OF THE PROBLEM OF OPTIMAL MAINTENANCE OF UNMANNED AERIAL VEHICLES IN CONDITIONS OF A SHORTAGE OF VEHICLES

Denys Medynskyi[1]
medynskyid@gmail.com

Iryna Borets[1]
boretc.irina@gmail.com

[1]*Department of Air Transportation Management*
National Aviation University
1 Lubomyra Guzara ave., Kyiv, Ukraine, 03058

Abstract

The ways to find the ways of solving the problem of optimal service for unmanned aerial vehicles in the conditions of vehicle shortage are explored.

In the study of theory of optimal control and discrete optimization insufficiently explored the problem of optimal graph coverage by the chains of a fixed pattern with the help of a description system of ordinary differential equations.

With the help of the graph theory investigated the optimization problem of the «Upper Cover» for unmanned aerial vehicles service in the conditions of vehicle shortage has been solved.

Study of the problem relates to the mathematical dependence of transport systems. It can be used for determination of the optimal ratio between the amount of Unmanned Aerial Vehicles and fuel (electricity) reserves used by the Unmanned Aerial Vehicles. The method of solving the problem of determining the weight of the arcs of the graph and the problem of constructing a chain is based on that dependence. Part of the problem is the task of docking. The combinatorial task of choosing a set of stations for servicing points of the initial unmanned aerial vehicles dislocation is based on the dependence mentioned. Effective method for solving the problem of optimal coverage of a graph for supply chains with constraints is developed on the bases of the dependence.

The proposed research methods can significantly reduce the cost of delivery of urgent goods using unmanned aerial vehicles. Perspective of further researches is studying of mathematical model of optimal servicing the delivery areas in the conditions of the lack of UAV. The constraints for practical purposes must have a group-theoretical approach to solving optimization problems and be reduced for the constructing an optimal cost matrix analysis. Algebraic approach is used, when there is a need in solving a large set of similar types of optimization problems with different constraints in the right hand side only.

It is possible to apply a heuristic algorithm for solving the problem of optimal UAV service. The problem means optimal coverage of a special graph with chains with restriction on the chain length.

Keywords: unmanned aerial vehicle, graph, drone, optimality criterion, algorithm solving method, phase coordinates, functions, supply chains.

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1. Introduction

In the process of solving a number of practical tasks connected with the UAVs control and the use of UAVs for servicing a particular amount of locations there is a need to determine the optimal ratio between the amount of objects, the amount of stations that are able to serve the UAV and the power capacity of drones. To figure out this problem let's address the issue of the optimal branching of the set of stations, located in a certain area, in groups, each of which is served using the «Gray Crow» UAV type. In this case, the trajectory and the order of the over flights along these stations should be determined, as well as the designated stations of the initial location of the drones stationed in the districts of the region.

2. Materials and methods

Let's try to develop an effective method of algorithms for solving the problem of optimal coverage of a special graph with restriction chains, chain length and total costs for the express cargo delivery [1]. The specificity of graph CZ reveals itself in the fact that the cost of the movement between the peaks is determined by the solution of the tasks of servicing the optimal routes in the auxiliary graph of transitions CZ [2].

The mathematical formulation of the problem will be considered by analyzing the already known methods of solving the problems of discrete optimization and optimal control.

While designing the UAV the problems related to the choice of engine parameters, as well as optimal modes of motion and trajectories arise.

3. Research results

There is the task, the purpose of which is the express cargo delivery (in cases, case boxes) to *N* destinations. One of the tasks is to determine the optimal ratio between the amount of UAVs and the fuel (power) reserves used by the UAVs [1].

The choice of these parameters, in turn, leads to the need to solve optimal service tasks at the unit locations and in the conditions of UAV shortage. The essence of the problem lies in the following: in areas (Mykolaiv region) – there is a certain set of service stations and first unit locations of drones at the designated coordinates.

For a particular drone variant, which is characterized by a certain payload mass with a caseside and a limited level of engine power, it is necessary to find such a distinction of service centres in areas with their designated start-up locations, in which delivery of all items requires the use of a minimum quantity of drones.

Moving along the specified trajectory, the drone uses two modes of engine operation. In this case, a proper trajectory for the express cargo delivery will have two sections:

1) the traffic area to the first station of unloading;

2) the over flight of all destinations. The second mode of the engine operation may be partially activated in the over flight and in the first area.

The first operation mode can't be used in the destination area of the over flight. In the process of the drone's flight over all destinations the express cargo delivery is carried out, and the dynamic parameters of the motion state changes too [3].

Unmanned Aerial Vehicles movement has to be organized in such a way that the stock of the battery in the flight area of all destinations would be minimal and would not exceed the predefined task.

Restrictions on the stock of the battery lead to impossibility of serving all destinations by only one of the UAVs. The choice of service stations for a UAV's flight over must be realized in a way enables attracting (maintaining) as many areas (regions) as possible. At the same time, a minimum amount of drones should be involved. The mathematical model of one UAV with M loads can be described in the following way.

The phase coordinates of the moving object (drone) $X_s(t)$, $S=i,...,n$ and the control $V_n(t)$, $n=i,...,m$ satisfy the linkage equation, which are considered as a system of ordinary differential equations from the calculated right parts:

$$x = f_{sq}\begin{pmatrix} t,\, x_{1q}(t), ..., x_{nq}(t), \\ V_{1q}(t), ..., V_{mq}(t) \end{pmatrix}, \quad (1)$$

$$S = 1,...,n,$$

$$q = 1,...,2l-1,\ \ l \le M,$$

interval determined $[t_0, T]$, M – the quantity of case boxes.

The functions $x_{sq}(t)$ are continuous in $[t_0, T]$, and the functions $V_{rq}(t)$ and are continuous in $[t_0, T]$ with the exception $f_{sq}(t, x, u)$ of points t_q, $t_0 \le t_q \le T$ in which they have breaks of the first kind [4].

Among $2l-1$ points of t_q, rupture of function f_{sq} and $V_{rq}-l$ points correspond to the breaking points of the control, and $|l-1|$ – the disconnection of case boxes.

Control $V_r(t)$ $V\_r(t)$ satisfies the constraint:

$$V(t) \in M, \tag{2}$$

where V is a closed set.

At points t_q the case-box disconnection is executed in the following ratio:

$$Q_\tau = \begin{bmatrix} x_{1q}(t_q), ..., x_{nq}(t_q), t_q, x_1, \\ q+1(t_q), ..., x_n, q+1(t_q) \end{bmatrix} = 0, \tag{3}$$

$$\tau = 1,...p \quad p \leq 2n+1.$$

The phase coordinates of the ends of the trajectory are linked by the dependencies:

$$V_\mu = \begin{bmatrix} x_{11}(t_0), ..., x_{n1}(t_0), x_1, \\ 2l-1(T), ..., x_n, 2l-1(T) \end{bmatrix} = 0, \tag{4}$$

$$\mu = 1,...p, \quad p \leq 2n+1.$$

The control $V_{rq}(t)$ and phase coordinates must be determined in such a way that the functionality, which determines the energy consumption (fuel) for the entire length of the trajectory and the flight section of all destination parameters, would be minimal.

$$J_0(x, \upsilon, t) = \sum\nolimits_{j=0}^{l-1} J_{j,j+1}(x, \upsilon, t) \to \text{min}, \tag{5}$$

$$J(x, \upsilon, t) = \sum\nolimits_{j=0}^{l-1} J_{j,j+1}(x, \upsilon, t) \to \text{min}, \tag{6}$$

where $J(x, \upsilon, t)$ is the functional energy expenditure (fuel) in j area (j=0) corresponds to the traffic area from the first division of the unloading movement and is determined by:

$$J_{j,j+1} = \int_{t_i}^{t_{i+1}} f_j^0(x, \upsilon, t) \mathrm{d}t. \tag{7}$$

For the task of optimal maintenance in the conditions of the shortage of UAVs – the reserves of energy (fuel) for the first mode of engines operation L and for the second L_0 are predefined, respectively $J_0(x, \upsilon, t)$ and $J(x, \upsilon, t)$ must satisfy the constraints:

$$J_0(x, \upsilon, t) \leq L + L_0, \tag{8}$$

$$J(x, \upsilon, t) \leq L. \tag{9}$$

The analysis of the ways of finding the solution of the problem of optimal service indicates that two stages are required [5]. At the first stage, all admissible set of initial locations and service stations, each of which has to be serviced by one type of UAV, must be constructed. In this case, it is necessary on the first try to solve the problems of the optimal equation with the discontinuities and to attract additionally the totals for which the functional cost will be as follows:

$$J_0(x, \upsilon, t) \leq L + L_0, \tag{10}$$

$$J(x, \upsilon, t) \leq L. \tag{11}$$

At the second stage of these admissible sets it is necessary to select such a set which will provide servicing of all items with a minimum number of UAVs, that is to say in the language of the theory of graphs it is necessary to solve the optimization problem of the «Upper Cover» [6].

Such approach enables finding the best way of solving the task, that is, the search of the solution is conducted under the condition of different simplifications. As a result, it will make possible the invention of an approximate solution, but for a wide range of output parameters, the patterns, which can be used in practice, will be identified.

For this purpose, in order to determine point 5 of the graph $CZ=[UZ,GZ,TR]$ it is necessary to select a sequence of arcs $\overline{CZ}(C_{z1},C_{z2},...,C_{zn})$, ($p$ may be much larger than $|CZ|$), hat provides:

$$\min\sum\nolimits_{C_z\in\overline{CZ}} S_{ij}=S, \tag{12}$$

with constraints:

– problem A:

$$\sum\nolimits_{C_z\in\overline{CZ}} V_{ij}^{l}=A^{l},\ \ l=1,2,...,t; \tag{13}$$

– problem $\hat{A}$:

$$\sum\nolimits_{C_z\in\overline{CZ}} V_{ij}^{l}. \tag{14}$$

Equals consistently $\left(A^{1},...,A^{t}\right)...,\left(A^{1},...,A^{t}\right)_{k}$;

– problem C:

$$\sum\nolimits_{C_z\in\overline{CZ}} V_{ij}^{l}=A^{l}. \tag{15}$$

It is necessary to reach the maximum number $\left(A^{1},...,A^{t}\right)_{i}$ in case:

1) $i\le M$, where M is a given whole number;

2) $S\le L+LO$;

3) $S_1=S-S_{12}\le L$, where L and LO are given numbers.

It is necessary to reach the maximum number in case:

– problem D:

$$\sum\nolimits_{C_z\in\overline{CZ}} V_{ij}^{l}. \tag{16}$$

The expenditures of energy (fuel) at service stations are linearly dependent on the length of a jagged line joining the delivery stations. Such assumption makes it possible to simplify substantially the time for solving the problem of optimal control.

Nowadays the optimization of the selection and order of overflying the service stations due to additional refinement of energy costs is relevant as it will ensure reducing the total expenses of the UAV for maintaining a given set of areas.

For this purpose there is a need to take into account:

1) dynamics of movement, which is described by differential equations (1);

2) unloading case boxes at a point in time t_k, that results in a discontinuity of the function $f_s, s=1,...,n.$

From the mathematical point of view, the problem of optimal maintenance in case of the absence of UAVs can be formulated as the problem of optimal coverage of a special graph by the chains of a given structure.

The indicative graph $G(X^0,X,C^0,C)$ is specified by the set of vertices

$$X^0=\left\{X_p^0\right\},\ \ p=1,...,k,\ \ X=\left\{X_i\right\},\ \ i=1,...,N,$$

the set of line

$$V^0=\left\{V_{ij}^0\right\},\ \ i=1,\ \ j=1,...,N,$$

$$V=\left\{V_{ij}\right\}=\left\{\left(x_i,x_j\right)\right\},\ \ i=1,\ \ j=1,...,N.$$

The graph must be covered with chains $Q_p^{l\upsilon}$ in such a way that the first point in each *V*-chain always belongs to the set X^0 and the others belong to the set L_0. The chains $Q_p^{l\upsilon}$ must satisfy the following constraints [7]:

1) the length *l* for each chain $Q_p^{l\upsilon}$ does not exceed $M, l\le M$;

2) each chain is modeled by a territory described by a system of differential equations:

$$x_{sq}=f_{sq}\left(x_{1q},...,x_{nq},V_{1q},...,V_{mq},t\right), \tag{17}$$

$$s=1,...,n,\ \ q=1,...,2l-1,$$

determined in the interval $\left[t_0,T\right]1$.

The functions $x_{sq}(t)$ are continuous in $\left[t_0,T\right]$, and the functions V_{rd} and $f_{sq}(x,\upsilon,t)$ are continuous in $\left[t_0,T\right]$ with the exception of $2l-1$ points of t_q, $t_0\le t_q\le T$, in which they have discontinuity of a function of the first type.

Among points $2l-1$ of the discontinuity of the function f_{sq} and $V_{rq}-l$ points correspond to the discontinuity points of control, and $\left[L-1\right]$ – the discontinuity of the function f_s and the ratio is applicable [8].

$$Q_\tau=\begin{bmatrix}x_{1q}\left(t_q\right),...,x_{nq}\left(t_q\right),t_q,x_1,\\ q+1\left(t_q\right),...,x_n,q+1\left(t_q\right)\end{bmatrix}=0, \tag{18}$$

$$\tau=1,...p,\ \ p\le 2n-1.$$

The phase coordinates of the ends of the *V*-chain $Q_p^{l\upsilon}-x_{s0}\left(t_{0\upsilon}\right)$, x_s, $2l-1\left(T_\upsilon\right)$ are described by the dependencies:

$$C\mu=\begin{bmatrix}x_{11}\left(t_{0\upsilon}\right),...,x_{n1}\left(t_{0\upsilon}\right),x_1,\\ 2l-1\left(T_\upsilon\right),...,x_n,2l-1\left(T_\upsilon\right)T_\upsilon\end{bmatrix}=0, \tag{19}$$

$$\mu=1,...p,\ \ p\le 2n-1.$$

The control $V_r(t)$ belongs to the bounded closed set *V*:

$$V(t)\in V. \tag{20}$$

Construction of a specific chain of the cost function:

$$J_0(x,\upsilon,t)=\sum\nolimits_{j=0}^{l-1}J_{j,j+1}(x,\upsilon,t)\to\min, \tag{21}$$

$$J(x,\upsilon,t)=\sum\nolimits_{j=0}^{l-1}J_{j,j+1}(x,\upsilon,t)\to\min. \tag{22}$$

Where $J_0(x,\upsilon,t)$ is the expenditure function of the *j* area that connects *j* and $j+1$ points of the territory.

The expenditure functions $J_0(x,\upsilon,t)$ and $J(x,\upsilon,t)$ must take the minimum value and meet the requirements:

$$J_0(x, \upsilon, t) \le L + L_0, \tag{23}$$

$$J(x, \upsilon, t) \le L, \tag{24}$$

and the length of the chain *l* goes to the maximum.

The coverage has to be provided in such a way that the amount of chains V_x covering the graph would be minimal

$$V_x \to \min. \tag{25}$$

Each arc is compared to the weight:

$$S(C_z \in Z), \tag{26}$$

$$S_{ij} = S(u_{zi}, u_{zi}), \quad z = 0,1 \tag{27}$$

and the vector of constraints

$$VZ = (V_2^1, V_z^t). \tag{28}$$

To determine the arcs of graph G_1, wit is necessary to choose the optimal arc sequence $\overline{CZ}(C_{z1}, C_{z2}, ..., C_{zn})$, $(n \ge m)$ which provides:

$$\min \sum\nolimits_{C_z \in CZ} S_{ij} = S^{\min}, \tag{29}$$

with constraints:

$$\sum V_{zij}^l = V^l, \quad l = 1,2,...,t. \tag{30}$$

Described $S^{\min}$ and will be the weight of the corresponding arc.

Having constructed the chain, let's exclude these vertices in graph *G*, belonging to set *X*, and if set *X* is not empty $X \ne 0$, then go to point 3 [12].

In the multiple formulation of the problem:

X^0 – the set of stations for the initial location of UAVs;

X – the set of service stations (areas).

Each *X* of these points is determined by its coordinates in the chosen coordinate system.

Graph edges – V_{ij} possible trajectory sections.

$Q_p^{l\upsilon}$ – the set involving the station of the initial disposition.

x_p^0 and l_υ – the service stations (areas) which represent a normal chain.

M – the amount of case boxes carried by one drone.

This cost matrix will characterize the graph GZ:

$$GZ = (VZ, CZ, MZ), \tag{31}$$

where GZ is the set of graph edges; MZ is the cost matrix of the order $n \times n$; VZ is the set of vertices $|VZ| = n$.

Solving the problem of finding the shortest path on the graph GZ for the constraints:

$$\sum C_z CCZ x_{ij}^k = \Delta x^k, \quad k = 1,...,t, \tag{32}$$

where $\Delta x^k = (\Delta x^1, ..., \Delta x^t)$ vector of predicting phase coordinates with a transient value from one point to another; t is the dimension of the vector of phase coordinates, which will determine the weight of any arc of graph *G*.

It has proposed to decompose the problem into two separate tasks [9]:

1) the task of determining the costs of movement between the vertices of graph *G* by solving the set of two-point optimal control tasks for the quantity of boundary values of the phase coordinates, the limit values of the vector of control of various useful loads of the drone;

2) the combinatorial task of choosing a set of points (areas) of service stations of the initial location.

To solve the first sub-problem, it is necessary to develop an efficient way for obtaining and economical memorizing a large array of district trajectories described by the differential equations (1) with different boundary conditions and different values of parameters (payload) for different boundary values of the control vector, as well as the method of describing two-point tasks [10].

The energy costs and displacement of the UAV from any of the *k*-points of the initial location to any of the *N*-points (areas) of service stations are specified by the matrix:

$$PV = \left\{ p_{ij}^{0} \right\}, \; i = 1,...,k, \;\; j = 1,...,N. \tag{33}$$

These costs can be determined regardless of the solution of the problem of optimal service.

At the first stage, the solution of fuel (energy) expenditures for a flight from one point to another is linearly dependent on the distance between the points and determined by the matrix:

$$P = \left\{ p_{ij} \right\}, \; i = 1,...,N, \;\; j = 1,...,N. \tag{34}$$

With this purpose let's suggest using together the task of an optimal equation and the task of constructing the smallest path on the graph GZ, which vertices are valid values of the control, and there is an arc between the vertices of the graph, if to assume the transition from one control to another within a certain period of time Δt.

4. The discussion of the results

In this task the sets of phase coordinates and control of the *V*-terminal are admissible. The completion of the cost matrix is based on:

– the complex nature of the movement between two vertices;

– the dependence of the trajectory of flight in the *i*-section on the traffic in the *i*-section;

– the change in the mass in terms of express cargo delivery in case boxes.

Let's describe one of the ways of constructing a cost matrix for a fictitious matrix.

The order of the cost matrix is determined by the number of elements of the set $V, |\upsilon| = n$. Let's construct a quadratic table with the size $n \times n$, where the sides in its column are counted by the elements of the set *V*. The sides correspond to the control with the set *V* of the initial for this drone $\Delta t = \tau$, and the final values correspond to the columns *V* [11].

At the intersection of the corresponding row and column, there is a number corresponding to the increase in costs Δj at this stage. It describes the costs required for the transitional value of the object from the initial state to the finite time τ. Moreover, each section of the table has certain data on the corresponding increments of phase coordinates [13].

Based on the above-mentioned simplifications, let's have chosen the following path of an approximate solution to the problem of optimal service in the conditions of the UAV.

The growth of phase coordinates is necessary for the transition from one point (district) to another, and this beginning is proportional to the entire distance.

Among all points let's choose point *A*, whose attainment is the most problematic. In the graph *G* state is corresponded with the value $A = \overline{x}$ which is identical to the minimum number of arcs whose weights do not exceed *L*.

From this point, let's select a set of service stations (points) for the vertices $Ax \in \overline{x}$, which belong to the *L*-locality. From this set $A \cup AV \cup AX$ it is necessary to choose a chain of maximum length, which contains not more than the *M*-vertex, vertex *A* and the initial vertex from the set AO. The energy costs of this over flight chain should not exceed $L + L_0$.

The main costs of the transition between service points (vertices of graph *G*) are determined by matrix *P*, they are approximate, and therefore even a slight error in the estimates of costs may entail a change in the quantity of points (areas) serviced by one UAV. The further construction of the set of service stations and the order of their over flight should be carried out by using a more accurate calculation of the cost for each chain.

In graph theories, the problem of constructing a chain with a maximum length can be formulated in the following way.

Let $G_1(AO, AX, A, AD)$ be a bound oriented graph, where AO is the set of vertices of graph G_1, such that $AO \in x^0$; AX – the set of vertices in graph G_1, such that $AO \in x$; AD – the set of the arcs of graph G_1, which is necessarily a part of chain $AO \in BX$.

The arcs $ad \in AD$ of the graph have received the additive constraints $V = (V_1, ..., V_t)$, which correspond to the increments of the phase coordinates necessary for the transition from one vertex to another.

There is a need to build the chain of maximum length $l \leq M$, which has vertex *A*, beginning at point $B \in AO$ and whose total weight does not exceed $L + L_0$, and the total weight of parts of the chain without the first arc does not exceed L.

This task is solved if the weight of the edges is determined with the help of a balanced and bound multi graph, $GZ = [UZ, GZ, TR]$ where VZ is the set of vertices $|UZ| = n$; GZ – the set of arcs $|GZ| = n$; TR – the set of the types of arcs.

The task is to find all shortest paths in an undirected graph with one positive restriction (**Fig. 1**).

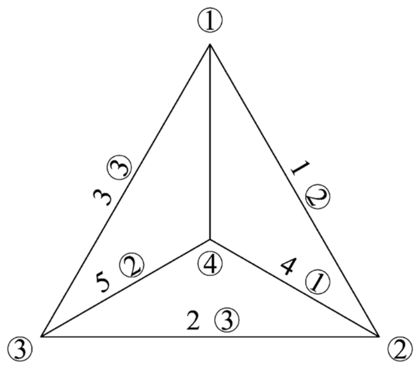

Fig. 1. Graph

Weight matrices S and limitations L have the form:

$$\begin{bmatrix} - & 1 & 3 & 6 \\ 1 & - & 2 & 4 \\ 3 & 2 & - & 5 \\ 6 & 4 & 5 & - \end{bmatrix}, \begin{bmatrix} - & 2 & 3 & 1 \\ 2 & - & 3 & 1 \\ 3 & 3 & - & 2 \\ 1 & 1 & 2 & - \end{bmatrix}.$$

Matrices corresponding to them K^S have the form:

$$\overset{L=1}{\begin{bmatrix} - & 1 & - & - \\ 1 & - & - & - \\ - & - & - & 5 \\ - & - & 5 & - \end{bmatrix}}, \overset{L=1}{\begin{bmatrix} - & - & - & 6 \\ - & - & - & 4 \\ - & - & - & - \\ 6 & 4 & - & - \end{bmatrix}}, \overset{L=1}{\begin{bmatrix} - & - & 3 & - \\ - & - & 2 & - \\ - & - & - & - \\ 3 & 2 & - & - \end{bmatrix}}.$$

Step 1. Let's pretend that $t = 0,\ M^0 = \|m_{ij}^0\|$,

$$m_{ij}^0 = \begin{cases} 0, \text{if } i = j, \\ \infty, \text{if } i \neq j, \end{cases} \quad t = \bar{t} = 0.$$

Step 2. $M = 24.$
Step 3. $M^t = \left\| m_{ij}^t \right\| = \left\| \infty \right\|, \quad t = 1,...,24.$
Step 4. K^1 set in accordance $T(1) = 1, \quad K^2 - T(2) = 2, \quad K^3 - T(3) = 3.$
Step 5. First iteration $t = 1.$
Step 6. $S = 1.$
Step 7. $t_1 = 0.$
Step 8–14. Multiply the matrix M^0 on the matrix K^1, let's obtain the matrix $M^1 = K^1.$
Step 15. $S = 2.$
Step 16. $S \leq 3,$ go to step 7.
Step 17. $\underline{m} = \min_{ij}, \ \left(m_{ij}^1\right) = \underline{1}.$

$$\begin{bmatrix} - & 0 & - & - \\ 0 & - & - & - \\ - & - & - & 4 \\ - & - & 4 & - \end{bmatrix}.$$

Step 18. $M^0 \neq M^1,$ go to step 24.
Step 24. $t = t + \underline{1} = 2.$
Step 25. $t \leq 24,$ go to step 6.

Second iteration
Step 6. $S = 1.$
Step 7. $t_1 = 2 - 1 > 0.$
Step 8–14. $M^2 = M^1 \otimes K^1.$

$$\begin{bmatrix} 12 & 10 & - & - \\ 10 & 8 & - & - \\ - & - & - & 5 \\ - & - & 5 & 8 \end{bmatrix}.$$

Step 15. $S = 2.$
Step 16. $S < 3,$ go to step 7.
Step 7. $t_1 = 2 - 2 = 0.$

$$\begin{bmatrix} 12 & 1 & - & - \\ 1 & 8 & - & - \\ - & - & - & 5 \\ - & - & 5 & 8 \end{bmatrix}.$$

Step 15. $S = 3.$
Step 16. $S \leq 3,$ go to step 7.
Step 7. $t_1 = 2 - 3 < 0,$ go to step 15.
Step 15. $S = 4.$
Step 16. $S > 3.$
Step 17. $m = 1.$
Step 18.

$$\begin{bmatrix} 11 & 0 & - & - \\ 0 & 7 & - & - \\ - & - & - & 4 \\ - & - & 4 & 7 \end{bmatrix}.$$

Step 19. $M^2 \neq M^0, \quad M^2 \neq M^1.$

Step 24. $t = 3$.

Step 25. $t < 24$, go to step 6.

Let's continue this process. For the final solution of the problem, it is necessary 15 iterations.

Step 5. $t = 15$.

Solving the problem with one positive constraint

Step 6–18. Obtain the matrix M^{15}.

Step 19. $M^{15} = M^{11} = M^{7}$, $\Theta = 15 - 11 = 4$, $\bar{t} = 11$, $\underline{t} = 11 - 4 = 7$.

Step 20. $\underline{t} = r^{1}$.

Step 21–23. $M^{8} = M^{12}$, $M^{9} = M^{13}$, $M^{10} = M^{14}$.

Thus, to solve the problem for any value of the constraint, it suffices to store eleven matrices. Let's depict the answer to the problem in the form of a diagram (**Fig. 2**).

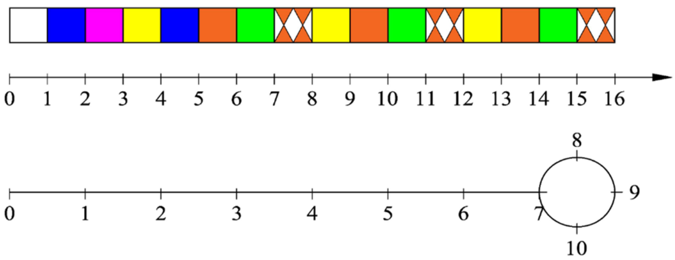

Fig. 2. Half ring diagram

5. Conclusions

In this article, the main attention has been paid to the development of the method for solving the problem of determining the weight of the arcs of graph G_1 and the task of building a chain, which part is also the docking task.

It is necessary to reach each of $\left(A^{1},...,A^{t}\right)$, $i = 1,...,k$ by defining the sequence of its attainment.

Quantities A^{l} – can be considerably larger than values of, A^{l}_{GZ} which are equal to the constraints of all arcs of the graph.

Problem A is the task of determining the weight of arcs in graph G_1.

Problem B is the sequential finding of the maximum number.

Problem C represents the finding of empty paths in the graph of length without moving M and satisfying the constraints – the task of building a chain in graph G_1.

Problem D is a special salesman task in graph, $GS(A,E)$ where A is the set of vertices (constraints, which are supposed to be achieved), E-arcs.

The proposed methods have to:

– obtain the optimal solution of the problem of covering a special graph with chains of a certain structure with constraints on the route section and total costs;

– build solutions of problems A, B, C, D for values which do not exceed the values of finite quantities, which are constrained by graph *GZ*, but not the values of the right parts of the constraints;

– obtain solutions for any specific sets of individual problems A, B, C, D through solving mass problems;

– reduce quantities of problems A, B, C, D to the values of the vector of constraints.

Advantages: algorithms allow to reduce significantly the delivery cost of urgent goods with the help of UAV.

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