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Uncertainty Quantification in Hydrodynamic Bearings

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Abstract

Although it is possible to imagine a strong sensitivity of hydrodynamic bearings performance to geometrical and fluid dynamic uncertainties, a small amount of scientific contributions have been found in literature about the use of uncertainty quantification techniques for the numerical modeling of bearings. In the present paper we aim at quantifying the effects of the aleatory uncertainty of some relevant input values on key parameters related to rotordynamic effects in turbomachinery, and in particular on the rotor thermal instability problem (e.g. the equilibrium position and the dynamic coefficients). A methodology is initially developed in order to study the propagation of the uncertainties in the numerical analysis of Tilting Pad Journal Bearings (TPJB). Due to the characteristics of the in-house finite element code TILTPAD considered for the UQ analysis, the Monte Carlo method has been selected among the possible approaches. The analysis here presented considers the effects of both manufacturing tolerances on the assembled bearing clearance and of the tolerances adopted for the characterization of the viscosity grade of the oil. The test case adopted for the analysis is the Kingsbury D-140 TPJB. Considering the individual variation of the selected parameters, it is possible to observe that the standard deviation (STD) of the the non-dimensional dynamic coefficients is up to 2.1% in case of viscosity variation and up to 9.1% in case of clearance variation. The STD of the frictional power losses is about 2.2% and 1.4% respectively. Considering the simultaneous variation of the selected parameters, it is possible to observe a STD of the non-dimensional dynamic coefficients comprised between 6.4% and 9.4%, while the STD of the frictional power losses is about 2.7%.

Keywords: Uncertainty Quantification; Tilting Pad Journal Bearing; Monte Carlo; Manufacturing Tolerance; Oil Viscosity

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Nomenclature

C	Damping coefficient $[N \cdot s/m]$
C_b	Assembled radial clearance $[m]$
K	Spring coefficient $[N/m]$
P_f	Frictional power $[W]$
x, y	Spatial coordinates $[m]$
ϵ	Relative eccentricity or thermal strain $[-]$
μ	Dynamic viscosity $[Pa \cdot s]$
σ	Standard deviation
$*$	Non-dimensional value

1. Introduction to uncertainty quantification

According to the work by Oberkampf *et al.* [1], six general phases are necessary to the process of modelling and simulation. Each phase can be a source of errors and uncertainties, as suggested by Roy and Oberkampf [2]. Hence, when high-fidelity simulation is necessary, all the sources of uncertainty must be identified and included somehow in the simulation. The AIAA committee suggests two different definitions for errors and uncertainties in a probabilistic framework [3]. An error is defined as "*a recognizable deficiency in any phase or activity of the modelling and simulation that is not due to lack of knowledge*". Uncertainty instead is defined as "*a potential deficiency in any phase or activity of the modelling process that is due to the lack of knowledge*". Uncertainty can be further classified between aleatory and epistemic, where the latter one represents the lack of knowledge of the physical model (and is therefore "reducible") while aleatory ("irreducible") uncertainty is connected to the physical variability within the system or its environment. The introduction of uncertainty in a deterministic simulation does not alter the steps of the process of modelling and simulation of a physical problem but adds some difficulties in each phase and asks for additional operations to be done in parallel with the usual ones. In particular three steps are needed in a uncertainty study: the definition, the propagation and the certification of the uncertainty. The first step consists in the identification and choice of the inputs affected by uncertainty and, consequently, the definition of their Probability Density Function (PDF). Various stochastic inputs can be considered, such as operating conditions (adopted by D'Ammaro and Montomoli [4] and by Carnevale *et al.* [5]) or geometrical features (considered by Montomoli *et al.* [6]). The second step consists in the propagation of the defined uncertainty within the numerical solver in order to identify the probability distribution of the selected system response quantities (SRQ). Once the quantities have been statistically evaluated it is possible to use them in a reliability assessment or in a validation procedure.

2. Monte Carlo method with response surface

Various methods are proposed in literature in order to approach uncertainty quantification (UQ). A well organized review is presented in the work of Montomoli *et al.* [7]. There are mainly three categories of UQ techniques: sampling based methods, quadrature techniques and intrusive techniques. The methods of the first group are computationally expensive but easy to be implemented and non intrusive, in the sense that it is not necessary to modify the numerical solver. Moreover, by means of improved sampling strategies (e.g. lattice based or latin hypercube samplings) it is possible to cut the computational cost. The methods of the second group are mainly based on polynomial chaos representation of the stochastic output. By means of quadrature formulas with orthogonal polynomials it is possible to obtain fast and accurate UQ analysis. Finally, intrusive techniques result to be accurate and fast but require a lot of work to modify the numerical models. The choice of the more suitable approach is strongly related both to the characteristics of the numerical code and to the complexity of the problem.

For the present work the in-house code TILTPAD developed at the Department of Industrial Engineering of the University of Florence is adopted. For a detailed description and validation of the code, which is beyond the scope of the present paper, see the work by Griffini *et al.* [8]. Here, only a few information is given. TILTPAD is a steady state finite-element method code for the thermo-hydrodynamic analysis of both plain and tilting pad journal bearings' performance. It is based on a 2D thin-film approach for the pressure field calculation, which is a typical simplification, and is equipped with a simplified 1D energy equation. It can also handle superlaminar regimes up to the onset of the fully turbulent condition. With such simplifications the code results to be an accurate and fast tool.

In this context, the Monte Carlo method has been selected due to its easy implementation to investigate a system affected by more that one uncertanty. As suggested by Ahlfeld *et al.* [9], the Monte Carlo method is coupled with response surfaces to reduce computational costs. Figure 1 shows a schematic of the classic Monte Carlo method (top) and of the Monte Carlo with response surface (bottom). In the first case the random generation of input conditions is obtained with the defined PDF. Then, a simulation is performed for each of the generated points in order to find the statistical distribution at the outlet of the system for the dependent variables of interest. The second method instead relies on a two phases approach: simulations are previously performed, covering the whole design space and defining the response surface, then the Monte Carlo method is used over this metamodel.

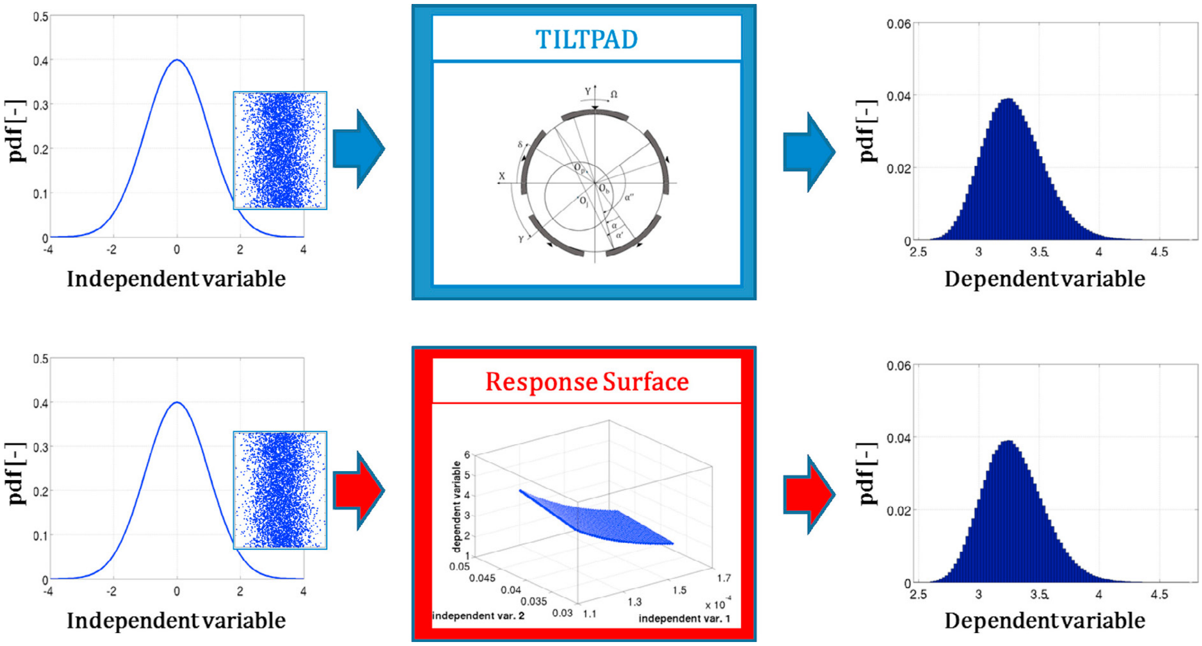


Fig. 1. Schematic of the Monte Carlo approach with the response surface.

3. Uncertainty definition

Hydrodynamic bearings are characterized by the small dimension of the meatus (with respect to shaft diameter), then the system can be highly affected by variations of both the operative conditions and the geometrical uncertanties. A comprehensive review about the identification of dynamic parameters in bearings is proposed by Tiwari *et al.* [10], where is underlined that many authors recognize the relevance of including accuracy estimation for the measured dynamic coefficients. Despite that, a small amount of works have been found in literature about UQ applied to the numerical modelling of these components. In the present work the effects of both manufacturing tolerances and tolerances adopted for the characterization of the oil are studied. In particular, among the various parameters having a role in the numerical modelling of the thermo-hydrodynamic problem, two are considered as aleatory parameters for the purpose of the present work: the radial clearance of the bearing (affected by manufacturing tolerances and by thermal, elastic and wear effects) and the dynamic viscosity of the oil (affected by the tolerance on the viscosity grade adopted to classify the oil and by the temperature).

It has to be noted that the dependence of both the variables on the temperature has to be considered in a wider sense with respect to what is usually done. The temperature uncertainty can be related to the operating conditions of the oil, to the average ambient temperature of the field where the machine is installed and, as highlighted in the work of De Jongh and Van Der Hoeven [11], to the temperature of the fluid processed by the machine. However, although temperature uncertainty can play a major role, in the present work a methodology for UQ analysis in hydrodynamic bearings is introduced and a preliminary evaluation is performed only on the base of the variables definition, e.g. on the base of clearance manufacturing tolerances and viscosity grade definition tolerances. The ISO tolerance system is considered to define the clearance manufacturing tolerances. Considering the nominal diameter of the hole, the nominal bearing clearance and the tolerances for shaft and holes, which are respectively adopted as $h6$ and $H6/7$ (data are referred to the catalogues available online of the main bearing producers), the tolerance range can be calculated. According to the tested bearing data the radial clearance range is as follows:

$$C_b = 125^{+32.5}_{-0}[\mu m] \tag{1}$$

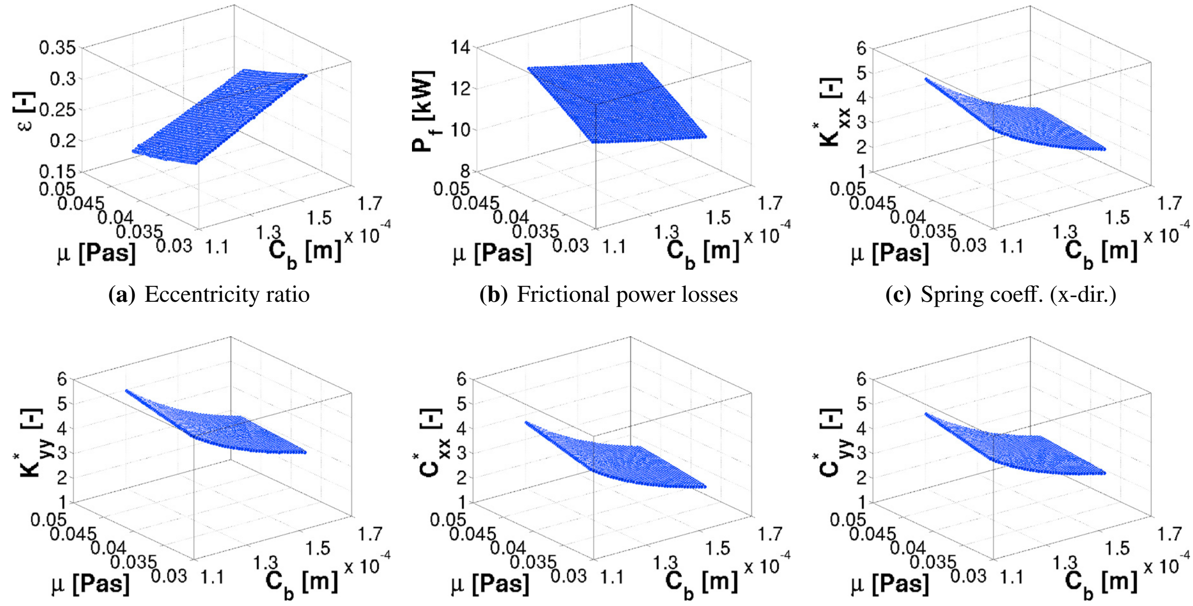
This result is obtained considering the $H7$ tolerance for the hole in order to test the worst case scenario. It has to be noted that the nominal clearance results to be the lower bound of the available range of assembled bearing clearance. This could be overcome since the industrial practice relies on some adjustment of the clearance when the system is assembled. Due to the high costs, that procedure is avoided most of the time. Hence, for the present work the adjustment is not considered and results will be remarkably affected by the choice of having a nominal clearance at the extreme of the tolerance range. The PDF of the tolerances is the main assumption of the work since there is no direct information about its realistic distribution. In particular, a normal distribution centered within the tolerance range is adopted resulting in a mean value of 141.25 μm. For the standard deviation a value of 5.4 μm (i.e., a 3.8% of the mean value) is selected in order to obtain the extremes of the tolerance range respectively at $\pm 3\sigma$. By means of such a choice, some of the tested values (the 0.3% of the tested points) result to be slightly outside of the tolerance domain.

The mineral oil adopted for the lubrication process is usually selected on the base of the viscosity grade (ISOVG). The proposed classification is based on the mean value of the kinematic viscosity (e.g. ISOVG-32 has a mean kinematic viscosity value of 32 mm^2/s at 40 $°C$). The oil adopted for the present analysis is an ISOVG-46 with a value of density of about 870 kg/m^3. According to the oil classification, the selected oil can have a kinematic viscosity between the range $41.4 \div 50.6$ mm^2/s and hence its dynamic viscosity range (used by the code) is within the range of $0.036 \div 0.044$ $Pa \cdot s$. Again, as done for the radial clearance, a normal distribution is considered with the mean value centered in 0.040 $Pa \cdot s$ and with a standard deviation of $\sigma = 0.0013$ $Pa \cdot s$ (i.e., a 3.3% of the mean value). The selection of the standard deviation implies again that the extremes of the tolerance range result exactly positioned at $\pm 3\sigma$.

The test case adopted for the analysis is a 140 mm commercial five pads tilting pad journal bearing in a load on pad configuration operated at 8000RPM, which has been presented in the work of Panara *et al.* [12]. Figure 2 reports the response surfaces adopted for the present analsysis. A test matrix has been performed in order to cover the entire surface determined by the two parameters variation. The field of simulation has been extended up to $\pm 4\sigma$ for each one of the parameters and has been covered by means of 41 testing points, resulting in 1681 simulations (accomplished in about 16 hours). The obtained surfaces give the response of the code in terms of eccentricity ratio, frictional power losses and non-dimensional dynamic coefficients with respect to the clearance and dynamic viscosity variations. A sketch of a 5 pads tilting pad bearing with the reference system adopted by TILTPAD is reported in the central box of Figure 1 (top).

4. The Monte Carlo analysis

Once the response surfaces are given for the parameters of interest, the Monte Carlo method is applied. This has been accomplished generating one million of random pairs of clearance and dynamic viscosity values, each one following its own normal distribution (hypothesized in the previous section), and then adopting the response surface to obtain the dependent results, which are statistically analysed in the following section. The effects of the uncertainty of the two parameters of interest have been initially evaluated for the selected dependent variables considering a simultaneous variation of the two parameters. Then, each single parameter is individually studied by means of the same approach with the objective, in this case, to evaluate their own weight. Referring to turbomachinery rotordynamics and in particular to the synchronous thermal instability, better known as Morton effect [13] [14], this kind of analysis



(a) Eccentricity ratio **(b)** Frictional power losses **(c)** Spring coeff. (x-dir.)

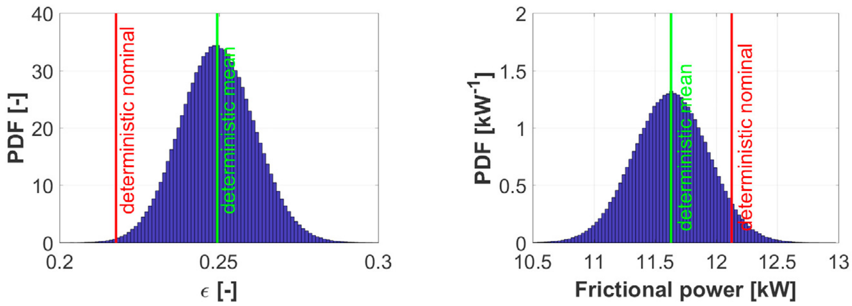
(d) Spring coeff. (y-dir.) **(e)** Damping coeff. (x-dir.) **(f)** Damping coeff. (y-dir.)

Fig. 2. Response surface from TILTPAD, adopted for the Monte Carlo approach.

is of great interest since it quantifies the effects of the uncertainties on some of its driving parameters (e.g. the dynamic coefficients). Particular attention has to be given to such a topic for all of the machines which lie between the marginally stable condition. In fact, for this particular condition the smallest change can result in an unstable working range.

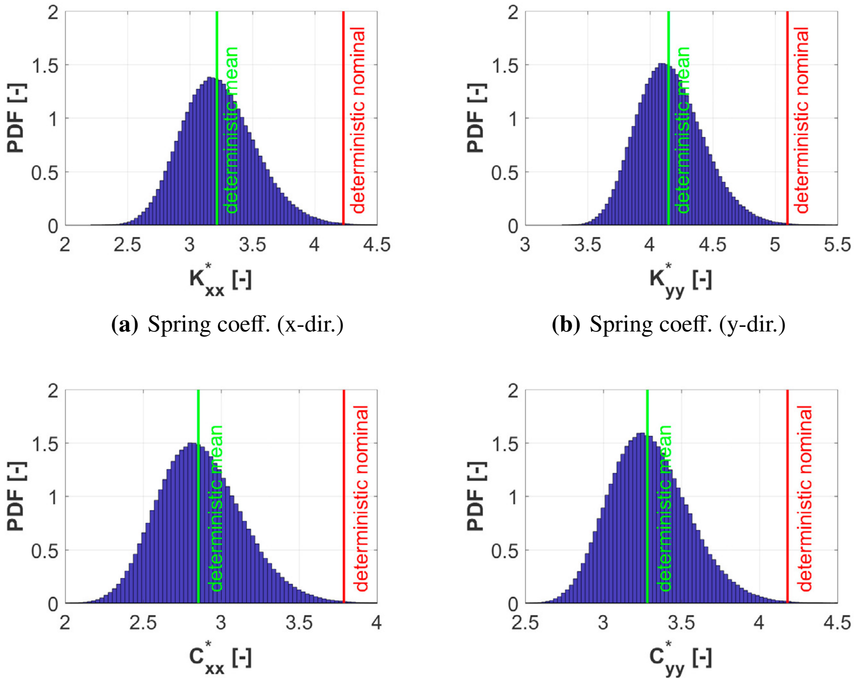
4.1. Multiple degrees of freedom

In terms of eccentricity ratio and frictional power losses, shown in Figure 3, a normal distribution is obtained with mean values respectively of about 0.250 kW and 11.6 kW and with standard deviations of about 4.7% and 2.7%. Such a behaviour (normal distribution) was expected since the response surface of relative eccentricity and frictional power losses resulted to be characterized by almost linear relations with respect to the independent variables (as shown in Figure 2(a) and Figure 2(b)). Concerning the non-dimensional dynamic coefficient, whose distributions are shown in Figure 4, something like a normal distribution is again obtained, although in this case a slight non-symmetry can be highlighted.



(a) Eccentricity ratio **(b)** Frictional power losses

Fig. 3. Results of the Monte Carlo approach with response surface.



(a) Spring coeff. (x-dir.) **(b)** Spring coeff. (y-dir.)

(c) Damping coeff. (x-dir.) **(d)** Damping coeff. (y-dir.)

Fig. 4. Results of the Monte Carlo approach with response surface.

For all of the analysed variables, shown in Figure 3 and Figure 4, the values of the deterministic calculations performed with the nominal and with the mean values of the clearance have been reported in red and green respectively. The choice not to consider the adjustment of the clearance and hence consider the nominal clearance as a lower bound drives a substantial difference between the results of the deterministic calculation obtained with the nominal value and the maximum probability value obtained with the UQ analysis. The maximum difference is found for the non-dimensional damping coefficient in the x-axis direction where an over-prediction of about 25.5% of the deterministic value with respect to the most probable one is highlighted. For sake of clarity, Table 1 summarizes the values related to the two deterministic calculations (mean and nominal) and to the maximum probability value of the obtained distributions. In case the mean value is adopted for the deterministic calculation, discrepancies are greatly reduced and a maximum difference of about 1.2% (evaluated with respect to the deterministic mean value) is obtained for the non-dimensional spring coefficient.

output	deter. mean	deter. nominal	max. probability
ϵ $[-]$	0.250	0.219	0.249
K^*_{xx} $[-]$	3.22	4.21	3.18
K^*_{yy} $[-]$	4.15	5.10	4.10
C^*_{xx} $[-]$	2.85	3.79	2.82
C^*_{yy} $[-]$	3.28	4.18	3.24
P_f $[kW]$	11.6	12.1	11.6

Table 1. Calculation results: deterministic calculation with mean clearance value, deterministic calculation with nominal clearance value and maximum probability value obtained with the uncertainty analysis.

The output of the dynamic coefficients is approximated to a normal distribution (neglecting the slight non-symmetry) and then it is possible to evaluate both the mean value and the standard deviation for each coefficient as shown in Table 2. Due to the imposed approximation, the values obtained as a mean value are slightly different from the maximum probability values obtained. The maximum value of the standard deviation is obtained for the damping coefficient with respect to the x-direction with a value of 9.4%. The standard deviation of the frictional power losses (not reported in Table 2) is about 2.7%.

	mean value [−]	standard deviation σ [%]
K^*_{xx}	3.23	9.0%
K^*_{yy}	4.17	6.4%
C^*_{xx}	2.87	9.4%
C^*_{yy}	3.30	7.7%

Table 2. Probability distribution analysis: mean value and standard deviation of the non-dimensional dynamic coefficients.

4.2. Single degree of freedom

Considering at first the case with uncertainty affecting the viscosity of the oil and with a constant value of the assembled bearing clearance (e.g. 125 μm) a normal distribution, at least in a first approximation, has been obtained for all of the evaluated variables. Results expressed in terms of mean value and standard deviation are proposed in Table 3. The sensitivity of the spring and damping coefficients to viscosity variations is lower than for the case with multiple degrees of freedom. In fact, a maximum standard deviation of 2.1% is found in Table 3 while in Table 2 the maximum standard deviation is 9.4%. Moreover, the uncertainty related to the dynamic viscosity mainly affects the frictional power losses with a 2.2% of standard deviation.

	viscosity uncertainty		clearance uncertainty	
	mean value	σ (STD) [%]	mean value	σ (STD) [%]
ϵ [−]	0.218	1.8%	0.249	4.3%
K^*_{xx} [−]	4.22	2.1%	3.23	8.8%
K^*_{yy} [−]	5.10	1.2%	4.16	6.3%
C^*_{xx} [−]	3.79	2.1%	2.87	9.1%
C^*_{yy} [−]	4.18	1.6%	3.30	7.6%
P_f [kW]	12.1	2.2%	11.6	1.4%

Table 3. Probability distribution analysis: mean value and standard deviation

When the case with uncertainty affecting the bearing clearance only is considered, distributions similar to the ones shown in Figure 3 and Figure 4 are obtained. Considering as a first approximation of normal distributions, as done in the case of simultaneous variations, the corresponding mean values and standard deviations are evaluated. Results of this latter analysis are reported in Table 3. The obtained values for the mean and standard deviations are close to the ones related to the simultaneous effect of viscosity and clearance, with a maximum of standard deviation obtained for the damping coefficient in the x-direction of about 9.1%. A reduction of the standard deviation of the frictional power losses is obtained (from a 2.7% to 1.4%) since this variable is more sensitive to the viscosity with respect than the clearance. According to the proposed analysis the dynamic coefficients result to be particularly affected by the clearance dimension uncertainty.

The impact of manufacturing processes on the aleatory uncertainty of the clearance dimension can be theoretically reduced using the appropriate manufacturing and mounting procedure. Despite that, it has to be reminded that the uncertainty level cannot be reset since there are other phenomena that define the clearance dimension, e.g. the thermal behaviour of the system and elastic deformations. In particular, the thermal behaviour may have a major role. In fact, as introduced in the work of De Jongh and Van Der Hoeven [11] a variation in the bearing clearance can be obtained due to the temperature of the gas evolving inside of the machine (e.g. a strong reduction was obtained for cryogenic applications under specific geometrical conditions). In the work of Ferron *et al.* [15] two clearance values are reported: the first one is related to a temperature of 20 °C and is of about 145 μm, the second one is related to a temperature of 45 °C and is of about 152 μm with a variation of about 4.8% with respect to the "cold" value. Hence, since the adjustment is done while the system is assembled (cold condition), when operated (hot condition) the clearance can assume sensibly different values.

5. Conclusions

This work presents the evaluation of the impact of aleatory uncertainty on key parameters of turbomachinery rotordynamic behaviour. A methodology based on the Monte Carlo method is developed. Responses surfaces have been employed to speed up the procedure. The analysis considers the effects of manufacturing tolerances and of tolerances adopted for the characterization of the viscosity grade of the oil. Two different Gaussian distributions for the assembled bearing clearance (mean value of 141.25 μm, standard deviation of 3.8%) and for the viscosity of the oil (mean value of 0.040 $Pa \cdot s$, standard deviation of 3.3%) are considered. The uncertainty propagation is considered for both the simultaneous and the individual variation of the input parameters. For the latter case, it is possible to observe that the standard deviation of the non-dimensional dynamic coefficients is up to 2.1 % in case of viscosity variation and up to 9.1 % in case of clearance variation, showing that the uncertainty on dynamic coefficient is mainly driven by manufacturing tolerance. The standard deviation of the frictional power losses is about 2.2 % in case of viscosity variation and 1.4 % in case of clearance variation, thus highlighting that frictional power losses is mainly driven by viscosity variations. Considering the simultaneous variation of the selected parameters, it is possible to observe a standard deviation of the non-dimensional dynamic coefficients comprised between 6.4 % and 9.4 %, while the standard deviation of the frictional power losses is about 2.7 %. The combination of the two parameters has hence a detrimental effect with an increase in the spread of the distribution for the non-dimensional dynamic coefficients. Results highlight the importance of uncertainty propagation in the evaluation of hydrodynamic bearing performance just from its basic formulation, i.e., based only on the definitions of clearance and viscosity grade of the oil.

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References

[1] Oberkampf W.L., DeLand S.M., Rutherford B.M., Diegert K.V., Alvin K.F. "Error and uncertainty in modeling and simulation." *Reliability Engineering and System Safety* (2002) 75(3):333–357

[2] Roy C.J., Oberkampf W.L. "A comprehensive framework for verification, validation, and uncertainty quantification in scientific computing." *Computer Methods in Applied Mechanics and Engineering* (2011) 200(25-28): 2131–2144

[3] AIAA Computational Fluid Dynamics Committee "Guide for the verification and validation of computational fluid dynamics simulations." *AIAA G-077-1998* (1998)

[4] D'Ammaro A., Montomoli F. "Uncertainty quantification and film cooling." *Computers and Fluids* (2013) 71:320–326

[5] Carnevale M., Montomoli F., D'Ammaro A., Salvadori S., Martelli F. "Uncertainty quantification: a stochastic method for heat transfer prediction using LES." *ASME Journal of Turbomachinery* 135:051021 (8 pages) (2013)

[6] Montomoli F., Massini M., Salvadori S. "Geometrical uncertainty in turbomachinery: tip gap and fillet radius." *Computers and Fluids* (2011) 46:362–368

[7] Montomoli F., Carnevale M., Massini M., D'Ammaro A., Salvadori S. "Uncertainty quantification in computational fluid dynamics and aircraft engines." *Springer* (2015)

[8] Griffini D., Salvadori S., Martelli F. "Thermo-hydrodynamic analysis of plain and tilting pad bearings." *Energy Procedia* (2016) 101:2–9

[9] Ahlfeld R., Montomoli F., Carnevale M., Salvadori S. "Autonomous uncertainty quantification for discontinuous models using multivariate Padé approximations." *ASME Journal of Turbomachinery* 140:041004 (10 pages) (2015)

[10] Tiwari R., Lees A.W., Friswell M.I. "Identification of dynamic bearing parameters: a review." *Shock and Vibration Digest* (2004) 36(2):99–124

[11] De Jongh F., Van Der Hoeven P. "Application of a heat barrier sleeve to prevent synchronous rotor instability." *27th, Turbomachinery Symposium Proceedings* 17–26 (1998)

[12] Panara D., Baldassarre L., Griffini D., Mattana A., Panconi S., Meli E. "Numerical prediction and experimental validation of rotor thermal instability." *44th Turbomachinery Symposium, College Station, TX* (2015)

[13] Morton P.G. "Some Aspects of Thermal Instability in Generators." *G.E.C. Internal Report No.S/W40 u 183* (1975)

[14] De Jongh F. "The synchronous rotor instability phenomenon - Morton effect." *37th, Turbomachinery Symposium Proceedings* (2008)

[15] Ferron J.J., Frene J.J., Boncompain R.R. "A study of the thermohydrodynamic performance of a plain journal bearing comparison between theory and experiments." *ASME. J. of Lubrication Tech.* 422–428 (1983)