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Unequal Polyomino Layers for Reduced SLL Arrays with Scanning Ability

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Abstract—Polyomino-based arrays allow to efficiently exploit the available array area with a regular element lattice, yet exhibit a nonuniform distribution of their phase centers, leading to superior electronic scanning capabilities. Yet polyomino arrays are usually implemented via polyomino of equal order, leading to uniform amplitude distribution and poor side lobe levels. In this contribution, a tiling of polyominoes of different orders is proposed to attain at the same time good scanning characteristics and side lobe level.

1. INTRODUCTION

Adopting a non-regular lattice or organizing the radiating elements into overlapped subarrays represent two effective strategies to minimize the number of controls in an array antenna. These solutions are mainly applicable in designs requiring a limited angular field of view as well as in designing wide band array. Recently there has been a renewed interest in this area for various applications ranging from spaceborne to ground antennas, from civil to military systems [1–5]. The key point in an irregular lattice is that of having a non-periodic disposition of the phase centers of the elements of the array, with benefits in scanning capabilities and grating lobes suppression. Yet the irregular element disposition is complex to handle from a technological point of view and leads to a very complex feeding network design. Furthermore, irregularly spaced elements often fill the available array area less effectively than elements in a regular lattice, hence decreasing antenna aperture efficiency.

An interesting solution to sparsify the positions of the phase centers maintaining at the same time a high aperture efficiency consists in filling the array aperture with a number of tiles characterized by a different orientation, most notably polyominoes [6, 7].

Tiling polyominoes is a complex matter, which attracted much attention in the past [8, 9]. The most popular placement algorithms rely on the circular placement method [10], approached with various kind of optimization strategies. The algorithm implemented in this paper is based on the circular placement, either in a conventional *outer* version, in which polyominoes are placed starting from the outer boundary of the domain going inwards, or in an original *inner* version, where polyominoes are placed starting from the center of the domain and going outwards.

While most polyomino-based array approaches use a large number of shapes, practical industrial realization is feasible only if a limited number of shapes is exploited, possibly only one shape, changing its orientation ([11] and references therein). Usually, in literature, polyominoes of equal order are used. This means that all polyominoes exhibit the same number of radiating element and, if tiles are fed with an equal power, the realized array is characterized by a uniform amplitude distribution.

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It is anyway well known that a nonuniform illumination of the elements of an array produces better side lobe level (SLL) characteristics, so polyomino tiles are unequally fed [11]. On the other hand, having all power amplifiers in the array equal leads to an easier and cheaper design. Hence, this paper addresses the problem of using a limited number of polyomino shapes of different orders, so as to fill a given aperture. All polyominoes are fed with the same input power, hence can be backed with identical amplifiers, but effectively generate a nonuniform amplitude distribution since this identical power is split onto a different number of elements within the polyomino: few in polyominoes of low order and several on those of high order.

The paper is organized as follows: Section 2 will sketch the placement algorithm; Section 3 will propose some array designs, comparing results attained with standard polyomino arrays using tiles of a single order.

2. PLACEMENT TECHNIQUE

Due to the limited number of polyomino shapes, which are used as stated in the introduction, a randomized exhaustive search strategy proves to be applicable and is here implemented.

Let us name P_n the set of polyominoes of order n. Let us name $L_n \subset P_n$ the subset of the polyominoes of order n, which holds all, and only the polyominoes we wish to use.

In our approach polyominoes will be placed in *rounds*, each round m ideally placing a ring, or layer, of polyominoes adjacent to the previous layer, placed at round $m-1$. Usually each round places polyominoes selecting them from a single subset L_n. In the proposed algorithm, on the other hand,in each round m polyominoes can be chosen from a subset L_n^m, which can be different at each step m.

At step $m = 0$ the *outer* algorithm selecting the four corners of the domain (the domain to be

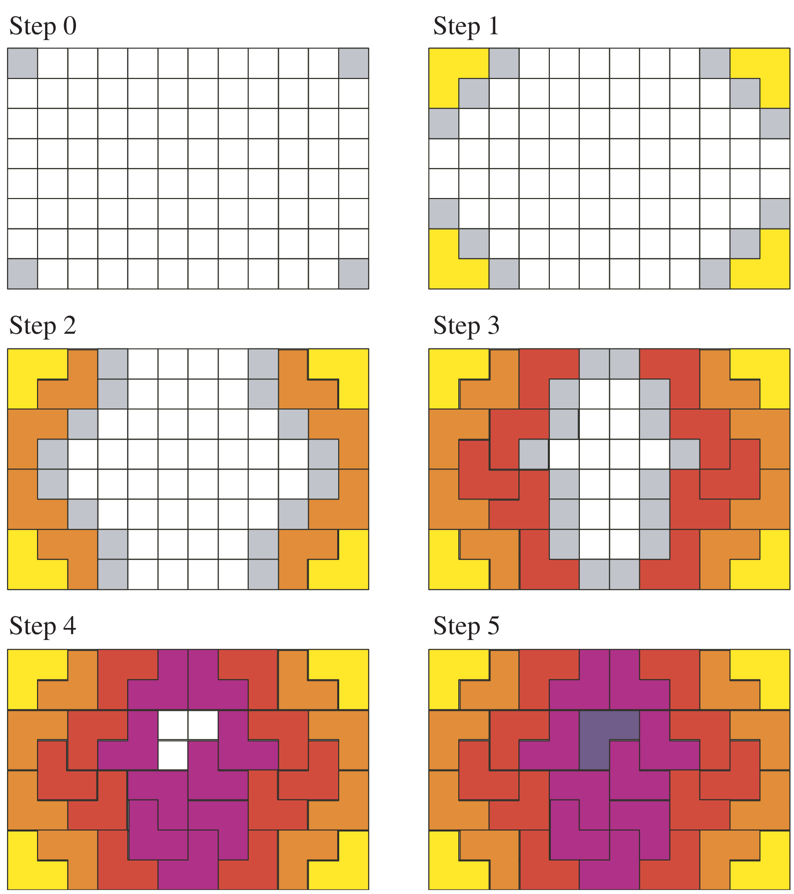

Figure 1. Example of filling of a 12×8 rectangle with L-shaped trominoes.

filled is ideally rectangular, since the application is to planar array, but this is not a limitation.) as perspective position for polyominoes (Fig. 1). Instead, for the *inner* algorithm, a polyomino is randomly selected and oriented, and placed in the center of the domain.

The outer strategy tends to leave some small hole in the center of the array if no polyomino can be placed to fit it, while the inner strategy tends to leave some larger holes in the outer boundary. The selection of the best strategy is hence up to the designer and his needs.

From this starting point onwards, the algorithm consists of the following steps:

1. Build the subset L_n^m of the allowable polyomino shapes to be placed at current round;
2. Build a list H^m of "hot" cells (in gray in Fig. 1) which are empty but adjacent to the part of the domain already filled by polyominoes;
3. Select a random cell $h \in H^m$.
4. Do an exhaustive search on any possible placement $\mathfrak{P}(p, h)$ of any polyomino $p \in L_n^m$ covering the selected cell h and not overlapping with an already existing polyomino (Fig. 2). All possible rotations, flips and translations in which a part of p overlaps on h are investigated;

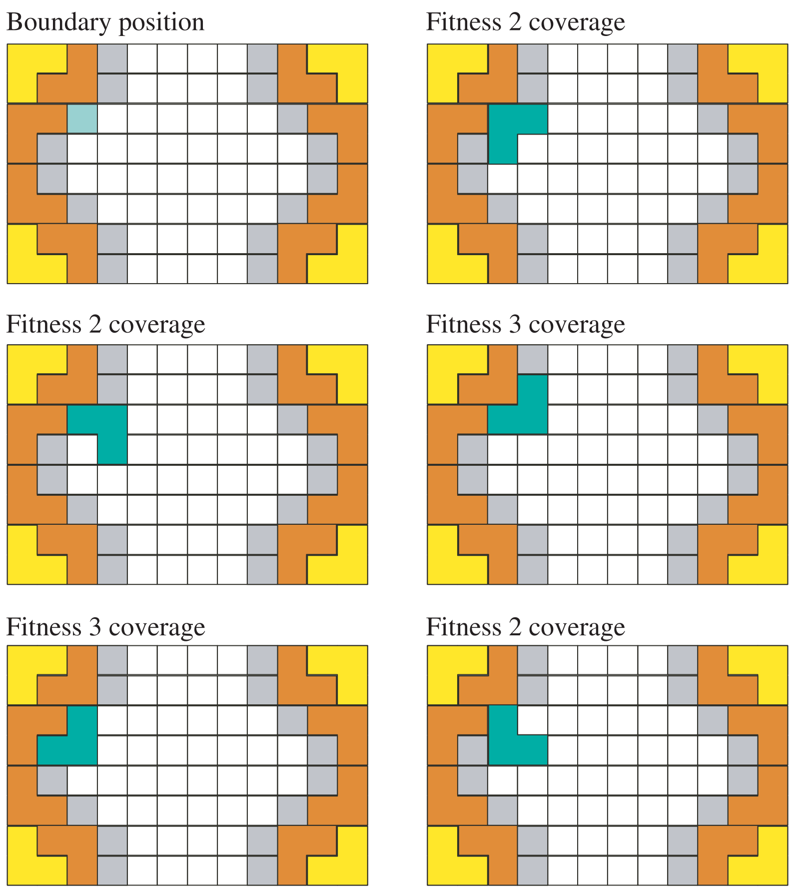

Figure 2. Placement strategy for a single L-shaped tromino on one possible cell in H^2.

5. Assign a fitness value to any possible in $\mathfrak{P}(p, h)$ on the basis of the number of edges p share with existing polyominoes (Fig. 2);
6. If a single element in $\mathfrak{P}(p, h)$ has a fitness higher than all the other, then it is selected, otherwise a random selection is performed among the element in $\mathfrak{P}(p, h)$ sharing the top fitness value.
7. The selected polyomino is placed at the selected position and orientation.
8. Since the newly placed polyomino can overlap with other cells H^m and not only with h, the set H^m is reduced by eliminating all those cells that are covered by the newly placed polyomino.

9. If $H^m \neq \emptyset$ then there are still uncovered "hot" cells: go to point 3;
10. If $H^m = \emptyset$ then the round is over. If there are still uncovered cells go to point 1, otherwise placement is ended.

The proposed arrays, in a nutshell, consists in the application of the aforementioned algorithm, varying L_n^m according to a given rule. In particular, in an *outer* placement, if polyominoes of larger size are used at the first rounds of the filling strategy, and polyominoes of smaller sizes are used as algorithm proceeds, then larger polyominoes will be placed at the outskirts of the array and smaller at the center. By providing the *same* feed power at the polyomino input port, then such power will be divided among a larger number of elements in the larger polyominoes at the outskirts, and among a smaller number of elements in the smaller central polyominoes, effectively achieving two objectives:

1. keep phase centers on an irregular lattice as for any polyomino-based array;
2. implement an array with element feed amplitude higher at array center and lower at array ends, so as to obtain lower side lobes.

The following section will present few synthesis example performed in this framework.

3. ARRAY SYNTHESIS

To demonstrate the effectiveness of the proposed strategy, several runs of the proposed algorithm have been performed and results presented here.

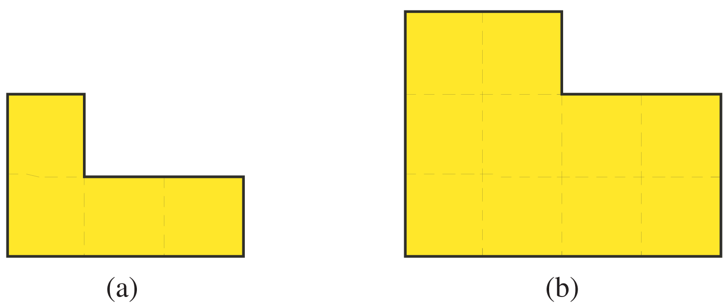

Figure 3. Selected polyominoes for array filling, (a) a tetromino and (b) a decomino.

Table 1. Polyomino array performances — Broadside.

	Array	SLL broadside	Filling factor
S_4	Only Tetrominoes	−13.00	92.50%
S_{10}	Only Decominoes	−13.39	97.50%
C_{i5}	Inner: 5 rounds of tetrominoes	−18.10	92.375%
C_{o5}	Outer: 5 rounds of decominoes	−17.14	98.375%
C_{i6}	Inner: 6 rounds of tetrominoes	−17.70	93.75%
C_{o6}	Outer: 6 rounds of decominoes	−20.23	97.75%
C_{i7}	Inner: 7 rounds of tetrominoes	−20.47	92.75%
C_{o7}	Outer: 7 rounds of decominoes	−18.72	98.0%
C_{i8}	Inner: 8 rounds of tetrominoes	−21.66	91.25%
C_{o8}	Outer: 8 rounds of decominoes	−18.38	97.5%
C_{i9}	Inner: 9 rounds of tetrominoes	−18.84	91.87%
C_{o9}	Outer: 9 rounds of decominoes	−16.24	97.6250
C_{i10}	Inner: 10 rounds of tetrominoes	−18.98	94.875%
C_{o10}	Outer: 10 rounds of decominoes	−13.34	97.625%

Keeping in mind that the lowest possible number of different shapes used, the cheaper the array to be realized will be, only two polyominoes are used, an L-shaped tetromino and an L shaped decomino (Fig. 3). Filling has been performed on a 40×40 elements square grid, both for outer and inner placement strategies, selecting a-priori a number of round to be filled with a given shape, and then completing filling with the other shape. In outer placement filling starts with the larger decomino, in inner filling it starts with the smaller tetromino, to have larger polyominoes at the outskirts.

Table 1 reports SLL levels attained, for broadside (in-phase) feeds, by varying the number of rounds with the initial shape. For comparison, also arrays comprising only tetrominoes or decominoes are reported. The two arrays comprising a single type of polyominoes exhibit an SLL of about −13 dB, which is typical for a uniformly fed array.

On the other hand, the other arrays, where power feeding each polyomino is distributed to four for elements in central polyominoes and to ten for elements in outer polyominoes presents a better SLL in

Table 2. S_4 Polyomino SLL in scanning.

		ϕ_0			
		0	45	90	135
	−15	−10.59	−12.09	−9.64	−10.60
	−10	−12.14	−12.24	−11.19	−11.71
	−5	−12.91	−8.87	−12.32	−12.31
θ_0	0	−13.00	−13.00	−13.00	−13.00
	5	−12.98	−12.40	−12.32	−12.30
	10	−12.16	−12.14	−11.17	−11.73
	15	−8.50	−12.22	−9.62	−10.60

Table 3. C_{o5} Polyomino SLL in scanning.

		ϕ_0			
		0	45	90	135
	−15	−14.82	−7.57	−15.69	−15.00
	−10	−15.90	−15.76	−16.14	−15.68
	−5	−16.53	−16.36	−16.50	−16.19
θ_0	0	−17.14	−17.14	−17.14	−17.14
	5	−16.55	−16.38	−16.53	−16.16
	10	−15.95	−15.82	−16.21	−15.64
	15	−15.10	−15.10	−15.94	−15.07

Table 4. C_{o6} Polyomino SLL in scanning.

		ϕ_0			
		0	45	90	135
	−15	−15.05	−18.39	−17.39	−15.79
	−10	−18.35	−19.12	−18.38	−17.11
	−5	−20.07	−19.74	−19.54	−11.86
θ_0	0	−20.23	−20.23	−20.23	−20.23
	5	−20.15	−19.74	−19.55	−18.46
	10	−19.28	−19.00	−18.38	−11.10
	15	−13.20	−14.68	−17.40	−15.77

broadside direction. It is important to note that the filling strategy does not perfectly cover the area, but filling factors are never lower than 90%, and sometimes as high as 98%.

To show the effectiveness of the design the SLL is evaluated for varying scanning angles. Tables 2 to 4 report the SLL for the S_4 reference configuration, and for two of the best configurations, namely C_{o5} and C_{o6}. Scanning is performed by assuming for the phases those of a sparse array whose elements are located at the feeding points (black squares in Fig. 4) of each polyomino, amplitude being equal for all feeding signals. While it is apparent that all designs have a degradation in SLL levels as the beam scans far from broadside direction, the overall better behavior of C_{o5} and C_{o6} configurations with respect to S_4 is apparent.

Finally, Fig. 4 shows the C_{o6} array, and Figs. 5 and 6 show two of the theoretical patterns obtained. For this polyomino configuration, Table 5 shows additional comparisons, with an equivalent uniformly fed array, and with a Tseng-Cheng (TC) [12] array exhibiting a −20 dB SLL, matching the SLL of the Polyomino array. It is apparent that both the uniform array and the TC have comparable beamwidth and scanning capabilities, yet only the TC has an acceptable SLL level. On the other hand, the TC array exhibits a complex weighting of feeding amplitudes, with a theoretical dynamics of 10^{11}, which effectively means that some of the 1600 elements are indeed not fed at all. The key point anyway is that the polyomino array has only 274 feeding points, that is it needs only 274 amplifiers and phase shifters, while the other two arrays needs 1600 amplifiers and phase shifter each.

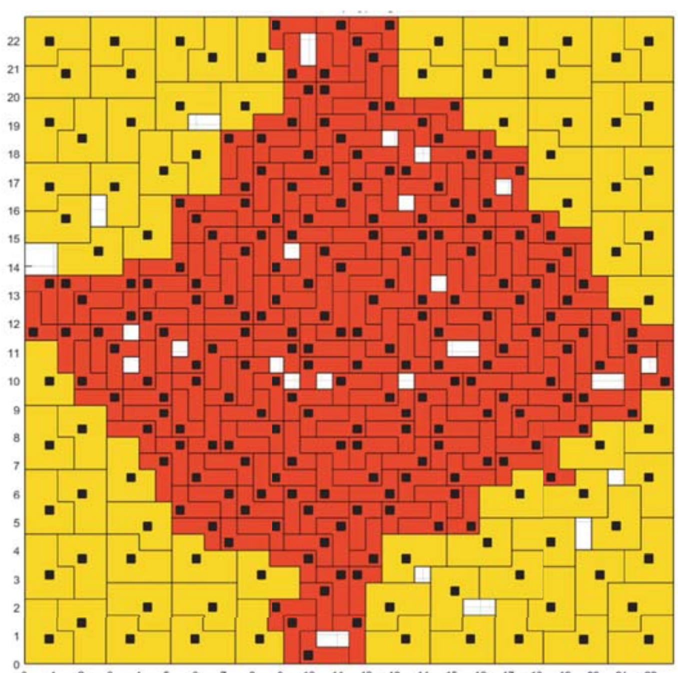

Figure 4. C_{o6} array layout, black squares represent the phase centers of the polyominoes, tetrominoes are in orange, decominoes in yellow.

Table 5. C_{o6} Polyomino comparison.

	C_{o6} Polyomino	Uniform	Tseng-Cheng
Feed Points	274	1600	1600
$\theta_{3\,\mathrm{dB}}$ Broadside	2.25°–2.25°	2.21°–2.21°	2.36°–2.36°
SLL @ Broadside	−20.23	−13.28	−20
$\theta_{3\,\mathrm{dB}}$ @ $\theta_0 = 10°$, $\phi_0 = 45°$	2.26°–2.25°	2.24°–2.27°	2.39°–2.36°
SLL@ $\theta_0 = 10°$, $\phi_0 = 45°$	−18.23	−13.28	−20

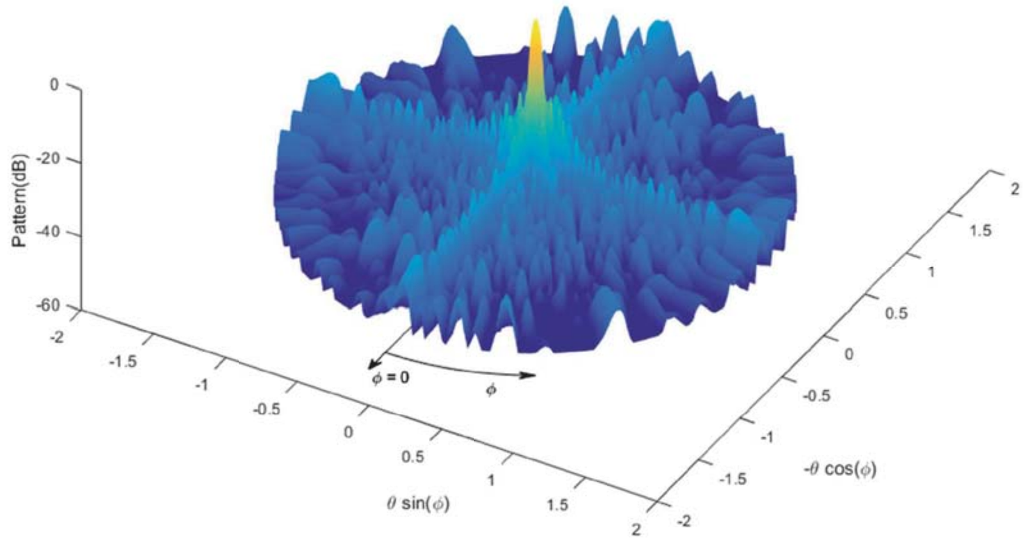

Figure 5. C_{o6} pattern for $\theta_0 = 0$, $\phi_0 = 0$ beam pointing (broadside).

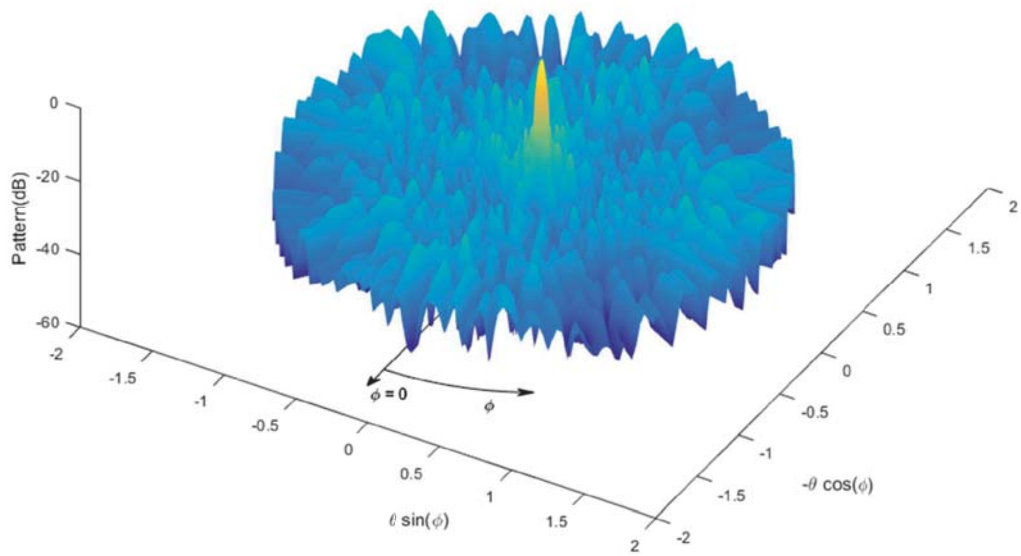

Figure 6. C_{o6} pattern for $\theta_0 = 10°$, $\phi_0 = 45°$ beam scanning.

4. CONCLUSIONS

A polyomino tiling algorithm conceived for placing polyominoes chosen from user-defined variable subset has been developed. Algorithm is suitable for concentrating smaller polyominoes close to the center and larger on the outskirts of the domain, to attain an amplitude tapering in arrays. This way both the advantages of standard polyominoes in scanning capabilities and those of non-uniformly fed arrays, but with all equal amplifiers, for what concerns side lobe levels are obtained.

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