# Turbulent flow-field effects in a hybrid CFD-CRN model for the prediction of $NO_x$ and CO emissions in aero-engine combustors

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### Abstract

The paper presents a numerical study where a hybrid CFD-Chemical Reactor Network (CRN) approach is used to predict pollutant emissions in a tubular combustor for aero-engine applications. A fully-automated clustering of the simulated flow field with the generation of a reactor network representative of the main flow features is exploited. Similar cells are detected and grouped using a two step approach, the first one based only on aerodynamic criteria for turbulent flows followed by a chemical refinement based on mixture fraction. A formulation for turbulent diffusion fluxes is introduced in the reactor code to model species and energy exchanges between reactors. Three different operating conditions are studied for which measured  $NO_x$  and CO are available. Results highlight the importance of including turbulent diffusion in the network solution. The accurate prediction of pollutant emissions at different load points confirms that CFD-CRN is a valid and flexible approach for preliminary assessment of aero-engine combustor emissions in the design phase. *Keywords:* Chemical Reactor Network, CFD; combustor, aeroengine, Gas Turbine, emissions, spray flame, liquid fuel, PSR, PFR,  $NO_x$ 

### **1. Introduction**

One of the main targets for the next generation of civil aero-engines is the abatement of engine pollutant emissions, in particular  $NO_x$ , to meet the stringent regulations to be implemented in the near future. The most prominent way to achieve the compliance is represented by lean burn technology.

Therefore, huge efforts have been put in developing injection strategies (i.e. lean direct injection systems) that create a lean burning mixture directly inside the combustion chamber by improving the rate of spray evaporation and fuel air mixing.

An example of such a technology is the so-called PERM injector developed by GE-Avio. The PERM injector (Partially Evaporating and Rapid Mixing), investigated in this paper, is a double swirler airblast atomizer developed to achieve partial evaporation inside the inner duct and rapid mixing within the combustor, optimizing the location and stability of the flame. Further details about the PERM injector can be

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Nomenclature						
$\dot{m}$	Mass Flow Rate $[kg/s]$	P	Pressure [bar]			
$\phi$	Equivalence Ratio[-]	PERI	M Partial Evaporation and Rapid Mixing			
au	Residence Time $[s]$	PFR	Plug Flow Reactor			
CFD	Computational Fluid Dynamics	PSR	Perfectly Stirred Reactor			
CRN	Chemical Reactor Network	RTD	Residence Time Distribution			
EDC	Eddy Dissipation Concept	$T_3$	Air Inlet Temperature $[K]$			
EI	Emission Index $[g/kg_{fuel}]$	V	Volume $[m^3]$			

 $_{12}$  found in [1] and [2].

Combustor design process requires a rapid and accurate estimate of the main performance and emission indices. To this end, Computational Fluid Dynamic (CFD) and in particular Reynolds Averaged Navier Stokes (RANS) approaches have been used extensively as a standard predictive tool for combustion applications at industrial levels. Even if RANS approaches remain a valid choice to provide fast indications [3, 4, 5], the accurate prediction of pollutants emissions requires the use of detailed chemical reaction mechanisms thus leading to very high computational costs.

Recently the use of tabulated chemistry methods such as ISAT [6] or FGM [7], allows handling detailed chemistry limiting CFD computational costs. However, the usual assumption of considering the flame in the thin laminar flamelet regime (high Damkholer numbers) wrinkled by turbulence does not allow to capture low Damkholer number reactions, such as those involved in  $NO_x$  formation. Some formulations have been proposed to fix this aspect [8], [9], but a general solution is far to be available.

In last years, a new family of approaches was conceived, based on the use of CFD in conjunction with Chemical Reactor Networks (CRNs).

Several applications of equivalent reactor network to lean-premixed combustors, diffusion flames or Rich
Quench Lean combustors for aero-engines can be found in literature [10, 11, 12].

The hybrid approach was first introduced by Ehrhardt et al. [13] and it is based on three main steps: first, 28 a CFD simulation of the reactive flow field is performed using a global chemical reaction scheme. The CFD 29 results are then post-processed applying a set of global criteria to separate the combustor in chemically and 30 physically homogeneous zones. The cells that satisfy the same criteria are clustered together to form the zones 31 of the reactor network. A Perfectly Stirred Reactor (PSR) or a Plug-Flow Reactor (PFR) is associated to 32 each zone according to the local flow conditions. The links and the exchanges of the main physical quantities 33 between the reactors are established by computing the mass fluxes between adjacent zones. Lastly the CRN 34 obtained is solved with a detailed chemical reaction mechanism to obtain an accurate prediction of pollutant 35

36 emissions.

Falcitelli et al. [14] defined a general algorithm to construct the CRN based on equivalence ratio and temperature. The procedure was applied to industrial furnaces [15, 16] and boilers for power generation [17, 18, 19]. An optimized procedure to split the reactive flow field into homogeneous zones is presented by Fichet et al. [20] which applied the procedure to a gas turbine combustor in order to model  $NO_x$  formation with about 400 chemical reactors.

<sup>42</sup> Monaghan et al. [21] employed this approach to study the pathways of formation of NO and NO<sub>2</sub> in a <sup>43</sup> methane-air diffusion flame. They identified six macro-zones with a criterion based on equivalence ratio. A <sup>44</sup> further refinement is realised based on temperature leading to a final network of 1114 PSRs. Recently the <sup>45</sup> procedure has been extended to confined swirling flames [22] and to gas turbines [23, 24, 25].

In their study Novosselov et al. [26] defined five main regions in the combustor: a main flame zone, a pilot flame zone, the center and the dome recirculation zones and the gas burn-out region. The network is then refined to obtain the final configuration of 31 chemical reactor elements representing the different flow and reaction zones of the combustor.

The advantage of having few elements in the network (some tens) is that it is possible to maintain a physical interpretation of each reactor: looking at the distribution of the main quantities in the network, i.e.  $NO_x$  formation rate, it is easier to obtain design indications to reduce emissions. On the other hand, in a network with a limited number of elements a significant error can be introduced if the reactors are linked only by the computed convective mass fluxes between adjacent zones from CFD. In this case, as it will be shown in this paper, the contribution of turbulent diffusion cannot be neglected and its physical and robust implementation is mandatory to provide correct results.

Finally, it is worth mentioning available commercial solutions for CFD-CRN procedure such as ANSYS 57 Fluent reactor network solver [27]. Clustering procedure is not based on aerodynamics and does not aim at 58 maintaining a physical meaning of each identified zone. It is based on temperature and mixture fraction and 59 a limited number of additional custom-field variables can be exploited to improve the clustering (e.g. spatial 60 coordinates or turbulent kinetic energy). Convective flows through reactors are computed from CFD using 61 a standard approach while diffusion fluxes are neglected. Only PSR reactors are employed. Energy equation 62 is not solved and the temperature in each reactor can be either fixed at a constant value derived by CFD or 63 calculated from the equation of state, retrieving pressure from the CFD solution. 64

The aim of the present work is the study of pollutant emissions of an aero-engine combustor at different operating conditions, representative of real flight operations, exploiting an integrated CFD-CRN modelling approach. A fully automated routine for the CFD postprocess and the network generation is developed for a tubular combustor with a swirling jet undergoing vortex breakdown, though it can be easily adapted to annular combustors. The splitting criteria are based on flow and mixing quantities only. Variables such as temperature or species concentrations that are directly influenced by the CFD simplified mechanism are avoided. The target is the generation of a CRN with few reactors to keep the physical interpretation of each <sup>72</sup> branch of the network. The network is then solved in an in-house code, REACT, developed by Andreini and <sup>73</sup> Facchini [12] and further improved in Andreini et al. [28]. In the present work, the original code has been <sup>74</sup> upgraded with the inclusion of turbulent diffusion. The implementation is then validated and emphasis is <sup>75</sup> put on its crucial role for the correct prediction of species and heat fluxes in the reactor network.

Direct comparisons of the results with available measured emissions at the combustor outlet shows en couraging results.

The paper is organized as follows: the investigated geometry is presented in Section 2. The methodology is then introduced starting from a description of the CFD setup (Section 3.1), the CFD post processing and network generation (Section 3.2) and the mathematical model of the reactor network as well as the implementation of turbulent diffusion in the REACT code (Section 3.3). Finally, in Section 4 the main assessment and results are presented before reporting the main conclusions.

## 83 2. Investigated combustor

The methodology is applied to study a lean spray flame generated by GE-Avio advanced PERM injection system in a laboratory test case.

The PERM injector, schematically represented in Figure 1 together with the investigated test-rig, is a 86 double radial co-rotating swirler where liquid fuel is mainly injected by a prefilming airblast scheme. A 87 film of fuel is generated over the inner surface of the lip that separates the two swirled flows. As the film 88 reaches the edge of the lip primary atomization occurs: fine droplets and rapid mixing are promoted by the 89 two co-rotating swirled flows generated by the double swirler configuration. To ensure a stable operation of 90 the lean burn system the ariblast injector is coupled with a hollow cone pressure atomizer (pilot injector) 91 located at centre of primary swirler, which generates a pilot flame to stabilize the combustion process in a 92 configuration usually referred to as "piloted airblast". 93

The PERM combustor rig was installed and tested at ONERA Palaiseau Center (see Figure 1). The combustor consists in a cylindrical flame tube with a length to diameter ratio L/D equal to 3.25. Air enters the combustion chamber through the swirled channels of the injector and through a slot located in the corner between the dome and the liner, which discharges the air flow used for the impingement cooling of the dome. Standard measurement of emissions (CO, NO<sub>x</sub>, UHC) at the combustor outlet were obtained at ONERA during NEWAC EU project and made available for this work by GE-Avio (for more details on the rig refer to [29]).

Three different operating conditions, respectively representative of Idle (Point 1), Cruise (Point 2) and Take-off (Point 3), have been investigated. The main operating parameters for the three cases are summarized in Tab. 1 where also the measured NO<sub>x</sub> and CO are reported in terms of Emission Index ( $EI_X$ ), defined as follows:

$$EI_X = 1000 \frac{\dot{m}_X}{\dot{m}_{fuel}} \qquad \left[\frac{g}{kg}\right] \tag{1}$$



Figure 1: Sketch of the studied tubular combustor equipped with a single PERM injector.

Test Point	P[bar]	$\phi$	$T_3[K]$	Pilot[%]	$EI_{CO}$	$EI_{NO}$
Point 1	5.3	0.599	613	20	14.23	3.59
Point 2	13.5	0.570	656	15	2.12	10.87
Point 3	22.4	0.520	811	15	0.91	26.16

Table 1: Operating parameters for the tested operating conditions.  $T_3$  is the inlet air temperature,  $\phi$  is the overall combustor outlet equivalence ratio and Pilot is the pilot to total fuel mass flow rate percentage.

# <sup>105</sup> 3. CFD-CRN Procedure

<sup>106</sup> A schematic representation of the CFD-CRN workflow is illustrated in Figure 2.

The procedure consists of three main steps. A CFD simulation of the combustor is performed at first. As a second step, the CFD solution is postprocessed to generate the chemical reactor network. In this phase, the computational cells are clustered into homogeneous regions associated to specific reactors of the network. Reactors properties and their flow connections are also computed in the post-process operation and used to define the network topology. Finally, the generated network is solved in a dedicated code, called REACT, with a detailed mechanism.

A detailed description of the three steps is given in the following sections.



Figure 2: Flow diagram of the coupled CFD-CRN procedure.

### 114 3.1. CFD simulations: numerical models

In the present study, RANS solutions are obtained using the commercial solver ANSYS CFX v16.2 [30]. Time-averaged Navier-Stokes equation are solved together with the additional scalar RTD, Residence Time Distribution, which represents the flow age since the instant of injection, and a set of passive scalars (tracers): a distinct tracer is injected from each air inlet to track the corresponding air stream. A coupled solver is used for pressure-velocity coupling and a second order accurate finite volume method is employed for all the equations.

Only a sector slice domain of  $1.5^{\circ}$  is simulated to reduce computational costs. The computational mesh counts about 60000 hexahedral cells with a single element in the azimuthal direction. The mesh was realized in order to have nearly isotropic volumes in the region where the flame is developing (up to L/D = 1.5). A sensitivity study on the mesh element size was carried out looking also at the effect on spray evolution. An hexahedral element of 0.4 mm was considered for the final setup which is below the size where appreciable differences can be observed: some details about the mesh sensitivity on a similar case are reported in [31] and [32].

A specified mass flow rate is assigned on all the air inlets. A simplified representation of the swirler vanes is realized, as depicted in Figure 3. Swirler air is injected by two distinct inlets located at the exit section of each vane row. The flow split between primary and secondary swirlers and flow directions is retrieved by preliminary simulations of the full 360° combustor, including the upstream plenum (see [2] for details). A third air inlet is included for the slot cooling. A constant static pressure is imposed at the domain outlet. At the lateral surfaces periodic boundary conditions are assigned while walls are considered adiabatic.

<sup>134</sup> Concerning the employed models, the  $k - \omega$  SST turbulence model [33] with a wall-function approach <sup>135</sup> is used together with the Eddy Dissipation Concept (EDC) combustion model [34]. A transport equation



Figure 3: Sketch of the considered computational domain.

is solved for each species present in the used skeletal reaction scheme and therefore complex mechanisms
are prohibitive for the high computational cost and numerical stiffness introduced. Finite rate kinetics is
considered by computing the source term as the minimum between a turbulent mixing term ad a standard
Arrhenius one.

JetA-1 fuel is modelled by a single species surrogate:  $C_{12}H_{23}$  whose transport properties for both liquid and vapor phases are computed accordingly to Rachner [35]. A two-step global mechanism for  $C_{12}H_{23}$  is employed, which consists in a first step for fuel oxidation into CO and  $H_2O$ , and a second step for CO oxidation into  $CO_2$ :

$$C_{12}H_{23} + 11.75O_2 \to 12CO + 11.5H_2O \tag{2}$$

$$CO + 0.5O_2 \rightarrow CO_2$$
 (3)

<sup>144</sup> Arrhenius coefficients for this scheme can be found in [36].

A two-way Eulerian-Lagrangian particle tracking approach is considered for the solution of spray dynamics 145 and two-phase flow physics. Pressure atomizer injection is realized injecting a statistically representative 146 population of parcels using a hollow cone model with a  $45^{\circ}$  cone angle. The main injection is modelled with a 147 surface injection at the lip tip where the primary breakup occurs. Particles are introduced with a  $0^{\circ}$  injection 148 angle and a temperature of 298 K. For the droplet size distribution, a Rosin Rammler probability density 149 function is used with a mean droplet size of 92.55  $\mu m$  and a spread parameter of 2. Secondary breakup 150 effects are considered through the well-known Taylor Analogy Breakup (TAB) model [37]. The dispersion of 151 particles due to turbulence in the fluid phase is included using the Random Walk model [38]. 152

### <sup>153</sup> 3.2. CFD post processing and network generation

CFD solutions provide information about flow field and chemistry, which are used to perform the zone splitting, build a CRN and compute reactors properties. The CFD post processing is based on a two-step approach: a first aerodynamic splitting and a further refinement based on mixture fraction.

### 157 1. Aerodynamic splitting

In this phase a first combustion zoning is performed by looking at typical aerodynamic quantities such as velocity, turbulence kinetic energy, turbulence length scales, RTD and tracers. The main regions associated to the combustor aerodynamics are then identified: principal inlet streams, high speed swirling jet, inner and corner recirculation regions, burnout region. In Figure 4 the obtained aerodynamic clustering is shown.

The criteria used to clustering cells, which are briefly presented in the following, are specified through User Defined Limits (UDL) to apply at flow variables (e.g.  $UDL_{T1}$  is a User Defined Limit for *Tracer1* concentration,  $UDL_{AV}$  is a User Defined Limit for the axial velocity, and so on). Acting on the ULDs it is possible to adapt the subdivision to the investigated case, maintaining the same aerodynamic structures.



Figure 4: Aerodynamic splitting obtained for the studied combustor equipped with a PERM injector.

168	• Air inlets regions are identified by high concentration of tracking scalars (e.g., Air $1 \rightarrow Tracer1 < 1$
169	$UDL_{T1}$ ).
170	• Jet and Peripheral Jet: they constitute the main flame zone and are identified by the highest
171	velocity in the domain (e.g., $Jet \rightarrow AxialVelocity > UDL_{AV}$ ).
172	• Main and Dome Recirculation are identified by negative axial velocity and high flow age
173	$(AxialVelocity < 0 \text{ and } RTD > UDL_{RDT}).$
174	• Flame Front and High Turbulence Kinetic Energy (HTKE) Regions are characterised by the high-
175	est levels of turbulence kinetic energy, due to the shears between the vortex breakdown and the
176	swirled jet (e.g., <i>Flame Front</i> $\rightarrow$ <i>TKE</i> > <i>UDL</i> <sub>TKE</sub> ). Intense reaction rates are found in here.
177	The reacting mixture feeds the inner recirculation and sustains the main flame.
178	• Near Wall, Burn Out and Final constitute the post-flame and burn out regions. Among the
179	remaining cells, they are distinguished by a different flow age. The Near Wall region, in fact, is
180	mainly fed by the "young" fast jet bypassing the inner recirculation.
181	2. Chemical splitting
182	The main features of the vortex breakdown stabilised flame are identified by this first clustering. A
102	further refinement is realized in this second phase based on mixture fraction, according to what proposed

further refinement is realized in this second phase based on mixture fraction, according to what proposed by Monaghan et al. [21]. In this step homogeneous cells from a chemical kinetic point of view are grouped.

Once the cell clustering is completed a reactor is associated to each region. Reactor properties (residence 186 time, volume etc.) and exchanged mass flows are computed. Based on this, connections are established 187 between reactors. 188

Looking at Figure 5 two different problems are now analyzed. Firstly, to correctly reproduce the flow 189

exchanges among recirculation regions, three zones are defined: the recirculation zone with negative axial velocity (PSR-2), one zone that collects the mass flow coming from the former (PSR-1a) and a third one that 191

delivers back mass flow to the recirculation region (PSR-1b). 192

190



Figure 5: Example of region splitting to account for flow recirculation. Red line highlights a region boundary aligned with streamlines.

Secondly, if the interface of two adjacent regions is aligned with the flow streamlines we run into the 193 limit situation where the computed mass flow will be zero. In this situation species and energy exchanges 194 are mainly governed by turbulent diffusion. Establishing connections in the CRN based on convective mass 195 flows alone this contribution is missed and the fluxes are not correctly represented. This is a key observation 196 that points out the importance of including such a contribution in the CRN solution and that gives reason 197 to the development introduced in REACT code (see section 3.4). 198

#### 3.3. The reactor network model 199

Once the cell clustering is completed a chemical reactor is associated to each region. The in-house code 200 REACT is used to solve the network. The code is based on CHEMKIN II libraries ([39]) and handles 201 both PSR and PFR models, which are both implemented with two different concepts. The first concept 202 considers standard ideal micro-mixing which does not allow to take into account liquid fuel injection, while a 203 second approach, proposed by Prior et al. [40], allows to model liquid fuel evaporation and non-ideal mixing 204 associated to it. Further details about numerical tool can be found in Andreini et al. [28]. In the present work 205

only ideal PSR and PFR reactors are used and the liquid fuel injection is not modelled inside the network.

The reactor network is created employing PSR in all the identified regions except for the burn-out ones close to the outlet (post-flame) where PFRs are used to complete the combustion. The choice of PSRs is particularly justified in the primary zone of the combustor, which typically includes large recirculation regions and requires sufficient residence time and high turbulence levels.

Appropriate residence times for each reactor are selected based on CFD computed flowfield. Following the approach proposed by Fichet et al. [20], a transport equation is solved in the CFD simulation for a passive scalar, RTD [s], which represents the mean age of the flow. A source term is introduced in RTD transport equation that increases its value of 1 each second, thus representing the linear dependence of the fluid age on time.

In each region the mean residence time ( $\Delta RTD$ ) is computed as the average of the pass-through times of the streamlines on the region itself:

$$\Delta RTD = \frac{\oint \dot{m}RTD}{\frac{\oint \dot{m}}{2}} \tag{4}$$

The solved equations for PSRs are the conservation of the k-th species and energy:

$$\frac{dY_k}{dt} = \frac{dY_k}{\tau} - \frac{dY^{(in)_k}}{\tau} + \frac{\dot{\omega}_k W_k}{\rho} + S_k^{diff}$$
(5)

$$\frac{dT}{dt} = \frac{1}{\tau c_p} \sum_{k=1}^{K} \left[ Y^{(in)_k} \left( h^{(in)_k} - h_k \right) \right] - \frac{1}{\tau c_p} \sum_{k=1}^{K} W_k h_k \dot{\omega}_k + Q_Y^{diff} + Q_T^{diff}$$
(6)

where  $Y_k$  is the mass fraction the k-th species,  $Y^{(in)_k}$  the inlet mass fraction,  $\tau$  the reactor residence time,  $\rho$  the average density of the reactor mixture,  $\dot{\omega}_k$  the chemical source term of the k-th species with molecular weight  $W_k$ , T is the temperature,  $c_p$  the average specific heat at constant pressure for the reactor mixture and  $h_k$  is the enthalpy.  $S_k^{diff}$  in Eq. 5 and  $Q_Y^{diff}$  and  $Q_T^{diff}$  in Eq. 6 are related to turbulent diffusion. For their expressions please refer to the dedicated section 3.4.

In a PFR, purely convective one-dimensional flow is assumed. Species and energy equations are solved along with the mass and momentum conservation in the following form:

$$u\frac{\partial Y_k}{\partial x} = \frac{\dot{\omega}_k W_k}{\rho} \tag{7}$$

$$\bar{c_p}\frac{\partial T}{\partial x} + u\frac{\partial u}{\partial x} + \sum_{k=1}^{K} h_k \frac{\partial Y_k}{\partial x} = 0$$
(8)

where x is the direction of the flow and u is the axial velocity.

For the two type of reactors, temperature is the main quantity driving chemical reactions and its wrong prediction can lead to inaccurate pollutant evaluations, especially for  $NO_x$  emissions. In REACT code, the reactor temperature can be either fixed at the value obtained by reference CFD calculation or computed by solving energy conservation inside the code. In contrast to most literature approaches, in the present work the energy conservation is solved in the network. In this way the predicted temperature and its effects on intermediate and dissociation reactions on the temperature are taken into account to obtain more accurate values for both temperature and species concentrations.

A chemical reaction mechanism based on a single element surrogate for Jet-A fuel, i.e.  $C_{12}H_{23}$ , involving 16 species and 39 reactions is used in the present work [41]. It includes both the thermal [42] and prompt [43] pathways for NO<sub>x</sub> formation. Similar results have been obtained testing other mechanisms, i.e. Leeds scheme for  $C_{10}H_{22}$  (see [44]) and the reaction set reported in [45] for the same surrogate species. Results are not reported here as the sensitivity to different surrogate species and different mechanisms is out of the scope of the present research.

The continuity, species and energy conservation equations for each reactor, which is assumed to be adiabatic, are solved in REACT. The solution for the first reactor of the network is subsequently passed to the adjacent reactors and the reciprocal interaction between them is established by the exchanged mass flow. This iterative process is considered converged when the maximum residual among temperature and species is less that  $10^{-6}$ .

### 245 3.4. Diffusion fluxes modelling in chemical reactors network

The implementation of turbulent diffusion in the CRN solving procedure is a key development of the present research. The inclusion of diffusion fluxes in the CFD-CRN coupling is crucial to correctly model the interactions between adjacent regions, as observed in the part dedicated to the cell clustering (see section 3.2).

In some applications (e.g. [21]) the turbulent diffusive mass flow between two reactors is calculated through the indirect use of a Peclet number (Pe) as follows:

$$\dot{m}^{diff} = \frac{\dot{m}^{adv}}{Pe} \tag{9}$$

where  $\dot{m}^{adv}$  is the computed convective mass flow.

With a null convective mass flow, i.e. when the interface between two regions in the CFD is aligned with the flow streamlines (see Figure 5), such an approach would return a zero diffusion flux. Therefore, a more physics-based formulation of turbulent diffusion is introduced in REACT code for PSRs reactors.

In the typical equations for PSRs (eqs. 5 and 6) the source terms  $S_k^{diff}$ ,  $Q_Y^{diff}$  and  $Q_T^{diff}$  are introduced, where  $S_k^{diff} = \frac{\dot{m}^{diff}}{\rho}$  represents the mass balance for species k due to turbulent diffusion. Generally, in a CFD RANS context based on eddy viscosity turbulence models, this contribution can be related to the gradient of the mean value of species mass fraction:

$$\dot{m}_{k}^{diff} = AD_{T} \frac{\partial Y_{k}}{\partial x_{i}} = A \frac{\mu_{T}}{Sc_{T}} \frac{\partial Y_{k}}{\partial x_{i}}$$
(10)

where A is the area of exchange,  $D_T$  the diffusion coefficient, defined as the ratio of the turbulent viscosity  $\mu_T$  and the turbulent Schmidt number  $Sc_T$ .

262 When implemented in REACT code, the mass flux of the k species is computed as follows:

$$\dot{m}_{k}^{diff} = \sum_{j}^{N_{reactors}} = A_{ij} \frac{\mu_{T}}{Sc_{T}} \frac{Y_{j,k} - Y_{i,k}}{Dist_{ij}}.$$
(11)

 $\frac{Y_{j,k}-Y_{i,k}}{Dist_{ij}}$  is the difference between the mass fractions of the *i*-th and the *j*-th reactors while  $Dist_{ij}$  is a characteristic length.

Solving for the *i*-th reactor, the diffusion fluxes with all the other  $N_{reactors}$  reactors are evaluated. The contact area  $A_{ij}$  and  $\mu_T$  are computed from the CFD, at each interface. The  $Sc_T$  number is assumed constant (typically in the range of 0.7-0.9) or it can be computed from the CFD as well. If *i*-th and *j*-th reactors are not connected, the computed area  $A_{ij}$  will be zero and so the diffusion flux. The sum of the diffusion fluxes of all the *k* species is null ( $\sum \dot{m}_{diff,k} = 0$ ). Therefore, including turbulent diffusion, the species are redistributed within the domain but the global mass balance of the reactor will not change.

The energy equation for PSR is also modified to account for heat diffusion which is computed with the following general expression:

$$q_i = \rho \sum_{k=1}^{K} h_k Y_k V_{k,i} - \lambda \frac{\partial T}{\partial x_i}$$
(12)

where  $V_{k,i}$  is a characteristic diffusion velocity and  $\lambda$  the thermal conductivity. Accordingly, two contributions are included in eq. 6:

- $Q^{diff}$  taking into account the flux of enthalpy due to species diffusion;
- $Q^T$  taking into account the heat diffusion related to temperature gradients.
- 277 In the REACT code implementation they are expressed as follows:

$$Q_{i}^{diff} = \sum_{k=1}^{K} \sum_{j=1}^{N_{reactors}} m_{k,i,j}^{diff} [max\left(0, sign\left(m_{k,i,j}^{diff}\right)\right) C_{p_{j}}T_{j} -min\left(0, sign\left(m_{k,i,j}^{diff}\right)\right) C_{p_{i}}T_{i}]$$

$$(13)$$

278 and

$$Q_i^T = \sum_{j=1}^{N_{reactors}} A_{i,j} \frac{\mu_T}{Sc_T} \frac{T_j - T_i}{Dist_{ij}}$$
(14)

As for the species diffusion, the area  $A_{ij}$  and the average  $\mu_T$  are computed automatically during the CFD post process routine while the  $Sc_T$  number is assumed constant and equal to 0.8.

### 281 4. Results

In this section the results obtained when applying the CFD-REACT procedure to study the combustor equipped with a PERM injector, are presented.

Firstly, the CFD results for the three tested conditions are shown and the main features of the PERM flame are described. Successively, the assessment of the implementation of diffusion fluxes in REACT is presented and finally the results obtained solving the CRNs for the test points are discussed.

287 4.1. CFD Results



Figure 6: Main flow structures generated by PERM injector

The flow-field generated by the PERM injector is shown in Figure 6. The typical flow structures of swirl stabilized burners are observed. The breakdown of main swirling jet originates a large inner recirculation region and a smaller one in the external corner. The flame is expected to be stabilized by hot gases in the shear layer between the jet and the inner recirculation. These exhaust gases deliver a continuous source of ignition.

Temperature and mixture fraction distributions obtained for the three test points are shown in Figure 7. Temperature is normalized as follows:

$$T_{norm} = \frac{T - T_3}{T_{max} - T_3}$$
(15)

where  $T_{max}$  is the maximum flame temperature observed in the three conditions.

The flame shape and anchoring are highly influenced by the evaporation of droplets and vapor fuel mixing but the general structure of the flame confirms what expected by the analysis of the flowfield: a lifted partially premixed is observed, with the main flame front located in the low-velocity high-turbulence region at main recirculation zone edge. Increasing the pressure (from test Point 1 to test Point 3), the evaporation rate is enhanced and particles partially evaporate within the injector and in the first part of the jet, as shown in Figure 8. Increased particle mass sources are observed closer to the injection point. Consistently, an earlier



Figure 7: Normalized temperature and mixture fraction contours obtained for the simulated test points.



Figure 8: Predicted evaporation for the three test points at different pressure.

evaporation leads to a more uniform mixture fraction distribution and smoother temperatures which can beseen in Figure 7.

# 304 4.2. Assessment of the network generation and diffusion fluxes

Before presenting the final results for the three operating conditions, a general assessment of the two step clustering process and a comparison between results obtained before and after introducing the diffusion fluxes in the solution are reported here for the high pressure case (Point 3).

- In Figure 9 the contour plots named "CFD" superimpose the clustered volume to the CFD field. Contour
- <sup>309</sup> plots "Mean Value CFD" is obtained averaging (weighting with mass) the CFD computed distributions over each region.



Figure 9: Comparison of temperature and mixture fraction distributions between CFD, zone-averaged CFD values and REACT results with and without diffusion.

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Comparing the two plots, it is possible to state if the adopted refinement is representative, before freezing the network and solve for it in REACT. The air-fuel mixing and the temperature levels are well reproduced by the adopted clustering. In mixture fraction contours, the rich regions of pilot and main injections are detected as well as the mixing with the main air along the jet. Mixture fraction and temperature levels of the corner recirculation are correctly reproduced while a colder inner recirculation is predicted. Despite this slight discrepancy, the accuracy obtained with the proposed clustering is retained sufficient to proceed with the CRN solution.

Results without accounting for turbulent diffusion are shown in Figure 9 ("REACT No Diffusion" in 318 the picture). Clearly both mixing and temperatures in the network are not fully representative. The fuel 319 is injected mainly in pilot and lip regions, where fuel evaporation is predicted by CFD. Therefore, the 320 corresponding reactors are rich. However, the fuel-air mixing along the jet is not correctly modelled. As 321 the jet outer surface is aligned with the streamlines, the computed convective mass flow is small and the 322 fuel cannot go towards the corner recirculation. It is instead transported along the jet towards the inner 323 recirculation, which becomes rich. Consequently, temperature distribution in Figure 9 shows hot inner 324 recirculation and secondary HTKE regions. Temperature stratification along the radius in the final part 325 of the combustor is also missed and a uniform distribution is predicted eventually altering post flame  $NO_x$ 326 formation. 327

Introducing the turbulent diffusion fluxes, the distributions in Figure 9 ("REACT Diffusion") are ob-328 tained. The higher the difference of species concentration between two adjacent reactors, the more intense 329 the diffusion is. Therefore, the fuel mainly flows from the rich part of the jet to the corner recirculation, 330 which is then enriched. Fuel is also taken out of the main recirculation and secondary HTKE that gets 331 leaner. Mixture fraction distribution in the network is clearly improved. This could be further adjusted 332 acting either on turbulent Schmidt number or on the characteristic distances between reactors. Both can be 333 used as network tuning parameters. Such a tuning is reasonable if the number of reactors is limited so that a 334 physical understanding is kept while adjusting the fluxes. A colder main recirculation is seen in temperature 335 distribution in Figure 9 while temperature of the corner region is slightly increased. The fuel preheating due 336 to temperature gradients is observed for pilot and main injection. The double effect of preheating the fuel 337 and of a mixture fraction closer to the stoichiometric one leads to high temperatures in the flame front. In the 338 CFD this is observed at the root of the flame. This suggests possible improvements for future development 339 of the zoning criteria such as splitting the flame front into more sub-zones. Temperature stratification in the 340 burn-out region is now captured by the network. 341

Despite a jet region with higher temperature levels and a colder dome recirculation, both mixture fraction and temperature distributions in the network can be considered a good representation of the initial CFD fields. The same network generation strategy is then applied to study the remaining test Points 1 and 2 at lower operating pressure and to compute pollutant emissions.

### 346 4.3. Comparison between the investigated test points

In this section, the comparison between measured and predicted NOx and CO, when accounting for diffusion fluxes, is presented for the three investigated test points.



Figure 10: Temperature and mixture fraction distributions obtained in REACT for the investigated test points.

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Mixture fraction and temperature distributions are shown in Figure 10 and can be directly compared to CFD results reported in Figure 7. The effect of pressure on the mixing process is captured by the CFD-CRN approach. The enrichment of pilot and lip injection regions with pressure is well represented in the network. The rich jet observed for Point 1, due to later evaporation of the fuel droplets, as well as the enrichment of the inner recirculation are reproduced. Thanks to the implemented turbulent diffusion fluxes, fuel is also present in the corner region.

The temperature increase in the inner recirculation and inner post-flame region at low pressure is captured. The intensification of the root of the flame is also reproduced even if such effect is spread over a larger region, the flame front.

From the same picture it can be seen that a critical region to be represented is the corner recirculation. The fuel is injected in the network following the evaporation pattern from CFD (see Figure 8). At low pressure, the fuel vapor is carried to the corner recirculation by both convective and diffusion fluxes. On the contrary, at high pressure the fuel evaporates close to the injector and follows a way that does not allow its convective transport to such a region. Using the same tuning for the diffusion fluxes results in a richer corner zone at low pressure and a leaner zone at high pressure. A case-dependent tuning could improve the predictions.

In Figure 11 measured and predicted values of CO and NO<sub>x</sub> at outlet are shown in terms of Emission Index. The predicted trend for NO<sub>x</sub> varying the pressure is well captured and a good matching of experimental data is obtained at all the test points.



Figure 11: Measured and predicted  $NO_x$  and CO emissions for the investigated test points.

In Figure 12 the distribution of the source term of  $NO_x$  is shown for the investigated cases. Consistently 368 with the temperature profiles in Figure 10, the formation of  $NO_x$  is strongly related to hot regions in the 369 flame for all three cases. In the investigated flame, in fact,  $NO_x$  are mainly produced via thermal pathway, 370 through the Zeldovich mechanism, which is driven by temperature levels. At low pressure,  $NO_x$  are mainly 371 produced in the inner recirculation and post flame regions. Increasing pressure, at Point 2 and Point 3 the 372 region of high production is moved to the flame front region and the inner recirculation. As seen in Figure 373 10, an ongoing reaction is predicted in the corner recirculation, which is different to what predicted by CFD 374 (see Figure 7). Nevertheless, low temperatures do not lead to  $NO_x$  production, as observed in Figure 12. 375

In general, long residence times in hot regions are critical for  $NO_x$  formation. NO maps in Figure 12 suggest that a reduced  $NO_x$  production could be achieved if part of the fuel is by-passing the inner recirculation, flowing along the fast jet directly to post-flame zone. This could be obtained modifying the PERM injection design such as the injected liquid droplet are no longer trapped in the main recirculation. The present study indeed provided indications for some of the main modifications introduced in the most recent designs of PERM injector.

Concerning CO emissions at the outlet, reported in Figure 11, a good agreement is obtained for Point 2



Figure 12: NO source term distributions obtained in REACT for the investigated test points.

and Point 3. CO at the outlet tends to reach values close to chemical equilibrium and therefore, when pressure is reduced CO increases. At the same time, lower flame temperatures induce local quenching of the reaction in the primary zone, which leads to higher CO production. The trend is well represented by the network model. However, the predicted increase is lower than in experiments. Deviations from the measured values can be ascribed to the wall thermal treatment in the model, where combustor walls are assumed adiabatic. This implies the prediction of higher temperatures and the partial messing of quenching effects. This is more evident for Point 1 where measured exhaust CO is above equilibrium values.

### <sup>390</sup> 5. Conclusions

In the present work, a hybrid CFD-CRN approach is used to study pollutant emissions (NO<sub>x</sub> and CO) 391 of an aero-engine combustor at different pressures. Numerical simulations of an experimental combustor, 392 equipped with a single GE-Avio PERM injector, are carried out for three different operating points, repre-393 sentative of Idle, Cruise and Take-off conditions. CFD solutions provide information for the reactor network 394 construction. Aerodynamic-based clustering criteria are defined to realise a first macro-clustering of the CFD 395 domain. Successively, to make the final network more representative from a chemical point of view, further 396 refinements on mixture fraction are introduced. The final networks count a limited number of reactors that 397 allow maintaining a physical interpretation of each of them. To model in a correct way the species and 398 energy fluxes in the reactor network, turbulent diffusion fluxes are implemented in the existing REACT code, 399

used to solve the network with a detailed chemical reaction mechanism. Results confirms that the used 400 splitting criteria are general and can be applied at different pressure levels. The necessity to properly model 401 turbulent diffusion is highlighted as well as the great improvement introduced accounting for it. An accurate 402 prediction of the concentrations of  $NO_x$  and CO is obtained at all the pressure levels, though CO deviates 403 from experiments at low loads, due to cooling effects at the combustor walls that are not included in the 404 model. Species and temperature are reasonably reproduced in the CRN, considering the limited number of 405 reactors employed. Further improvements could be introduced with a more refined clustering and with a 406 case-dependent calibration of diffusion fluxes. The approach has great potential as it allows for fast evalua-407 tions of emissions in reacting system and to provide indications for improvements of combustor and injection 408 system design. 409

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