Time history analysis of slender masonry towers: a simplified Bouc & Wen approach

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SUMMARY. Considering the need of simplified but effective methods to assess the seismic response of existing masonry towers, the paper investigates a simplified approach based on an equivalent single degree-of-freedom Bouc & Wen model. As a prototype of masonry towers, a cantilever masonry beam is analysed assuming that the first modal shape governs the whole dynamic behaviour. With this hypothesis the paper discusses the identification of the Bouc & Wen model parameters in order to reproduce the cantilever masonry beam hysteretic response. The results of the simplified Bouc & Wen approach are compared with the results of finite element (FE) simulations to discuss the effectiveness of the method.

1 INTRODUCTION

Several simplified approaches have been recently proposed to assess the structural response of masonry structures under seismic loading. In particular, a one-dimensional deterministic numerical model to compute time histories of the non-linear response of slender masonry towers was proposed by Lucchesi and Pintucchi [1]. The main mechanical characteristics of the material in each section along the tower were taken into account by means of a non-linear elastic constitutive law in terms of generalized stress and strain. The material was assumed as no-tensile resistant (NTR) with limited compressive strength. A three-dimensional fibre model was proposed by Casolo [2] [3] to describe the dynamic response of slender masonry towers and employed to perform vulnerability analysis. The structure consisted of a set of fibres aligned with the vertical axis of the tower, and the constitutive behaviour of each single fibre was assumed hysteretic with damage. The uncertainties deriving from both the non-linear behaviour of masonry and the loadings were taken into account employing a statistical approach and the combined effects of the most relevant factors that rule the structural response (viscous damping, height, strength, stiffness, strain softening and hysteretic dissipating characteristics) were estimated. The response of slender masonry walls, under turbulent wind, was analysed by Betti *et al.* [4] by means of an approach based on the modal reduction. The material was assumed as NTR, and the mechanical properties distribution were assumed as deterministic.

All the mentioned studies showed that the non-linear behaviour of masonry is characterized by a hereditary nature, hence the restoring force that describe the masonry structure mechanical behaviour cannot be simply described as a function of the instantaneous displacement or acceleration due to hysteresis phenomena. In this respect, a plethora of studies and illustrative applications have shown the high flexibility of the hysteretic model proposed by Bouc [5], and subsequently improved by Wen [6] [7]. The model has the advantage of the computational simplicity (since to describe the hysteretic behaviour only one auxiliary non-linear differential equation is needed) and it has been extensively employed to describe a wide range of hysteretic behaviours like degradation of stiffness and strength [8]. The growing interest about this model is testified by the ever increasing number of researches, and applications, that employ such a model. A comprehensive review of such studies has been recently discussed by Ismail et al. [9].

As masonry structures may exhibit such kind of hysteretic behaviour, the paper discusses the employability of the Bouc & Wen model as an efficient approach to account for the non-linear hysteretic phenomena that develop in masonry during seismic loading. As a clarifying example a masonry cantilever beam is analysed, and the paper discusses on the identification of the Bouc & Wen parameters.

2 THE BOUC & WEN MODEL

The dimensional differential equation that describes the mechanical behaviour of the equivalent single degree of freedom (SDOF) oscillator can be written in the following form:

$$\begin{gathered} m\ddot{x}(\mathrm{t}) + c\dot{x}(\mathrm{t}) + kg(\mathrm{t}) = f(\mathrm{t}) \\ g(\mathrm{t}) = \alpha x(\mathrm{t}) + (1-\alpha)z(\mathrm{t}) \end{gathered} \tag{1}$$

where m denotes the mass of the system, c is the viscous linear damping coefficient, k is the stiffness of the system and $f(\mathrm{t})$ is the external excitation. $x(\mathrm{t})$ denotes the one degree of freedom displacement and the overdots represent its derivative with respect to time. $kg(\mathrm{t})$ is the non-linear restoring force that, according to the Bouc & Wen model, is assumed as a linear combination of a linearly elastic force and a history-dependent term: $k\alpha x(\mathrm{t})$ represents the elastic component (instantaneous response) while $k(1-\alpha)z(\mathrm{t})$ represents the hysteretic one (which depends on the past history of stresses and strains). α is a relation between the final and the initial stiffness ($0 < \alpha < 1$) while $z(\mathrm{t})$ is the hysteretic displacement, obtained as solution of the following differential equation:

$$\dot{z}(\mathrm{t}) = \dot{x}(\mathrm{t})\{A - |\, z\,|^{n} \cdot [\beta + \gamma sgn(\dot{x})sgn(z)]\} \tag{2}$$

where $sgn(\bullet)$ denotes the signum function. The differential eqs (1)-(2) contain five non-dimensional unspecified time-invariant parameters that can generate a broad range of different hysteresis loops. In particular, according to the above representation, the set of parameters which control the shape and the size of the hysteresis loop to be identified is composed by $\{A, \alpha, n, \beta$ and $\gamma\}$. Remaining parameters to be identified to completely characterize the mechanical behaviour of interest are the three dimensional parameters $\{m, c,$ and $k\}$.

With respect to the first set, several studies have been conducted over the Bouc & Wen model to quantify the importance of each parameter on the overall response of different hysteretic structures, and to classify these parameters accordingly. The manner in which the parameters of the Bouc & Wen model influence the shape of the hysteresis loop was, for instance, recently deepened by Ikhouane et al. in [10]. Shortly: the exponential n rules the smoothness of the transition from the linear to the plastic behaviour (small values of the parameter n correspond to smooth transition from elastic to post-elastic region, whereas for large values of n the Bouc & Wen model approaches a bilinear model); the parameter A is related to the initial stiffness; the parameter β characterizes the cycles amplitude, while the parameter γ rules the unloading path.

The parameters β and γ control the size and shape of the hysteretic loop, and consequently the dissipated energy. In case of low dissipation (narrow hysteresis) of the system, the parameter β

assumes low values, i.e. tends to zero. Previous studies have demonstrated that the parameters of the Bouc & Wen model are functionally redundant [11].

3 THE REFERENCE CANTILEVER MASONRY BEAM

As reference case study a 10 m wide, 40 m high and 1 m thick cantilever masonry beam was analysed. A numerical model of the masonry beam was built with the FE code ANSYS by means of 8-node isoparametric solid finite elements having dimensions of 1.0×1.0×1.0 m. The final three-dimensional model consisted of 400 *solid65* elements and about 2,640 degrees-of-freedom (DOFs). To reproduce the non-linear masonry behaviour a smeared crack approach was employed, combining a plasticity criterion with a failure surface. The employed plasticity criterion was the Drucker-Prager one, originally proposed for geo-materials. To account for cracking and crushing failure modes the Drucker-Prager plasticity criterion was combined with the Willam-Warnke failure criterion (originally proposed for concrete). The proper combination of the two models allows for an elastic-brittle behaviour in case of biaxial tensile stresses or biaxial tensile-compressive stresses with low compression level. On the contrary, the material behaves as elasto-plastic in case of biaxial compressive stresses or biaxial tensile-compressive stresses with high compression level. As a result masonry is modelled as an isotropic medium with plastic deformation, cracking and crushing capabilities.

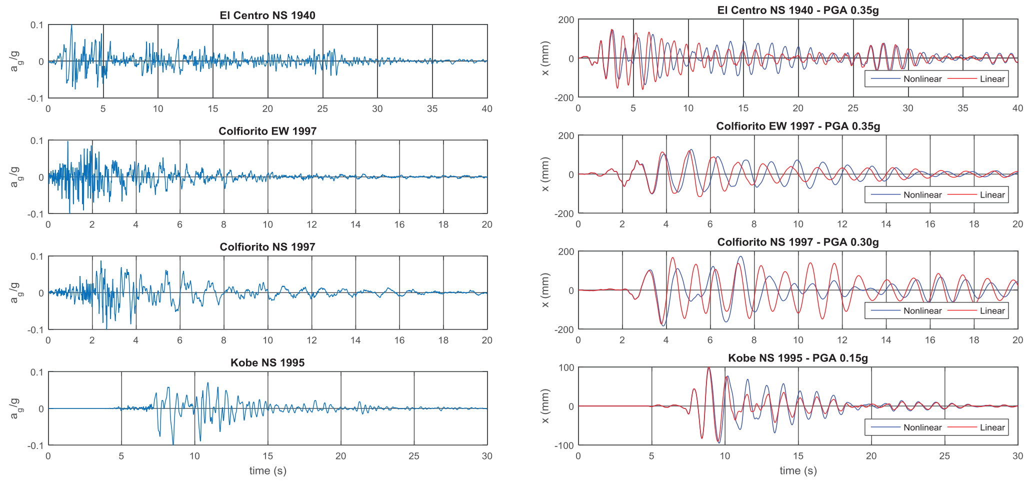

Figure 1: Natural ground accelerations (scaled at PGA=0.1g) - *left*; Time-histories of top displacement (comparison between linear and non-linear response) - *right*.

The ANSYS model was employed to analyse the behaviour of the cantilever beam under seismic loads through non-linear analyses in the time domain; furthermore, modal and static analyses were also performed.

In particular, the time histories were computed under the action of four different natural ground acceleration records. These accelerograms, scaled at PGA = 0.1g, are shown on the left side of Figure 1. On the right, Figure 1 reports the time-histories of the results (displacement of the centre of mass of the top section) as obtained with the 4 ground accelerations, comparing the non-linear response with the linear one. Results of the FEM analyses were assumed as reference to discuss the identification of the parameters of the equivalent SDOF Bouc & Wen model.

4 BOUC & WEN MODEL PARAMETERS IDENTIFICATION

The results of the modal analyses performed by means of ANSYS show that the dynamic behaviour of the cantilever beam can be modelled accurately enough by means of its first modal shape. Therefore an equivalent non-linear SDOF hysteretic system can be defined, whose degrading stiffness can be determined by means of the performed non-linear numerical analyses.

The identification of the (non-dimensional) Bouc & Wen parameters was the concern of a plethora of researches, and several methods have been proposed in literature based on simulated or experimental input/output data. According to Ismail et al. [9] and Ortiz et al. [12], the procedures employed in literature to identify the parameters of the Bouc & Wen model can be classified into two major families: a) methods based on the minimization of a cost function and b) methods based on non-linear filtering. Herein it is proposed to identify the Bouc & Wen model parameters through a two-steps procedure. In a first step, the non-linear static response of the masonry cantilever beam acted upon by a static horizontal load is employed to assess a few of the Bouc & Wen parameters. In a second step, the results of the ANSYS time-history analyses are employed as reference to estimate the remaining parameters. In addition, several scenarios are considered to discuss the efficiency of the procedure, and the effectiveness of the approximation is evaluated through comparison between the time-history obtained with the equivalent SDOF Bouc & Wen oscillator and the time-history evaluated with the reference FE model.

The whole set of parameters to be identified is composed by following terms: $\{m, c, k, A, \alpha, n, \beta$ and $\gamma\}$. Among the Bouc & Wen parameters that must be identified, it can be shown [13] [14] that parameter A in Eq. (2) can be considered redundant, and henceforth in the following it will be set as unitary (A=1) without loss of generality. The parameter k, that represents the tangent stiffness of the equivalent oscillator, can be obtained (having assumed the parameter A unitary, and substituting the initial conditions $z(0)$=0) by deriving the non-linear restoring function $kg(t)$ with respect to the displacement x, obtaining:

$$\{z(0) = 0; A = 1\} \quad \Rightarrow \quad k_\mathrm{i} = k_\mathrm{t}\,|_{\mathrm{z=0}} = k[\alpha + (1-\alpha)] = k \tag{3}$$

The post-yield stiffness, k_f, according to Marano and Greco [8], is given as $k_\mathrm{f} = \alpha k$, hence the following relation asymptotically holds:

$$\dot{z} = 0 \quad \Rightarrow \quad k_\mathrm{t} = k_\mathrm{f} = \alpha k \quad \Rightarrow \quad \alpha = \frac{k_\mathrm{f}}{k_\mathrm{i}} \tag{4}$$

When the maximum displacement is reached, and the unloading process begins, following expression holds for the unloading initial stiffness k_u:

$$\left.\begin{array}{r} z = \hat{z} = 1/(\gamma+\beta)^{1/n} \\ sgn(z) = 1;\ sgn(\dot{x}) = -1 \end{array}\right\} \Rightarrow k_\mathrm{u} = k\left\{\alpha + (1-\alpha)\left[1 - \frac{\beta-\gamma}{\beta+\gamma}\right]\right\} \tag{5}$$

and consequently following relation holds:

$$\frac{\beta-\gamma}{\beta+\gamma} = \frac{1-(k_\mathrm{u}/k)}{1-\alpha} = \frac{k_\mathrm{i}-k_\mathrm{u}}{k_\mathrm{i}-k_\mathrm{f}} \tag{6}$$

Finally, the elastic limit displacement can be expressed, according to Cunha [13], as:

$$\beta + \gamma = x_Y^{-n} \tag{7}$$

Therefore, only one parameter, n, or alternatively β or γ, remains undetermined. It is noteworthy to observe that the parameter n influences the transition from the elastic to the post-elastic behaviour and the distance of the unloading path from the first loading, while the ratio $\alpha_{\beta\gamma}=\beta/\gamma$ affects the transitions from the loading curve to the unloading one. In fact, analysing eq. (3), when the end of the first loading branch is reached following conditions hold: $\dot{x} > 0 \Rightarrow sgn(\dot{x}) = 1$. Subsequently, when passing from the loading to unloading branch, it happen that: $\dot{x} < 0 \Rightarrow sgn(\dot{x}) = -1$ while variation of parameter z is infinitesimal and its signum is unchanged and equal to 1. Consequently, the tangent stiffness at the beginning of the unloading branch is equal to:

$$\begin{aligned} k_{t0}^{s} &= k\{\alpha + (1-\alpha)[1-(\beta-\gamma) \mid z \mid^{n}]\} = k\left\{\alpha + (1-\alpha)\left(1-\frac{\beta-\gamma}{\beta+\gamma}\right)\right\} \\ &= k\left\{\alpha + (1-\alpha)\left(1-\frac{\alpha_{\beta\gamma}-1}{\alpha_{\beta\gamma}+1}\right)\right\} \end{aligned} \tag{8}$$

having assumed that $z \approx \hat{z} = [1/(\beta+\gamma)]^{1/n}$. Based on the ratio between β and γ the hysteretic loop assumes a bulge shape (β<γ) or a slim-S one (β>γ).

The above discussed Bouc & Wen model parameters are hence optimized in order for $x(t)$ to approximate the displacement of the centre of mass of the top section of the cantilever beam. The results (load vs. displacement) of the static non-linear analyses performed through the FE model by applying a monotonically increasing/decreasing horizontal displacement at the top section of the masonry beam, in such a way to obtain the first-loading curve and the subsequent unloading curve was first taken into consideration to approximate the system behaviour with the Bouc & Wen model. Taking into account that the reference FE shape exhibits a slight, but clearly visible, hysteretic behaviour (a slim-S one), it is expected to obtain value of β greater than γ.

Table 1 : Bouc & Wen parameters identification through static non-linear analyses.

	k_i (N/mm)	x_Y (mm)	n	α	β	γ
1st	5654	90	4.0	0.1395	$1.523 \cdot 10^{-08}$	$6.646 \cdot 10^{-12}$
2nd	5654	90	5.0	0.1395	$1.693 \cdot 10^{-10}$	$2.290 \cdot 10^{-14}$
3rd	5654	90	6.0	0.1395	$1.881 \cdot 10^{-12}$	$9.600 \cdot 10^{-17}$

The mixed two-step identification procedure combines, as mentioned, the results coming from the static non-linear analysis with the one obtained with time-history numerical analyses.

The static non-linear numerical analysis (the cyclic loading-unloading curve) was employed to have a first assessment of a few of the Bouc & Wen parameters: the initial stiffness k_i = 5654 N/mm, the ratio $\alpha = k_f / k_i = 0.1395$ and the limit elastic displacement x_Y = 90 mm. Several scenarios were considered analysing different values of the exponential n.

As a first attempt, the value of the exponent n was assumed equal to 4. With this assumption the hysteresis loop shown with a red line in Figure 2 (left) is obtained. The hysteresis loops obtained with n=5 is reported in Figure 2 (right). It is possible to observe that all the choices reproduce correctly the transition from the elastic to the post-elastic branch together with the unloading one; in addition also the energy dissipation cycle is quite close to the reference one (the one obtained with the FE code). The obtained values of the identified Bouc & Wen parameters are reported in Table 1. It is remarkable to observe that the ratio $\alpha_{\beta\gamma}=\beta/\gamma$ ranges between 10^4 and 10^5.

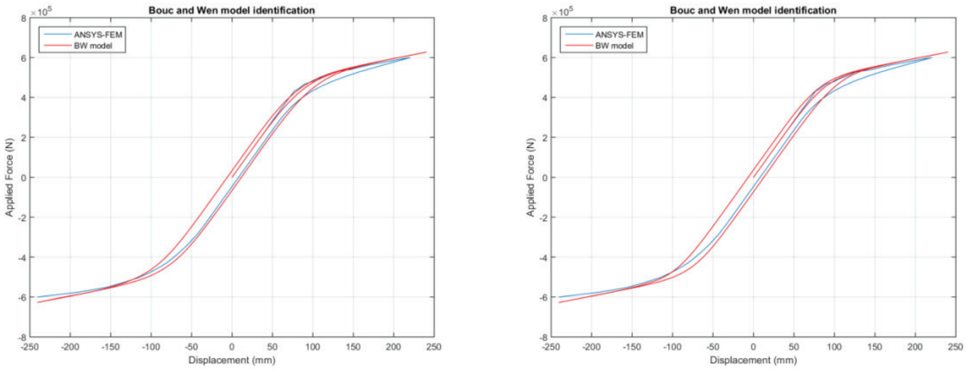

Figure 2: Comparison of the identified Bouc & Wen model with the ANSYS results (1st scenario: n=4 - *left*; 2nd scenario: n=5 - *right*).

The identified values were preliminary assumed as fixed, and the remaining ones (the mass m and the damping c of the equivalent systems reported in eq. (1)) were estimated by means of the optimization of the time-history response of the Bouc & Wen model with respect of the simulated time-history responses obtained with the FE model. To perform this optimization, following error function (to be minimized over the whole time-history) was considered:

$$\varepsilon(\boldsymbol{q}) = \int_0^T \frac{\left|w\big(x_{\mathrm{R}}(t)\big) \cdot \big(x_{\mathrm{BW}}(t|\boldsymbol{q}) - x_{\mathrm{R}}(t)\big)\right|}{\left|w\big(x_{\mathrm{R}}(t)\big) \cdot x_{\mathrm{R}}(t)\right|} dt \tag{9}$$

where $x_{\mathrm{R}}(t)$ denotes the FE model whole time-history response, $x_{\mathrm{BW}}(t|\boldsymbol{q})$ indicates Bouc & Wen model displacement response (which is function of a vector $\boldsymbol{q}$ that collects the model parameters as next specified) and w was a weighting function, defined as follows:

$$w(x_{\mathrm{R}}) = \frac{exp[\mathrm{K_w}\, |x_{\mathrm{R}}(t)|/(|x_{\mathrm{R}}(t)|)_{\max}]}{exp(\mathrm{K_w})} \tag{10}$$

The optimal values were obtained through the direct search Nelder-Mead algorithm which provides the unconstrained minimum of the given cost function: $\min_{\boldsymbol{q}\in R^N}\{\varepsilon(\boldsymbol{q})\}$.

The whole parameters to be identified where collected as follows: $\boldsymbol{q} = [n, m, k,\ \gamma, \beta, \alpha, \xi]$, and to perform the optimization several scenarios were considered as next discussed.

4.1 *Scenario 1*

According to the static non-linear analysis, in this first scenario, the parameter α was assumed fixed and equal to 0.1395. The damping ratio was assumed, in accordance with the simulations, equal to 0.035. The parameters n was chosen to assume three discrete values, between 4 and 6. Hence the following subset of the whole parameters was considered for optimization: $\boldsymbol{q} = [\, m, k, \gamma, \beta]$. The obtained results are summarized in Table 2. It is noteworthy to observe that the mass and the stiffness identified through the optimization are such as to cause that the circular frequency ω (or correspondingly the frequency *f*) of the Bouc & Wen model to match the first frequency of the cantilever beam (i.e. instead of the initial stiffness of the system, the ratio between the mass and the stiffness is maintained constant). The errors between the reference FE simulations and the Bouec & Wen model are reported in Table 3 where it is possible to observe a good agreement between the simplified approach and the reference time-histories. A comparison between the Bouc & Wen and FE displacement time-histories is reported on the left side of Figure 3 (case n=5).

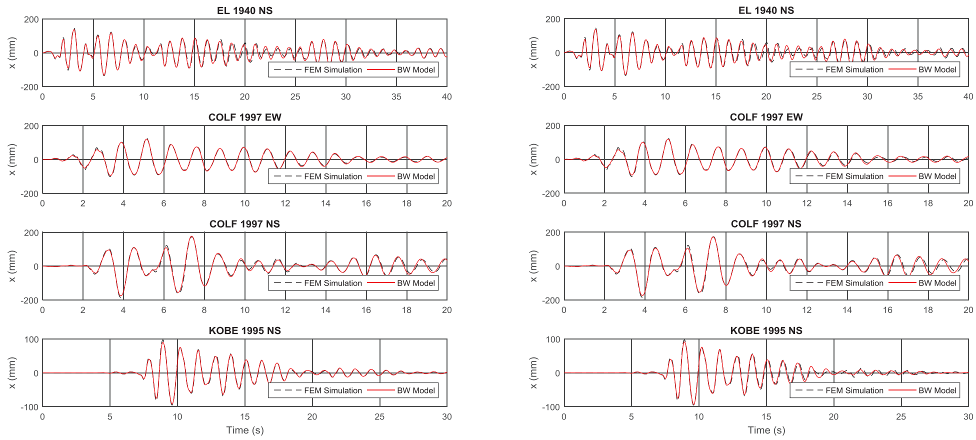

Figure 3: Displacement time history: comparison between Bouc & Wen identified model and ANSYS results (1st scenario - *left*; 2nd scenario - *right*).

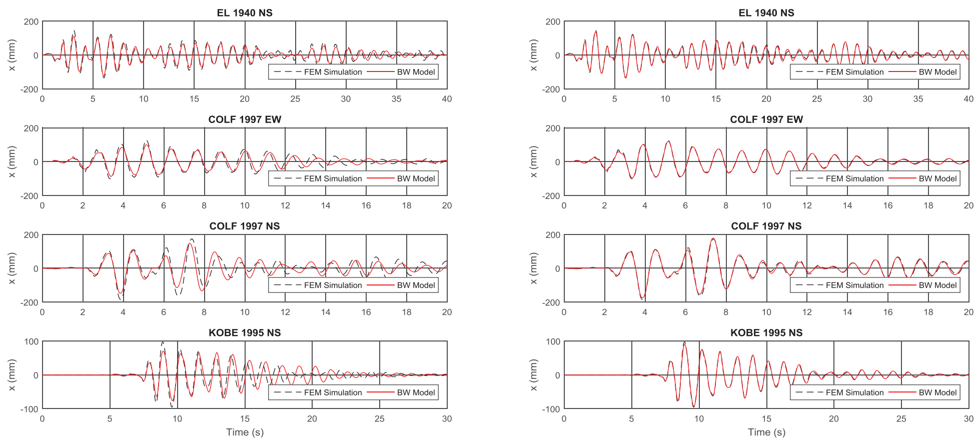

Figure 4: Displacement time history: comparison between Bouc & Wen identified model and ANSYS results (3rd scenario - *left*; 4th scenario - *right*).

4.1 *Scenario 2*

Taking into account the results of the first scenario, a second round of analyses was performed where again the parameters α and the damping ratio were assumed fixed. In this case, however, the mass of the system was also assumed fixed, and equal to the modal mass of the first modal shape of the cantilever beam (448.32 ton). The optimization was performed to identify remaining parameters including the stiffness of the system k. In this scenario, hence, the following subset of parameters was considered: $\boldsymbol{q} = [\,k, \gamma, \beta]$. The optimization algorithm allows to obtain, regardless of the value of the parameter n, a stiffness k that is almost the same (for different values of the parameters β and γ) and corresponds to the fixed numerical frequency (Table 2). The errors, reported in Table 3, have about the same magnitude of the previous scenario (refinement of the results can be obtained by employing more refined optimization algorithms), and the results are illustrated on the right side of Figure 3 (case n=4).

Table 2 : Bouc & Wen parameters identification.

n [-]	m [ton]	k [N/mm]	γ [-]	β [-]	α [-]	ξ [-]	f [Hz]
1st scenario							
4.0	16062.16	510234.05	5.02E-12	2.00E-07	0.1395	0.035	0.90
5.0	3159.85	95882.96	7.06E-17	2.43E-09	0.1395	0.035	0.88
6.0	711.43	22123.46	3.98E-19	7.00E-11	0.1395	0.035	0.89
2nd scenario							
4.0	448.32	14155.87	4.15E-12	4.93E-08	0.1395	0.035	0.89
5.0	448.32	14086.69	1.90E-17	8.24E-10	0.1395	0.035	0.89
6.0	448.32	13942.98	2.12E-20	1.22E-11	0.1395	0.035	0.89
3rd scenario							
4.0	611.21	16310.93	6.65E-12	1.52E-08	5.00e-07	0.035	0.82
5.0	618.83	16291.32	2.29E-14	1.69E-10	1.17e-05	0.035	0.82
6.0	710.58	18544.34	9.60E-17	1.88E-12	1.00e-07	0.035	0.81
4th scenario							
4.0	448.32	13640.00	4.62E-13	3.61E-08	0.1395	0.035	0.88
5.0	448.32	13640.00	1.48E-16	5.84E-10	0.1395	0.035	0.88
6.0	448.32	13640.00	3.29E-18	9.24E-12	0.1395	0.035	0.88

4.1 *Scenario 3*

As a third scenario following set of parameters to be optimized was considered: $\boldsymbol{q} = [\,m, k, \alpha]$. Parameter n was again selected to assume three discrete values (between 4 and 6), while the parameters β and γ were assumed (depending on the value of n) as obtained to reproduce the simulated hysteretic cycle obtained with the static non-linear analyses. The damping ratio was fixed equal to 0.035. The optimization provides low values of the parameter α and the identified parameters are not able to match the simulated results with an error that is varying between 10 and 20%. This is also visible on the left side of Figure 4 (case n=4).

4.2 *Scenario 4*

Based on previous results the mass m was assumed fixed an equal to the modal mass of the cantilever beam, while the stiffness k was assumed in order to reproduce the first frequency of the beam: $k = m \cdot (2\pi f)^2$. The parameter α was set equal to 0.1395 and the damping ratio was assumed, in accordance with the simulation, equal to 0.035. Following subset of parameters was optimized: $\boldsymbol{q} = [\,\gamma, \beta]$ and the final identified parameters are resumed in Table 2 (the errors are reported in Table 3). The responses of the FE reference simulations are compared with the equivalent Bouc & Wen single-degree-of-freedom system on the right side of Figure 4 (case n=4) where it is possible to observe a good agreement between the simulated results and the results offered by the equivalent oscillator.

Table 3 : Errors between Bouc & Wen and FE results.

n [-]	Err [%] El Centro	Err [%] Colfiorito NS	Err [%] Colfiorito EW	Err [%] Kobe
		1st scenario		
4.0	9.7130	5.2780	5.0680	5.8459
5.0	7.4523	2.8731	4.1121	2.5553
6.0	11.1698	6.5450	5.1559	6.4270
		2nd scenario		
4.0	9.0593	4.5953	4.8952	5.1020
5.0	10.7304	6.1384	5.1911	6.4944
6.0	11.1758	6.5496	5.1599	6.4359
		3rd scenario		
4.0	11.1496	15.0307	17.1116	18.1631
5.0	12.1281	15.8073	17.9665	18.8614
6.0	12.8697	16.4302	18.4721	19.4324
		4th scenario		
4.0	5.4769	2.6095	4.3007	2.2910
5.0	7.2209	2.7678	4.1750	2.4826
6.0	8.6983	3.9407	4.4200	3.6299

The analysis of the results obtained with the reported scenarios allows a few comments. The vector of parameters to be identified to build the equivalent Bouc & Wen oscillator is composed by following terms: : $\boldsymbol{q} = [n, m, k, \gamma, \beta, \alpha, \xi]$. The first parameter, the exponent n can be selected indifferently as 4, 5 or 6. The parameter α can be efficiently evaluated as the ratio between the post-yielding and the pre-yielding stiffness of the system (that can be evaluated trough a cyclic static non-linear analysis of the system at hand). The mass m of the equivalent system can be assumed equal to the modal mass, and the stiffness can be evaluated directly in order to replicate the first frequency of the system in undamaged. The only two parameters that need to be evaluated are hence the parameters β and γ (that control the size and shape of the hysteretic loop). Nevertheless in the analysed scenarios it results that the obtained values of the parameter γ are very low, hence further investigations will be aimed to assess the errors obtained if this parameter is assumed directly equal to zero.

5 CONCLUSIONS

The paper discussed on a simplified SDOF approach for the time-history analysis of masonry towers analysing as a clarifying example a masonry cantilever beam. The method, based on the assumption that the dynamic response of such class of structures is mainly ruled by its first modal shape, proposes a SDOF Bouc & Wen hysteretic model to reproduce the dynamic structural response. The identification of the parameters needed to build the SDOF oscillator is analysed in depth, and a two-stages identification procedure is proposed: static non-linear analyses are employed to assess a few of the model parameters; remaining one are assessed to minimize the error between the estimated displacement and the one obtained in a reference FE model of the structure. The results show that the parameters that need to be identified through the comparison with the reference time-history are mainly two: β and γ. According to the results, the identified

SDOF Bouc & Wen system match quite well (errors are lower than 5%) the simulated system response when acted upon 4 different natural ground accelerometers. In the paper, an intentionally simple optimization algorithm was employed to identify these parameters.

Future development of the research will investigates optimization functions that analyse also the difference between the dissipated energy of the equivalent Bouc & Wen model with the one dissipated by the reference system. In addition, further improvements of the research will take also into account the analysis of the response of cantilever masonry beams with varying slenderness.

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