Squeezing a Thermal Mechanical Oscillator by Stabilized Parametric Effect on the Optical Spring

A. Pontin,[1,2] M. Bonaldi,[3,4] A. Borrielli,[3,4] F. S. Cataliotti,[5,6,7] F. Marino,[7,8] G. A. Prodi,[1,2] E. Serra,[1,9] and F. Marin[5,6,7,*]

[1]*Istituto Nazionale di Fisica Nucleare (INFN), Gruppo Collegato di Trento, I-38123 Povo, Trento, Italy*
[2]*Dipartimento di Fisica, Università di Trento, I-38123 Povo, Trento, Italy*
[3]*Institute of Materials for Electronics and Magnetism, Nanoscience-Trento-FBK Division, 38123 Povo, Trento, Italy*
[4]*INFN, Gruppo Collegato di Trento, Sezione di Padova, 38123 Povo, Trento, Italy*
[5]*Dipartimento di Fisica e Astronomia, Università di Firenze, Via Sansone 1, I-50019 Sesto Fiorentino (FI), Italy*
[6]*European Laboratory for Non-Linear Spectroscopy (LENS), Via Carrara 1, I-50019 Sesto Fiorentino (FI), Italy*
[7]*INFN, Sezione di Firenze*
[8]*CNR-INO, L.go Enrico Fermi 6, I-50125 Firenze, Italy*
[9]*Department of Microelectronics and Computer Engineering/ECTM/DIMES, Delft University of Technology,
Feldmanweg 17, 2628 CT Delft, The Netherlands*

We report the confinement of an optomechanical micro-oscillator in a squeezed thermal state, obtained by parametric modulation of the optical spring. We propose and implement an experimental scheme based on parametric feedback control of the oscillator, which stabilizes the amplified quadrature while leaving the orthogonal one unaffected. This technique allows us to surpass the −3 dB limit in the noise reduction, associated with parametric resonance, with a best experimental result of −7.4 dB. While the present experiment is in the classical regime, in a moderately cooled system our technique may allow squeezing of a macroscopic mechanical oscillator below the zero-point motion.

A recent major breakthrough in experimental quantum mechanics is the possibility of preparing macroscopic systems close to their fundamental quantum state. In particular, micro-and nanooscillators have been recently cooled down to an occupation number close to unity or even below it [1–5]. While remaining in a thermal state, such systems display peculiar quantum properties such as asymmetric modulation sidebands induced in a probe field [6]. A further interesting development would be the creation of a qualitatively different quantum state, for instance, a mechanical squeezed state. To this purpose, possible techniques are backaction evading measurements [7–10] and degenerate [11,12] or nearly degenerate [13–15] parametric modulation. Mechanical oscillators operate in the degenerate parametric regime when their spring constant is modulated at twice the oscillator resonance frequency. In such a condition, the response of the oscillator to an external excitation acting close to resonance is enhanced, until the parametric modulation depth reaches a threshold marking the birth of self-oscillations (parametric resonance) [16]. More precisely, the response is amplified in the quadrature of the motion in phase with the parametric modulation, and deamplified in the orthogonal quadrature ($\pi/2$ quadrature). Therefore the distribution of fluctuations in the phase plane caused by stochastic excitation is squeezed and, in particular, its variance is reduced below its free-running value in the $\pi/2$ quadrature. As a consequence, the parametric effect can be used to produce quadrature squeezed states of a macroscopic

oscillator, similarly to what is commonly obtained for the electromagnetic field in optical parametric oscillators [17,18]. This effect has already been demonstrated for thermal oscillators [11,12,15], and is expected even for the quantum noise [13,14]. However, since the amplified quadrature evolves into self-oscillations for an excitation strength approaching the threshold, the corresponding noise reduction in the $\pi/2$ quadrature, monotonic with the parametric excitation, is limited to −3 dB. This is a general feature of parametric squeezing [17,18]. Recent proposals to surpass this limit are based on continuous weak measurements and a detuned parametric drive [13], or unbalanced sidebands modulation [19]. A recent experiment [15] shows, indeed, that the uncertainty in the knowledge of the oscillator trajectory in the phase space (localization) is squeezed with a minimal variance reduced by −6.2 dB with respect to that of a free thermal oscillator. The authors also suggest that, using the information on the oscillator position in an appropriate feedback loop, even the confinement of the oscillator in a strongly (>3 dB) squeezed state could be obtained, though such a result has not yet been demonstrated [20].

In this work, we report on the observation of the confinement of a micro-oscillator in a squeezed thermal state, obtained by parametric modulation of the optical spring constant [21–23]. We also propose and apply an experimental scheme based on parametric feedback that, stabilizing the amplified quadrature without influencing the

orthogonal one, allows us to surpass the >3 dB barrier on noise reduction, with a best experimental result of -7.4 dB.

Our optomechanical system is composed of a low-deformation micro-oscillator [24,25] with a high-reflectivity coating, working as the end mirror in a high Finesse (half linewidth $\kappa/2\pi = 2.3$ MHz) Fabry-Perot cavity. The oscillator has resonance frequency $\omega_m/2\pi = 128\,960$ Hz, mechanical quality factor $Q = \omega_m/\gamma_m = 16\,000$, and effective mass $m = 1.35 \times 10^{-7}$ kg. In the presence of an input field, the intracavity power depends on the detuning $\Delta = \omega_L - \omega_c$ between the laser and the cavity resonance, and actually on the cavity length L_c. As a consequence, the micro-oscillator feels a position-dependent radiation pressure that can be described as the effect of an additional optical spring [21]. The delay in the intracavity field buildup gives an imaginary component in the spring constant, which modifies the damping coefficient $\gamma_{\rm eff}$ of the optomechanical system. The complex optical spring constant is $m|G|^2\omega_m\Delta/[(\kappa + i\omega)^2 + \Delta^2]$, where G is the effective optomechanical coupling constant with $|G|^2$ proportional to the intracavity power. For the case of our interest (bad cavity limit $\kappa \gg \omega_m$, small detuning $\Delta \ll \kappa$, and $\omega \approx \omega_m$) the expression can be simplified introducing the quantities $K_{\rm opt} \approx (m|G|^2\omega_m/\kappa^2)\Delta$, $\gamma_{\rm opt} \approx (2K_{\rm opt}/m\kappa)$, and writing the effective susceptibility as $\chi_{\rm eff}^{-1} = m(\omega_{\rm eff}^2 - \omega^2 - i\omega\gamma_{\rm eff})$ with $\gamma_{\rm eff} = \gamma_m + \gamma_{\rm opt}$ and

$$\omega_{\rm eff} = \sqrt{\omega_m^2 - K_{\rm opt}/m} \simeq \omega_m - \frac{|G|^2}{2\kappa^2}\Delta. \tag{1}$$

To our purpose, it is useful to underline that (a) the frequency shift is approximately proportional to the detuning and, therefore, the optomechanical resonance can be tuned by adjusting the frequency of the drive laser, (b) the damping as well depends on the detuning; therefore, by varying the working point we can chose the effective resonance width, and (c) in the bad cavity limit, the shift in the resonance frequency is larger than the variation in its width; thus, the latter can be neglected when considering small variations of Δ around the working point.

Our setup is sketched in Fig. 1 and more details are given in the Supplemental Material [28]. Two laser beams derived from the same Nd:YAG source are sent to the cavity. The first one (probe beam) is used in a Pound-Drever-Hall (PDH) scheme [26] to obtain a signal proportional to the detuning. The PDH signal is used to lock the laser to the cavity resonance and to measure the oscillator displacement $x(t)$. For the measurement, the PDH signal is sent to a double-phase digital lock-in amplifier whose outputs are acquired for the reconstruction of the motion of the oscillator. The second beam (control beam, with $|G|^2 \simeq 6 \times 10^{12}$ Hz) has a frequency shift with respect to the probe, allowing a variable detuning. This second beam is used to set and control the optical spring. An additional electro-optic power modulator in its path allows us to
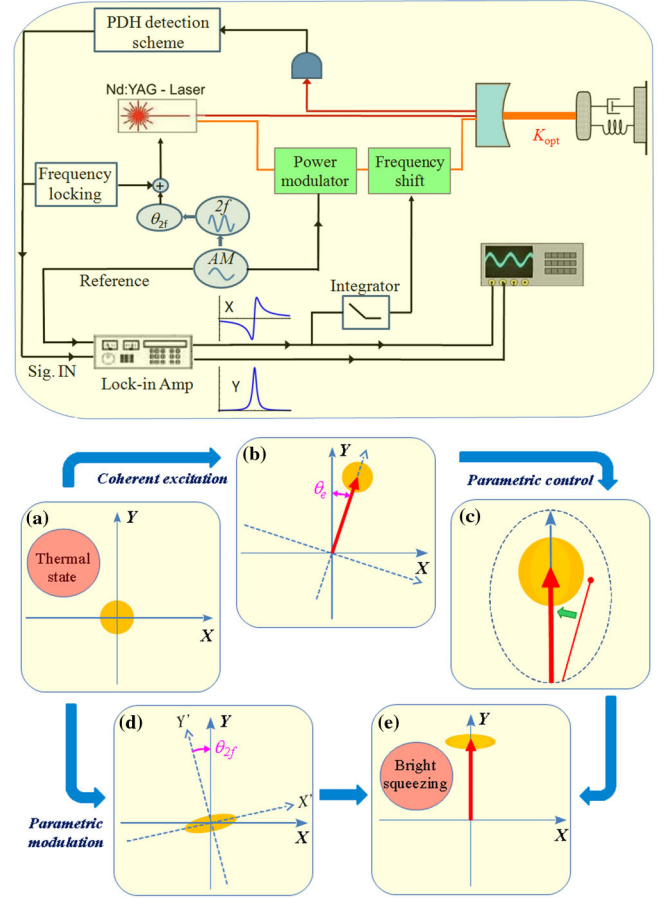

FIG. 1 (color online).   Upper panel: scheme of the experimental setup. Lower panel: sketch of the experimental techniques applied to the oscillator to obtain the bright squeezed state (e) from the thermal state (a).

produce a sinusoidal modulation [amplitude modulation (AM)] in the radiation pressure. In summary, AM drives the oscillation; slow adjustments to the control beam frequency tune the effective mechanical frequency, and phase lock the mechanical oscillator with a definite phase; the frequency modulation $2f$ produces the parametric forcing that squeezes the oscillator fluctuations.

The motion of the oscillator can be decomposed into two quadratures $X(t)$ and $Y(t)$ in an arbitrary rotating frame at frequency ω_0, according to $x(t) = X(t)\sin\omega_0 t + Y(t)\cos\omega_0 t$. For the optomechanical oscillator at temperature T, by choosing $\omega_0 = \omega_{\rm eff}$, $X(t)$ and $Y(t)$ are Gaussian, stochastic, independent variables (see the sketch in Fig. 1(a) and the experimental measurements in the left panel of Fig. 2) with null average, variance $\langle X^2\rangle = \langle Y^2\rangle = \sigma_0^2 = k_B T_{\rm eff}/m\omega_{\rm eff}^2$ where the effective temperature is $T_{\rm eff} = T\gamma_m/\gamma_{\rm eff}$, and Lorentzian spectral densities

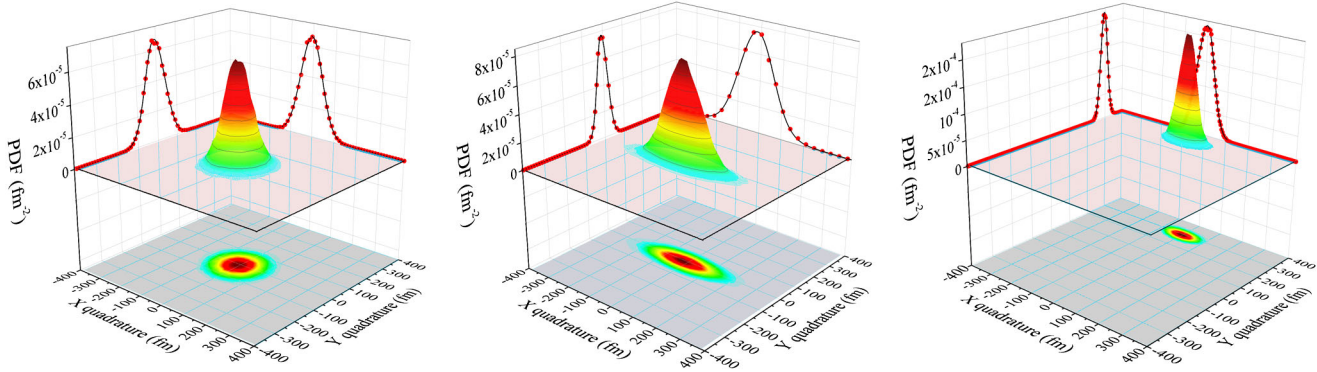$$S_X = S_Y = \sigma_0^2 \frac{\gamma_{\rm eff}}{\omega^2 + \gamma_{\rm eff}^2/4}. \tag{2}$$

FIG. 2 (color online). Phase space probability density functions (PDFs) for the three configurations named, respectively, (a),(d), and (e) in the lower panel of Fig. 1: from left to right, thermal oscillator (a) at the effective temperature $T_{\text{eff}} \simeq 15$ K ($\gamma_{\text{eff}}/2\pi = 110$ Hz); parametrically squeezed oscillator (d), with a parametric gain $g = 0.83$; squeezed oscillator with coherent excitation and frequency feedback (e), with a parametric gain $g = 5.4$.

The addition of a coherent excitation of amplitude F_e at frequency ω_e (produced, in our case, by the *AM* oscillator) shifts the distribution in the phase plane $X - Y$ by a vector rotating with angular frequency $\omega_e - \omega_0$, and, in particular, if $\omega_e = \omega_0$ by a constant vector with Cartesian components $\bar{X}_e = F_e \operatorname{Re}(\chi_{\text{eff}}(\omega_0)e^{i\theta_e})$ and $\bar{Y}_e = F_e \operatorname{Im}(\chi_{\text{eff}}(\omega_0)e^{i\theta_e})$, where θ_e is the phase between the excitation (i.e., the modulation in the intracavity power) and the detection [Fig. 1(b)]. Once $\theta_e = 0$ (this is experimentally performed by tuning the lock-in reference phase), $\bar{X}_e$ vs ($\omega_0 - \omega_{\text{eff}}$) has a dispersive shape that can be used as the error signal in a parametric feedback loop. This loop acts on the detuning of the control beam in order to correct ω_{eff} and keep it fixed at ω_0. The distribution in the phase plane displays now a nearly symmetric two-dimensional Gaussian shape centered at $\bar{Y}_e$ on the y axis. Since the frequency ω_{eff} is now locked to ω_0, it can be chosen at will and, as a consequence, we can also choose the effective resonance width γ_{eff}. Moreover, frequency instabilities (thermal drifts and slow fluctuations due to the effect of intracavity radiation) are corrected and, even more important, we have a precise phase reference of the coherent component of the oscillator motion.

Let us now analyze qualitatively the effect of the feedback on the two quadratures. If $\bar{Y}_e^2 \gg \langle (Y - \bar{Y}_e)^2 + X^2 \rangle$, we can define a time-dependent oscillator phase $\phi = \arctan X/Y \simeq X/\bar{Y}_e$, with $\langle \phi \rangle = 0$ and $\langle \phi^2 \rangle \ll 1$, and an instantaneous angular frequency $\omega_0 + \dot{\phi}$. The control loop acts by correcting $\dot{\phi}$: pictorially, it rotates the fluctuating vector (X, Y) forcing it to point to the y direction [Fig. 1(c)]. As long as $\phi \ll 1$, the feedback reaction just influences the X quadrature, while Y remains free.

We now move to the most important part of this work, the parametric squeezing. A modulation of the spring strength is just obtained by modulating the detuning of the control beam. The modulation signal is derived from a copy of the reference oscillator at ω_0, frequency doubled and phase shifted by θ_{2f}. For a free-running optomechanical

oscillator (i.e., switching off the coherent excitation and frequency feedback), the expected variances in the two quadratures X' and Y', now referred to the phase of the parametric modulation, are [12]

$$\sigma_{X'}^2 = \langle X'^2 \rangle = \frac{\sigma_0^2}{1 - g}, \qquad (3)$$

$$\sigma_{Y'}^2 = \langle Y'^2 \rangle = \frac{\sigma_0^2}{1 + g}, \qquad (4)$$

where g is the parametric gain, proportional to the depth of the parametric modulation, and we have identified Y' with the previously mentioned $\pi/2$ quadrature. The spectral densities maintain a Lorentzian shape, with width multiplied, respectively, by $(1 - g)$ and $(1 + g)$. In the $X - Y$ plane, we find an elliptical probability distribution rotated by an angle [Fig. 1(d)] that can be set to zero by tuning θ_{2f}, thus setting $X' \equiv X$ and $Y' \equiv Y$ (the experimental PDFs are shown in the central panel of Fig. 2). The variance $\sigma_{X'}^2$ clearly diverges for $g \to 1$, giving the upper limit $\sigma_{Y'}^2 < 0.5\sigma_0^2$ (the mentioned -3 dB limit reduction). Switching on the coherent modulation just shifts the ellipse center to $(0, \bar{Y}_e/(1 + g))$ [Fig. 1(e)]. The configuration in the phase plane is now equivalent to that of an optical field with bright squeezing [27]. However, we remark that the fluctuations along x still increase with g and the squeezing remains limited to 3 dB. On the other hand, by activating the parametric feedback we can depress the parametric amplification and prevent the divergence of $\langle X^2 \rangle$. As a consequence, the parametric gain g can now be increased above unity (see the right panel of Fig. 2). S_X is deformed and depends on the electronic servo loop, but the standard deviation of X is maintained close to its thermal value. The crucial issue is that the Y quadrature remains free. The fluctuations on Y maintain, indeed, a Gaussian distribution and S_Y keeps a Lorentzian shape, as shown in Fig. 3. The Y variance $\sigma_Y^2 = \langle (Y - \bar{Y}_e)^2 \rangle$ is further reduced below the -3 dB barrier, continuing to follow Eq. (4).
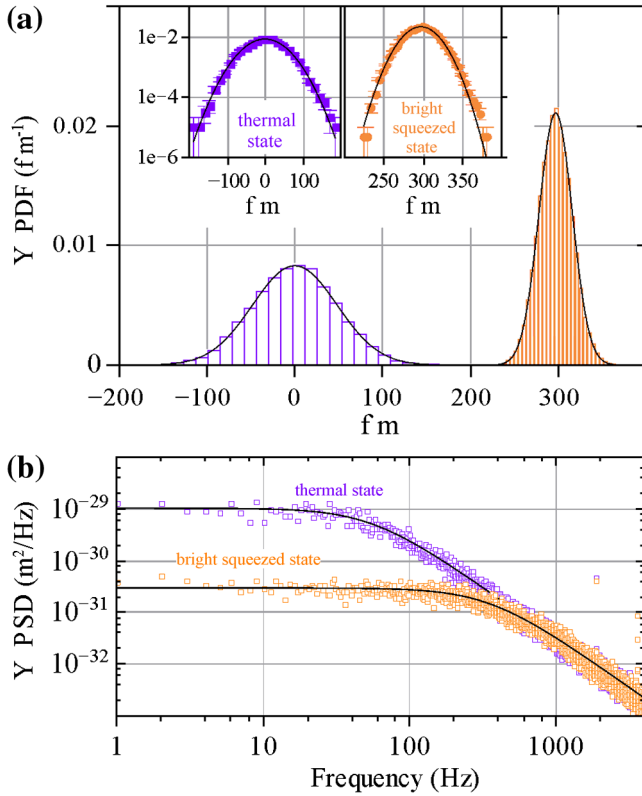
(a)

(b)

FIG. 3 (color online).   Upper panel: experimental PDFs of the Y quadrature. Violet (dark gray) histogram: thermal oscillator at $T_{\text{eff}} \simeq 15$ K. Orange (light gray) histogram: squeezed oscillator (with coherent excitation and parametric feedback) with $g = 5.9$. Solid lines show the Gaussian fitting functions. In the inset the same histograms are shown in logarithmic scale, with statistical error bars (see the Supplemental Material [28]). Lower panel: corresponding power spectral densities (PSD) of the Y quadrature, with Lorentzian fitting functions.

This is shown quantitatively in Fig. 4, where we plot the variances for the X and Y quadratures, normalized to their free-running value in the absence of parametric modulation, for two experimental configurations: the parametrically squeezed oscillator, and the system with coherent excitation and parametric feedback. In both cases, the amplitude of the coherent excitation has been adapted during the measurement in order to keep a constant value of the coherent component in the oscillator motion, i.e., a constant $\langle Y \rangle \simeq \bar{Y}_e$, compensating the parametric deamplification. This value is $\langle Y \rangle \simeq 300$ fm, i.e., at least 6 times larger than the standard deviation of the thermal distributions. This ensures, together with the stabilization of the X quadrature, that the condition $\phi \ll 1$ is satisfied. The solid lines in Fig. 4 are given by the expressions $1/(1-g)$ and $1/(1+g)$ [see Eqs. (3) and (4)] with $g = V_{2f}/V_{th}$, where V_{2f} is the amplitude of the modulation sent to the laser frequency controller, and the threshold V_{th} is obtained by fitting Eq. (4) to the variance of Y. The maximum noise reduction is -7.4 ± 0.2 dB, limited by the appearance of instabilities

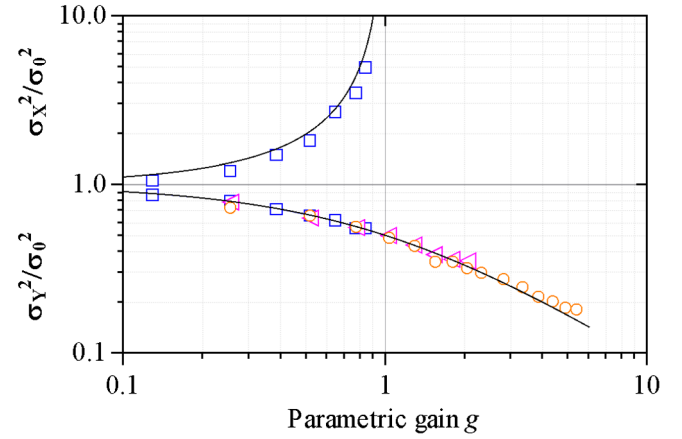FIG. 4 (color online).   Normalized measured variances of the X and Y quadratures (see text), as a function of the parametric gain g. Squares: parametric modulation without coherent excitation and parametric feedback: $\omega_{\text{eff}}/2\pi \equiv \omega_0/2\pi = 127\,400$ Hz ($\gamma_{\text{eff}}/2\pi = 160$ Hz, $T_{\text{eff}} \simeq 10$ K at $g = 0$). Circles: parametric modulation in the presence of coherent excitation and parametric feedback for $\omega_{\text{eff}}/2\pi \equiv \omega_0/2\pi = 127\,400$ Hz ($\gamma_{\text{eff}}/2\pi = 160$ Hz); triangles: $\omega_{\text{eff}}/2\pi \equiv \omega_0/2\pi = 128\,000$ Hz ($\gamma_{\text{eff}}/2\pi = 110$ Hz, $T_{\text{eff}} \simeq 15$ K at $g = 0$). Solid lines represent the theoretical curves $1/(1-g)$ and $1/(1+g)$.

in the control loop (servo bumps). An optimization of the control loop, not yet performed, would likely allow a wider working range and stronger squeezing.

In summary, we have described three original experimental results. The first one is the parametric excitation and squeezing obtained by modulation of the optical spring (similar recent proposals are described in Refs. [22,23]). The second result is the frequency locking and phase stabilization of the optomechanical oscillator using feedback on the optical spring constant (parametric feedback) and, thus, on the effective mechanical oscillation frequency. The addition of a known coherent component $\langle Y \rangle$ to the motion of the oscillator establishes a correspondence between quadrature fluctuations (on X and Y) and, respectively, phase and amplitude fluctuations. This allows us to limit the effect of phase locking to just one quadrature, at first order (quadratic fluctuations must, however, be included in a complete quantum description). As the third issue, we have demonstrated noise reduction and quadrature confinement below the -3 dB barrier. All these results can be reproduced in a large variety of optomechanical devices, including systems with reduced effective mass, which can be operated at a low occupation number [1–5]. Parametric feedback is based on a low-fidelity measurement whose backaction, influencing both quadratures, sets a limit to the achievable noise reduction. However, a weak measurement (with sensitivity well below the standard quantum limit) is sufficient to confine the (classical) motion of the Y quadrature, giving a limited detrimental effect. The ultimate achievable squeezing is just

limited by technical effects (e.g., detector noise, dynamic range of the measurement, and nonlinearity of the optomechanical oscillator). In the scheme implemented in the work of Ref. [15] the low-fidelity estimate must trace back the evolution of the classical oscillator trajectory, in order to extract information on the oscillator position surpassing its standard limit. In our case, the combination of coherent excitation and parametric feedback loop sets *a priori* the reference frame and confines the oscillator in a squeezed state. Remarkably, the low-fidelity measurement is just necessary on the (not-squeezed) X quadrature. Therefore such a measurement is not critical, thus avoiding the problem of optimal filtering [14,15]. Finally, we remark that, due to the strong obtainable squeezing, the starting system can be a moderately cooled oscillator (with an occupation number significantly above unity) that can even be reached in the bad cavity configuration exploited in this work. As a consequence, our scheme can be efficiently exploited to produce a macroscopic mechanical oscillator in a bright squeezed state, opening the way to further studies of quantum phenomena in macroscopic systems.

---

*marin@fi.infn.it

[1] A. D. O'Connell *et al.*, Nature (London) **464**, 697 (2010).
[2] J. D. Teufel, T. Donner, D. Li, J. W. Harlow, M. S. Allman, K. Cicak, A. J. Sirois, J. D. Whittaker, K. W. Lehnert, and R. W. Simmonds, Nature (London) **475**, 359 (2011).
[3] J. Chan, T. P. M. Alegre, A. H. Safavi-Naeini, J. T. Hill, A. Krause, S. Gröblacher, M. Aspelmeyer, and O. Painter Nature (London) **478**, 89 (2011).
[4] E. Verhagen, S. Deléglise, S. Weis, A. Schliesser, and T. J. Kippenberg, Nature (London) **482**, 63 (2012).
[5] T. P. Purdy, R. W. Peterson, P.-L. Yu, and C. A. Regal, New J. Phys. **14**, 115021 (2012).
[6] A. H. Safavi-Naeini, J. Chan, J. T. Hill, Thiago P. Mayer Alegre, A. Krause, and O. Painter, Phys. Rev. Lett. **108**, 033602 (2012).
[7] V. B. Braginsky, Y. I. Vorontsov, and K. S. Thorne, Science **209**, 547 (1980).
[8] A. A. Clerk, F. Marquardt, and K. Jacobs, New J. Phys. **10**, 095010 (2008).
[9] J. B. Hertzberg, T. Rocheleau, T. Ndukum, M. Savva, A. A. Clerk, and K. C. Schwab, Nat. Phys. **6**, 213 (2009).
[10] M. R. Vanner, J. Hofer, G. D. Cole, and M Aspelmeyer, Nat. Commun. **4**, 2295 (2013).
[11] D. Rugar and P. Grütter, Phys. Rev. Lett. **67**, 699 (1991).
[12] T. Briant, P. F. Cohadon, M. Pinard, and A. Heidmann, Eur. Phys. J. D **22**, 131 (2003).
[13] A. Szorkovszky, A. C. Doherty, G. I. Harris, and W. P. Bowen, Phys. Rev. Lett. **107**, 213603 (2011).
[14] A. Szorkovszky, A. C. Doherty, G. I. Harris, and W. P. Bowen, New J. Phys. **14**, 095026 (2012).
[15] A. Szorkovszky, G. A. Brawley, A. C. Doherty, and W. P. Bowen, Phys. Rev. Lett. **110**, 184301 (2013).
[16] L. D. Landau and E. M. Lifshitz, *Mechanics* (Pergamon, New York, 1976), Chap. 5, p. 80.
[17] D. F. Walls and G. J. Milburn, *Quantum Optics* (Springer-Verlag, Berlin, 2008).
[18] M. J. Collett and C. W. Gardiner, Phys. Rev. A **30**, 1386 (1984).
[19] A. Kronwald, F. Marquardt, and A. Clerk, arXiv:1307.5309.
[20] Following Ref. [15], we call localization the identification of the location in the phase plane, distinguished by the confinement in a chosen limited (squeezed) region.
[21] V. B. Braginsky, M. L. Gorodetsky, and F. Ya. Khalili, Phys. Lett. A **232**, 340 (1997); B. S. Sheard, M. B. Gray, C. M. Mow-Lowry, D. E. McClelland, and S. E. Whitcomb, Phys. Rev. A **69**, 051801(R) (2004).
[22] A. Mari and J. Eisert, Phys. Rev. Lett. **103**, 213603 (2009).
[23] A. A. Rakhubovsky and S. P. Vyatchanin, Phys. Lett. A **377**, 1317 (2013).
[24] E. Serra, A. Borrielli, F. S. Cataliotti, F. Marin, F. Marino, A. Pontin, G. A. Prodi, and M. Bonaldi, Appl. Phys. Lett. **101**, 071101 (2012).
[25] E. Serra, A. Bagolini, A. Borrielli, M. Boscardin, F. S. Cataliotti, F. Marin, F. Marino, A. Pontin, G. A. Prodi, M. Vannoni, and M. Bonaldi, J. Micromech. Microeng. **23**, 085010 (2013).
[26] R. W. P. Drever, J. L. Hall, F. V. Kowalski, J. Hough, G. M. Ford, A. J. Munley, and H. Ward, Appl. Phys. B **31**, 97 (1983).
[27] F. A. M. de Oliveira, and P. L. Knight, Phys. Rev. Lett. **61**, 830 (1988); R. Paschotta, M. Collett, P. Kürz, K. Fiedler, H. A. Bachor, and J. Mlynek, Phys. Rev. Lett. **72**, 3807 (1994).
[28] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.112.023601 for a detailed description of the experimental setup and additional information on data analysis.