

# Separation of particles from suspensions using transverse force field: a mass transport analysis

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**Abstract** We report the calculation of the efficiency for particle separation processes occurring in suspension under transverse force field in a circular duct. The calculation has been performed for a laminar fluid flow directed toward the duct vertical axis. The force field acting on the suspended particles only, is orthogonal to the duct. In the dimensionless calculations occurs a dimensionless group,  $Ca$ , which provides a tool for the dimensioning of the purification section.

## List of symbols

$a$	$\sqrt{\frac{d^2}{4} - y^2}$
$Ca$	Dimensionless group defined as $L v_f / v_{\max} d$
$C_{i0}$	Concentration at time $t = 0$
$C_{in}$	Inlet concentration
$C_{out}$	Outlet concentration
$C_t$	Concentration at time $t$
$d$	Purification duct diameter
$ds$	Differential surface elements
$F$	Force field
$i$	Imaginary unit
$k$	Generic mobility of the particle
$L$	Length of the purification duct
$\vec{n}$	Normal vector
$r$	Particle radius

$v_{av}$	Average vertical velocity
$v_h$	Horizontal velocity of the particle
$v_{\max}$	Maximum vertical velocity
$v_y$	Vertical velocity
$w$	Integration variable for the surface element
$W$	Integration domain for the particle flow trough the duct wall
$X$	Horizontal coordinate which define the free region of the particle

## Greek symbols

$\eta$	Dynamic viscosity of the fluid
$\pi$	pi Greek
$\tau_h$	Horizontal transit time
$\tau_v$	Vertical transit time
$\Phi_{in}$	Inlet particle flow
$\Phi_{out}$	Outlet particle flow
$\omega$	Integration dummy variable
$\Omega$	Domain corresponding to the fraction of the duct cross section not yet purified
$\partial\Omega_L$	Left boundary of the domain $\Omega$
$\partial\Omega_R$	Right boundary of the domain $\Omega$
$\Gamma$	Duct full cross-section domain
$\sim$	Denotes dimensionless variables

## 1 Introduction

We report the analysis of the mass transport problem of a particle suspension in a moving fluid under the influence of a homogeneous transverse force field acting on the particles only. The purpose of our investigation is the determination of the purification efficiency for a variety of processes based on the directional transfer of impurities originated by the application of magnetic field and crossed

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currents. Such methods have been demonstrated to be effective for the purification of conductive melts [1–3]. The application of these techniques to the purification of liquid indium has been recently reported [1, 4]. The purification effect has been interpreted in [4] in terms of the existence of small aggregates of contaminants, insoluble in the melt, in the nanometre range. Such heterogeneities, show lower electrical conductivity compared to the bulk of the melt. Then a current flowing through the bulk is inhomogeneous around the aggregates. The application of a magnetic field orthogonal to current flowing in the duct hence produce an apparent Lorentz force capable of moving the aggregates. It has been proved [4] that the particles hitting the wall, stick on it, reducing the contaminants concentration in the liquid. The calculation does not require any hypothesis on the nature of the driving force, hence the derived equations show general validity.

## 2 Theory

We consider a fully developed laminar flow as shown in Fig. 1. The force field acting on the particles is homogeneous and orthogonal respect to the fluid flow occurring along the circular duct axis. The particles are assumed to be spherically shaped and homogeneous in the radius. Our calculations neglects the effect of the random diffusion motion and of the gravity force. Such assumptions are justified only in the case of the application of a strong transverse force. Further we assume that the particles hitting the duct wall stick on it and disappear without modifying the domain contour. Such conditions are in agreement with what previously found in [1, 2, 4]. Under these conditions the motion of the aggregates is purely hydrodynamic and in the case of the separation generated by the presence of the transverse force, the drift velocity of the aggregates is inversely proportional to viscous drag.

We quantify the efficiency of the separation process in terms of a purification coefficient,  $K$ , defined as the ratio between the contaminants outlet and inlet flow:

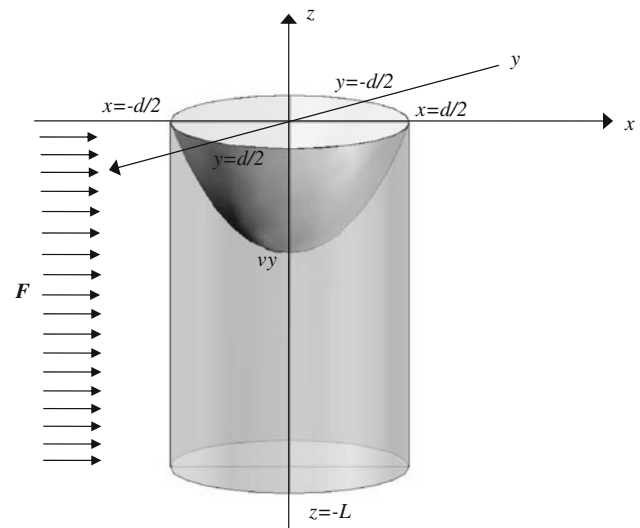
$$K = \frac{\Phi_{out}}{\Phi_{in}}. \quad (1)$$

For  $K = 0$  a total purification is achieved, while for  $K = 1$  no separation occurs. The product of the inlet concentration for the purification coefficient returns the outlet concentration.

$$C_{out} = KC_{in}, \quad (2)$$

$\Phi_{in}$  is given by:

$$\Phi_{in} = \iint_{\Gamma} cv_v dx dy. \quad (3)$$



**Fig. 1** The geometry of the purification duct. The sketch also shows the homogeneous transverse force field  $F$  acting and the fluid flow velocity profile  $v_y$

In order to calculate it we set the centre of the duct cross section as the origin of the frame of reference. Hence the duct is a circular basis cylinder described by Eq. 4:

$$x^2 + y^2 = \frac{d^2}{4}. \quad (4)$$

Assuming a laminar flow, the vertical velocity profile is given by Eq. 5 (Fig. 1):

$$v_v = \frac{4v_{max}}{d^2}(x^2 + y^2) - v_{max}. \quad (5)$$

Inserting Eq. 5 into Eq. 2 and integrating over the whole duct cross section according to Eq. 3 gives the inlet flow:

$$\Phi_{in} = -\frac{\pi d^2 v_{max} C_{in}}{8}. \quad (6)$$

The purification coefficient is a dimensionless quantity and does not depend on the scaling choice of the variables. Here it is convenient to operate with dimensionless variables. Hence we scale our system to  $d$  for the length,  $v_{max}$  for the velocity and  $C_{in}$  for the concentration. The dimensionless inlet flow is then:

$$\tilde{\Phi}_{in} = -\frac{\pi}{8}. \quad (7)$$

Determining the outlet flow requires some mathematical workout. The concentration profile at the end of the purification section has to be found. Here its derivation starts.

The velocity of the particles due to the transverse force is expressed as follows:

$$v_h = kF. \quad (8)$$

The hydrodynamic mobility for spherical particles is given by:

$$k = (6\eta\pi r)^{-1}, \tag{9}$$

hence  $v_h$  is:

$$v_h = (6\eta\pi r)^{-1}F \tag{10}$$

Equation 10 gives the transverse component of the motion of the particles. This motion results in an abrupt change of the concentration from 0 to  $C_{in}$  at a given, and for the moment unknown, point  $X$  which is a point of the boundary of the concentration profile. Equating the horizontal and vertical transit times we get Eq. 11.

$$-\frac{L}{v_{av}} = \frac{X+a}{v_h}, \tag{11}$$

It is worthwhile recalling that  $a$  depends on  $y$ , as showed in the list of symbols. So Eq. 11 is our basis for deriving the collection of the  $X$  points. In order to do that we have to determine the vertical average velocity  $v_{av}$ :

$$v_{av} = \frac{1}{X+a} \int_{-a}^x v_v d\omega, \tag{12}$$

which integration lead to Eq. 13:

$$v_{av} = \frac{4v_{max}}{d^2(X+a)} \left( \frac{X^3}{3} - a^2X - \frac{2}{3}a^3 \right), \tag{13}$$

substituting Eq. 7 in Eq. 5 and after some algebraic workout we get:

$$-\frac{Lv_h}{dv_{max}} = \frac{4}{d^3} \left( \frac{X^3}{3} - a^2X - \frac{2}{3}a^3 \right). \tag{14}$$

Here, the implicit form of the concentration profile is given. We also notice that the equation is dimensionless. The first member is a measure of the ratio between the longitudinal and the transverse transit times.

$$\tau_v = \frac{3}{2} \frac{L}{v_{max}}, \tag{15}$$

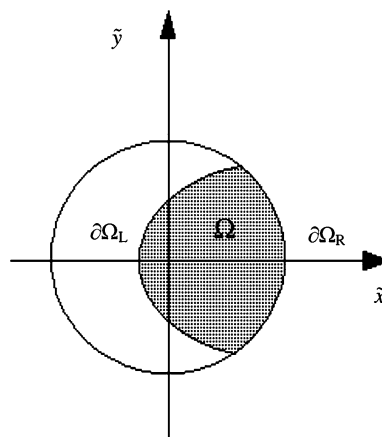
$$\tau_h = \frac{d}{v_h}. \tag{16}$$

From now on we will refer to the first member of Eq. 14 as  $Ca$ . Scaling the second member of Eq. 14 to  $d$  leads to Eq. 17:

$$\tilde{X}^3 - 3\tilde{a}^2\tilde{X} - 2\tilde{a}^3 + \frac{3}{4}Ca = 0. \tag{17}$$

The explicit form is derived solving Eq. 17 and selecting the well behaved solution.

$$\tilde{X} = \frac{1 - i\sqrt{3}\tilde{a}^2}{(-8\tilde{a}^3 + 3Ca + \sqrt{3}\sqrt{-16\tilde{a}^2Ca + 3Ca^2})^{1/3}} + \frac{1}{4}(1 + i\sqrt{3}) \times (-8\tilde{a}^3 + 3Ca + \sqrt{3}\sqrt{-16\tilde{a}^3Ca + 3Ca^2})^{1/3}. \tag{18}$$



**Fig. 2** The dashed domain  $\Omega$  is the part of the cross section where the particle concentration is  $C_{in}$ , elsewhere the concentration is 0.  $\partial\Omega_L$  and  $\partial\Omega_R$  are the left and right boundary of the domain  $\Omega$ . The dimensionless outgoing particles flow at a given cross section is the integral of the dimensionless form of Eq. 5 over  $\Omega$

Such a solution is the left boundary  $\partial\Omega_L$  (Fig. 2) of the dimensionless concentration profile. The right boundary  $\partial\Omega_R$  is given by the dimensionless duct perimeter Eq. 4:

$$\tilde{x}^2 = \tilde{a}^2. \tag{19}$$

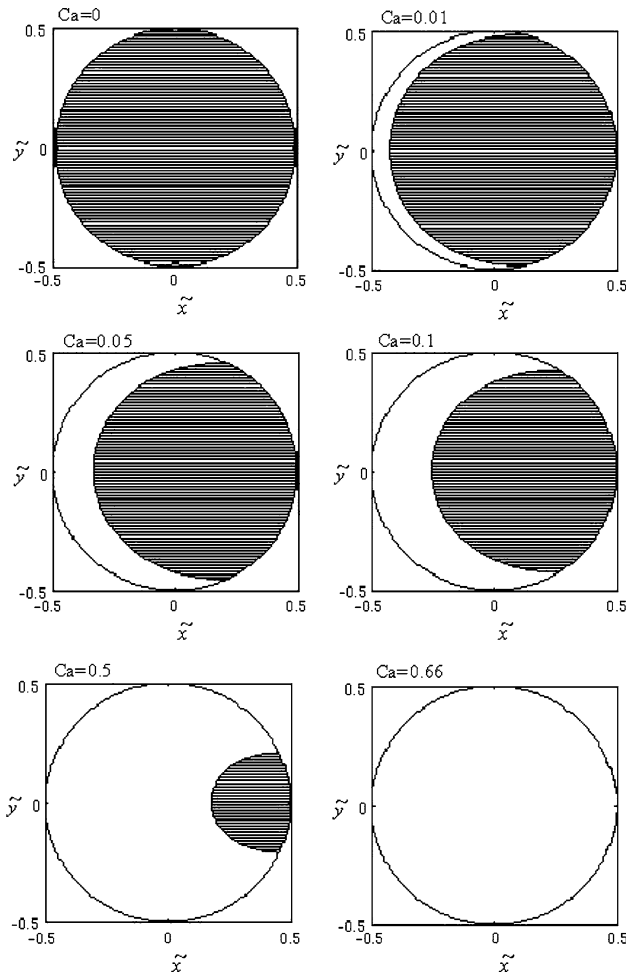
The domain  $\Omega$  enclosed between the two boundaries is the area which has not been purified yet (Fig. 2). A sketch showing the dependence of  $\Omega$  on the dimensionless group  $Ca$  is shown in Fig. 3. For  $Ca = 0$  there is no transverse force acting on the particles; hence the concentration profile of the aggregates is homogeneous trough the whole duct length. For  $Ca > 0$ , the aggregates start moving toward the wall, while at the same time, they move vertically for the forced convection. As the transverse force field increases the transverse velocity increases and the purification become more efficient as it is evident from the reduction of the cross hatched area corresponding to the different values of  $Ca$  which is reported in Fig. 3. For  $Ca = 2/3$  we have that  $K = 0$  and the purification is complete. This value of  $Ca$  depends on the scaling choice for the velocity which has been scaled against the average velocity for a fully developed parabolic flow in a circular duct [5].

The intersection between the right and the left boundaries of the domain  $\Omega$  are:

$$\tilde{X} = \sqrt[3]{\frac{3}{16}Ca}, \tag{20a}$$

$$\tilde{Y} = \pm \sqrt{\frac{1}{4} - \left(\frac{3}{16}Ca\right)^{2/3}}. \tag{20b}$$

The calculation of the outgoing flow requires the introduction of Eq. 18 in the integration limits, leading to



**Fig. 3** Cross-section of the purification duct. The cross-hatched areas are the  $\Omega$  domains for six  $Ca$  values

heavy-duty calculations. We choose to avoid such an integration. In fact, for the mass conservation we have:

$$\tilde{\Phi}_{out} = \tilde{\Phi}_{in} - \tilde{\Phi}_w. \tag{21}$$

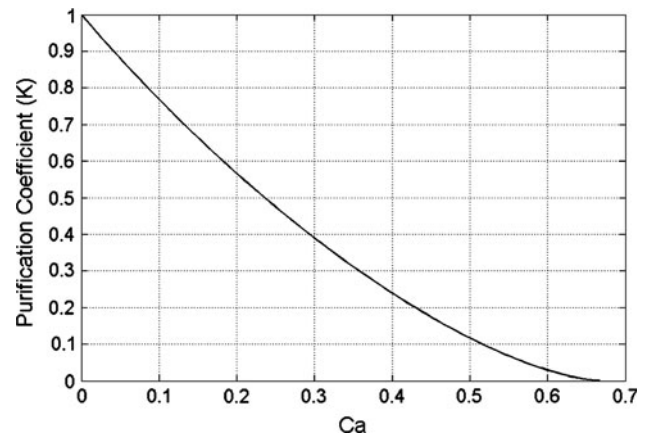
Hence the purification coefficient can be calculated as follows:

$$K = 1 - \frac{\tilde{\Phi}_w}{\tilde{\Phi}_{in}}. \tag{22}$$

The purification coefficient  $K$  is then evaluated from the particle flow across the duct wall surface. This is given by Eq. 23.

$$\tilde{\Phi}_w = \iint_w \tilde{c}\tilde{v}_h \cdot \tilde{n}ds. \tag{23}$$

With the aid of some physical consideration the integration of Eq. 23 is straightforward. According to our assumptions, we have a constant horizontal particles velocity. For the Gauss theorem the surface flow gets



**Fig. 4** Dependence of the purification coefficient  $K$  on the dimensionless group  $Ca$

contribution from the component of the surface which is orthogonal to the horizontal velocity only. Hence the integral of Eq. 23 it is equivalent to the integral of the horizontal velocity over the projection of the  $\Omega$  right boundary over the  $y-z$  plane. The resulting integral is:

$$\tilde{\Phi}_w = 2 \int_0^{Ca} \sqrt{\frac{1}{4} - \left(\frac{3}{16}w\right)^{2/3}} dw. \tag{24}$$

Solving it returns Eq. 25:

$$\tilde{\Phi}_w = \sqrt{2 - 2^{1/3}3^{2/3}Ca^{2/3}} \left( -\frac{3^{1/3}Ca^{1/3}}{8 \cdot 2^{5/6}} + \frac{3Ca}{8\sqrt{2}} \right) + \frac{1}{8} \text{ArcSin} \left[ \left(\frac{3}{2}\right)^{1/3} Ca^{1/3} \right]. \tag{25}$$

Inserting Eq. 25 into Eq. 22 finally gives the value of the purification coefficient, which is a function of  $Ca$  only. Figure 4 reports the complete dependence of  $K$  from  $Ca$ .

Assuming a 3 mm duct, with a length of 1 m, a maximum inlet flow velocity of 1 mm/s and a drift velocity of the impurities of  $10^{-3}$  mm/s, the resulting  $Ca$  value is 0.33 providing a  $K$  of 0.35. Such values are typical of the experiments described in [4].

### 3 Conclusions

We derived a formula for the determination of the purification efficiency for the particle separation processes occurring in suspension under transverse force field acting on the suspended particles only. Results have been described in terms of a dimensionless group  $Ca$  which provides a straightforward tool for the interpretation of the results and the dimensioning the purification sections. The main finding of our work is a criterion for the design of the devices for the purification of semiconductor melt, such

as indium or silicon, based on the directional transfer of the impurities originated by the application of magnetic field and crossed currents described in [1–3].

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