FRACTIONAL LOWER-ORDER STATISTICS FOR EFFICIENT ADAPTIVE TEMPORAL AND SPATIAL METHODS IN NON-GAUSSIAN ENVIRONMENT

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Abstract

This paper addresses the problem of blind equalization for digital communications using constant modulus signals in the presence of heavy-tailed additive channel noise. We compare the performance of a temporal filter antenna system with the ones obtained with an array of sensors used at the receiver to copy the information sequence. First, we demonstrate the negative effects of channel noise to the original CMA cost function in terms of reliability and convergence. Then, we introduce a new CMA criterion for both temporal and spatial systems based on the fractional lower-order statistics (FLOS) of the received data. We perform an analytical study of the properties of the new cost function and we illustrate its convergence behavior through computer simulations.

Introduction

Adaptive channel equalization is an effective tool for the antenna receiver for estimating the information sequence in severe interference backgrounds. As a result, the problem of linear channel distortion or multipath suppression has been the focus of considerable research in the signal processing and communications communities.

The Constant Modulus Algorithm (CMA) was studied by Treichler and Larimore [7] who analyzed its performance in terms of capture and lock behavior. Initial CMA studies considered only the temporal diversity at the receiver. The development of advanced division access techniques has made the concept of spatial diversity worth-pursuing. As a consequence, directional array antenna beamformers have taken the place of omnidirectional antennas. In this context, Gooch and Lundel introduced the so-called constant modulus array, which exploits the constant modulus properties of the communication signal of interest to steer a beam in the direction of the information sequence while placing nulls in the directions of interferences [1].

Most of the theoretical work on blind equalizers based on the CM criterion typically exploit higher (than second) order statistics or second-order cyclostationary statistics of the channel output signal. For this reason these methods have focused on the case where the channel noise is assumed to follow the Gaussian model. The Gaussian assumption is frequently motivated because it often leads to mathematically tractable solutions. However, algorithms designed under the Gaussian assumption exhibit various degrees of performance degradation, depending on the non-Gaussian nature of the environment. For some applications, such degradation is so strong to compromise irreparably the information transmitted over such impulsive channels. In these cases, there is a point break after which is not possible to recover the desired signal. As we will show, for the Constant Modulus Algorithm class, the presence of heavy-tail nature noise, even though not much impulsive, (i.e. realistic near Gaussian values), lead to a total lost of performance if used with methods optima under the Gaussianity assumption.

Indeed, experimental results have been reported where electromagnetic noise in urban mobile-radio channels is heavy-tailed in nature and cannot be modeled by means of Gaussian or other exponential-tailed distributions [4, 5]. In addition, impulsive channels appear in telephone lines, underwater acoustic communications (ice-cracks), atmospheric environments (thunderstorms), and mobile communications.

Hence, there is a need to use more general and realistic non-Gaussian models and design efficient equalization techniques that take into account the possible heavy-tail nature of the data, and simultaneously work well in good Gaussian channels. Our work is devoted to the development of a novel constant modulus method which makes use of temporal filter diversity and array signal processing system for robust performance in the presence of interference/noise environments that can be modeled according to the alpha-stable law. We compare the performance of a temporal FIR filter and an antenna array model with complex coefficients which both use a fractional lower-order statistics on computing the adaptation iterative law in presence of impulsive noise and Gaussian channels.

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Symmetric Alpha-Stable Statistics for Heavy-Tailed Noise

Man-made as well as natural physical processes can generate interferences containing noise components that are impulsive in nature. In modeling this type of signals the *symmetric alpha-stable* ($S\alpha S$) distribution provides an attractive theoretical tool. It was proven that under broad conditions, a general class of heavy-tailed noise follows the stable law [6].

The $S\alpha S$ class of distributions is best defined by its characteristic function:

$$\varphi(\omega) = \exp(\jmath\delta\omega - \gamma|\omega|^{\alpha}), \quad (1)$$

where α is the *characteristic exponent* restricted to the values $0 < \alpha \leq 2$, δ ($-\infty < \delta < \infty$) is the *location parameter*, and γ ($\gamma > 0$) is the *dispersion* of the distribution. The dispersion parameter γ determines the spread of the distribution around its location parameter δ, much in the same way that the variance of the Gaussian distribution determines the spread around the mean. The characteristic exponent α is the most important parameter of the $S\alpha S$ distribution and it determines the shape of the distribution. The smaller the characteristic exponent α is, the heavier the tails of the alpha-stable density. It is this heavy-tail characteristic that makes the alpha-stable densities appropriate for modeling noise that may be impulsive in nature. We should also note that the stable distribution corresponding to $\alpha = 2$ coincides with the Gaussian density.

The appeal of $S\alpha S$ distributions as statistical models derives from some important properties. They: (i) naturally arise as limiting processes via the Generalized Central Limit Theorem; (ii) possess the stability property and share many features with the Gaussian density such as unimodality, symmetry with respect to the location parameter, bell-shape; (iii) all but Gaussian distributions possess finite moments of order p only when p is strictly less than α: $E|X|^p < \infty$ for $p < \alpha$. Strictly related with this last property, the main concept to point out for this class of distribution is the introduction of fractional-lower order moments as the only possible tool capable to digitally process the analyzing data.

CMA class: temporal and spatial analysis

The constant modulus family of blind equalizers is based on a cost function that assigns a penalty to deviations in the modulus of the controller's complex output signal. The cost function is given by

$$J_{pq}^{CM} = E[\,|\,||\,y(n)\,||^{p} - \delta\,|^{q}\,] \quad (2)$$

where $E[\cdot]$ denotes statistical expectation, $y(n)$ is the controller output at time n, p and q are positive integers, and δ is a constant greater than zero related to the constant modulus signal [2].

The most famous member of this family is the Constant Modulus Algorithm (CMA) for which both parameters p and q are equal to two. Once pointed out the cost function J, it is possible to approach the specific problematic under different point of view. Briefly speaking, a first temporal analysis was performed by Treichler *et al.* [7], where the channel diversity referred mainly to the temporal domain and was produced by sampling the received analog signal in time. But spatial diversity has been used by employing an array of sensors at the receiver [1], showing the benefit of a spatial sampling of the received signal.

Assuming the two different signal representation, it is possible to describe the two distinct analysis. Let start with the temporal approach where a N FIR filter whose complex coefficients $\mathbf{w}(n)$ are iteratively adjusted. The vector of data in the delay line of the filter is $\mathbf{x}(n) = [x(n), \ldots, x(n-N+1)]^T$, which give a complex filter output as

$$y(n) = \mathbf{w}^H(n)\mathbf{x}(n) \quad (3)$$

where the signal $x(n)$ is the received signal at the receiver at time n resulting of the channel distortion and multipath effects. Consider now an array of N equispaced sensors, which receive signals generated by Q sources located at $\vartheta_1, \ldots, \vartheta_Q$. Assuming the signal bandwidth to be narrow as compared to the inverse of the travel time across the array, it follows that, by using a complex envelop representation, the array output can be expressed as:

$$\mathbf{x}(t) = \mathbf{A}(\vartheta)\mathbf{s}(t) + \mathbf{n}(t), \quad (4)$$

where $\mathbf{x}(t) = [x_1(t), x_2(t), \cdots, x_N(t)]^T$ is the array output vector; $\mathbf{s}(t) = [s_1(t), s_2(t), \cdots, s_Q(t)]^T$ is the signal vector received by the reference sensor of the array; $\mathbf{A}(\vartheta)$ is the $N \times Q$ *steering matrix*, whose rth column vector $\mathbf{a}(\vartheta_r)$ is $[1, e^{-\jmath 2\pi(d/\lambda)sin\vartheta_r}, \ldots, e^{-\jmath(N-1)2\pi(d/\lambda)sin\vartheta_r}]^T$ and $\mathbf{n}(t) = [n_1(t), \ldots, n_N(t)]^T$ is the noise vector. The input-output relation at the controller is the same as (3). The CMA attempts to minimize the cost function shown in (2) by following the path of steepest descent.

Under the hypothesis of sinusoid incident signals for the temporal analysis, assuming $\Gamma(\omega)$ the matrix of column vectors containing the shift carriers of the corresponding tone, it is possible to find a mathematical form for the complex gains of the adaptive filter for each frequencies, for both spatial and temporal dimensionality, which is [7, 1]

$$\bar{\mathbf{v}}(n+1) = \begin{cases} E\left[\Gamma^H(\omega)\mathbf{w}(n+1)\right] & \text{temporal analysis} \\ E\left[\Lambda^H(\vartheta)\mathbf{w}(n+1)\right] & \text{spatial analysis} \end{cases} \quad (5)$$

where the bar means the expected value. In the case of orthogonal signals, it is possible to find a closed form:

$$\bar{\mathbf{v}}(n+1) = \left[\mathbf{I} - \mu \begin{pmatrix} \bar{q}_{11} & 0 \\ 0 & \bar{q}_{22} \end{pmatrix}\right] \bar{\mathbf{v}}(n) \quad (6)$$

where, despite of a multiplication factor of N for the temporal analysis,

$$\bar{q}_{11} = |A_1|^2\left(|\bar{v}_1(n)|^2 + 2|\bar{v}_2(n)|^2\right) \qquad (7)$$

$$\bar{q}_{22} = |A_2|^2\left(2|\bar{v}_1(n)|^2 + |\bar{v}_2(n)|^2\right) \qquad (8)$$

where we supposed a FIR filter and an antenna array system with $N = 2$ taps.

The FLOS-CM Time-Space Algorithms

The main characteristic associated with the classical CMAs is that they involve fourth-order moments of the signal. In the presence of heavy-tailed noise, the use of second- or higher-order statistics in effect amplifies the noise. For such cases, we propose a new cost function that considers information of constant modulus regarding the communication signals and uses FLOS to mitigate the impulsive noise component, as we pointed out describing the $S\alpha S$distributions. The new cost function has the expression

$$J_{p,q}^{FLOS-CMA} = E\left[\left|\, \| y(n) \|^{(p-1)} y(n) - \delta y(n) \right|^q\right] \qquad (9)$$

where $y(n)$ is the system output and δ is a real correlated to the constant signal modulus (actually, is the constant amplitude to the pth-1 power). The pair $(p,\ q)$ takes values, possibly fractional, strictly between 0 and α, where α is the characteristic exponent of the alpha-stable distribution that best describes the statistics of the noise vector $\mathbf{n}(t)$. Furthermore, the product $p \cdot q$ must be less than α. It is important to point out the reasons why we choose to define such fractional cost function. Basically, our tools are a transmitted signal which has constant modulus and an additive noise which can be impulsive in nature: the gradient function takes into account both properties, since at the convergence, when the amplitude should converge to the constant value, the argument of the function tends to zero. Besides, in order to compensate the heavy tailed noise, a fractional lower-order moment in the iterative law has been introduced.

First, we develop the recursive update formula for the weights in the temporal dimension using the principle of steepest descent to minimize the proposed cost function in (9). For the iterative law of the complex weights vector $\mathbf{w}(n)$, see as reference the paper [3]. Following the same steps as for the CMA, starting with the equation (5) in order to obtain (6), it is possible to write an analogous formula for the FLOS methods. In this case the equations

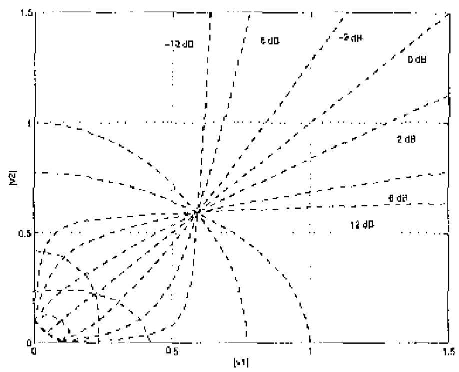


Figure 1: Boundary curves with SIR trajectories defining the lock and capture zones for the FLOS-CMA.

(7) and (8) become

$$\bar{q}_{11} = |A_1|^2\Big(p\bar{P}^{(p-2)}(|\bar{v}_1(n)|^2 + p|\bar{v}_2(n)|^2) + \qquad (10)$$

$$-\ (1+p)\bar{P}^{(\frac{p-1}{2}-1)}\left(|\bar{v}_1(n)|^2 + (\frac{p+1}{2})|\bar{v}_2(n)|^2\right)+1\Big)$$

$$\bar{q}_{22} = |A_2|^2\Big(p\bar{P}^{(p-2)}(p|\bar{v}_1(n)|^2 + |\bar{v}_2(n)|^2) + \qquad (11)$$

$$-\ (1+p)\bar{P}^{(\frac{p-1}{2}-1)}\left((\frac{p+1}{2})|\bar{v}_1(n)|^2 + |\bar{v}_2(n)|^2\right)+1\Big)$$

where P is the total output power: $P = |v_1(n)|^2 + |v_2(n)|^2$.

Naturally, the convergence behavior of the FLOS-CM algorithms is an important issue when more that one constant modulus signals are present. It is possible to address the lock and capture properties for the array antenna system of the new criterion by finding the expression of the curve boundaries that divide the two zones in positive lock zone and positive capture zone. Figure 1 shows the boundary curves for the FLOS-CMA and the trajectory curves for different values of Signal to Interference Ratio. Comparing it with the analogous for Gaussian model in [7] it is possible to see a very close relation.

Similar studies can be carried out for the temporal analysis in case of sinusoid signals in terms of the output vector $\bar{\mathbf{v}}(n)$. This approach permit to opportunely initialize the weights vector in order to have a priori knowledge of the algorithm in terms of lock (desired signal) and capture (interference signal) behavior. One difference between the two parallel analysis is the capability to discriminate this two lock and capture zones for the temporal system only if the incoming signals are tones. However, in general the signals involved during a radio communication transmission are not sinusoids, so is not possible to find an equivalent expression for the vector $\bar{\mathbf{v}}(n)$ as (5) for the temporal analysis. Adversely, in the case of antennas array it is always possible to find an expression as described in (5) due to the presence of another information term carried out with the signals represented by the angular spatial arrival direction. This is an important point which can explain the diversity in the bit error rate curves shown in the simulation result section.

Experimental Results

In this section, we test and validate the new FLOS-CM temporal-spatial adaptive algorithms and compare their performance with that of the conventional CM algorithms in a noisy environment. An interest problem for the definition of the new cost function expressed by the equation (9), is the choice of the couple (p, q). The way to choose the values (p, q) is obliged to satisfy the relation: $p \cdot q < \alpha$ due to the characteristic of the impulsive noise. Once followed this condition, a set of possible numbers can be chosen. A criterion which can be used to make the decision is to chose the (p, q) couple which gives us a cost function J with better performance. Taking as example the case of $\alpha = 1.5$, the possible ensemble of values is $\{(1.1, 1.1), (1.1, 1.2), (1.1, 1.3), (1.2, 1.2), (1.2, 1.1), (1.3, 1.1)\}$, considering the first decimal approximation only. In Figure 2 it is shown a zoom of the smallest curve for $(p, q) = (1.1, 1.3)$ expanses in the bigger scale corresponding to the values $(p, q) = (1.3, 1.1)$. It is so shown, that the couple $(p, q) = (1.1, 1.3)$ provide better performance for the adaptive algorithm because it gives more smooth cost function, with less curvature and more linear trend. All the other possible values have a bigger slope than the $(p, q) = (1.1, 1.3)$ case anyway, explaining why this choice is the best one. Basically, the best choice is the ones which reaches a cost function as regular as possible. This property can help to lead in a stable solution, in which once we obtain the desired response, we most probably will stay near, with very small oscillation movements.

Let consider two independent transmitted signals impinging on the array from directions $\theta = [30°, -40°]$. The desired signal is supposed to be the first signal coming from $30°$ deg direction. The second signal is a delayed version of the desired one. A power control system is supposed working at the receiver. The number of snapshots available to the array is $M = 100,000$, $M = 500,000$ or $M = 1,000,000$ depending on the expected BER and we performed 10 Montecarlo runs. We plot the Bit Error Rate versus the SNR of the two different systems for both the original CMA and the proposed FLOS-CMA algorithms: the ones with temporal analysis and the second with spatial processing (cf. Figure 3). In Figure 3 (a) the noise component is modeled as an alpha-stable process with $\alpha = 1.85$, i.e., the noise is fairly close to Gaussian. Actually, this value has been measured for some particular communication transmissions. Figure 3 (b) is the bit error rate of a Gaussian channel.

Figure 3 demonstrates that occurrences of noise outliers

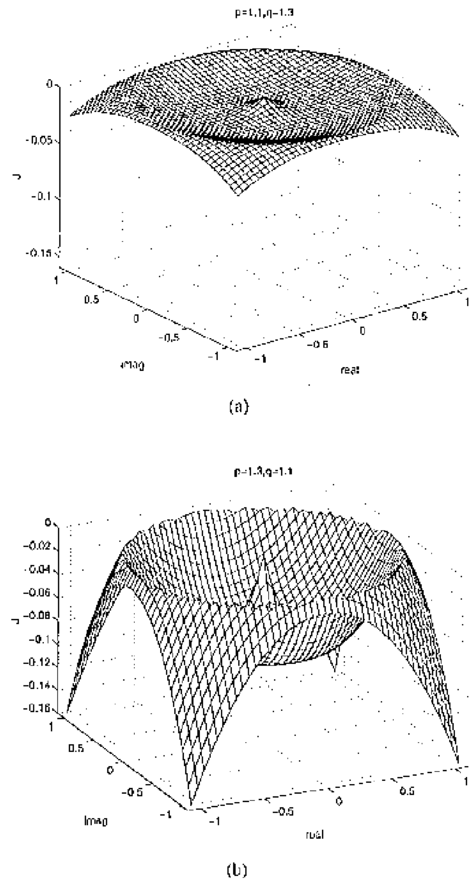
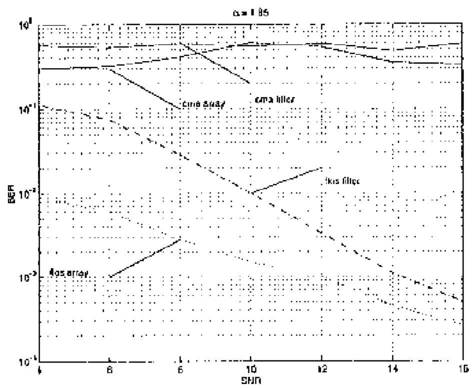


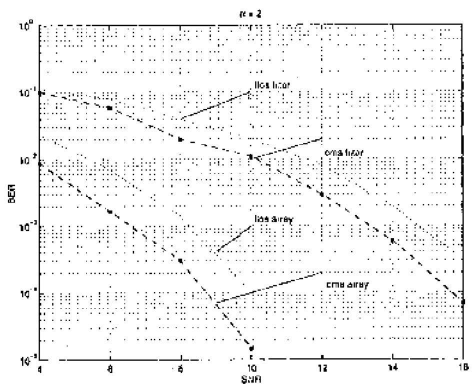
Figure 2: FLOS-CMA Cost Function, (p,q) = (1.1,1.3) (a) (p,q) = (1.3,1.1) (b).

during the adaptation, have an adverse affect to the learning curve of the original CMA method. In fact, even a relative low impulsiveness of $\alpha = 1.85$ has a disruptive effect on the convergence behavior of the classical CM algorithms, both in the temporal and spatial base. Briefly speaking, as an impulse occurs, the classical CM algorithms loose their convergence properties and they become not able anymore to recover the transmitted information sequence. On the other hand, the proposed FLOS-CMA cost function can suppress the noise components and results in a much deeper bit error rate curve. The gain that we can reach between the array model and the FIR system can be even of 10 dB for low SNR $(4, 6, 8)$ and becomes smoother as the SNR increases (few dB).

However, in presence of Gaussian noise channels, the CM algorithms outperform the FLOS methods ranging from values between 2 and 8 dB. As we can see from the curves, the antenna array system at the receiver shows a consistent improvement in terms of number of bit correctly received due to a further diversity that such model



(a)



(b)

Figure 3: Bit error rate curves for the filter and the array receiver system. (a): $\alpha = 1.85$, (b): $\alpha = 2$.

implement. Besides, as pointed out in the previous section, the impossibility to define the two lock and capture zones in the case of the N dimensionality FIR filter for all radio communications signals strongly bounds the performance results of the proposed FLOS-CM algorithm in the case of temporal analysis.

Conclusions

We proposed a new method for blind equalization of communication signals using a constant modulus criterion based on fractional lower-order statistics. The introduced FLOS-CM Algorithms exploit the constant modulus property of the signal of interest and uses the heavy-tailed noise suppression capabilities of FLOS to recover the information sequence in a temporal and spatial point of view. The capability of an array system to steer a beam in the direction of the signal while suppressing interference and noise is resulting in a better performance in terms of bit error rate. Besides, the directional of arrival information allows the algorithm to divide the signal space in the capture and lock zone. Such property, which is not always possible with a FIR omnidirectional filter, provides a further improvement in the BER curves.

The main advantage of the proposed method is its robustness in the presence of various noise environments. Truly, by changing the parameters p and q in the criterion in (9) we obtain a class of FLOS-based CM Algorithms which provide considerable flexibility that can be useful for optimization purposes in the presence of non stationary noise environments. Besides, we showed a similar behavior of CMA and FLOS algorithms in presence of Gaussian channels.

The proposed method developed using FLOS has approximately the same computational complexity as the existing CMA methods. The additional computational load is due to the need for calculating a fractional power ($p < 2$) rather than a square power. The technique can be used in commercial communication applications in which impulsive channels tend to produce large-amplitude interferences and sharp noise spikes more frequently than what is expected from Gaussian channels.

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