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## The Unconscious as an Ultrametric Set

To my *Icarus* (Jazz)

*This paper illustrates the recontextualization of a formal theory from the domain of cognitive science to psychoanalysis. Starting from an approach driven by information theory, a description of mental space in terms of a peculiar topographical structure is derived. This “ultrametric” structure can be seen to fit the constraints of primary process thinking, as presented by Matte Blanco in his essays on bi-logic. The author’s reformulation of the distinction between primary and secondary processes in topological terms leads to a speculative proposal of “quasi-symbols” as possible objects of mental life. The paper also seeks to capture some aspects of the crucial interplay between the phenomenological experience of personal change and the proposed conceptual advance in theorization.*

The theme of my writing is an observation, and the search for meaning that came after it. As I am so used to scientific English, I feel I should apologize for the personal style of this text, which may even be too intimate at some points. Let us go step by step, however.

My observation concerns a homology, an identity in the deep structure of two ways of thinking about the mind, which may appear very distant at a first glance.

My field is cognitive neuroscience; recently, I have been working on a method aimed at investigating “What Remains of Memories” (Lauro-Grotto, Borgo, Piccini, and Treves 1997)—what remains of memories when age or illness deprives us of our tools to remember. Don’t be misled by my opening. We are dealing here with a quantitative method based on Shannon information theory (Shannon 1948). This type of analysis is commonly used in decoding single cell recordings. When an animal is trained to identify a set of visual stimuli, the activity

traces of the cells involved in the recognition process can be interpreted in terms of the degree of similarity among pairs of recordings in such a way that one can get an idea of the structure of the set of neural representations that code for the stimuli. I am not fond of laboratory animal studies, but I find that the method is very intriguing as it allows us to access some very deep characteristics of mental images. Thus, I devoted myself to the development of a version of the task that can be employed with behavioral data, in the classic format of a paper and pencil neuropsychological test.

The stimuli I present are pictures of famous people from several decades, and they must be classified in a grid of semantic dimensions, such as nationality and profession. When the subject fails to recognize the picture, he or she is nevertheless required to provide the most plausible classification. By analyzing the confusion matrix—that is, the matrix presenting both correct and incorrect classifications—it is possible to derive a quantitative index providing an estimate of the extent of the relationships of similarity that the subject is able to perceive in the set of stimuli. This is an indirect estimate of the richness of the conceptual structure underlying the classification, and it is obtained from the way in which it is reflected in the experimental data. The index we derive is called the *metric content* of the mental representations; metric content manifests itself in the actual behavior of the subject as recorded in the confusion matrix.<sup>1</sup>

My results are mainly based on work with patients suffering from neurodegenerative diseases such as Alzheimer's dementia, but the same method has been applied to track the evolution of semantic representations through different phases of life, from youth to senescence. Along the entire life span, semantic memory—the mental lexicon in which our knowledge of the world is stored and organized—is slowly enriched by new incoming information, while the network of links and associations among concepts gets more and more entangled. Not surprisingly, there is a simultaneous increase of information and of the metric content. It may be less trivial to find that, in dementia, when neural resources do progressively and inexorably shrink, the loss of information content is paralleled by a marked increase in the metric content of the representations. It is as though the

semantic network tends to become increasingly dominated by similarities and less sensitive to differences. The mind, I might propose, having less space at its disposal in order to note down its knowledge, say, about dogs and cats, resorts to keeping a common image for the two concepts and gets rid of all the distinctive features that would keep the two concepts apart. A new “dog-cat” super-class is thus created, and in this class all the individual attributes are either shared or get lost.

The tendency to generate clusters of concepts that in turn become more and more homogeneous in their inner structure can be described mathematically as a transition from a metric space to an *ultrametric* one. Both metricity and ultrametricity are topological properties, in the sense that they define “who is close to whom” in a spatial representation of a given set.

In a metric semantic space, a concept such as “cat” would be closer to the concept “dog” than to either “butterfly” or “car” (maybe with an unexpected discontinuity if we think of the *gatto delle nevi* or “snow cat,” an Italian expression meaning a machine to make ski trails, just to remind ourselves of how creative language is!). If a semantic space has an ultrametric structure, on the contrary, all the items are organized in a hierarchical sequence of clusters of increasing generality, while inside each cluster all the concepts are equidistant from each other, as is the case for the vertices of an equilateral triangle. In an ultrametric set, if two concepts are found to belong to the same cluster they become *ipso facto* indistinguishable; to some extent, they are interchangeable with each other and with the whole cluster as well. This strange state of affairs has the remarkable consequence that proper parts and the whole of the set are to be considered as equivalent to each other.

“Remarkable consequence,” I have just written, which is vaguely surprising to me as for many years I simply ignored it. This was until I came across a quotation from Matte Blanco in a book by Carli and Paniccia called *Analysis of the Request* (2003). In italics, on the penultimate line on page 38, I found the expression “sets of symmetry” (*sacche di simmetria*). My former heart of a theoretical physicist gave a sudden start. Indeed, there was a time when I too shared the belief that “symmetry” and “symmetry breaking” would be powerful words able to reduce to order the state of the world.

I sought out the texts by Matte Blanco, *The Unconscious as Infinite Sets* (1975), first of all, and a few months later the original version of *Thinking, Feeling, and Being* (1988). These books contain a brilliant reformulation of the epistemological foundations of classical psychoanalytic theory from Freud to Melanie Klein. Even more fascinating to me, they offer a taxonomy of mental life, such as it manifests itself both in clinical practice and in the individual experience of the author.<sup>2</sup>

According to Matte Blanco, two opposite and apparently irreducible and contradictory ways of being do coexist in mental life: the asymmetric or heterogeneous mode, following the rules of classical reasoning, and the symmetric or homogeneous mode, which can be described as a system of logic operating on the basis of two fundamental principles.

The first principle, known as the *generalization principle*, states that:

The system Ucs. treats an individual thing (person, object, concept) as if it were a member or element of a set or class which contains other members; it treats this class as a subclass of a more general class, and this more general class as a subclass or subset of a more general class, and so on. (Matte Blanco 1975, 38)

The second principle, known as the *symmetry principle*, holds: “The system Ucs. treats the converse of any relation as identical with the relation. In other words, it treats asymmetrical relations as if they were symmetrical” (38).

Matte Blanco shows us how to derive from these two principles alone the whole set of features that, according to the first Freudian topography, characterize unconscious mental activity—i.e., the absence of mutual exclusion and negation, displacement, condensation, atemporality, and the substitution of the inner for the external world. Every mental phenomenon, from abstract thinking to emotion, seems to be marked by the co-presence in various proportions of the symmetric and the asymmetric modes: the mind is biological in structure.

Now, here comes my observation: the structural unconscious, in the way it is reformulated by Matte Blanco, the symmetric mode—all this is homologous to an ultrametric structure. The

generalization principle reflects the hierarchical arrangement in which all the stimuli (or concepts) are perceived as belonging to classes, and the classes are clustered into super-classes of increasing generality. Finally, a single omni-comprehensive class is generated.

The symmetry principle reflects the property of the ultrametric organization according to which all the elements of a given class are equidistant from each other, and all of them are placed at the same distance from any other element of any different class. This formulation is appropriately captured in a corollary of the second principle, stating that “when the principle of symmetry is applied there can be no relations of contiguity between the parts of a whole” (Matte Blanco 1975, 40).

As I have already pointed out, distance in a semantic space can be read out as a measure of the difference among concepts. In an ultrametric set, all the concepts are at the same distance from each other, all the possible “positions” are to be considered as equivalent, and there is no way to distinguish one position from the other.

Let us try to refine our intuition of this alien situation by considering a single ultrametric class comprising three concepts: within ordinary Euclidean space we can imagine them to be placed at the vertices of an equilateral triangle. An ultrametric structure could be considered to be isomorphic to an ordinary metric structure of two-dimensional space (the equilateral triangle) only if we were to assume that every vertex of this triangle is found at exactly the same distance from every other point of the ordinary space. Maybe this is too much even for our intuition!

However, we know that something similar can actually be experienced in finite space when we look at a very distant three-dimensional structure and we perceive it as though it were a single point. Symmetrization of relationships can therefore be described as a transition from a metric to an ultrametric conceptual organization. Deployment, as Matte Blanco terms the inverse phenomenon, amounts to the opposite shift, from an ultrametric to a metric structure: in the course of deployment, therefore, there is an apparent increase of the available information and contradictory aspects come to be evident as well.

For a long time I have been wondering what this might mean. Is what I see in my neurological patients perhaps a “reversion to the unconscious,” and does this structural reorganization open a pathway that runs from memory to oblivion? In other words, is someone daring enough to take seriously the notion that unconscious mental activity might correspond to a conceptualization in ultrametric space? To this day, I am at a loss to understand why such an elegant way of thinking about the human mind as bi-logic is not a more commonly shared heritage among psychologists, to say nothing about philosophers of mind.

At present, the cognitive sciences seem to be devoted to perpetuating an endless and unproductive conflict between the symbolic paradigm and the subsymbolic neurocomputational paradigm. The human mind as a pure symbol processor is a strictly computational mind, an encapsulated mind to some extent, while for many neurophysiologists the mind is basically a pattern associator, mainly driven by external data. The sole choice appears to be between, on the one hand, the complete absence of friction of pure symbols, which can be manipulated effortlessly and whose purchase on reality can be ignored by syntax, and, on the other hand, fixed-point dynamics, which arrive at generalizations by extracting common features. In fixed-point dynamics, similar inputs always produce more or less the same output. In either case, there is no unconscious anywhere, if you do not restrict unconscious activities to subliminal phenomena or background noise in the brain, out of the spotlight of consciousness itself.

I find this state of my discipline to be quite depressing, and I often share the mood that J. A. Fodor so poignantly expresses with the words of Eeyore: “Still snowing [*Nevica ancora*]. And it’s freezing cold. At least we haven’t had an earthquake recently” (2001, 9).<sup>3</sup> However, if neither the symbolic nor the subsymbolic paradigm provides a satisfactory solution, perhaps in the end it might not be such a bad thing if the mind were a bit of a stranger to itself. As in any relationship, perceiving “otherness” is the ineluctable condition for starting the quest . . .

Even harder for me to cope with, moreover, is the “fundamental antinomy of human beings and the world” that, according to Matte Blanco (1988), the bi-logical structure of the mind

would imply.<sup>4</sup> Why is this? Why should the mind appear to us as something so irreducibly different from any other natural phenomenon?

From the dawn of life, experience engraves and models our inner landscapes, and perhaps what I have called semantic memory is no more than such a landscape. Experience unfolds in time, and time revolves everything, changing opposites into one other. Emotional experience, relational experience is contradictory: it creates tensions in any structure devoted to its reception. Individual memory of past events, episodic and autobiographical in nature, can be depicted as a thread that is entangled in nodes and webs, a one-dimensional line that, due to bending and folding, becomes like a shimmering surface by which we catch a glimpse of our precarious identity and of our knowledge of the world.

Or maybe this is the picture I had in my mind while I was inquiring about the meaning of my metric content estimates, by aligning a visual association with the formalized concept. This numerical index would therefore remain in effect for the outcome of the folding process, the torsion the thread undergoes, or the extent of the packing on a hypothetical bundle of experiences . . .

Prompted by this picture, I began to consider the possibility that ultrametricity could be used as a means to generate mental representations that hold in their inner structure all the contradictory aspects of experience and present a smooth surface allowing a kind of “easy handling” by mental processes.

I would call these hypothetical items of mental life *quasi-symbols*—as this is what they are, at least of my mental life.<sup>5</sup> In them the contradictory content of experience is condensed, so that the past remains pliable enough to allow for memory and narration, and the future retains enough freedom for imagination to wander.

Such is dreaming, such is the creative mental image, and such the intimate nature of words. If I say *mother*, my mind rings out with the ultimate good and bad of presence and absence . . . In the crucible of emotional life our experience grows dense in the form of our personal inner language, in the existential lexicon that we may perhaps have learned to interpret through psychoanalysis, but which, nevertheless, we have been using for so long in daily communication among ourselves.



Its peace made with itself, the mind unfolds her wings  
in reality, it lets the sun gently dry them and breathes deeply,  
preparing for the next bound . . . Let's go!

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### Notes

1. See Lauro-Grotto (2006). This paper describes the theoretical background of the method and its operationalization in the form of a neuropsychological test. The trend of the metric content in aging and dementia is explored in Ciaramelli, Lauro-Grotto, and Treves (2007).
2. A very clear introduction to the work of Matte Blanco can be found in Rayner and Tuckett (1998).
3. Fodor, of course, is alluding to the melancholy donkey in A. A. Milne's *Winnie the Pooh*, written in 1924. "Nevica ancora" is also the subtitle of the essay that Fodor derived from his so-called Italian Lectures, a series of talks the philosopher gave at the San Raffaele Institute in Milan in the summer of 1998. There he discusses the stakes and the limits of what he calls the "New Synthesis" in the cognitive sciences. This paradigm, according to Fodor, combines aspects of Chomsky's innatism and neo-Cartesianism with neo-Darwinian elements and the fundamental assumption of massive modularity in order to provide a comprehensive account of mental functions. *Hélas!* This very promising explanatory scheme is shipwrecked on the reef of the lack of constraints on abductive thinking—that is, reasoning from effects back to causes—or what is known as the "frame problem."
4. The third chapter of Matte Blanco's book is entirely devoted to this theme; nevertheless, in the same chapter an opening can be found to a future perspective in which bi-logic could be reformulated in terms of a unitary super-logic that unfolds in symmetric and asymmetric modes of being. The redescription of bi-logic in terms of metricity and ultrametricity should be seen as moving in this direction as both are mathematical properties of the topology of space. Ultrametricity has also been introduced in theoretical physics by Giorgio Parisi (Mézard, Parisi, and Virasoro 1987) in order to describe spin glass systems, a type of complex system not very different from neural networks. A Hopfield neural network does in fact show a spontaneous transition from a metric to an ultrametric regime when the storage capacity limit of the system is exceeded. Not only might it therefore be the case that a unique conceptualization could express the characteristic of bi-logic, but we might also have a putative neural model for it.
5. To my knowledge there is no model of any physical system that can express metric properties at the global level while conforming to an ultrametric organization at the local level, as I would claim should be the case for *quasi-symbols*.

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