Analysis and Compensation of CTI Effects on CCD Transversal Filter Response

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Abstract—Using an approximation generally satisfied in practical applications, an expression for the deviation of the CCD transversal filter transfer function due to the device CTI is obtained.

Then a very simple algorithm for the coefficient modification is derived to compensate for the CTI effects, which are thus reduced by more than an order of magnitude.

Computer simulations finally show the validity of the obtained results.

I. Introduction

Filtering with charge-coupled devices (CCD's) has received considerable attention in recent years for many applications [1], [2]. Although recursive and nonrecursive filters can be realized, CCD transversal filters have been emphasized since they do not require feedback amplifiers with very accurately controlled gains. However, CCD transversal filters are affected by some sources of errors, and one of the major contributions to their nonideal behavior comes from the charge transfer loss, i.e., from the fraction ϵ of the stored signal sample that remains behind at any transfer step [2]. A charge-transfer inefficiency (CTI) ϵ different from zero causes the actual frequency response of the realized filter to deviate from the nominal one, designed for ideal delay elements.

In this paper, by making an approximation normally justified in practical applications, the deviation of CCD transversal filter response caused by the CTI is explicitly obtained. This analysis is then used to derive a simple algorithm for the compensation of such errors. Computer simulations are also reported showing the validity of the analysis and the effectiveness of the proposed compensation algorithm.

II. Deviation of the Filter Response Caused by CTI

It is well known [2] that the effects of the charge-transfer inefficiency in a CCD structure can be analyzed, replacing each delay element (CCD cell) z^{-1} with the circuit model of Fig. 1, which has the input–output transfer function

$$H_d(z) = (1-\epsilon)\frac{z^{-1}}{1-\epsilon z^{-1}}. \tag{1}$$

Normally, the CTI value ϵ is assumed constant for each cell.

An FIR discrete filter of length N and coefficients h_k, designed to have a desired transfer function

$$H_0(z) = \sum_{k=0}^{N-1} h_k z^{-k} \tag{2}$$

when realized using a CCD device with a CTI value ϵ, gives rise to an actual transfer function $H(z)$ expressed by, recalling (1),

$$H(z) = \sum_{k=0}^{N-1} h_k [H_d(z)]^k = \sum_{k=0}^{N-1} h_k z^{-k} \frac{(1-\epsilon)^k}{(1-\epsilon z^{-1})^k}. \tag{3}$$

In many applications, the values of ϵ and N are such that their product $N\epsilon \ll 1$. In this case, the following approximation holds, at least in a region of the z plane containing the unit circle $|z| = 1$:

$$\left(\frac{1-\epsilon}{1-\epsilon z^{-1}}\right)^k \simeq 1 - k\epsilon(1-z^{-1}). \tag{4}$$

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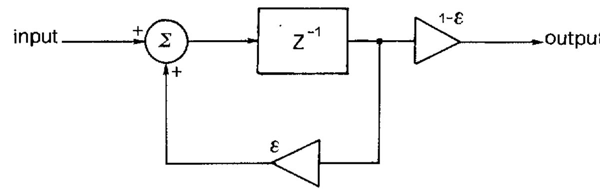


Fig. 1. Circuit model for a CCD cell.

Inserting this approximation into expression (3) of the actual transfer function $H(z)$, we obtain

$$H(z) \simeq \sum_{k=0}^{N-1} h_k z^{-k}[1 - k\epsilon(1-z^{-1})]$$

$$= H_0(z) + \epsilon(z-1)\frac{dH_0(z)}{dz} \tag{5a}$$

$$= H_0(z)\left[1 + \epsilon(z-1)\frac{d \log H_0(z)}{dz}\right]. \tag{5b}$$

These relations express how the nominal filter transfer function $H_0(z)$ is modified by a CTI ϵ different from zero and, taking $z = e^{j\omega}$, they can be used to evaluate the amplitude error $\Delta A(\omega)$ and the phase error $\Delta\varphi(\omega)$ of the filter frequency response due to the device CTI.

For example, with simple algebra, we obtain for a delay line the expressions

$$\Delta A_{DL}(\omega) \simeq N\epsilon(1-\cos\omega) \tag{6a}$$

$$\Delta\varphi_{DL}(\omega) \simeq N\epsilon \sin\omega \tag{6b}$$

which are in close accordance with performed computer simulations and with results already reported in the literature [2], [3].

Similar formulas can be evaluated for other types of filters, including wide-band differentiators and Hilbert transformers. For every considered filter, a very close agreement has been found between the expressions derived from (5) and the amplitude and phase errors obtained by computer simulation of the CTI effect on the frequency response.

III. Simple Coefficient Modification Algorithm

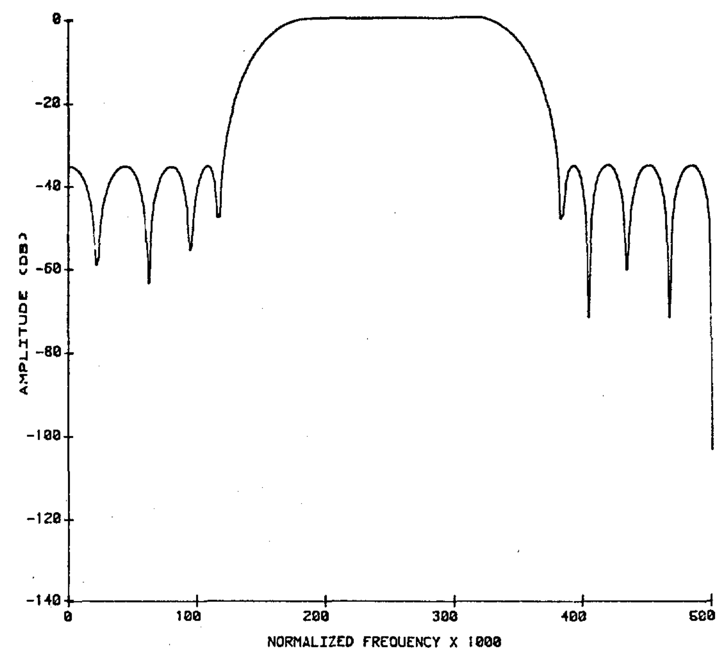
When the CTI value can be predicted with sufficient accuracy, its effects can, in principle, be eliminated or drastically attenuated. This can be achieved by using a suitable equalizer filter in cascade with the CCD filter as proposed in [3] for a delay line or modifying the weighting coefficients h_k to new suitable values h'_k. It is well known [4], [5] that, for the latter approach, in general, the new coefficients can be obtained iteratively by the relation

$$h'_k = \frac{1}{(1-\epsilon)^k}\left[h_k - \sum_{j=1}^{k} h'_{k-j}\binom{k-1}{j}\epsilon^j(1-\epsilon)^{k-j}\right],$$

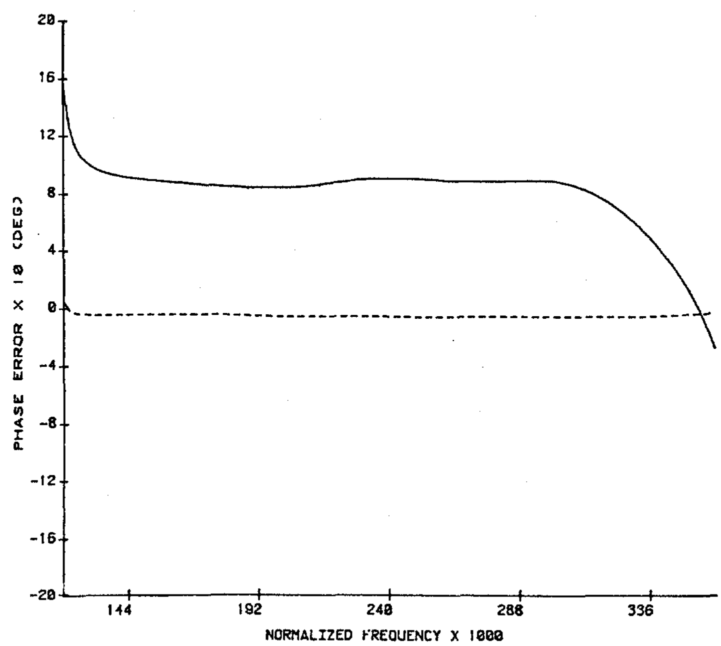
$$k = 1, \cdots, N-1$$

$$h'_0 = h_0. \tag{7}$$

In the approximation stated in the previous section, it is possible to derive a much simpler relation for the new coefficients h'_k. Their z-transform $H_1(z)$ has to satisfy the equation, from (5a),



(a)



(b)

Fig. 2. CTI effects for an equiripple bandpass linear-phase filter with $N = 32$ and $\epsilon = 10^{-3}$. (a) Filter frequency response. (b) Amplitude error without (continuous line) and with (dashed line) compensation.

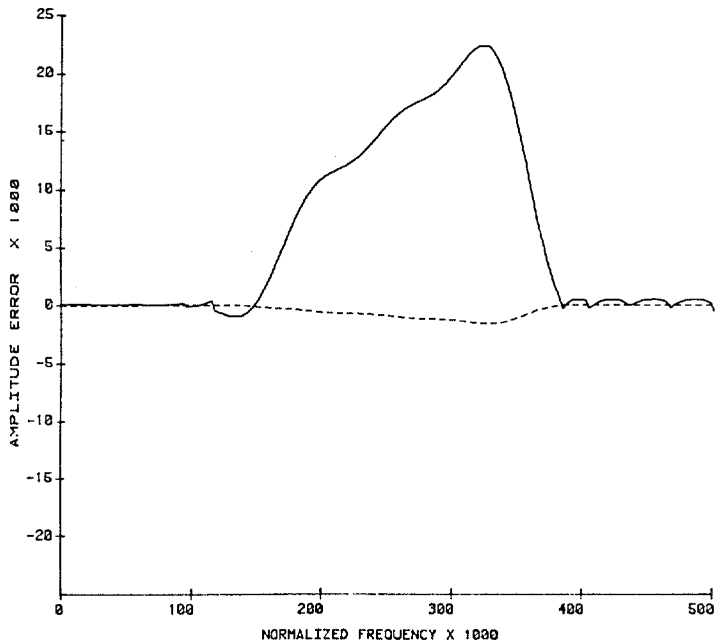
$$H_1(z) + \epsilon(z - 1)\frac{dH_1(z)}{dz} = H_0(z) \tag{8}$$

where $H_0(z)$, given by (2), is the desired filter transfer function. The inverse z-transformation of (8) is the difference equation

$$h'_k = \frac{1}{1 - k\epsilon}\left[h_k - \epsilon(k - 1)\, h'_{k-1}\right] \tag{9}$$

which, starting with $h'_0 = h_0$, iteratively gives the coefficients h'_k of the compensated filter. It should be observed that the relation (9) is very close to that derived from (7) when only the first term of the summation is retained, following the assumption that $N\epsilon \ll 1$.

Fig. 2 shows the amplitude and phase errors with and without the coefficient compensation (9) for an equiripple linear-phase bandpass filter of length $N = 32$ (a commonly available number of delay elements for CCD's) and for a transfer inefficiency $\epsilon = 10^{-3}$. It is interesting to observe that the amplitude error without compensation is maximum in the transition



(c)

Fig. 2. (c) As in (b) for the phase error in the passband of interest.

bandwidth close to the boundaries of the filter passband. This behavior can easily be fully justified from (5). Fig. 2 also shows that the coefficient compensation (9) reduces the amplitude and phase errors by more than an order of magnitude. The residual errors are to be attributed to the terms neglected in the approximation (4) and to the fact that the charge-transfer loss actually transforms an FIR filter into an IIR one, as can be seen from the model of Fig. 1 and from relation (3).

IV. Conclusions

In many applications of CCD transversal filtering, the CTI ϵ and the filter length N are such that $N\epsilon << 1$. In this case, the general expressions (5) for the transfer function deviation caused by the device CTI have been obtained. From these relations, the amplitude and phase errors of the filter frequency response can be derived in a straightforward way. The accordance between the computed errors and those obtained from computer simulations and/or those already known in the literature was verified for many filters.

Furthermore, the simple modification algorithm (9) of the filter coefficients derived from the preceding analysis has proved to be quite effective in reducing the amplitude and phase deviations by more than an order of magnitude. The simplicity of the compensation algorithm (9) is, in principle, particularly attractive when the coefficients of a CCD transversal filter are externally evaluated and/or modified by means, for example, of a microprocessor and supplied to a multiplying digital-to-analog converter to perform the filtering operation [2].

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Confidence Bounds for Signal-to-Noise Ratios from Magnitude-Squared Coherence Estimates

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Abstract—**Coherence is used frequently to determine the degree to which one observed voltage is related to another observed voltage. Typically, in practice, these observables are degraded by system noise that is often independent, white, and Gaussian. Often, in measuring coherence, the interest is to determine the fraction of the observed power that is due to coherent signals and the fraction that is due to the uncorrelated noise floor. The term "signal" as used here describes a component of voltage of interest to an observer. With accurate coherence estimates, uncorrelated noise power can be separated from coherent signal power. Therefore, the concern in this article is with the accuracy of signal-to-noise ratio (SNR) calculations made from magnitude-squared coherence (MSC) estimates. Use is made of work by Carter and Scannel [1] in which they determine confidence bounds of MSC estimates for stationary Gaussian processes. Their results are used in this article to derive corresponding confidence bounds for SNR calculations without recourse to the complicated details of the underlying SNR statistics.**

Discussion

The magnitude-squared coherence (MSC), or the coherence function as it is sometimes called, between two stationary pro-

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