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Blind adaptive MUD with silence listening[1]

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Abstract

This paper presents a blind multi-user detector with an additional adaptive branch which is designed to correct the wrong estimate of the wanted user's signature caused by a multi-path channel. In order to reach this goal without training sequences, the knowledge of the time periods of silence, i.e. the time periods when the desired user is not transmitting any signal in the channel, is an additional information that is supposed to be available to the receiver. © 1997 Elsevier Science B.V.

Zusammenfassung

Dieser Artikel stellt einen blinden Multi-User-Detektor vor. Er beinhaltet einen zusätzlichen adaptiven Zweig, der zur Korrektur einer, aufgrund von Mehrwegeausbreitung falschen Schätzung der gewünschten Benutzerkennung entworfen wurde. Um dieses Ziel ohne Trainingssequenzen zu erreichen, wird angenommen, daß dem Empfänger die Information über Ruhepausen, also die Zeiträume, in denen der Sender kein Signal in den Kanal sendet, als eine zusätzliche Information zur Verfügung steht. © 1997 Elsevier Science B.V.

Résumé

Cet article présente un détecteur multi-utilisateurs aveugle ayant une branche adaptative additionnelle conçue pour corriger l'estimation fausse de la signature de l'utilisateur désiré causée par un canal multi-chemins. Dans le but d'atteindre cet objectif sans séquence d'apprentissage, la connaissance des périodes temporelles de silence, c'est à dire des périodes temporelles durant lesquelles l'utilisateur désiré ne transmet aucun signal sur le canal, est une information additionnelle supposée être disponible au niveau du récepteur. © 1997 Elsevier Science B.V.

Keywords: CDMA; Multi-user detection; Adaptive receiver; Multipath channel

1. Introduction

In wireless communication systems, the CDMA protocol used for the multiple access to the shared medium is considered the optimal choice for a better utilization of the communication medium, and also to fight the unwanted effects induced by multi-path. Nevertheless, the presence of a strong interferer results often in an unacceptable performance degradation of the CDMA receiver. This phenomenon is called "near-far effect" because this problem often arises when the interfering user is located at a shorter distance from the receiving station than the user we want to demodulate.

The near-far effect can be dramatically reduced by demodulating all users at the same time. This

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well-known technique is called multi-user detection (MUD).

Several papers [4, 6, 3] in the literature suggest some adaptive approaches to multi-user detection with various degree of blindness with respect to the knowledge of the channel characteristics and of the adopted signals.

The main advantages of a blind adaptive MUD receiver can be found in the simplicity and compactness of the corresponding practical implementation, especially for the handset design. Moreover, the adaptive behavior may be particularly useful when the multi-path phenomenon is present and when the received signal is subject to time-varying distortions.

The weakest aspect found in these techniques is the need of the knowledge of a close approximation of the signal adopted by the user whose information is seeked.

In the literature the correction of the received signal estimation is often accomplished by using training sequences. Part of the overall capacity of the channel is thus used by the training sequences, and this reduces the information rate available to the users.

For this reason it is important to develop a technique capable to follow the variations inducted by the multi-path channel; on the other hand, it is also advisable that the impact of the new adaptation ability does not steal capacity from the communication resource.

This paper presents an adaptive multi-user detector which tries to estimate the effective signal received by the user by listening to the channel when the user is not transmitting. The only requirements are the *transmitted* waveform of the user of interest, and the state of this user, as explained later in Section 3.

The resulting detector is expected to behave well even when the time varying channel changes continuously the shape of the waveform assigned to the user of interest.

The present paper is organized as follows. Section 2 describes the general structure of a blind adaptive linear multi-user receiver. The *silence listening* mechanism is defined in Section 3, while Section 4 describes the adaptive implementation of the proposed receiver. In Section 5 there are the assumptions made for the simulation of the receiver, and the commented results. The conclusive remarks can be found in Section 6.

2. The receiver structure

Recent works [4–6] have shown that multi-user detection exerts a radical improvement on the performance of a CDMA receiver. An attractive model of a blind adaptive MUD receiver has recently been presented by Madhow, Honig and Verdú [3]; this is a model for which only the knowledge of the signature waveform of the desired user, along with the timing of all the users, are required.

The structure of the receiver is shown in Fig. 1, where the receiver signal is

$$y = \sum_{k=1}^{K} A_k b_k s_k + \boldsymbol{n}, \tag{1}$$

where K is the number of users, A_k the received amplitude of the kth user's signal, b_k the antipodal binary information from the kth user, s_k the received signal from the kth user, here represented as a member of a Hilbert space $\mathscr{H}$, $\boldsymbol{n}$ is the projection onto $\mathscr{H}$ of a white Gaussian stochastic process with zero mean and variance equal to σ^2.

We are interested in the first user's information.

Due to multi-path, only an estimate of the first user's signal is available at the receiver: say $\tilde{s}_1$.

The linear MUD receiver is composed by two components, the estimate of the desired user's signal $\tilde{s}_1$, and an orthogonal component $\boldsymbol{x}_1$ dedicated to the suppression of the interferers.

For a binary, antipodal and equiprobable signaling, the estimated information is given by a threshold detector which follows the decorrelator, as shown in Fig. 1.

If a reliable estimate of the first user's signal is available, i.e. $\tilde{s}_1 = s_1$, then $\boldsymbol{x}_1$ may be adaptively modified in order to reduce the so-called *mean output energy* defined as

$$\mathrm{MOE}[\boldsymbol{x}_1] = E[\langle y, \tilde{s}_1 + \boldsymbol{x}_1 \rangle^2], \tag{2}$$

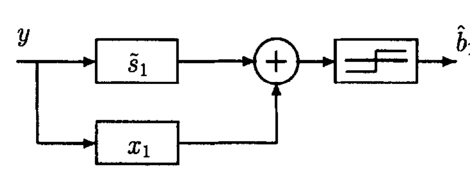

Fig. 1. The general structure of the MUD receiver.

where $E[\cdot]$ is the expectation value and the operator $\langle\alpha,\beta\rangle$ is the scalar product defined over the Hilbert space chosen to represent the signal considered in the transmission.

The local minimization of the MOE is proven to yield the same canceller as the one given by the minimization of the quantity $\mathrm{MSE}(\boldsymbol{x}_1)$ defined as

$$\mathrm{MSE}[\boldsymbol{x}_1] = E[(A_1 b_1 - \langle \boldsymbol{y}, \boldsymbol{s}_1 + \boldsymbol{x}_1\rangle)^2]. \tag{3}$$

So, the principal result obtained by Honig, Madhow and Verdú is that it is not necessary to know the data in order to implement a gradient descent algorithm for the minimization of the mean-square error.

When the transmitted waveform from user 1 is not exactly known, the adaptation rule of $\boldsymbol{x}_1$ needs to be modified in order to prevent the cancelation of the desired signal. The blind adaptation rule, as described in [3], is modified with an additional constraint on the so-called *surplus energy*, defined as

$$\chi = \|\boldsymbol{c}_1\|^2 - 1, \qquad \boldsymbol{c}_1 = \tilde{\boldsymbol{s}}_1 + \boldsymbol{x}_1. \tag{4}$$

By imposing a value of χ in the blind adaptation rule, a lower degradation of the *signal-to-interference ratio (SIR)* is achieved, even in the presence of an estimation error of the desired user's signal.

3. The silence listening

The inspection of the common characteristics of the information flowing in wireless systems reveals the substantially discontinuous nature of the information flow. In voice channels over 30% of time is not used to transmit any information. In data channels discontinuances are also present, depending on the nature of the connected system.

We shall now prove that in CDMA systems, the knowledge of a user's silence period is useful to the correct estimate of that user's signature waveform. Unlike the training sequences, which may be also used to correct that estimate, silence periods are always present during the transmission; so the algorithms that control them may be easily integrated in the communication devices.

In the present paper, the *silence/talk* status of the desired user is assumed to be known.

We may think of a possible arrangement in the transmission protocol like the one represented in Fig. 2.

In this case the talk/silence command is issued by the base station towards all the mobile users. This command derives from a packet reservation control channel and may be used to adjust the data rates in multi-rate access wireless system.

Using such a scheme, the base station does not need to acquire the status of the users, since it itself forces the states transitions.

Another possible implementation of the talk/silence control is represented in Fig. 3.

This time each single user informs asynchronously the base station about its state. In this case an activity speech detector, integrated in the handset of the mobile user, may be used to generate the silence flag, which is then transmitted to the base station.

In both the proposed schemes the *silence/not silence* information is a very slow signal if compared to the spreading signatures, and may be transmitted in a very

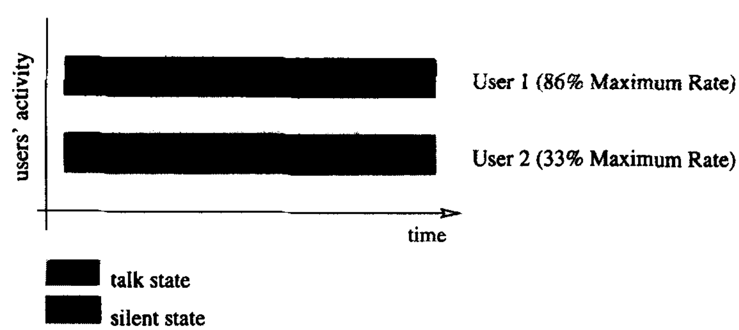

Fig. 2. A possible multi-rate talk/silence access protocol.

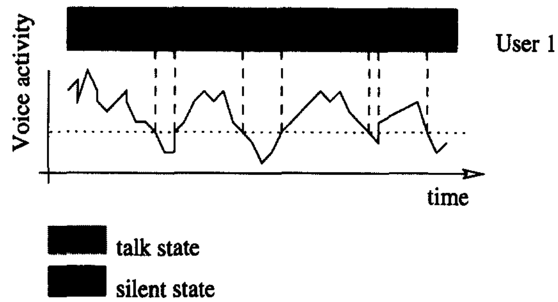

Fig. 3. A talk/silence control based on the activity detection.

narrow portion of the medium spectrum without negligible loss of capacity.

Let us consider the up-link channel of a CDMA communication system with K users sharing the same medium.

In this section we classify the signals involved in the transmission of the desired user's information as follows:

$\hat{s}_1$ is the *transmitted waveform* from user 1; the uncorrupted version of the signal associated with the desired user.

s_1 is the *received waveform* from user 1. In a multipath environment it represents the distorted signal derived from $\hat{s}_1$.

$\tilde{s}_1$ is the signature waveform employed by the receiver for demodulation. It represents *the estimate of the received waveform* from the desired user.

Our target is the demodulation of the information from the first user, whose state is considered to be known at the receiver of the base station BS.

Two possible states are considered with respect to the first user as shown in Fig. 4.

TALK corresponding to a received signal as

$$y = \sum_{k=1}^{K} A_k b_k s_k + n.$$

SILENCE with a received signal as

$$y_s = \sum_{k=2}^{K} A_k b_k' s_k + n.$$

In order to fight against the wrong estimation of the first user's signal caused by channel distortions such as multi-path, an adaptive algorithm which influences the desired user's signal estimation $\tilde{s}_1$ is derived.

We look at the quantity called *mean silence output energy* defined as

$$\mathrm{MSiE}[\tilde{s}_1] = E[(\langle y_s, \tilde{s}_1\rangle + \langle y, x_1\rangle)^2]. \tag{5}$$

As shown in Appendix A, in the low-noise region ($\sigma \to 0$) the mean silence output energy is equal to

$$\mathrm{MSiE}_{\sigma\to 0}[\tilde{s}_1] = \sum_{k=2}^{K} A_k^2 \langle \tilde{s}_1, s_k\rangle^2. \tag{6}$$

The local minimization of such a quantity, along with the constraint of a unitary norm of $\tilde{s}_1$ ($\|\tilde{s}_1\| = 1$), yields a correction on the estimate of $\tilde{s}_1$ towards a signal less sensitive to the interferent components of the received signal. It should be noted that $\mathrm{MSiE}[\tilde{s}_1]$ by itself does not have a minimum in $\tilde{s}_1 = s_1$, so that the derived algorithm does not tend to improve the estimation of $\tilde{s}_1$ towards s_1. It tends instead to decrease the components of $\tilde{s}_1$ that lie along the interfering

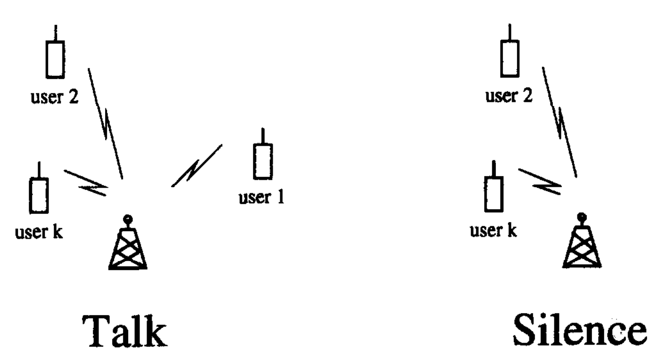

Fig. 4. The two possible states of user 1.

signals. Moreover, stronger interferers contribute to the MSiE value more than weaker ones.

At the end of the adaptation performed during the *silent state*, the resulting s_1 signal, chosen among those unitary normed signals orthogonal to x_1, will have its component reduced along the interferers. This will improve the capability of the receiver to reject the interfering signals and will result in a higher signal-to-interference ratio.

It is important to point out that the minimization of MSiE is a local one, with its initial value of $\tilde{s}_1$ being the uncorrupted signal assigned to the user of interest, namely $\hat{s}_1$.

If for any reason the adaptive algorithm falls in the region of attraction of another signal equally orthogonal to the interference signal, but without any component along s_1, the receiver will no longer be able to recover the information. A technique is studied to prevent this problem; for example the amount of corrections performed during silence periods can be limited as soon as an attractor boundary is reached.

In the practical realization of the proposed receiver, the above-mentioned correction in the estimate of $\tilde{s}_1$ is performed when the user of interest is known to be in the *silent state*. Instead, during the *talk state*, the adaptive mechanism of $\tilde{s}_1$ is 'frozen' while it is running the adaptive branch of the canceler x_1.

From the definition of y and y_s it is clear that those signals are not available at the same time, and so the MSiE could not be computed. With a little artifice, anyway, it is possible to compute the MSiE by a stored estimation of the quantity $\langle y, x_1 \rangle$ during the silence periods.

4. The design of the receiver

The structure of the proposed receiver is shown in Fig. 5.

The stochastic gradient descent algorithm for adaptivity of $x_1[i]$ is fully described in [3], so only the final formulation is reported here.

For the adaptation rule of $\tilde{s}_1$, the expression of $\mathrm{MSiE}[\tilde{s}_1]$ is recalled

$$\begin{aligned}\mathrm{MSiE}[\tilde{s}_1] &= E[(\langle y_s, \tilde{s}_1 \rangle + \langle y, x_1 \rangle)^2] \\ &= E[\langle y_s, \tilde{s}_1 \rangle^2] + E[\langle y, x_1 \rangle^2] \\ &\quad + 2E[\langle y_s, \tilde{s}_1 \rangle \langle y, x_1 \rangle]. \end{aligned} \tag{7}$$

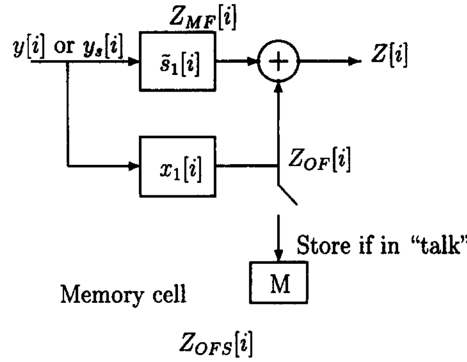

Fig. 5. The proposed receiver.

Its unconstrained stochastic gradient ($\tilde{s}_1$ being the variable vector) is

$$\begin{aligned}\nabla\mathrm{MSiE}[\tilde{s}_1] &= 2\langle y_s, \tilde{s}_1 \rangle y_s + 2\langle y, x_1 \rangle y_s \\ &= 2(\langle y_s, \tilde{s}_1 \rangle + \langle y, x_1 \rangle) y_s. \end{aligned} \tag{8}$$

The component of (8) orthogonal to x_1 is

$$2(\langle y_s, \tilde{s}_1 \rangle + \langle y, x_1 \rangle)(y_s - \langle y_s, x_1 \rangle x_1), \tag{9}$$

so the adaptation rule for $\tilde{s}_1$ is

$$\begin{aligned}\tilde{s}_1[i] &= \tilde{s}_1[i-1] \\ &\quad -\mu(\langle y_s[i], \tilde{s}_1[i-1] \rangle + \langle y[i], x_1[i-1] \rangle) \\ &\quad \times (y_s[i] - \langle y_s[i], x_1[i-1] \rangle x_1[i-1]). \end{aligned} \tag{10}$$

From now onwards we will call

$$\langle y[i], \tilde{s}_1[i-1] \rangle = Z_{\mathrm{MF}}[i], \tag{11}$$

$$\langle y[i], x_1[i-1] \rangle = Z_{\mathrm{OF}}[i], \tag{12}$$

$$\langle y_s[i], x_1[i-1] \rangle = Z_{\mathrm{OFS}}[i]. \tag{13}$$

It should be noticed that the quantity $\langle y, x_1 \rangle$ is not available at the same time as $\langle y_s, x_1 \rangle$. For that reason, during the 'talk' period, the quantity $\langle y, x_1 \rangle$ is continuously stored in a memory cell called $Z_{\mathrm{OFS}}[i]$. When a state transition occurs at i^*, the last value of $Z_{\mathrm{OFS}}[i^*]$ is taken as an estimate of $Z_{\mathrm{OFS}}[i]$ for $i > i^*$.

The overall adaptation rules for each state of the receiver are:

TALK

$$\boldsymbol{x}_1[i] = \boldsymbol{x}_1[i-1](1-\mu_x v_x) - \mu_x Z[i] \times (y[i] - Z_{\mathrm{MF}}[i]\tilde{s}_1[i-1]), \quad (14)$$

$$\tilde{s}_1[i] = \tilde{s}_1[i-1] \quad \text{(holding...)}, \quad (15)$$

$$Z_{\mathrm{OFS}}[i] = Z_{\mathrm{OF}}[i] \quad \text{(storing...)}. \quad (16)$$

SILENCE

$$\tilde{s}_1[i] = \tilde{s}_1[i-1] - \mu_s(Z_{\mathrm{MF}}[i] + Z_{\mathrm{OFS}}[i]) \times (y_s[i] - Z_{\mathrm{OF}}[i]\boldsymbol{x}_1[i-1]), \quad (17)$$

$$\boldsymbol{x}_1[i] = \boldsymbol{x}_1[i-1] \quad \text{(holding...)}, \quad (18)$$

$$Z_{\mathrm{OFS}}[i] = Z_{\mathrm{OFS}}[i-1] \quad \text{(holding...)}, \quad (19)$$

where

μ_s, μ_x are the adaptation steps of the two adaptive branches: signature (s) and canceller (x). When the stochastic processes involved in the reception can be considered stationary, the algorithm is conducted to the solution [2] if $\mu_{s,x} = \alpha/i$, with $\alpha \in \mathscr{R}^+$. In practice, a residual variation of the parameters in the receiver is tolerated in order to be able to follow the non-stationary conditions of a real channel. In the simulations we obtained convergence after a reasonable number of steps ($\approx$1000 symbols), if $\mu_{s,x}$ follows the design rules described by Honig, Madhow and Verdú in [3];

v is the Lagrange multiplier responsible for the upper bound to the *surplus energy* as described in [3].

The exponential decay step was also tested to provide a more stable convergence under stationary conditions of the stochastic processes. In those simulations the decay law is present in both the adaptive branches and the time constant is set to a value that is 200 times the average duration of each state.

For those simulations the step evolution is defined by

$$\mu(i) = \mu_0 \mathrm{e}^{-\tau i}, \quad (20)$$

where μ_0 is the initial value of the adaptation step; and τ is the decay factor.

Under non-stationary conditions of the channel the exponential decay step must be completed with a lower bound limiter in order to follow the channel variation. An implementation of this lower bound limiter with a time-varying channel is under study at the time of writing.

5. The simulation testbed

The performance of the receiver, and the gain obtained by silence listening are evaluated by computer simulations. The simulated CDMA transmission system is characterized by:

1. DS-CDMA 31-chips Gold Sequence [1] for the wanted user, unitary energy of the desired user's spreading sequence. If $\hat{s}_1$ for $t \in [0, T_b]$, is the spreading sequence assigned to the first user, the received signal corrupted by multi-path is
$$s_1(t) = \frac{\hat{s}_1 + a\hat{s}_1(t - T_b/2)}{\|\hat{s}_1 + a\hat{s}_1(t - T_b/2)\|},$$
with a being the *multi-path index*. The received signal from user 1 is thus
$$b_1(i)s_1(t - iT_b), \quad t \in [iT_b, (i+1)T_b].$$
The self-interference effect is modeled like an additional interfering user which uses a shifted version of the signal assigned to it:
$$A_{K+1}b_{K+1}(i)s_{K+1}(t - iT_b) = ab_{K+1}(i)\hat{s}_1(t + T_b/2 - iT_b), \quad t \in [iT_b, (i+1)T_b], \quad (21)$$
where a is again the *multi-path index*.
2. K interferent users, each with a different 31-chip Gold sequence and an amplitude A_k with ($k = 2, \ldots, K$). Each signal waveform received from the interfering users is supposed to have

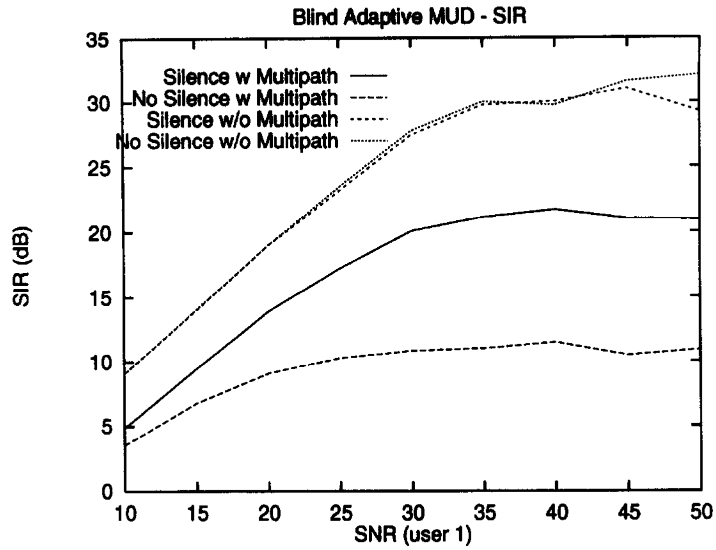

Fig. 6. SIR versus SNR for user 1.

unitary energy. The received signal from the kth interferer is thus

$$A_k b_k(i) s_k(t - iT_b), \quad t \in [iT_b, (i+1)T_b],$$
$$k = 2, \ldots, K. \tag{22}$$

3. Additive white Gaussian zero-mean noise process with variance σ^2.

The performances are computed in terms of signal-to-interference ratio, defined as the ratio (in dB) between the power of the information-bearing signal and the power of the interfering signal which passes through the linear receiver, as shown in the following formula:

$$\mathrm{SIR} = \frac{\langle s_1, c_1 \rangle^2}{\|c_1\| \sigma^2 + \sum_{k=2}^{K+1} A_k^2 \langle s_k, c_1 \rangle^2}. \tag{23}$$

In the simulations neither time-variant multi-path, nor loss of synchronization between the transmitters and the receiver have been considered. At the time of this writing, the ability of the receiver to follow deep fades, is being tested, and the results will be presented as soon as possible. Extensive simulations of the proposed CDMA receiver have been conducted using the software Ptolemy (version 0.6)[1] running on a Spark 20 machine.

Table 1
Simulation parameters

Number of 'real' interfering users (K)	7
$A_k \quad k = 2, \ldots, K$	$\sqrt{10}$
Talk/silence periods ratio	1 : 1
Multi-path index (a)	1.0
μ_x (initial value)	10^{-3}
μ_s (initial value)	10^{-3}
μ decay factor	1/2000
v (surplus energy limiter)	1.0

The multi-path environment considered in the simulations is stationary so an exponential decay law for the adaptation step has been used first.

Fig. 6 shows the values of the SIR of the proposed receiver versus the SNR of the first user's signal.

The transitions from talk to silence are performed every 10 symbols of the first user. The simulation parameters are shown in Table 1.

[1] The Ptolemy has been developed by the University of Berkeley and is freely available under the GNU license.

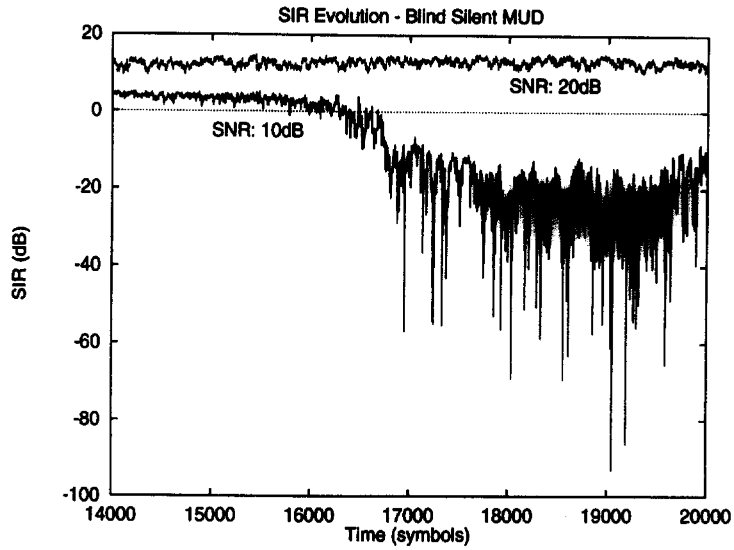

Fig. 7. A *false lock* occurrence.

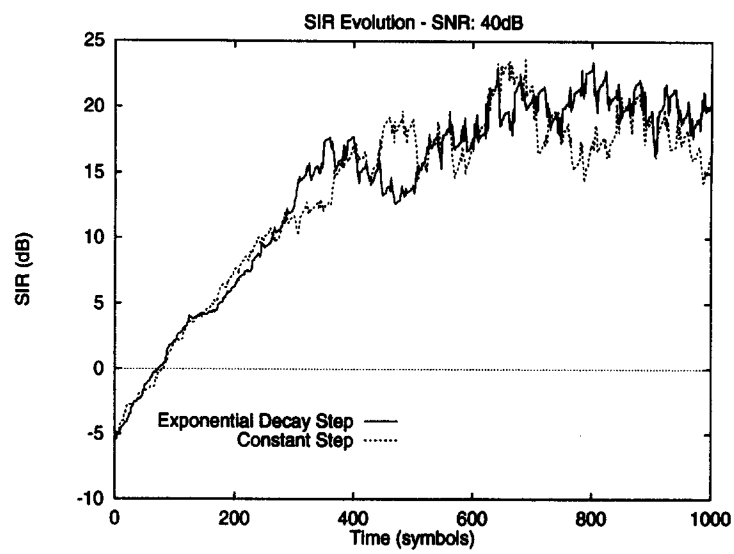

Fig. 8. Constant step versus exponentially decreasing step: starting segment.

The two curves labeled 'Silence w Multi-path' and 'No Silence w Multi-path' represent, respectively, the performance of the receiver with and without the silence adaptation algorithm.

The two curves labeled 'Silence w/o Multi-path' and 'No Silence w/o Multi-path' represent the performance in case of a reception not corrupted by any multi-path effects, that is with $a = 0$. As one might expect the impact of the silence listening on the performance is negligible when the multi-path distortion is not present (small dots and short dashes in the graph).

When the multi-path is present, we notice a significant increase in the SIR level if the silence listening adaptation is switched on (continuous line).

In Fig. 7 we can see the major defect of our adaptation algorithm: when the SNR is sufficiently low, the risk of an erroneous estimate of the information bearing signal grows. Here the adaptation algorithm

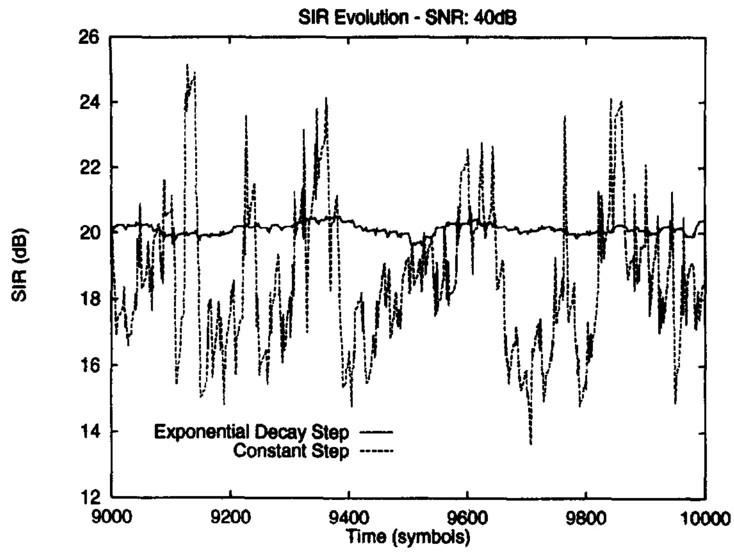

Fig. 9. Constant step versus exponentially decreasing step: late segment.

falls in the attractor of a signal which does not contain any component along the original first user's one.

Two possible solutions are under study to limit the occurrences of this false lock:

- to impose an elastic constraint on the evolution of $\tilde{s}_1$, so the adaptation of $\tilde{s}_1$ is continuously recalled towards $\hat{s}_1$.
- to reset the adaptation algorithm after a fixed amount of adaptation steps.

The last figures (Figs. 8 and 9 show a comparison between an exponential decay adaptation step and a constant step.

Fig. 8 shows the initial segment of the time evolution of the SIR value. The presence of a decay law in the adaptation step does not modify the behavior of the adaptive algorithm if the time constant of the decay law is consistently larger than the mean convergence time.

As time elapses (Fig. 9) the exponential step performs a more steady convergence to the value of the SIR. It is however unable to follow all the possible variations of the channel, once the convergence has been reached.

A constant step performs a fast convergence as well, but it is more sensitive to false locks if the SNR is low.

6. Conclusions

In communication systems, where discontinuous information flow through the shared medium, it is possible to improve the robustness of the receiver against the distortion effects due to multi-path, by the listening of the silence periods of the desired user.

Though the proposed receiver is based on some assumptions that are valid in the high SNR regions, no particular limitations are imposed to the power of the interfering users.

Appendix A

Here the expression for the mean output silence energy is derived. Let us start from the definition of MSiE as defined in (5):

$$E[(\langle y_s, \tilde{s}_1\rangle + \langle y, x_1\rangle)^2]$$

$$= E[\langle y_s, \tilde{s}_1\rangle^2 + \langle y, x_1\rangle^2 + 2\langle y_s, \tilde{s}_1\rangle\langle y, x_1\rangle]$$

$$= E\left[\left(\sum_{k=2}^{K} A_k b_k' \langle s_k, \tilde{s}_1\rangle\right)^2\right.$$

$$+\left(\sum_{k=1}^{K} A_k b_k \langle \boldsymbol{s}_k, \boldsymbol{x}_1 \rangle\right)^2\Bigg]$$

$$+2\sum_{k=2}^{K}\sum_{h=1}^{K} A_k A_h b_k' b_h \langle \boldsymbol{s}_k, \tilde{\boldsymbol{s}}_1 \rangle \langle \boldsymbol{s}_h, \boldsymbol{x}_1 \rangle].$$

Now, considering only the terms that will survive after the $E[\,\cdot\,]$ operator, this equation amounts to the following:

$$E\left[\sum_{k=2}^{K} A_k^2 \langle \boldsymbol{s}_k, \tilde{\boldsymbol{s}}_1 \rangle^2 + \cdots + \sum_{k=1}^{K} A_k^2 \langle \boldsymbol{s}_k, \boldsymbol{x}_1 \rangle^2 + \cdots\right]$$

$$= A_1^2 \langle \boldsymbol{s}_1, \boldsymbol{x}_1 \rangle^2 + \sum_{k=2}^{K} A_k^2 (\langle \boldsymbol{s}_k, \tilde{\boldsymbol{s}}_1 \rangle^2 + \langle \boldsymbol{s}_k, \boldsymbol{x}_1 \rangle^2).$$

Now, considering the parts which vary with $\tilde{\boldsymbol{s}}_1$, this amounts to

$$\sum_{k=2}^{K} A_k^2 \langle \boldsymbol{s}_k, \tilde{\boldsymbol{s}}_1 \rangle^2. \qquad (24)$$

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