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Zero Convergent Solutions for a Class of p-Laplacian Systems

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Abstract. We present some results about the existence of positive decaying solutions for a class of systems of two second order coupled nonlinear equations with p-laplacian operator, $p > 1$. In addition the generalized Emden-Fowler type systems are considered and necessary and sufficient conditions are given in order for the system to have regularly and/or strongly decaying solutions. The results here presented complete the ones in [10].

MSC 2000. 34B15, 34C11.

Keywords. p-laplacian operator, regularly decaying solutions, strongly decaying solutions, fixed point theorems.

1 Introduction

In this contribution we present some asymptotic results for a system of two coupled nonlinear equations with the 1-dimensional p-laplacian operator $\Psi_p(u) = |u|^{p-2}u$, $u \in \mathbb{R}$, of the form

$$\begin{aligned} &[r(t)\Psi_p(x')]' = -F(t,x,y) \\ &[q(t)\Psi_k(y')]' = G(t,x,y) \end{aligned} \tag{S}$$

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This is in the final form and completes recent results by the authors.

where we assume $p, k > 1$ and

(i) $r,\ q : [0,\infty) \mapsto (0,\infty)$ continuous,

(ii) $F,\ G\ :\ [0,\infty) \times (0,\delta] \times (0,\delta] \mapsto (0,\infty)$ continuous, with δ a suitably small positive constant,

(iii) $\exists f,\ g : [0,\infty) \times (0,\delta] \mapsto (0,\infty)$ continuous s.t. $F(t,u,v) \le f(t,v)$, $G(t,u,v) \le g(t,u)$, $\forall (t,u,v) \in [0,\infty) \times (0,\delta] \times (0,\delta]$.

Systems of type (1) comes out in the study of the existence of radial solutions for nonlinear coupled elliptic systems with p-laplacian $\Delta_p u = \mathrm{d}iv(|\nabla u|^{p-2}\nabla u)$

$$\Delta_p u = F(|x|,u,v)$$
$$\Delta_k v = G(|x|,u,v)$$

in exterior domains $E_a = \{x \in \mathbb{R}^N : |x| \ge a\}$, $N \ge 2$, $p > 1$, $k > 1$. Systems of partial differential equations of this form have been object of increasing interest in the last years, due to their relevance in applied sciences, especially in plasma physics, biomathematics, chemistry and, in general, in reaction-diffusion problems. Some relevant examples can be found in [2], [4], [5], [6], [7] and in the book [3] to which we address the interested reader.

The particular case of an ordinary differential system of two coupled generalized Emden-Fowler equations of the form

$$\begin{aligned}(p(t)|x'|^{\alpha-1}x')' &= \varphi(t)y^{\lambda}\\ (q(t)|y'|^{\beta-1}y')' &= \psi(t)x^{\mu}.\end{aligned} \tag{2}$$

has been recently studied in [8], [9], [12], with regard to existence of positive decreasing solutions, but the sign condition on the nonlinear terms is opposite to our case. The results here presented are therefore both more general and complementary to the corresponding ones in [8], [9],[12]. As both the nonlinear terms in (2) have positive sign, the dynamics which is described is completely different from our case; further (2) seems to be more close to the case of a single equation

$$[r(t)\Psi_p(x')]' = f(t,x).$$

that has been considered, for instance, in [1], [11] (see also the references therein).

Here we want to present some existence results which complete the ones obtained by the authors in [10]; in particular system (1) is more general than the one considered in [10] since the nonlinear terms F and G in (1) depend on both the unknowns x and y. No assumption is done on the continuity and on the boundedness of F and G in a neighborhood of $v = 0$ and of $u = 0$ respectively, thus we can treat the regular case and the singular case at the same time, and the forced case as well. For sake of completeness we will also quote the necessary and sufficient conditions that can be derived for a Emden-Fowler type system, whose proofs can be found in [10].

We end this section stating some definitions.

In this contribution we deal with the existence of *decaying solutions* of (1), i.e. solutions (x, y) such that x and y are eventually positive nonincreasing and $x(\infty) = y(\infty) = 0$. If (x, y) is a decaying solution of (1), then the first quasiderivative of x, $x^{[1]} := r(t)\Psi_p(x'(t))$ is eventually negative decreasing and the first quasiderivative of y, $y^{[1]} := q(t)\Psi_k(y'(t))$ is eventually negative increasing. Thus they admit limit as $t \to \infty$ and $-\infty \le x^{[1]}(\infty) < 0$, $-\infty < y^{[1]}(\infty) \le 0$. A decaying solution (x, y) is called *regularly decaying* if $x^{[1]}$ is bounded and $y^{[1]}$ tends to a (negative) nonzero limit, *strongly decaying* if $x^{[1]}$ is bounded and $y^{[1]}$ tends to zero. The set of regularly decaying solutions will be denoted by D_R and the set of strongly decaying solutions with D_S. In the next section we will find minimal conditions in order that (1) has solutions in the class D_R and in D_S, using both a topological approach (the Shauder-Tychonoff fixed point theorem in a Frechét space) and integral inequalities. This method allows us also to obtain an asymptotic estimate of the convergence rate of the solutions.

Finally, we close this section by giving two necessary conditions that can be easily proved (see [10]); the first one will be always assumed in the following, the second one will be assumed only in the existence results in D_R:

– If (1) has at least one *decaying solution* then the condition

$$\int_0^\infty \Psi_{p^*}\left(\frac{1}{r(t)}\right) dt < \infty \tag{H1}$$

is satisfied, where p^* is the conjugate numbers of p, i.e. $1/p + 1/p^* = 1$.

– If (1) has at least one *regularly decaying solution* then conditions (3) and

$$\int_0^\infty \Psi_{k^*}\left(\frac{1}{q(t)}\right) dt < \infty \tag{H2}$$

are satisfied, where k^* is the conjugate numbers of k.

2 Existence results in D_R and in D_S for (1)

Concerning the existence of solutions in the class D_R, the following result holds.

Theorem 1. *Assume* (3), (4) *and*

1. *there exist a positive continuous functions φ_1 on $[0, \infty)$, a nonnegative continuous function φ_2 on $[0, \infty)$ and a monotone positive continuous function $\hat{f}$ on $(0, \delta]$ such that for $(t, u) \in [0, \infty) \times (0, \delta]$*

$$\begin{gathered} f(t,u) \le \varphi_1(t)\hat{f}(u) + \varphi_2(t), \\ \int_0^\infty \varphi_2(t)\, dt < \infty, \quad \int_0^\infty \varphi_1(t) \hat{f}\left(\int_t^\infty \Psi_{k^*}\left(\frac{1}{q(\tau)}\right) d\tau\right) dt < \infty; \end{gathered} \tag{5}$$

2. *there exist a positive continuous functions γ_1 on $[0,\infty)$, a nonnegative continuous functions γ_2 on $[0,\infty)$ and a monotone positive continuous function $\hat{g}$ on $(0,\delta]$ such that for $(t,u) \in [0,\infty) \times (0,\delta]$*

$$\begin{gathered} g(t,u) \leq \gamma_1(t)\hat{g}(u) + \gamma_2(t), \\ \int_0^\infty \gamma_2(t)\,dt < \infty, \quad \int_0^\infty \gamma_1(t)\hat{g}\left(\int_t^\infty \Psi_{p^*}\left(\frac{1}{r(\tau)}\right)d\tau\right)dt < \infty. \end{gathered} \tag{6}$$

Then (1) has solutions in the class D_R.

Proof. The argument is a slight generalization of the one employed in [10] in the proof of Theorem 1, in which the particular case of a nonforced system is considered. We denote by $C[t_0,\infty)$ the locally convex space of all continuous functions defined on $[t_0,\infty)$ with the topology of uniform convergence on any compact subintervals of $[t_0,\infty)$. Thus $C[t_0,\infty)$ is a Fréchet space. We choose $t_0 \geq 0$ such that

$$\begin{aligned} &\int_{t_0}^\infty \varphi_2(t)\,dt \leq \frac{1}{4}, \quad \int_{t_0}^\infty \varphi_1(t)\hat{f}\left(\int_t^\infty \Psi_{k^*}\left(\frac{1}{q(\tau)}\right)d\tau\right)dt \leq \frac{1}{4}, \\ &\int_{t_0}^\infty \gamma_2(t)\,dt \leq \frac{1}{4}, \quad \int_{t_0}^\infty \gamma_1(t)\hat{g}\left(\int_t^\infty \Psi_{p^*}\left(\frac{1}{r(\tau)}\right)d\tau\right)dt \leq \frac{1}{4}. \end{aligned} \tag{7}$$

Let

$$m = \max\left\{\int_{t_0}^\infty \Psi_{k^*}\left(\frac{1}{q(\tau)}\right)d\tau; \quad \int_{t_0}^\infty \Psi_{p^*}\left(\frac{1}{r(\tau)}\right)d\tau\right\}$$

and, without loss of generality, assume $2m \leq \delta$.
Consider the set $\Omega \subset C[t_0,\infty) \times C[t_0,\infty)$ given by

$$\begin{aligned} \Omega = \Big\{ &(u,v) \in C[t_0,\infty) \times C[t_0,\infty) \text{ such that:} \\ &\int_t^\infty \Psi_{p^*}\left(\frac{M_1}{r(\tau)}\right)d\tau \leq u(t) \leq \int_t^\infty \Psi_{p^*}\left(\frac{M_2}{r(\tau)}\right)d\tau, \\ &\int_t^\infty \Psi_{k^*}\left(\frac{N_1}{q(\tau)}\right)d\tau \leq v(t) \leq \int_t^\infty \Psi_{k^*}\left(\frac{N_2}{q(\tau)}\right)d\tau \Big\} \end{aligned}$$

where $M_i, N_i, i = 1,2$ are four suitable positive constants. On this set we define the operator T with values in $C[t_0,\infty) \times C[t_0,\infty)$, by

$$\begin{aligned} T(u,v) &= (T_1(u,v), T_2(u,v)) \\ T_1(u,v)(t) &= \int_t^\infty \Psi_{p^*}\left[\frac{1}{r(s)}\left(M_2 - \int_s^\infty F(\tau,u(\tau),v(\tau))\,d\tau\right)\right]ds, \\ T_2(u,v)(t) &= \int_t^\infty \Psi_{k^*}\left[\frac{1}{q(s)}\left(N_1 + \int_s^\infty G(\tau,u(\tau),v(\tau))\,d\tau\right)\right]ds. \end{aligned}$$

We have to prove that T is continuous and maps Ω into a compact subset of Ω.

First we consider the case $\hat{f}$ and $\hat{g}$ nondecreasing on $(0,\delta]$; for this case let $M_1 = N_1 = 1/2$, $M_2 = N_2 = 1$.
(i) $T(\Omega) \subset \Omega$. The positivity of F and G immediately implies, for every $(u,v) \in \Omega$

$$T_1(u,v)(t) \le \int_t^\infty \Psi_{p^*}\left(\frac{1}{r(s)}\right) ds, \quad T_2(u,v)(t) \ge \int_t^\infty \Psi_{k^*}\left(\frac{1/2}{q(s)}\right) ds.$$

The assumptions on F and G, together with (5) and (7), imply that for any $(u,v) \in \Omega$ it holds

$$\int_{t_0}^\infty F(t,u(t),v(t))\,dt \le \int_{t_0}^\infty \varphi_1(t)\hat{f}\left(\int_t^\infty \Psi_{k^*}\left(\frac{1}{q(\tau)}\right) d\tau\right) dt + \int_{t_0}^\infty \varphi_2(t)\,dt \le \frac{1}{2},$$
$$\int_{t_0}^\infty G(t,u(t),v(t))\,dt \le \int_{t_0}^\infty \gamma_1(t)\hat{g}\left(\int_t^\infty \Psi_{p^*}\left(\frac{1}{r(\tau)}\right) d\tau\right) dt + \int_{t_0}^\infty \gamma_2(t)\,dt \le \frac{1}{2}.$$

These inequalities imply

$$T_1(u,v)(t) \ge \int_t^\infty \Psi_{p^*}\left(\frac{1/2}{r(s)}\right) ds, \quad T_2(u,v)(t) \le \int_t^\infty \Psi_{k^*}\left(\frac{1}{q(s)}\right) ds,$$

and (i) follows.
(ii) $T(\Omega)$ is relatively compact. Since $T(\Omega) \subset \Omega$, functions in $T(\Omega)$ are equibounded. The equicontinuity of functions in $T(\Omega)$ easily follows by observing that, in virtue of the above estimates, for any $(u,v) \in \Omega$ it holds

$$0 \le -(T_1(u,v))'(t) \le \Psi_{p^*}\left(\frac{1}{r(t)}\right), \qquad 0 \le -(T_2(u,v))'(t) \le \Psi_{k^*}\left(\frac{1}{q(t)}\right).$$

(iii) T is continuous in $\Omega \subset C[t_0,\infty) \times C[t_0,\infty)$. Let $\{(u_n,v_n)\}$, $n \in \mathbb{N}$, be a sequence in Ω which uniformly converges on every compact interval I of $[t_0,\infty)$ to $(\bar{u},\bar{v}) \in \Omega$. In view of the assumption on F and of the upper bound (5), the Lebesgue dominated convergence theorem and the uniform convergence on I of the sequence $\{F(t,u_n,v_n(t))\}$ imply that the sequence $\left\{\int_t^\infty F(\tau,u_n(\tau),v_n(\tau))\,d\tau\right\}$ uniformly converges to $\int_t^\infty F(\tau,\bar{u}(\tau),\bar{v}(\tau))\,d\tau$ on I. Analogously, the upper bound

$$0 \le \Psi_{p*}\left[\frac{1}{r(t)}\left(1 - \int_t^\infty F(\tau,u_n(\tau),v_n(\tau))\,d\tau\right)\right] \le \Psi_{p*}\left(\frac{1}{r(t)}\right)$$

allows us to apply again the Lebesgue dominated convergence theorem to the sequence

$$\left\{\Psi_{p*}\left[\frac{1}{r(t)}\left(1 - \int_t^\infty F(\tau,u_n(\tau),v_n(\tau))\,d\tau\right)\right]\right\}.$$

It follows that the sequence $\{T_1(u_n,v_n)\}$ uniformly converges on I to $T_1(\bar{u},\bar{v})$, that is the continuity of T_1. The argument for the continuity of T_2 is quite the same and (iii) is proved.

Since (i), (ii), (iii) are satisfied, the Schauder-Tychonoff theorem implies that the operator T has a fixed point $(z,w) \in \Omega$. It is easy to show that (z,w) is solution of (1) on $[t_0,\infty)$ and $(z,w) \in D_R$.

In the case $\hat{f}$ and $\hat{g}$ nonincreasing on $(0,\delta]$ the assertion can be proved by using an argument analogous to that given in the first part of the proof and choosing $M_1 = N_1 = 1$, $M_2 = N_2 = 3/2$. Finally, when $\hat{f}$ is nondecreasing and $\hat{g}$ nonincreasing on $(0,\delta]$ or vice-versa, it is sufficient to choose $M_1 = N_2 = 1$, $M_2 = 3/2$, $N_1 = 1/2$ or $N_1 = M_2 = 1$, $M_1 = 1/2$, $N_2 = 3/2$ respectively. The details are left to the reader. □

Other existence results in the class D_R can be found in [10] (Th. 1 and Prop. 1) in case that the nonlinearities in (1) satisfy some additional assumptions.

Remark 2. The assumptions in Theorem 1 are quite general and Theorem 1 can be applied also to systems with singular forcing terms; an interesting case is when $F(t,u,v) = \varphi(t)u^{\lambda_1}v^{-\lambda_2}$ and/or $G(t,u,v) = \psi(t)u^{-\mu_1}v^{\mu_2}$, with $\lambda_1, \mu_2 \geq 0$ and $\lambda_2, \mu_1 > 0$.

With regards to existence of solutions of (1) in the class D_S, here we treat only the case in which $F(t,u,v)$ and $G(t,u,v)$ are both singular in a neighborhood of $v = 0$ and $u = 0$ respectively. The remaining cases, i.e. F and G both regular functions or one regular and the other singular in a neighborhood of $v = 0$ and $u = 0$ respectively, can be obtained with minor changes from the results in [10] (Th. 2, Th. 3, Prop. 2). The details are left to the reader. To state the following existence, we need a further assumption in addition to (i)-(iii).

Theorem 3. *Assume*
(iv) $\exists h : [0,\infty) \times (0,\delta] \mapsto (0,\infty)$ *continuous s.t.* $G(t,u,v) \geq h(t,u)$, $\forall (t,u,v) \in [0,\infty) \times (0,\delta] \times (0,\delta]$.

Let (3) *be satisfied and suppose that there exist two positive continuous functions* γ_1, φ_1 *on* $[0,\infty)$*, two nonnegative continuous functions* γ_2, φ_2 *on* $[0,\infty)$*, three nondecreasing positive continuous functions* $\hat{h}$, $\hat{g}$, $\hat{f}$ *on* $(0,\delta]$ *and two positive constants* $0 < N < M$*, such that for* $(t,u) \in [0,\infty) \times (0,\delta]$*:*

$$h(t,u) \geq \gamma_1(t)\hat{h}(u) + \gamma_2(t), \quad g(t,u) \leq \gamma_1(t)\hat{g}(u) + \gamma_2(t),$$
$$f(t,u) \leq \varphi_1(t)\hat{f}(u) + \varphi_2(t)$$
$$\int_0^\infty \gamma_2(t)\,dt < \infty, \qquad \int_0^\infty \varphi_2(t)\,dt < \infty,$$
$$\int_0^\infty \Psi_{k^*}\left(\frac{D_N(t)}{q(t)}\right)dt < \infty, \quad \int_0^\infty \varphi_1(t)\hat{f}\left(\int_t^\infty \Psi_{k^*}\left(\frac{C_M(s)}{q(s)}\right)ds\right)dt < \infty,$$

where

$$D_N(t) = \int_t^\infty \left[\gamma_1(s)\hat{g}\left(\int_s^\infty \Psi_{p^*}\left(\frac{N}{r(\tau)}\right)d\tau\right) + \gamma_2(s)\right]ds$$
$$C_M(t) = \int_t^\infty \left[\gamma_1(s)\hat{h}\left(\int_s^\infty \Psi_{p^*}\left(\frac{M}{r(\tau)}\right)d\tau\right) + \gamma_2(s)\right]ds.$$

Then (1) has solutions in the class D_S.

Proof. Choose $t_0 \geq 0$ such that

$$\int_{t_0}^{\infty} \left[\varphi_1(t)\hat{f}\left(\int_t^{\infty} \Psi_{k^*}\left(\frac{C_M(s)}{q(s)}\right) ds\right) + \varphi_2(t)\right] dt \leq M - N.$$

Let

$$m_1 = \max\left\{\int_{t_0}^{\infty} \Psi_{p^*}\left(\frac{M}{r(t)}\right) dt, \int_{t_0}^{\infty} \Psi_{k^*}\left(\frac{D_N}{q(t)}\right) dt\right\}$$

and, without loss of generality, suppose $2m_1 < \delta$. Define the set Ω as follows

$$\Omega = \Bigg\{(u,v) \in C[t_0,\infty) \times C[t_0,\infty) \text{ such that:}$$
$$\int_t^{\infty} \Psi_{p^*}\left(\frac{N}{r(\tau)}\right) d\tau \leq u(t) \leq \int_t^{\infty} \Psi_{p^*}\left(\frac{M}{r(\tau)}\right) d\tau,$$
$$\int_t^{\infty} \Psi_{k^*}\left(\frac{C_M(\sigma)}{q(\sigma)}\right) d\sigma \leq v(t) \leq \int_t^{\infty} \Psi_{k^*}\left(\frac{D_N(\sigma)}{q(\sigma)}\right) d\sigma\Bigg\}$$

Consider the operator $\hat{T}: \Omega \mapsto C[t_0,\infty) \times C[t_0,\infty)$, $\tilde{T}(u,v) = (\tilde{T}_1(u,v), \tilde{T}_2(u,v))$, given by

$$\tilde{T}_1(u,v)(t) = \int_t^{\infty} \Psi_{p^*}\left(\frac{1}{r(s)}\left(M - \int_s^{\infty} F(\tau,u(\tau),v(\tau))\, d\tau\right)\right) ds,$$
$$\tilde{T}_2(u,v)(t) = \int_t^{\infty} \Psi_{k^*}\left(\frac{1}{q(s)}\int_s^{\infty} G(\tau,u(\tau),v(\tau))\, d\tau\right) ds.$$

Clearly $\tilde{T}_1(u,v)(t) \leq \int_t^{\infty} \Psi_{p^*}\left(\frac{M}{r(\tau)}\right) d\tau$ for every $(u,v) \in \Omega$, $t \geq t_0$. The following inequalities hold for $s \geq t_0$:

$$\int_s^{\infty} F(t,u(t),v(t))dt \leq \int_s^{\infty} f(t,v(t))dt \leq \int_s^{\infty} \big(\varphi_1(t)\hat{f}(v(t)) + \varphi_2(t)\big)dt$$
$$\leq \int_{t_0}^{\infty} \left[\varphi_1(t)\hat{f}\left(\int_t^{\infty} \Psi_{k^*}\left(\frac{C_M(s)}{q(s)}\right) ds\right) + \varphi_2(t)\right] dt \leq M - N;$$
$$\int_s^{\infty} G(t,u(t),v(t))dt \leq \int_s^{\infty} g(t,u(t))dt \leq \int_s^{\infty} \big(\gamma_1(t)\hat{g}(u(t)) + \gamma_2(t)\big)dt$$
$$\leq \int_s^{\infty} \left[\gamma_1(t)\hat{g}\left(\int_t^{\infty} \Psi_{p^*}\left(\frac{N}{r(s)}\right) ds\right) + \gamma_2(t)\right] dt = D_N(s);$$
$$\int_s^{\infty} G(t,u(t),v(t))dt \geq \int_s^{\infty} h(t,u(t))dt \geq \int_s^{\infty} \big(\gamma_1(t)\hat{h}(u(t)) + \gamma_2(t)\big)dt$$
$$\geq \int_s^{\infty} \left[\gamma_1(t)\hat{h}\left(\int_t^{\infty} \Psi_{p^*}\left(\frac{M}{r(s)}\right) ds\right) + \gamma_2(t)\right] dt = C_M(s).$$

From the above estimates it follows that the operator $\tilde{T}$ maps Ω into itself. To apply Tychonov fixed point theorem, it is sufficient to show that $\tilde{T}(\Omega)$ is relatively compact and $\tilde{T}$ is continuous in Ω. The argument is similar to that given in the proof of Theorem 1 and the details are left to the reader. Then the operator $\tilde{T}$ has at least one fixed point (z, w). It is easy to show that (z, w) is solution of (1) on $[t_0, \infty)$ and both components of the fixed point are positive in each interval of $[t_0, \infty)$: so $(z, w) \in D_S$. □

Remark 4. Unlike Theorem 1, that can be applied to both the regular case and the singular one (see Remark 2), Theorem 3 is for the singular case only. Existence results for the regular case can be found in [10] and applied to (1) with minor changes. This fact depends on the structure of the operator $\tilde{T}$ whose fixed points are singular solutions of (1).

The conditions stated in Theorem 1 and in Theorem 3 are sharp conditions for existence of solutions of (1) in D_R and in D_S respectively, since these conditions becomes also necessary in case of a forced Emden-Fowler type system

$$\begin{aligned} [r(t)\Psi_p(x')]' &= -\varphi_1(t)\Psi_\mu(y) - \varphi_2(t), \quad \mu \neq 1 \\ [q(t)\Psi_k(y')]' &= \gamma_1(t)\Psi_\nu(x) + \gamma_2(t), \quad \nu \neq 1, \end{aligned} \tag{8}$$

where we assume $p, k > 1$, $\mu, \nu \neq 1$, $\varphi_1, \gamma_1 : [0, \infty) \mapsto (0, \infty)$ continuous, $\varphi_2, \gamma_2 : [0, \infty) \mapsto [0, \infty)$ continuous, and $\varphi_2, \gamma_2 \in L^1(0, \infty)$. The following result holds, whose proof can be found in [10]

Theorem 5 ([10] Th. 3, Th. 4, Prop. 3).

- *The Emden-Fowler forced system* (8) *has solutions in the class* D_R *if and only if conditions* (3), (4) *and*

$$\int_0^\infty \varphi_1(t) \left(\int_t^\infty \Psi_{k^*}\left(\frac{1}{q(\tau)} \right) d\tau \right)^{\mu-1} dt < \infty$$
$$\int_0^\infty \gamma_1(t) \left(\int_t^\infty \Psi_{p^*}\left(\frac{1}{r(\tau)} \right) d\tau \right)^{\nu-1} dt < \infty$$

  are satisfied.
- *The Emden-Fowler forced system* (8) *has solutions in the class* D_S *if and only if conditions* (3) *and*

$$\int_0^\infty \varphi_1(t) \left(\int_t^\infty \vartheta(s)\, ds \right)^{\mu-1} dt < \infty$$

  are satisfied, where

$$\vartheta(t) = \Psi_{k^*}\left[\frac{1}{q(t)} \int_t^\infty \left(\gamma_1(s) \left(\int_s^\infty \Psi_{p^*}\left(\frac{1}{r(\tau)} \right) d\tau \right)^{\nu-1} + \gamma_2(s) \right) ds \right].$$

Comments and examples that illustrate the previous result can be found in [10].

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