2012

Free Convection Along a Vertical Wavy Surface in a Nanofluid

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Bachelor of engineering in Mechanical Engineering

Acharya Nagarjuna University, India

June, 2008

Submitted in partial fulfillment of requirements for the degree

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

at the

CLEVELAND STATE UNIVERSITY

December, 2011

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ACKNOWLEDGEMENTS

I would like to thank my academic advisor and thesis committee chairperson Dr. Rama Subba Reddy Gorla for his invaluable guidance, encouragement and time during the period of this work. I appreciate his knowledge in the field and his willingness to share it with me by engaging in new research projects. His appetite for research and his kind and helpful nature always inspire me to work hard.

I would like to thank my thesis committee members Dr. Asuquo B. Ebiana and Dr. Majid Rashidi for their encouragement and valuable time to review my thesis.

I sincerely appreciate love and help of all my friends specially Kaustubh, Lokamanya, Sowmya Reddy, Ramya Sree, Prabu and Uday who encouraged and helped me a lot in downtimes during this phase of my life.

I would like to thank my mother, father and brother for allowing me to realize my own potential. I am thankful to almighty God for his grace, which is more than enough for each new day!

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DEEPAK RAVIPATI

ABSTRACT

The study of this paper is to introduce a boundary layer analysis for the fluid flow and heat transfer characteristics of an incompressible nanofluid along a vertical wavy surface in a nanofluid. The Resulting transformed governing equations are solved numerically by an implicit finite-difference scheme (Keller-Box method). The results are presented for the major parameters including the wave amplitude α, buoyancy ratio parameter N_r, Brownian motion parameter N_b, Thermophoresis parameter N_t and Lewis number L_e. A systematic study on the effects of the various parameters of the local frication factor, surface heat transfer rate (Nusselt number) and mass transfer rate (Sherwood number) characteristics is carried out. The Obtained results are presented graphically.

Keywords: Nanofluid, Free Convection and Wavy Surface.

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NOMENCLATURE

D_B	Brownian diffusion coefficient
D_T	Thermophoretic diffusion coefficient
L_e	Lewis number
N_r	Buoyancy Ratio
N_b	Brownian motion parameter
N_t	Thermophoresis parameter
U	Reference velocity
C_p	Specific heat at constant pressure
C_{fx}	Local skin-friction coefficient
f	Dimensionless stream function
g	Acceleration due to gravity
G_r	Grashof number
k (T)	Thermal conductivity
N_u	Local Nusselt Number
P_r	Prandtl number
q_w	Heat flux at the surface
T	Temperature of the fluid in the boundary layer
T_∞	Temperature of the ambient fluid
T_w	Temperature at the surface

u, v The dimensionless x and y- component of the velocity

$\hat{u}, \hat{v}$ The dimensional $\hat{x}$ and $\hat{y}$ component of the velocity

x, y In the direction along and normal to the surface

Greek Symbols:

β Volumetric coefficient of thermal expansion

ψ Stream function

α Amplitude of the wavy surface

η Non-dimensional similarity variable

τ Shear stress

ρ Density of the fluid

ν_∞ Reference kinematic viscosity

μ Viscosity of the fluid

μ_∞ Dynamic viscosity of the ambient fluid

θ Dimensionless temperature function

σ(x) Surface profile function defined in (1)

∅ Nano-particle volume fraction

ρ_p Nano-particle mass density

ρ_f Fluid density

ρ_{c_f} Heat capacity of fluid

Subscript:

W Wall conditions

∞ Ambient temperature

X Differentiation with respect to x

Superscript:

$'$ Differentiation with respect to η

CHAPTER I

INTRODUCTION

The study of convective heat transfer in nanofluids is gaining a lot of attention. The nanofluids have many engineering applications in the industry since materials of nanometer size have unique physical and chemical properties. The term nanofluids refers to a liquid containing a dispersion of submicron solid-liquid composite materials consisting of solid nanoparticles or nanofibers with sizes typically 1-100 nm suspended in liquid. Modern technology makes it possible to produce ultra fine metallic or nonmetallic particles of nanometer dimensions, which makes revolutions in heat transfer enhancement methods. Considering a very small particle size and their small volume fraction (<1% volume fraction) of Cu nanoparticles with ethylene glycol or carbon nanotubes dispersed in oil is reported to increase the inherently poor thermal conductivity of the liquid by 40% and 150%, respectively [1,2]. Also the relatively large surface area of nanoparticles increases the stability and reduces the sedimentation of nanoparticles. A more dramatic improvement in that transfer efficiency is expected a results decreasing the particle size in a suspension because heat transfer takes place at the surface of the particles. High concentrations (>10%) of particles is required to achieve such enhancement in case of conventional particle-liquid suspensions. In general problems of rheology and stability are amplified at high concentration, precluding the widespread use of conventional slurries as heat transfer fluids. In some cases, the observed enhancement in thermal conductivity of nanofluids is the order of magnitude larger than predicted by well-established theories. Other perplexing results is that rapidly evolving field include a surprising strong temperature dependence of thermal conductivity [3] and a three-fold higher critical heat flux compared with the base fluids [4]. The feasibility of nanofluids in nuclear applications is made by improving the performance of any water-cooled nuclear system with heat removal limited has been studies by kim et al. [5] at the Massachusetts Institute of technology(MIT) is exploring the nuclear applications of nanofluids, specifically in Main reactor coolant for pressurized water reactors (PWRs), Coolant for the emergency core cooling system(ECCS) of both PWRs and boiling water reactors and coolant for in-vessel retention of the molten core during severe accidents in high power-density light water reactors.[6].

Nanofluids can be utilized where straight heat transfer enhancement is very important as in most Heat transfer applications such as industrial cooling applications, smart fluids, nuclear reactors, extraction of geothermal power and other energy sources. Other applications, such as nanofluid coolant, nanofluid in fuel, Brakes and other vehicular nanofluids. Electronic applications such as cooling of microchips, microscale fluidic applications. Biomedical applications such as nanodrug delivery, cancer therapeutics, cryopreservation, nanocryosurgery, sensing and imaging. Review of experimental studies indicates that nanofluids have the potential to conserve 1 trillion Btu of energy for U.S. industry by replacing cooling and heating water with nanofluid. For the U.S. electric power industry, using nanofluids in closed-loop cooling cycles could save about 10–30 trillion Btu per year (equivalent to the annual energy consumption of about 50,000–150,000 households). The related emissions reductions would be approximately 5.6 million metric tons of carbon dioxide; 8,600 metric tons of nitrogen oxides; and 21,000 metric tons of sulfur dioxide [7]. There is a growth in the use of colloids which are nanofluids in the biomedical industry for sensing and imaging purposes. This is directly related to the ability to design novel materials at the nanoscale level, alongside recent innovations in analytical and imaging technologies for measuring and manipulating nonmaterials. This has led to the fast development of commercial applications which uses a wide variety of manufactured nanoparticles. The production, use and disposal of manufactured nanoparticles will lead to discharges in air, soils and water systems. Negative effects are likely, so quantification and minimization of these effects on environmental health is necessary. True knowledge of concentration and physicochemical properties of manufactured nanoparticles under realistic conditions is important to predicting their fate, behavior and toxicity in the natural aquatic environment. The aquatic colloid and atmospheric ultrafine particle literature both offer evidence as to the likely behavior and impacts of manufactured nanoparticles [8].

In geothermal power, energy extraction from the earth's crust involves high temperatures around 5000C to 10000C, nanofluids can be employed to cool the pipes exposed to such high temperatures. When drilling, nanofluids can serve in cooling the machinery and equipment working in environments with high temperatures. Nanofluids could be used as a working fluid to extract energy from the earths core [9], However previous research has concentrated on the problem of natural convection for the case of flat plates, on the other hand, few studies have been carried out to examine the effect of geometric complexity, such as irregular surfaces, the convection heat transfer. That is because complicated boundary conditions or external flow fields are difficult to deal with. However, the prediction of heat transfer form an irregular surface is of fundamental importance, and is encountered in several heat transfer devices, such as flat-plate solar collectors and flat-plate condensers in refrigerators. For example, in cavity wall insulating systems, grain storage installations geothermal and industrial applications, such as the dispersion of chemical contaminants through water-saturated soil, the migration of moisture through air contained in fibrous insulations heat radiator in industry. Irregularities frequently occur in the process of manufacture. Moreover, surfaces are sometimes intentionally roughened to enhance heat transfer because the presence of rough surfaces disturbs the flow and alters the heat transfer rate. Despite of many investigations done in irregular surfaces, these investigations have only concentrated on the problems of free convection along a vertical wavy surface in nanofluids.

CHAPTER II

LITERATURE REVIEW

Heat transfer problems have several engineering applications such as thermal energy storage, crude oil extraction, geothermal energy recovery, ground water pollution and flow through filtering media. Described below are a few studies which show work done in the area of the geometric complexity, such as irregular surfaces, on the convection heat transfer in nanofluids. That is because complicated boundary conditions or external flow fields are difficult to work with. However, the prediction of heat transfer from an irregular surface is of fundamental importance, and is encountered in several heat transfer devices, such as flat-plate solar collectors and flat-plate condensers in refrigerators. Moreover, surfaces are sometimes intentionally roughened to enhance heat transfer for the presence of rough surfaces disturbs the flow and alters the heat transfer rate. However, all of the previous studies considered only the case of flat plate or simple two-dimensional bodies, and few have been on wavy surfaces.

Ghosh and Yao [10] have studied laminar free convection along a semi-infinite vertical wavy surface. This is a model problem for the investigation of heat transfer from roughened surfaces in order to understand heat transfer enhancement. In many applications of practical importance, however, the surface temperature is non-uniform. In this note, the case of uniform surface heat flux rate, which is often approximated in practical applications and is easier to measure in a laboratory, has been investigated. Numerical results have been obtained for a sinusoidal wavy surface. The results show that the Nusselt number varies periodically along the wavy surface. The wavelength of the Nusselt number variation is half of that of the wavy surface, while the amplitude gradually decreases downstream where the boundary layer grows thick. It is hoped that experimental results will become available in the near future to verify the results of this investigation.

Molla, Hossain and Yao [11] investigated the effect of internal heat generation /absorption on a steady two-dimensional natural convection flow of viscous incompressible fluid along a uniformly heated vertical wavy surface. The equations are mapped into the domain of flat vertical plate, and then solved numerically employing the implicit finite difference method, known as Keller-box scheme. Effects of the pertinent parameters, such as the heat generation/absorption parameter (Q), the amplitude of the waviness (α) of the surface and Prandtl number Pr on the rate of heat transfer in terms of the local Nusselt number (Nu_x), isotherms and the streamlines were discussed.

Hossain and Rees [12] studied the combined heat and mass transfer in natural convection flow from a vertical wavy surface. In this paper, effects of combined buoyancy forces from mass and thermal diffusion by natural convection flow from a vertical wavy surface have been investigated using the implicit finite difference method. The study was focused on the evolution of the surface shear stress, $f''(0)$, rate of heat transfer, $g'(0)$, and surface concentration gradient, $h'(0)$ with effect of different values of the governing parameters, such as the Schmidt number Sc ranging from 7 to 1500 which are appropriate for different species concentration in water (Pr = 7.0), the amplitude of the waviness of the surface ranging from 0.0 to 0.4 and the buoyancy parameter, w, ranging from 0.0 to 1.

A previously proposed transformation has been applied by Yao, L. S [13] to the natural-convection boundary layer along a complex vertical surface created from two sinusoidal functions, a fundamental wave and its first harmonic. The numerical results demonstrate that the additional harmonic substantially alters the flow field and temperature distribution near the surface. The conclusion that the averaged heat-transfer rate per unit-wetted wavy surface is less than that of a corresponding flat plate has been confirmed for this more complex surface. On the other hand, the total heat-transfer rates for a complex surface are greater than that of a flat plate. According to the numerical results the enhanced total heat-transfer rate seems to depend on the ratio of amplitude and wavelength of a surface.

Neild and Kuznestov [14] have studied the convective heat transfer in a nanofluid past a vertical plate. According to this study a model was used in which Brownian motion and thermophoresis are accounted with the simplest possible boundary conditions, namely those in which both the temperature and the nanoparticle fraction are constant along the wall. Using this, solution was obtained which depends on five dimensionless parameters, namely a Prandtl number P_r, a Lewis number L_e, a buoyancy-ratio parameter N_r, a Brownian motion parameter N_b, and a thermophoresis parameter N_t. They have explored the way in which the wall heat flux, represented by a Nusselt number Nu and then scaled in terms of - local Rayleigh number defined in the paper to produce a reduced Nusselt number, depends on these five parameters.

Chamkha and Khaled [15] have solved the problem of coupled heat and mass transfer by natural convection from a semi-infinite inclined flat plate in the presence of an external magnetic field and internal heat generation or absorption effects. Surface of the plate has a power- law variation of both wall temperature and concentration and is permeable to allow for possible fluid wall suction or blowing. The resulting governing equations were solved by using similarity transformation and solved numerically by implicit iterative and finite-difference scheme methods. They performed parametric study of all involved parameters and conducted a set of numerical results for the velocity and temperature profiles as well as the skin-friction parameter, average Nusselt number, and the average Sherwood number is illustrated graphically to show typical trends of the solutions.

Wang and Chen [16] studied mixed convection boundary layer flows of Non-Newtonian fluids over the wavy surfaces using the coordinate transformation and the cubic spline collocation numerical method. The effects of the wavy geometry, the buoyancy parameter and the generalized Prandtl number for pseudoplastic fluids, Newtonian fluids and dilatant fluids on the skin-friction coefficient, local and mean Nusselt numbers have been graphically studied. As per the results both higher generalized Prandtl numbers and buoyancy parameters were seen to enhance the influence of wavy surfaces on the local Nusselt number, irrespective of whether the fluids were Newtonian fluids or non-Newtonian fluids. Moreover, the irregular surfaces were having higher total heat flux than that of corresponding flats plate for any fluid.

Yang, Chen and Lin [17] applied Prandtl transformation method to study the natural convection of Non-Newtonian fluids along a wavy vertical plate in the presence of a magnetic field. A simple transformation was proposed, to transform the governing equations into the boundary layer equations, and solved numerically by the cubic spline approximation. A simple coordinate transformation was employed to transfer the complex wavy surface to a vertical flat plate for a constant wall temperature by the numerical method. The effects of the magnetic field parameter, the wavy geometry and the non-Newtonian nature of the fluids on the flow characteristics and heat transfer were discussed in detail. It was found that the action of the magnetic field decelerates the flow, does decreasing the Nusselt number.

Hossain and Pop [18] formulated the problem of the boundary layer flow and heat transfer on a continuous moving wavy surface in a quiescent electrically conducting fluid with a constant transverse magnetic field. The resulting parabolic differential equations were solved numerically using the Keller-box scheme. They presented the detailed results for the velocity and temperature fields, and also the results for the skin-friction coefficient and the local Nusselt number. Those results are given for different values of the amplitude of the wavy surface and magnetic parameter when the Prandtl number equals 0.7. They showed that the flow and heat transfer characteristics are substantially altered by both the magnetic parameter and the amplitude of the wavy surface

Hossain, Kabir and Rees [19] studied the effect of a temperature dependent viscosity on natural convection flow of viscous incompressible fluid from a vertical wavy surface has been investigated using an implicit finite difference method. They focused on the evaluation of the local skin-friction and the local Nusselt number. The governing parameters were, the Prandtl number, Pr, ranging from 1 to 100, the amplitude of the waviness of the surface, alpha, ranging from 0.0 to 0.4 and the viscosity variation parameter, epsilon, ranging from 0.0 to 6.

Kumari, Pop and Takhar [20] considered a theoretical analysis of laminar free-convection flow over a vertical isothermal wavy surface in a Non-Newtonian power-law fluid. First they casted the governing equations into a non dimensional form by using suitable boundary-layer variables that subtract out the effect of the wavy surface from the boundary conditions. Then they solved the boundary-layer equations numerically by a very efficient implicit finite-difference method known as the Keller-Box method. A sinusoidal surface was used to elucidate the effects of the power-law index, the amplitude wavelength, and the Prandtl number on the velocity and temperature fields, as well as on the local Nusselt number. The results obtained showed that the local Nusselt number varies periodically along the wavy surface. The wave-length of the local Nusselt number variation is half that of the wavy surface, irrespective of whether the fluid is a Newtonian fluid or a Non-Newtonian fluid.

Lakshmi and Sibanda [21] In their article, studied free convection of heat and mass transfer along a vertical wavy surface in a Newtonian fluid saturated Darcy porous medium by considering cross diffusion (namely the Soret and the Dufour effects) in the medium. The vertical wavy wall and the flow governing equations are transformed to a plane geometry case by using a suitable transformation. They then presented a similarity solution to the problem under the large Darcy–Rayleigh number assumption. The governing partial differential equations were reduced to a set of ordinary differential equations that are integrated using numerical methods to study the nature of the non-dimensional heat and mass transfer coefficients in the medium. The results are presented for a range of the flow governing parameters such as the diffusivity ratio parameter, the buoyancy ratio parameter, the Soret parameter, the Dufour parameter and the amplitude of the wavy surface.

Hossain, Kabir and Rees [22] in their paper presented the effect of a temperature dependent viscosity on natural convection flow of viscous incompressible fluid from a vertical wavy surface has been investigated using an implicit finite difference method. They have focused their attention on the evaluation of the local skin-friction and the local Nusselt number. The governing parameters are the Prandtl number, Pr, ranging from 1 to 100, the amplitude of the waviness of the surface α, ranging from 0.0 to 0.4 and the viscosity variation parameter, $\in$, ranging from 0.0 to 6.

In this study, Chen, Yang and Chang [23] used Prandtl's transposition theorem to stretch the ordinary coordinate-system in certain direction. The small wavy surface can be transferred into a calculable plane coordinate-system. The new governing equations of turbulent forced convection along wavy surface were derived from complete Navier–Stokes equations. A simple transformation was proposed to transform the governing equations into boundary layer equations for solution by the cubic spline collocation method. The effects such as the wavy geometry, the local skin-friction and Nusselt number were studied. The results of the simulation show that it is more significant to increase heat transfer with small wavy surface than plat surface

Kumar and Shalini [24] analyzed the natural convection heat transfer from a vertical wavy surface in a thermally stratified fluid saturated porous medium under Forchheimer based Non-Darcian assumptions. They reduced the governing equations to boundary layer equations based on non-similarity transformation deduced by scale analysis. Those simplified partial differential equations are solved numerically by a finite difference scheme following the Keller Box approach. Extensive numerical simulations are carried out for various values of wavelength-to-amplitude ratio of wavy vertical surface at different thermal stratification levels of a porous medium both under Darcian and Non-Darcian assumptions. Results of their study are compared with those available in literature. In the Darcian case, local heat fluxes along the wavy vertical surface are periodic with an oscillatory pattern of period, which is exactly half of the period of a vertical wavy surface. In the non-Darcian case, the local heat fluxes continue to be periodic but with a complex oscillatory pattern of period exactly the same as that of the vertical wavy surface. Increasing S or Gr^*or α leads to a fall in local Nusselt number.

Polidori, Fohanno and Nguyen [25] have studied the problem of natural convection flow and heat transfer of Newtonian alumina–water nanofluids over a vertical semi-infinite plate from a theoretical viewpoint, for a range of nanoparticle volume fractions up to 4%. The analysis is based on a macroscopic modeling and under the assumption of constant thermophysical nanofluid properties. They proposed semi-analytical formulas of heat transfer parameters for both the uniform wall temperature (*UWT*) and uniform heat flux (*UHF*) surface thermal conditions and found that natural convection heat transfer is not solely characterized by the nanofluid effective thermal conductivity and that the sensitivity to the viscosity model used seemed undeniable and plays a key role in the heat transfer behavior.

Mamun, Hossain and Gorla [26] investigated the natural convection laminar flow of a viscous incompressible fluid along a uniformly heated vertical wavy surface with temperature dependent viscosity and thermal conductivity. They considered the boundary layer regime when the Grashof number G_r is large. The appropriate transformation variables were used. The basic governing equations are first transformed to non-similar boundary layer form and then solved numerically employing the implicit finite difference method together with Keller-Box scheme. They presented the numerical results in terms of the streamlines and isothermals as well as fluid flow and heat transfer characteristics, namely, the local skin-frication coefficient and the local Nusselt number for a wide range of values of the viscosity and thermal conductivity variation parameter, surface waviness and the Prandtl number

Bachok, Ishak and Pop [27] have studied the steady boundary-layer flow of a nanofluid past a moving semi-infinite flat plate in a uniform free stream. Assumptions were made that the plate is moving in the same or opposite directions to the free stream to define the resulting system of nonlinear ordinary differential equations. Governing equations were solved using the Keller-Box method. The results are obtained for the skin-friction coefficient, the local Nusselt number and the local Sherwood number as well as the velocity, temperature and the nanoparticle volume fraction profiles for the governing parameters, namely, the plate velocity parameter, Prandtl number, Lewis number, the Brownian motion parameter and the thermophoresis parameter.

Xuan and Li [28] have experimentally investigated flow and convective heat transfer characteristics for Cu–water based nanofluids through a straight tube with a constant heat flux at wall. Results showed that the nanofluids give substantial enhancement of heat transfer rate compared to pure water.

Khan and Pop [29] have studied the problem of laminar fluid flow which results from the stretching of a flat surface in a nanofluid and investigated it numerically. The model they used for the nanofluid incorporates the effects of Brownian motion and thermophoresis and found solution which depends on the Prandtl number P_r, Lewis number L_e, Brownian motion number N_b and thermophoresis number N_t. They showed variation of the reduced Nusselt and reduced Sherwood numbers with N_b and N_t for various values of P_r and $L_e$in tabular and graphical forms and conclude that a reduced Nusselt number is a decreasing function while the reduced Sherwood number is increasing function of each values of the parameters L_e, L_e, N_b and N_t considered for study.

Koo and Kleinstreuer [30] have studied steady laminar liquid nanofluid flow in microchannels for conduction-convection heat transfer for two different base fluids water and ethylene glycol having copper oxide nanospheres at low volume concentrations. They conjugate the heat transfer problem for microheat-sinks solved numerically. They employed new models for the effective thermal conductivity and dynamic viscosity of nanofluids in light of aspect ratio, viscous dissipation and enhanced temperature effects for computation of the impact of nanoparticle concentrations in these two mixture flows on the microchannel pressure gradients, temperature profiles and Nusselt numbers.

CHAPTER III

NUMERICAL STUDY OF FREE CONVECTION ALONG A VERTICAL WAVY SURFACE IN A NANOFLUID

3.1 Mathematical formulation and analysis

We consider the steady free convection boundary layer flow past a vertical wavy plate placed in a nano-fluid. The co-ordinate system is chosen such that $\hat{x}$ -measures the distance along the wavy surface and $\hat{y}$ measures the distance normal outward, see figure (1). Here we consider a wavy surface that is described under the foregoing assumptions with introducing Overbeck-Boussinesq approximations, the equations governing the steady state flow can be written in two dimensional Cartesian coordinates($\hat{x}, \hat{y}$), in usual notation as

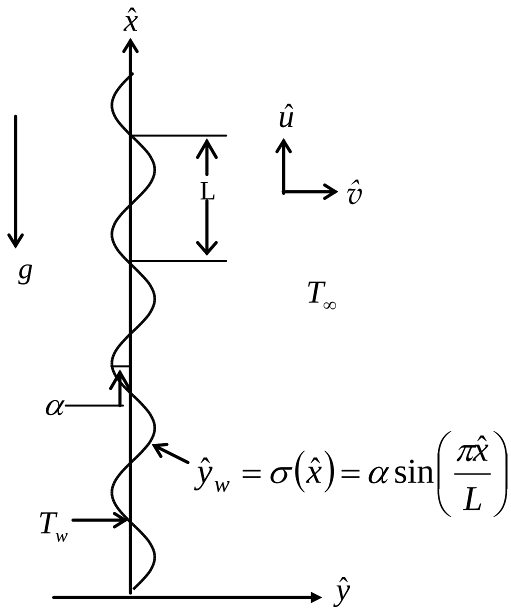

Figure 1: Physical model and coordinate system

$$\hat{y}_w = \sigma(\hat{x}) = \alpha \sin\left(\frac{\pi\hat{x}}{L}\right) \tag{1}$$

where L is half of the wavelength of the wavy surface. The geometry of the wavy surface and the two-dimensional Cartesian coordinate system are as shown in Figure1.

$$\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{v}}{\partial \hat{y}} = 0 \tag{2}$$

Momentum equation

$$\hat{u}\frac{\partial \hat{u}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{u}}{\partial \hat{y}} = -\frac{1}{\rho_{f_\infty}}\frac{\partial \hat{p}}{\partial \hat{x}} + \frac{\mu}{\rho_{f_\infty}}\frac{\partial^2 \hat{u}}{\partial \hat{y}^2} + g\beta(T - T_\infty)(1-\phi_\infty) - \frac{(\rho_p - \rho_{f_\infty})g(\mathrm{C} - \mathrm{C}_\infty)}{\rho_{f_\infty}} \tag{3}$$

$$\hat{u}\frac{\partial \hat{v}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{v}}{\partial \hat{y}} = -\frac{1}{\rho_{f_\infty}}\frac{\partial \hat{p}}{\partial \hat{y}} + \frac{\mu}{\rho_{f_\infty}}\frac{\partial^2 \hat{v}}{\partial \hat{y}^2} \tag{4}$$

Energy Equation

$$\hat{u}\frac{\partial T}{\partial \hat{x}}+\hat{v}\frac{\partial T}{\partial \hat{y}}=\alpha\frac{\partial T^2}{\partial \hat{y}^2}+\tau\left[D_B\frac{\partial C}{\partial \hat{y}}\frac{\partial T}{\partial \hat{y}}+\frac{D_T}{T_\infty}\left[\frac{\partial T}{\partial \hat{y}}\right]^2\right] \tag{5}$$

Concentration Equation

$$\hat{u}\frac{\partial C}{\partial \hat{x}}+\hat{v}\frac{\partial C}{\partial \hat{y}}=D_B\frac{\partial C^2}{\partial \hat{y}^2}+\frac{D_T}{T_\infty}\frac{\partial T^2}{\partial \hat{y}^2} \tag{6}$$

where $(\hat{x},\hat{y})$ are the dimensional coordinates in the vertical and horizontal directions, $(\hat{u},\hat{v})$ are the velocity components parallel to $(\hat{x},\hat{y})$, g is the acceleration due to gravity, $\hat{p}$ is the pressure of the fluid, μ (T) is the dynamic viscosity, ρ_f , μ and β are the density, viscosity, and volumetric volume expansion coefficient of the fluid while ρ_p is the density of the particles. D_B Brownian diffusion coefficient and D_T is the thermophoretic diffusion coefficient.

The associated boundary conditions are:

$$\hat{u}=0,\ \hat{v}=0,\ T=T_w \text{ at } \hat{y}=\hat{y}_w=\sigma(\hat{x})$$

$$\hat{u}=0,\ T=T_\infty,\ \hat{p}=p_\infty \quad \text{as } \hat{y}\to\infty$$

where T_w is the surface temperature, T_∞ is the ambient temperature of the fluid.

Following Yao (1983), we now introduce the following non-dimensional variables:

$$x=\frac{\hat{x}}{L},\quad y=\frac{\hat{y}-\sigma(\hat{x})}{L}Gr^{1/4},\ u=\frac{\rho L}{\mu_\infty}Gr^{-1/2}\hat{u},\quad p=\frac{L^3}{\rho \nu_\infty^2}Gr^{-1}\hat{p}$$

$$v = \frac{\rho L}{\mu_\infty} Gr^{-1/4} \left(\hat{v} - \sigma_x \hat{u}\right), \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \phi = \frac{C - C_\infty}{C_W - C_\infty} \tag{7}$$

On introducing the above dimensionless dependent variables into the equations (1) – (6)

The following dimensionless form of the governing equations are obtained at leading order in the Grashof number Gr

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{8}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \sigma_x Gr^{1/4}\frac{\partial p}{\partial y} + \left(1 + \sigma_x{}^2\right)\frac{\partial^2 u}{\partial y^2} + \left(1 + \sigma_x{}^2\right)\frac{\partial \theta}{\partial y}\frac{\partial u}{\partial y} + \theta - N_r \phi \tag{9}$$

$$\sigma_x\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) + \sigma_{xx}u^2 = -Gr^{1/4}\frac{\partial p}{\partial y} + \sigma_x\left(1 + \sigma_x{}^2\right)\frac{\partial^2 u}{\partial y^2} + \sigma_x\left(1 + \sigma_x{}^2\right)\frac{\partial \theta}{\partial y}\frac{\partial u}{\partial y} \tag{10}$$

By combining equations (7) and (8)

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \left(1 + \sigma_x{}^2\right)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma_x \sigma_{xx}}{1 + \sigma_x{}^2}u^2 + \frac{1}{1 + \sigma_x{}^2}\theta - N_r\frac{\phi}{1 + \sigma_x{}^2} \tag{11}$$

Energy Equation

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\left(1 + \sigma_x{}^2\right)\frac{\partial^2 \theta}{\partial y^2} + \frac{N_b}{\Pr}\left(\frac{\partial \theta}{\partial y}\right)\left(\frac{\partial \phi}{\partial y}\right) + \frac{N_t}{\Pr}\left(\frac{\partial \theta}{\partial y}\right)^2 \tag{12}$$

Concentration Equation

$$u\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y} = \frac{1 + \sigma_x{}^2}{v_\infty}\left[D_B\frac{\partial^2 \phi}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\Delta T}{\Delta \phi}\frac{\partial^2 \phi}{\partial y^2}\right] \tag{13}$$

Now we introduce the following transformations to reduce the governing equation to a convenient form:

$$\psi = x^{3/4} f(x,\eta), \quad \eta = x^{-1/4} y, \quad \phi = \phi(x,\eta) \;\; \theta = \theta(x,\eta) \tag{14}$$

Where η is the pseudo similarity variable and ψ is the stream-function that satisfies the equation and is defined by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{15}$$

Introducing the transformations given in equation (14) into the equations (11), (12) and (13) we have,

$$x\left[f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x}\right] = \left(1+\sigma_x^{\,2}\right)f''' + \frac{3}{4}ff'' - \left[\frac{1}{2} + \frac{x\sigma_x\sigma_{xx}}{1+\sigma_x^{\,2}}\right]'\left[f'\right]^2 + \frac{1}{1+\sigma_x^{\,2}}\theta - \frac{N_r}{1+\sigma_x^{\,2}}\phi \tag{16}$$

$$x\left[f'\frac{\partial \theta}{\partial x} - \theta'\frac{\partial f}{\partial x}\right] = \frac{1}{P_r}\left(1+\sigma_x^{\,2}\right)\theta'' + \frac{N_b}{\mathrm{P_r}}\theta'\phi' + \frac{N_t}{P_r}\theta'^2 + \tfrac{3}{4} f\;\theta' \tag{17}$$

$$x\left[f'\frac{\partial \phi}{\partial x} - \phi'\frac{\partial f}{\partial x}\right] = \left(1+\sigma_x^{\,2}\right)\left[\phi'' + \frac{N_t}{N_b}\theta''\right] + \frac{3}{4}P_r L_e f\phi' \tag{18}$$

Where prime denotes differentiation with respect to η and parameters are defined by

$$N_r = \frac{(\rho_p - \rho_{f\infty})(C_w - C_\infty)}{\rho_{f\infty}\beta(T_w - T_\infty)(1-\Phi_\infty)}$$

$$N_b = \frac{\mathcal{E}(\rho_{cp})D_{\boldsymbol{B}}(C_w - C_\infty)}{(\rho_{cf}) \propto}$$

$$N_t = \frac{\mathcal{E}(\rho_{cp})D_{\boldsymbol{T}}(T_w - T_\infty)}{(\rho_{cf}) \propto T_\infty}$$

$$P_r = \frac{V_\infty}{\propto}$$

$$G_r = \frac{(1-\Phi_\infty)L^3 g\beta\Delta T_w}{\mathrm{v}^2}$$

$$L_e = \frac{\alpha}{D_B} \tag{19}$$

The boundary conditions now take the following form:

$$f(x,0) = f'(x,0) = 0, \theta(x,0) = 1 \tag{20}$$

$$f'(x,\infty) = \theta(x,\infty) = 0$$

Solutions of the non-similar partial differential system given by 16, 17 and 18 subject to the boundary conditions (20) are obtained by using the implicit finite difference method developed by Keller (1978).

However, once we know the values of the function f and θ and their derivatives, it is important to calculate the values of the local Nusselt number (Nu_x), Sherwood number (Sh_x) , and skin-friction (C_{fx}) from the following relations

$$Nu_x = q_w\, x/k_\infty(T_w - T_\infty) \text{ and } C_{fx} = 2\tau_w/\rho U_\infty^2 \tag{21}$$

Where

$$q_w = -(k\hat{n}.\nabla T)_{y=0}, \tau_w = (\mu\hat{n}.\nabla\hat{u})_{y=0} \text{ and } U_\infty = \mu_\infty Gr^{1/2}/\rho L \tag{22}$$

here $\hat{n}$ is the unit normal to the surface. Using the transformation (14) Nu_x and C_{fx} take the following forms:

$$Nu_x(Gr/x)^{-1/4} = -\sqrt{1+\sigma_x^2}\theta'(x,0) \tag{23}$$

$$C_{fx}(Gr/x)^{1/4}/2 = \sqrt{1+\sigma_x^2}f''(x,0) \tag{24}$$

$$Sh_x\,(Gr/x)^{-1/4} = -\sqrt{1+\sigma_x^2}\,\emptyset'(x,0) \tag{25}$$

3.2. Numerical Solutions

Keller Box Method:

The resulting nonlinear system of the partial differentials equations is solved numerically by the Keller-Box method which is an implicit finite difference method. The method allows for non-uniform grid discretion and converts the differential equations into algebraic ones which are then solved using Thomas algorithm. Thomas algorithm is essentially the result of applying gauss elimination to the tri-diagonal system of equations. The number of grid points in both directions affects the numerical results. To obtain accurate results, a mesh sensitivity study was performed.

3.3 Results and discussions

The nonlinear ordinary differential Equations (16-18) were solved numerically to satisfy the boundary conditions (20) for parametric values of L_e(Lewis number), N_r (buoyancy ratio number), N_b(Brownian motion parameter) and N_t(Thermophoresis parameter) using implicit finite difference method (Keller-Box method).

The effect of various parameters on the free convection with heat and mass transfer along a vertical wavy surface is examined and discussed in this section. The parameters that the solution affected are the $N_r,\ N_t,\ N_b,\ L_e\ and\ \alpha$.

Figure 2-4 represents the effect of variations in the velocity, temperature and concentration for values of N_r, it is clear that as the Buoyancy ratio increases, the velocity decreases while the temperature and concentration increases. So, we conclude that Buoyancy has a minor effect on the particle diffusion.

Figure 5-7 represents the effect of variations in the velocity, temperature and concentration for values of N_t, it is clear that as the thermophoresis parameter increases, the velocity, temperature and concentration increases. N_t plays a strong role in determining the diffusion of heat and nanoparticle concentration.

Figure 8-10 represents the effect of variations in the velocity, temperature and concentration for values of N_b, it is clear that as the Brownian motion increases, the velocity and temperature increases while concentration decreases. Brownian diffusion promotes heat conduction and also reduces nanoparticle diffusion.

Figure 11-13 represents the effect of variations in the velocity, temperature and concentration for values of L_e. It is clear that as the L_e increases, the velocity increases while the temperature and concentration within the boundary layer decreases. L_e is the ratio of molecular thermal diffusivity to mass diffusivity. As L_e increases, thermal diffusivity dominates.

Figure 14-16 indicates the effect of variations in the surface wave amplitude in velocity, temperature and concentration. It is clear that as the α increases, the velocity decreases while the temperature and concentration increases. So, we conclude that much roughness of the surface results in an increase of viscous effect within the boundary layer and hence, the flow and thermal fluxes reduces.

Figure 17-19 represents the effect of variations in the local skin-frication coefficient, $C_{fx}(G_r/x)^{1/4}/2$, Nusselt number $Nu_x(G_r/x)^{-1/4}$ and Sherwood number $Sh_x(G_r/x)^{-1/4}$ for values of N_r, it is clear that as the N_r increases the Skin-frication coefficient, Nusselt number and Sherwood number decreases. So, we conclude that much value of the Buoyancy ratio, the Skin-frication coefficient, heat and mass transfer rate are decreasing.

Figure 20-22 represents the effect of variations in the local skin-frication coefficient, $C_{fx}(G_r/x)^{1/4}/2$, Nusselt number $Nu_x(G_r/x)^{-1/4}$ and Sherwood number $Sh_x(G_r/x)^{-1/4}$ for values of N_t, it is clear that as the N_t increases, the Skin-frication coefficient, Nusselt number and Sherwood number decreases. So, we conclude that much value of Thermophoresis parameter, the Skin-frication coefficient, heat and mass transfer rate are decreasing.

Figure 23-25 indicates the effect of Brownian motion parameter N_b on local skin-frication coefficient $C_{fx}(G_r/x)^{1/4}/2$, Nusselt number $Nu_x(G_r/x)^{-1/4}$ and Sherwood number, $Sh_x(G_r/x)^{-1/4}$, and it is clear that increasing the value of N_b, increases the Skin-frication coefficient and Sherwood number, where as Nusselt number decreases. So, we conclude that much value of Brownian motion parameter, the Skin-frication coefficient, mass transfer rate increasing and heat transfer rate decreasing.

Figure 26-28 indicates the influence of Lewis number L_e on local skin-frication coefficient, $C_{fx}(G_r/x)^{1/4}/2$, Nusselt number, $Nu_x(G_r/x)^{-1/4}$ and Sherwood number, $Sh_x(G_r/x)^{-1/4}$)for values of L_e, and it is clear that as the L_e increases the rate of heat transfer decreases, while the rate and amplitude of mass transfer rate and Skin-frication coefficient increases.

Figure 29-31 represents the effect of variations in the surface wave amplitude on local skin-frication coefficient, $C_{fx}(G_r/x)^{1/4}/2$, Nusselt number $Nu_x(G_r/x)^{-1/4}$ and Sherwood number, $Sh_x(G_r/x)^{-1/4}$ for values of α.We observe that, as the value of α increases ,the Skin-frication coefficient, Nusselt number and Sherwood number decreases. So we conclude that much roughness of the surface increasing, the Skin-friction coefficient, heat and mass transfer rate decreasing.

The Brownian motion and thermophoresis of nanoparticles increase the effective thermal conductivity of the nanofluid. Both Brownian motion and thermophoresis give rise to cross diffusion terms similar to soret and dufour cross diffusion terms that arise with a binary fluid.

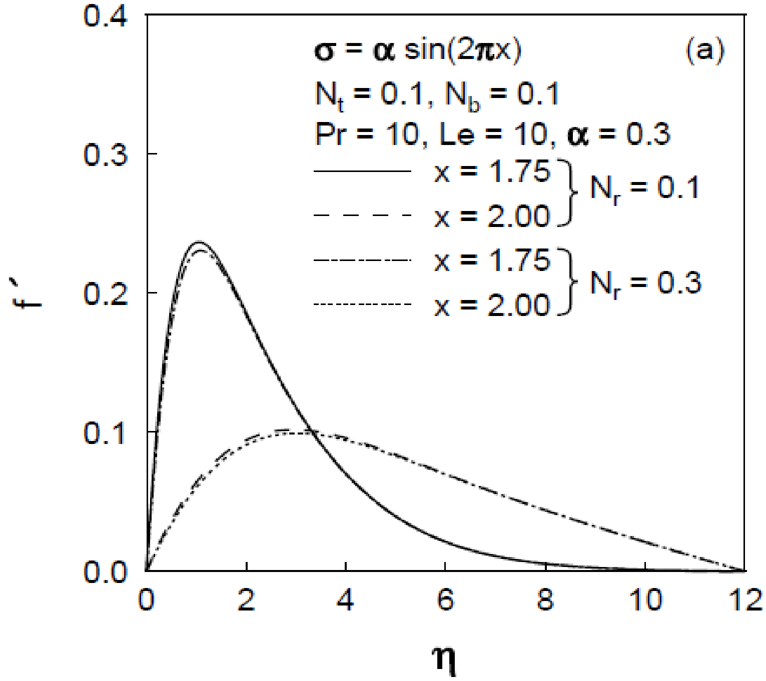

Figure 2 Effect of N_r on velocity profiles

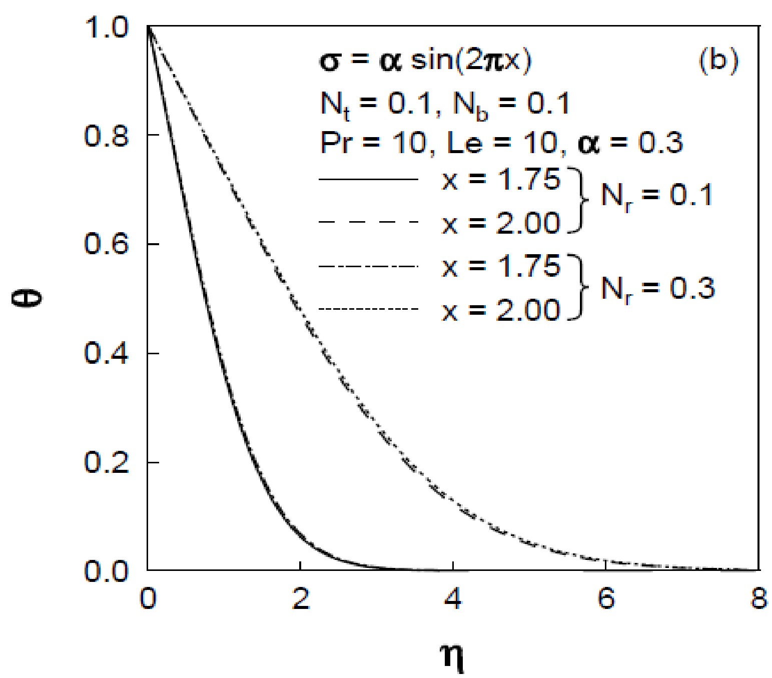

Figure 3 Effect of N_r on temperature profiles

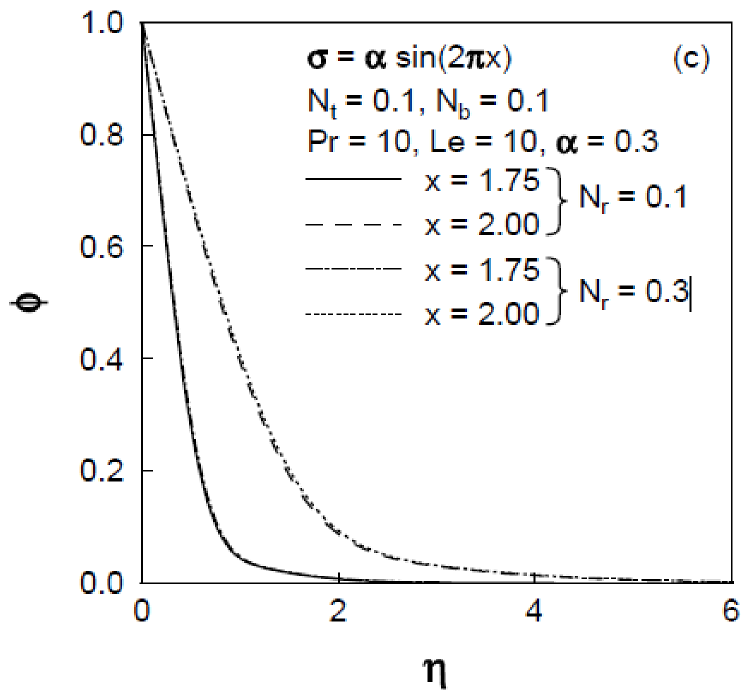

Figure 4 Effect of N_r on concentration profiles

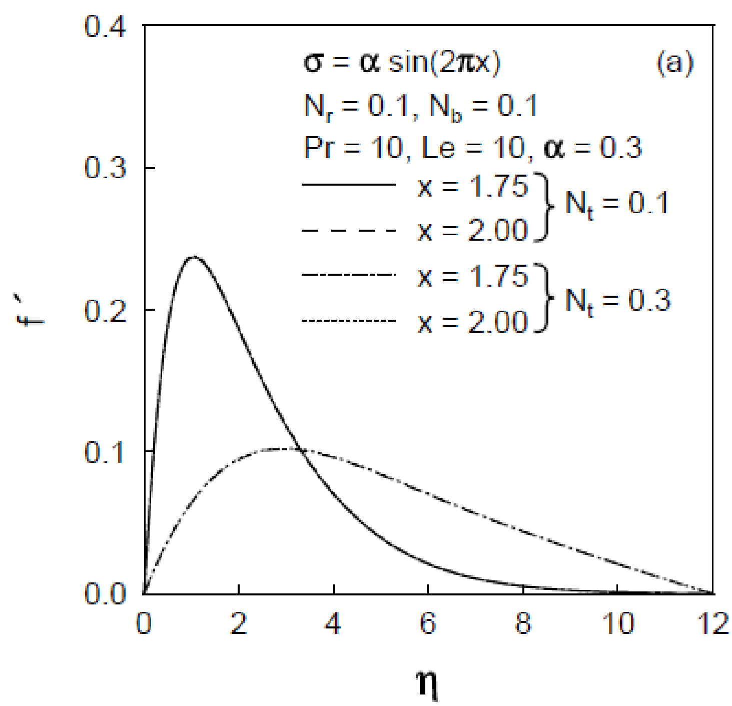

Figure 5 Effect of N_t on velocity profiles

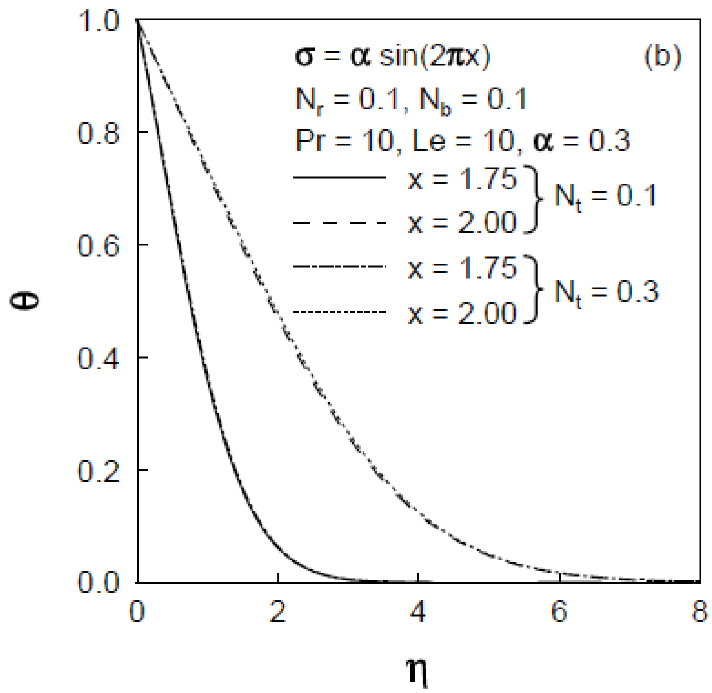

Figure 6 Effect of N_t on temperature profiles

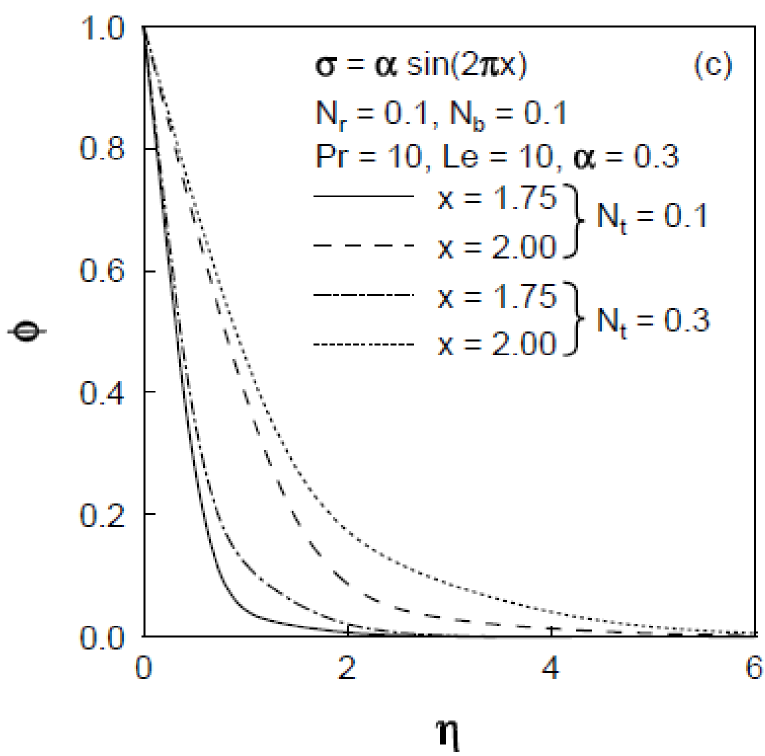

Figure 7 Effect of N_t on concentration profiles

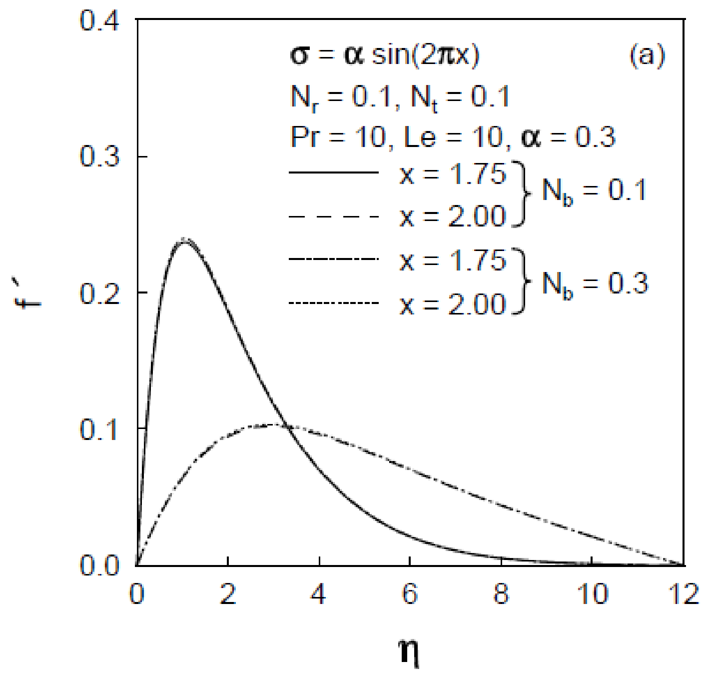

Figure 8 Effect of N_b on velocity profiles

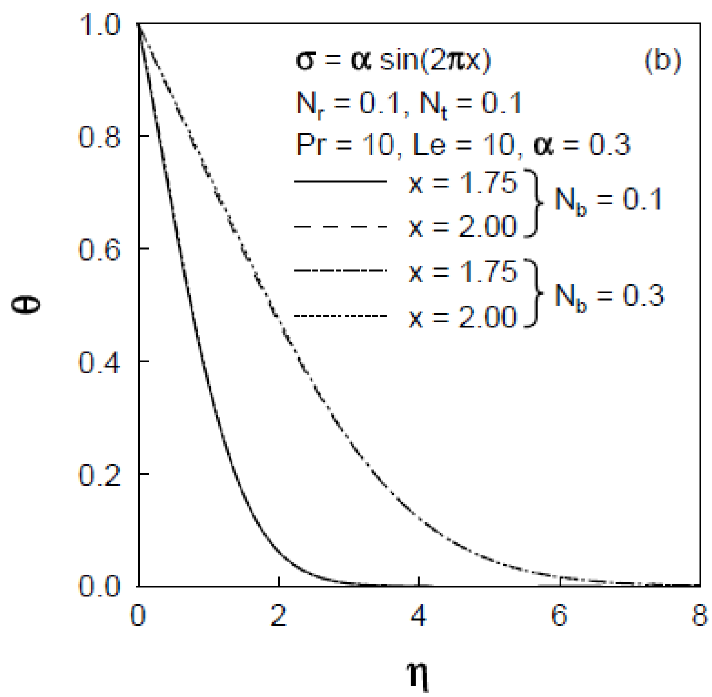

Figure 9 Effect of N_b on temperature profiles

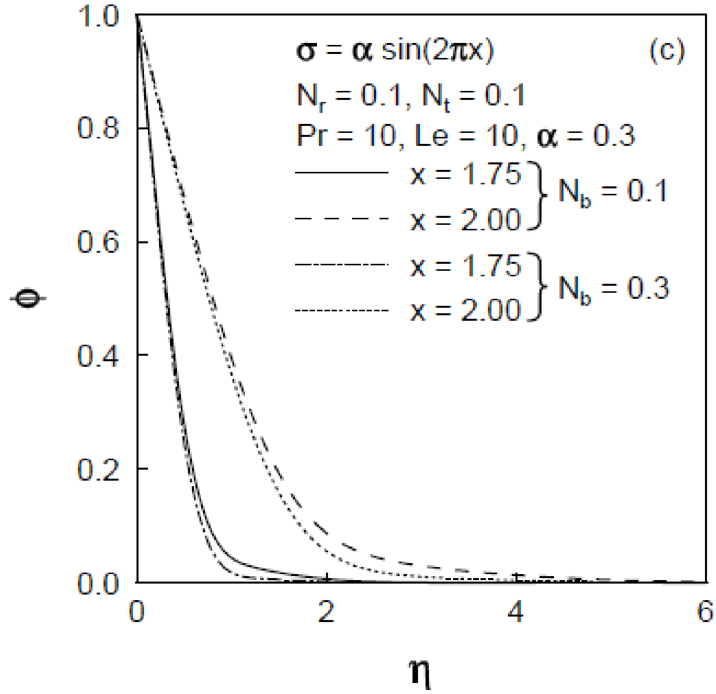

Figure 10 effect of N_b on concentration profiles

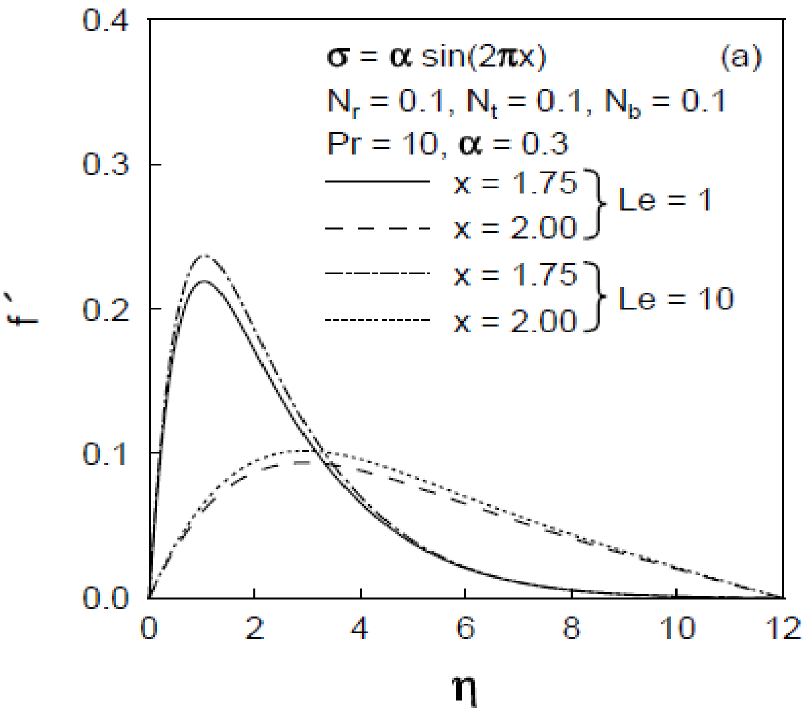

Figure 11 Effect of L_e on velocity profiles

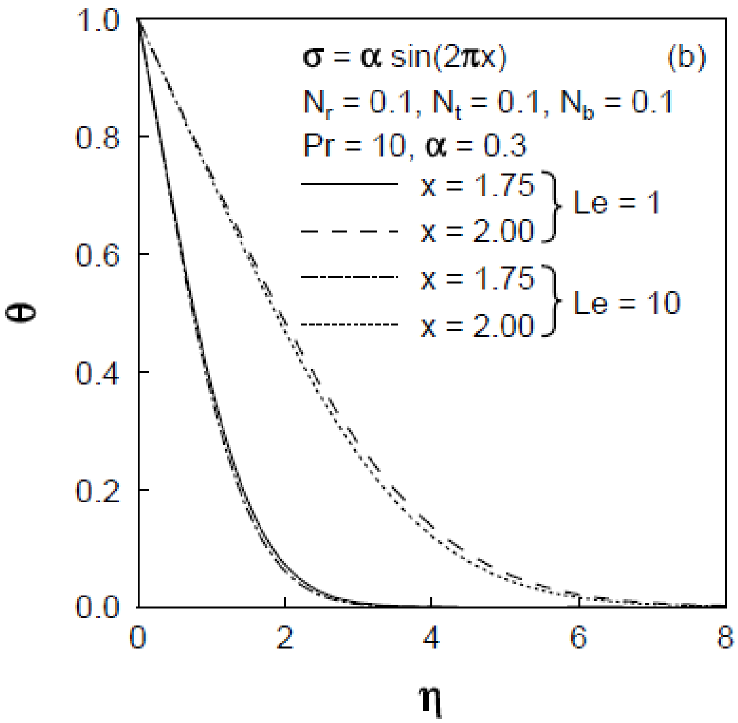

Figure 12 Effect of L_e on temperature profiles

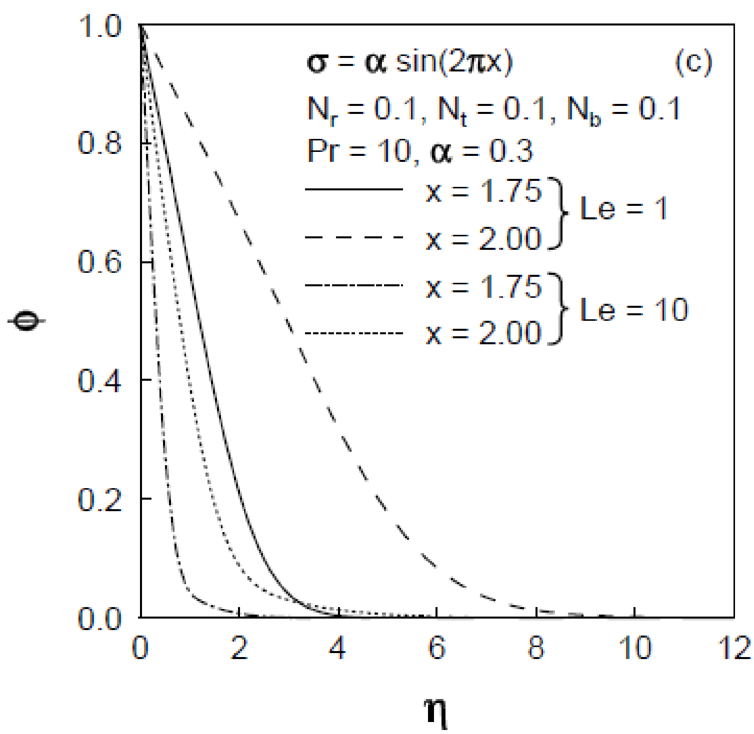

Figure 13 Effect of L_e on concentration profiles

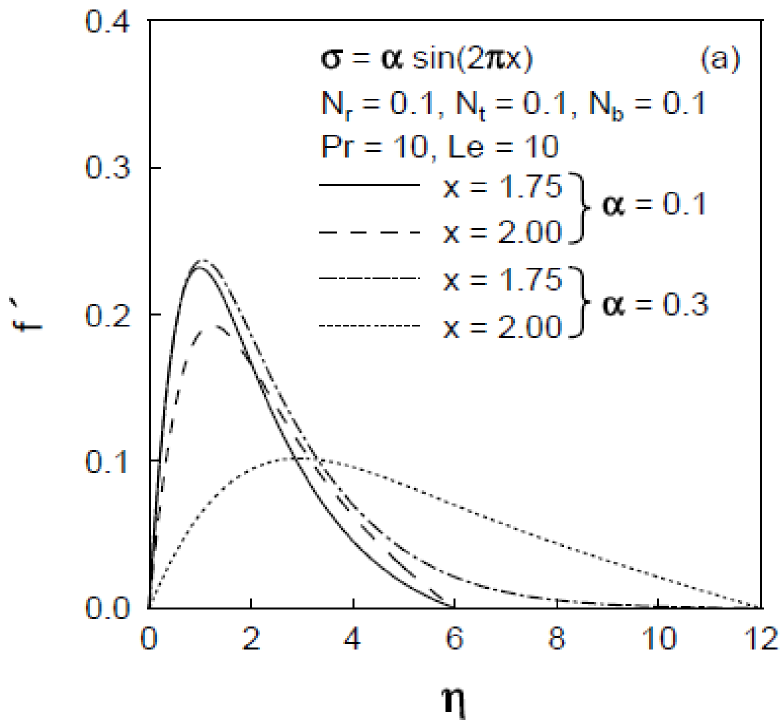

Figure 14 Effect of α on velocity profiles

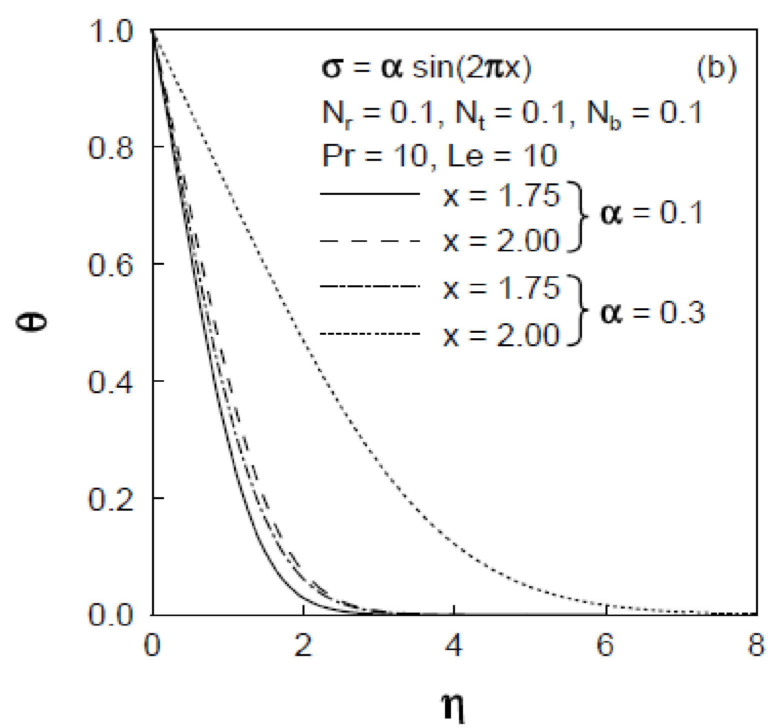

Figure 15 Effect of α on temperature profiles

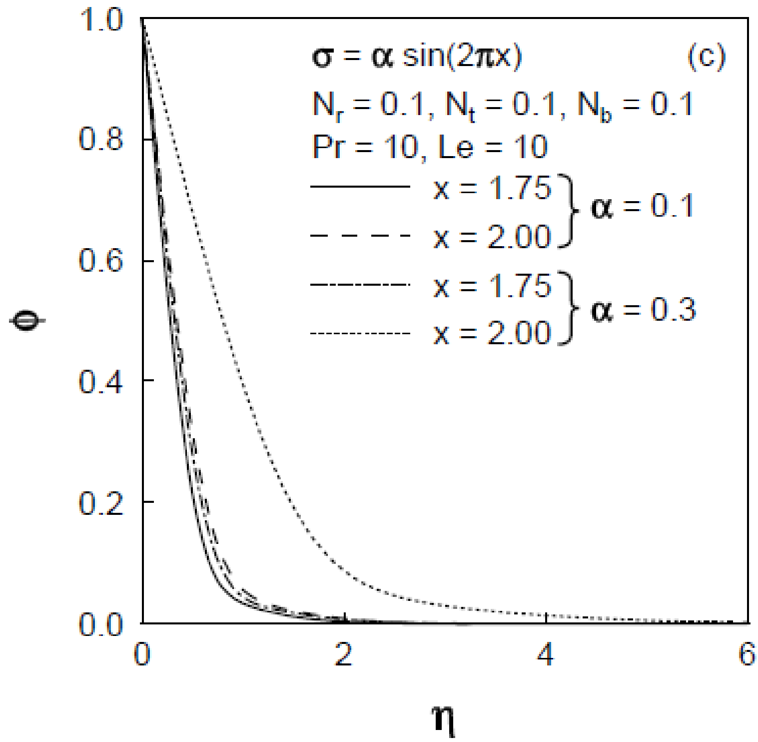

Figure 16 Effect of α on concentration profiles

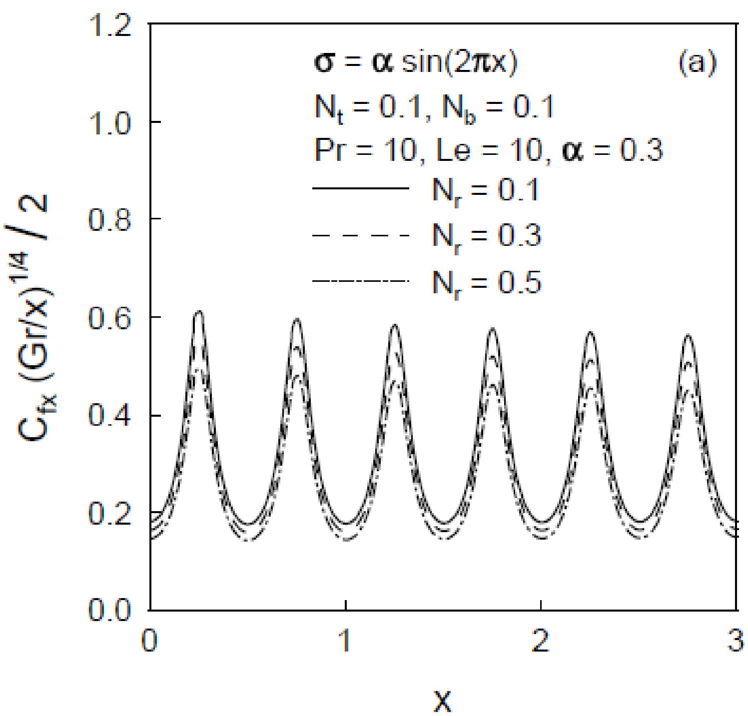

Figure 17 Effect of N_r on skin- frication coefficient

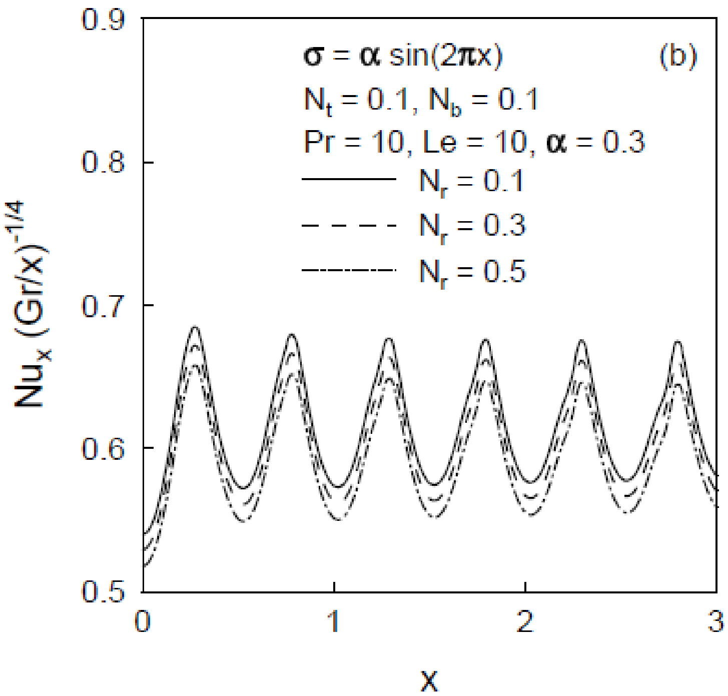

Figure 18 Effect of N_r on heat transfer rate

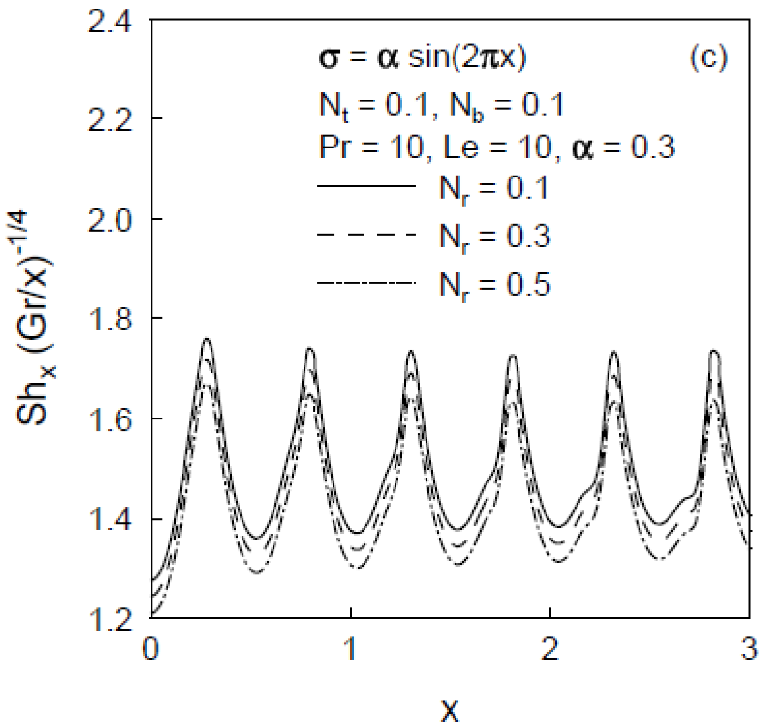

Figure 19 Effect of N_r on mass transfer rate

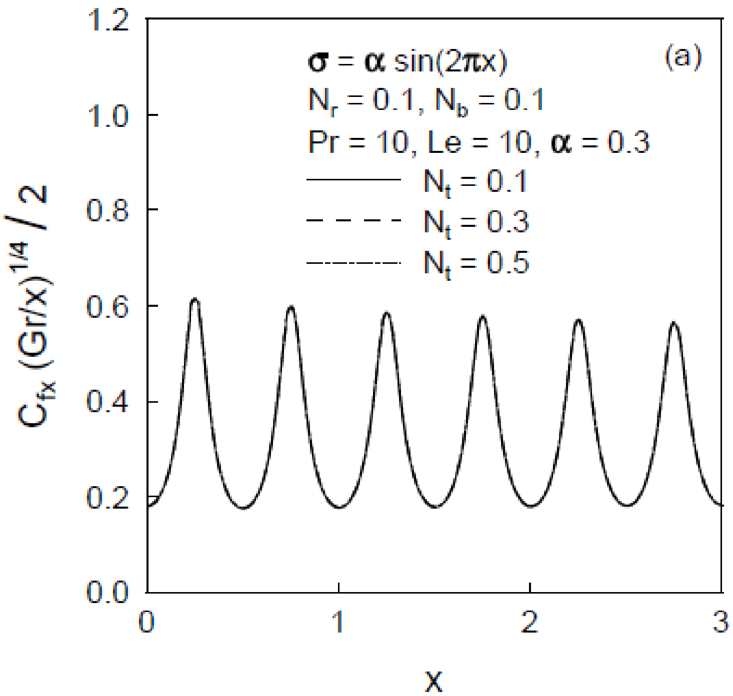

Figure 20 Effect of N_t on skin- frication coefficient

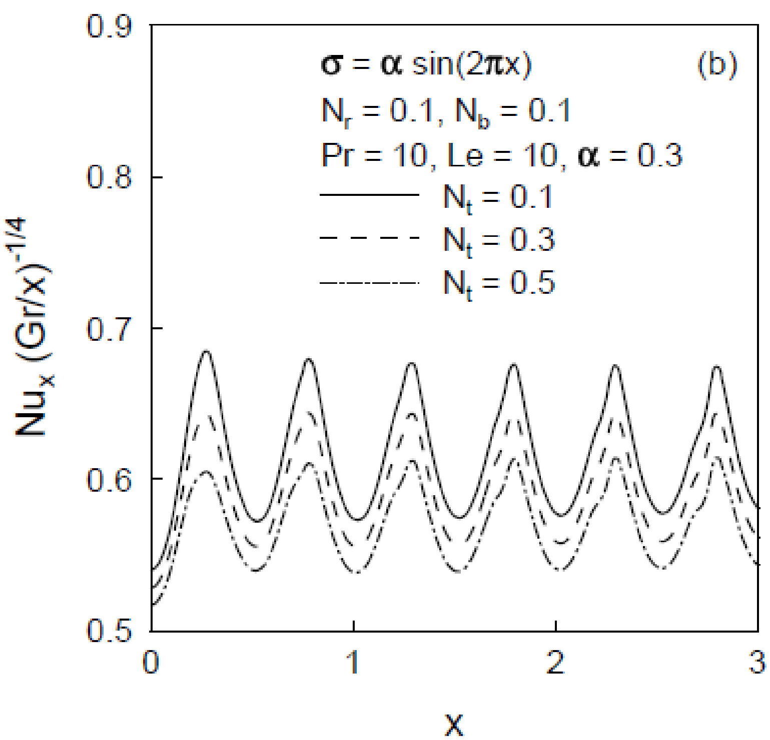

Figure 21 Effect of N_t on heat transfer rate

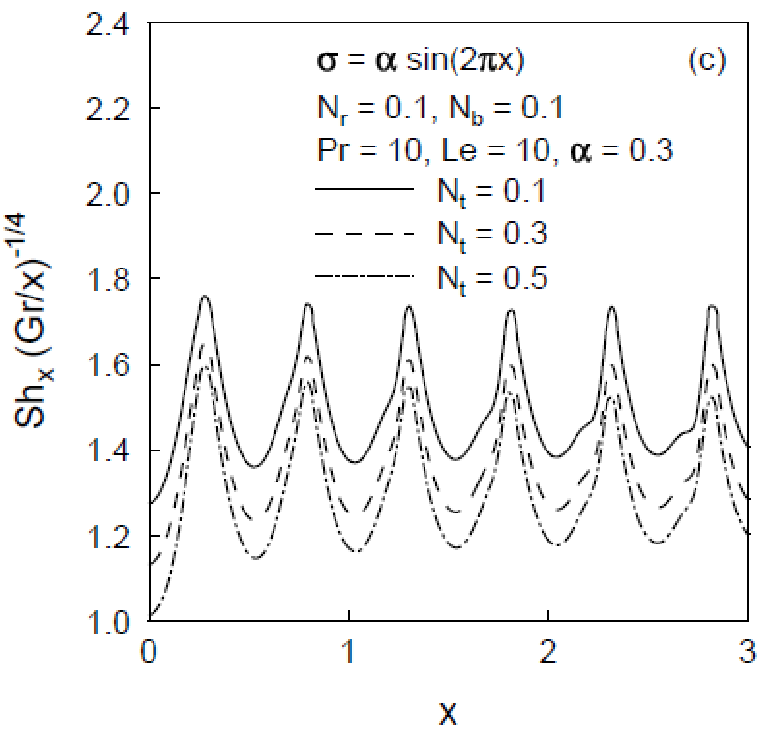

Figure 22 Effect of N_t on mass transfer rate

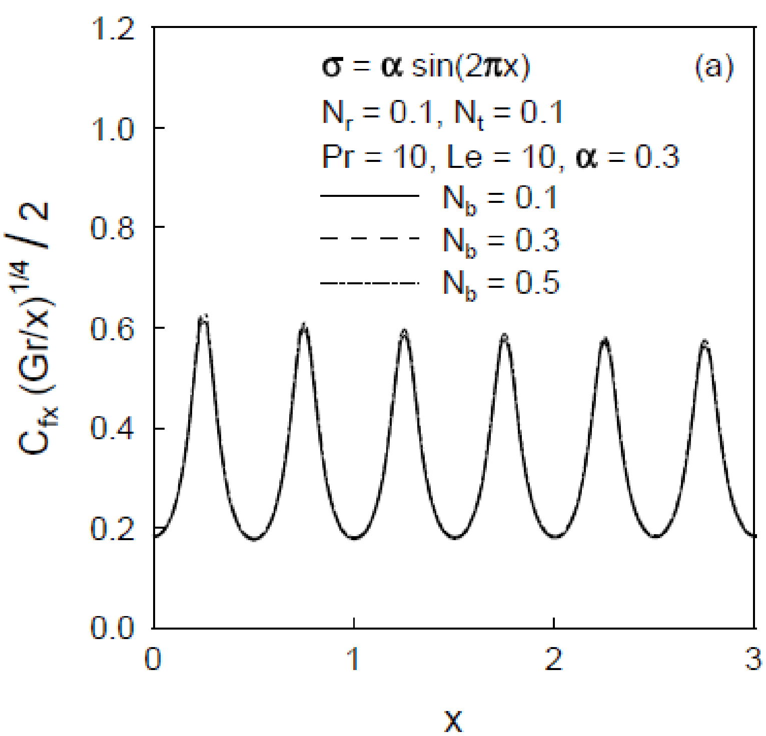

Figure 23 Effect of N_b on skin- frication coefficient

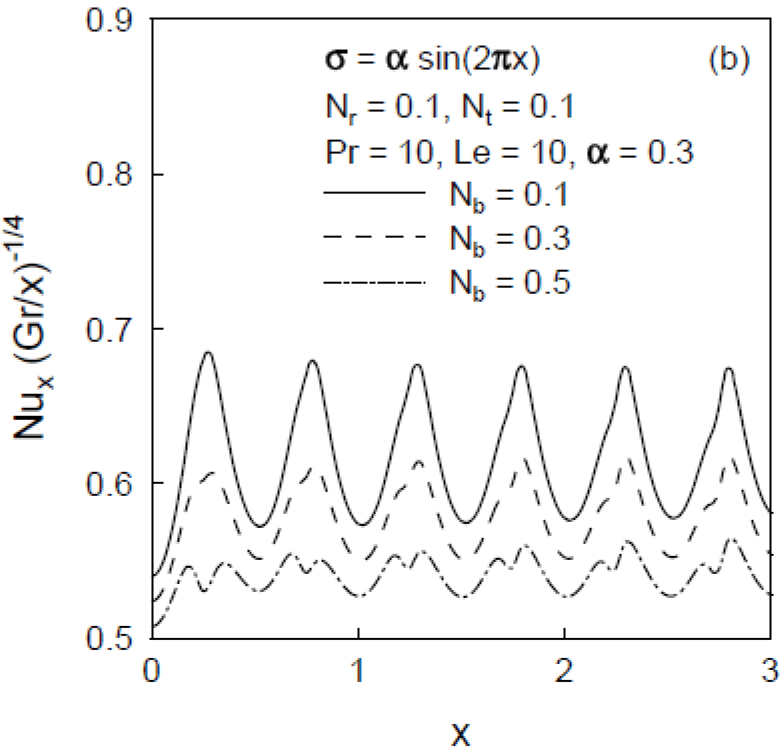

Figure 24 Effect of N_b on heat transfer rate

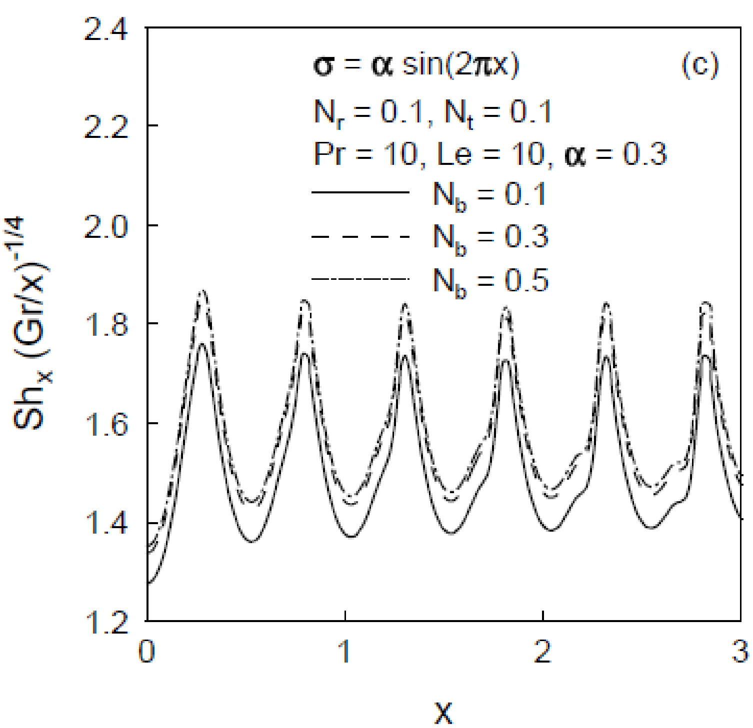

Figure 25 Effect of N_b on mass transfer rate

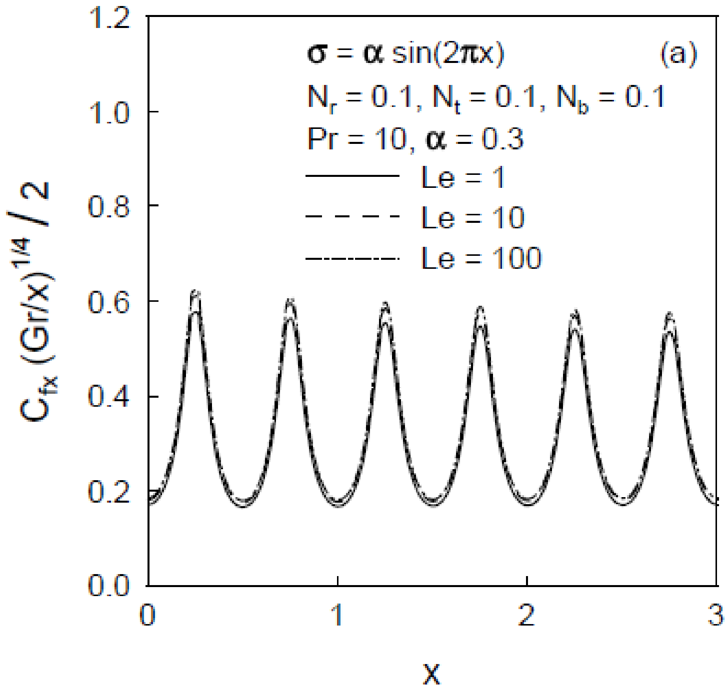

Figure 26 Effect of L_e on skin- frication coefficient

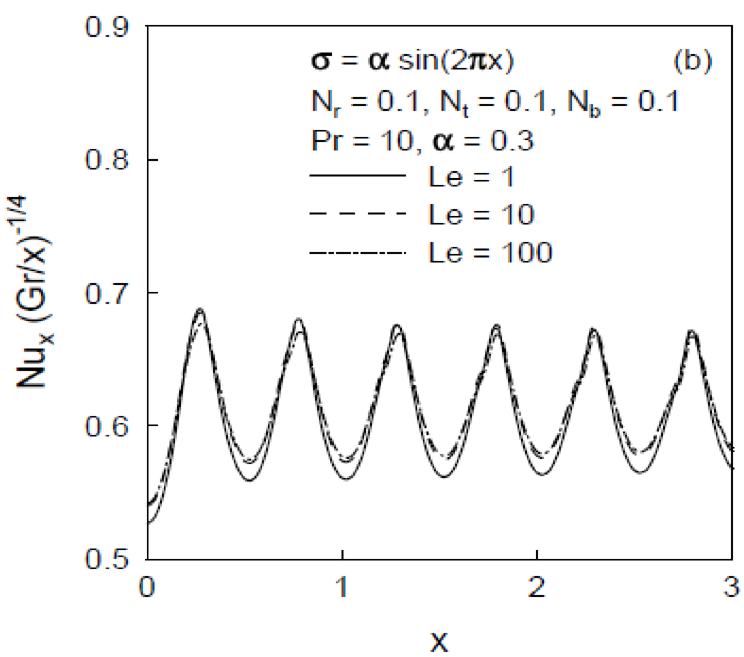

Figure 27 Effect of L_e on heat transfer rate

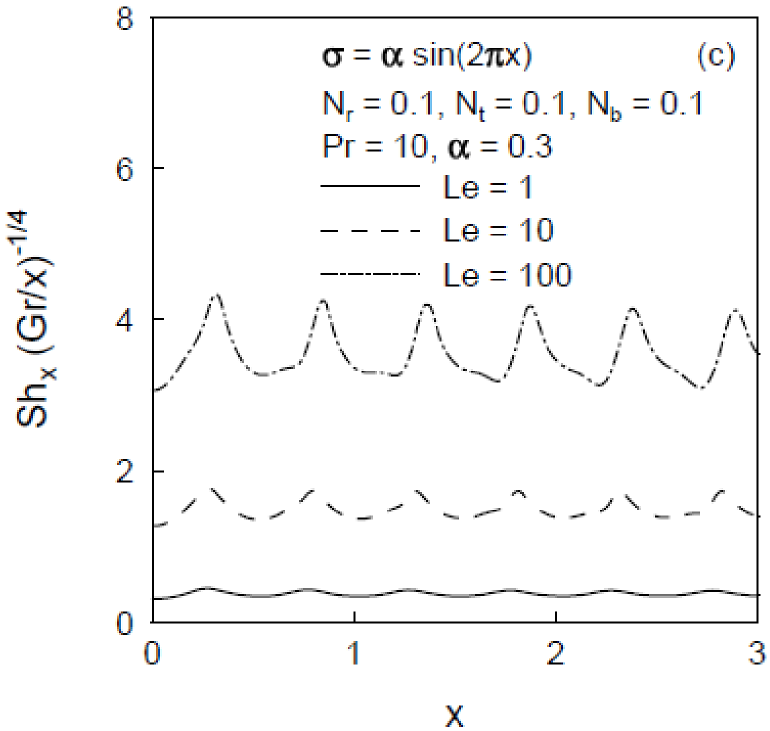

Figure 28 Effect of L_e on mass transfer rate

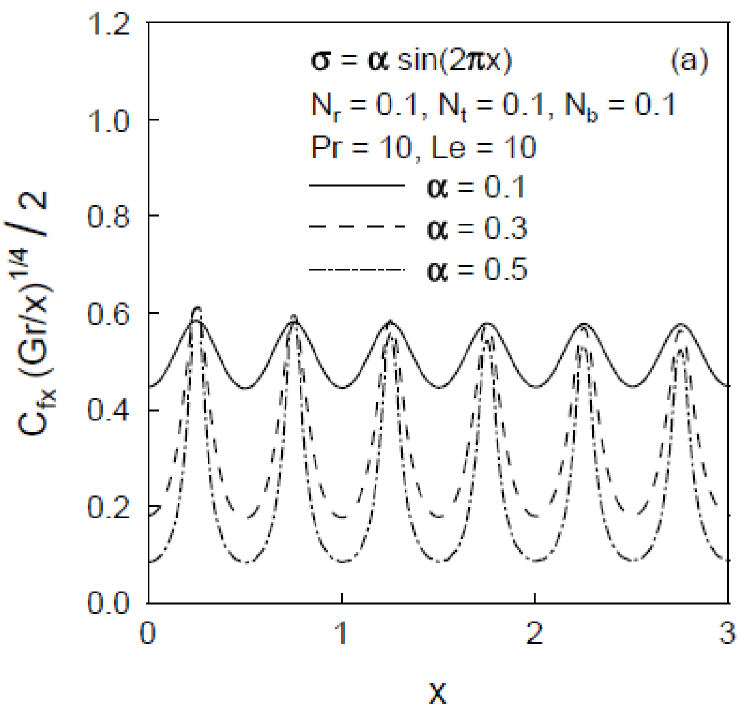

Figure 29 Effect of α on skin- frication coefficient

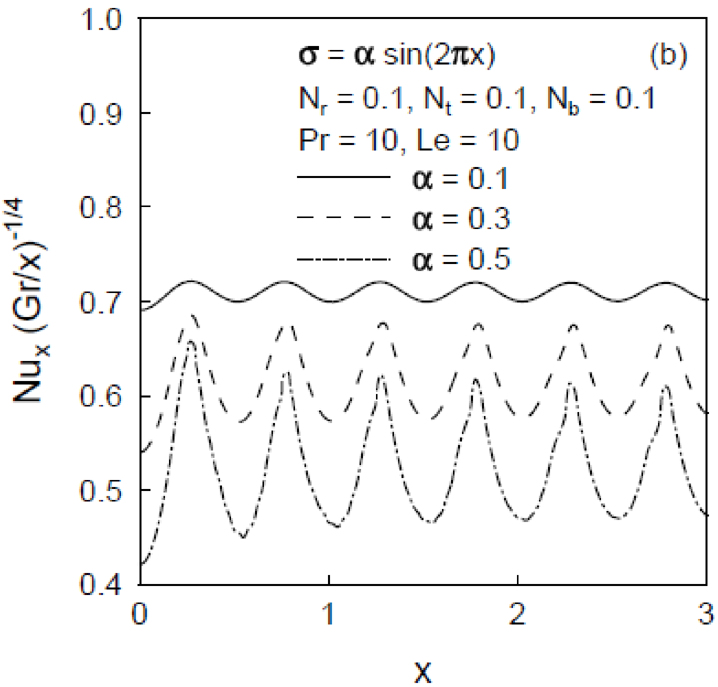

Figure 30 effect of α on heat transfer rate

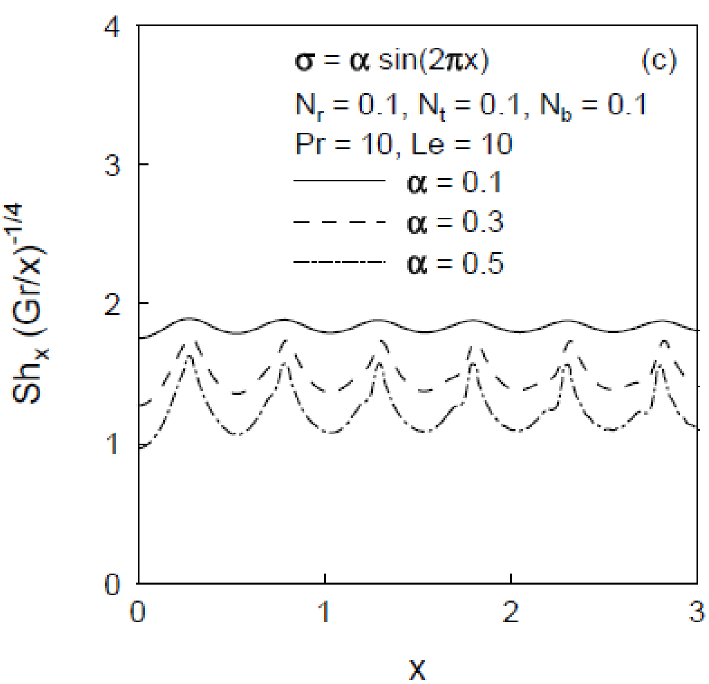

Figure 31 Effect of α on mass transfer rate

CHAPTER IV

CONCLUDING REMARKS

In this study, we presented a boundary layer analysis for free convection along a vertical wavy surface in a nanofluid. Numerical results for friction factor, surface heat transfer rate and mass transfer rate have been presented for parametric variations of buoyancy ratio parameter N_r, Brownian motion parameterN_b, thermophoresis parameter N_t and Lewis number L_e. The results indicate that as N_r and α increases, the friction factor, the heat transfer (Nusselt number) and mass transfer rates (Sherwood number) decreases. As N_b increases, the friction factor and surface mass transfer rates increases, where the surface heat transfer rates decreases. As N_t increases, the friction factor increases, where as mass transfer rates and heat transfer rates decreases. As L_e increase, the friction factor, mass transfer and heat transfer rates increases.

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