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MULTIPLEXED CONTROL OF SMART STRUCTURE USING PIEZOELECTRIC ACTUATORS

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Bachelor of Science in Mechanical Engineering

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Mumbai, India Sep, 2001

submitted in partial fulfillment of requirements for the degree

MASTERS OF SCIENCE IN MECHANICAL ENGINEERING

at the

CLEVELAND STATE UNIVERSITY

December, 2008

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ACKNOWLEDGMENTS

I would like to thank my advisor Dr. Hanz Richter, who provided essential support and assistance throughout my graduate career, and also for his guidance which immensely contributed towards the completion of this thesis. This thesis would not have been realized without his support. I would also like to thank Dr. Paul Lin and Dr. Lili Dong for being in my thesis committee. Thanks are also due to my family and friends who have encouraged, supported and inspired me.

MULTIPLEXED CONTROL OF SMART STRUCTURE USING PIEZOELECTRIC ACTUATORS

KUMAR S NALE

ABSTRACT

Active control of smart structures containing a large number of actuators and sensors presents a tradeoff between increased system performance and the cost and bulk of the required hardware and computational resources. A technique called multiplexed control offers advantages when software and hardware resources are scarce and performance specifications call for a large number of actuators and sensors. In structural control applications, in particular those using smart materials, it is often desirable to increase the number of actuators to enhance controllability.

The focus of this research is to demonstrate real-time multiplexing on the hardware side of the actively controlled structure. Multiplexing effectively reduces the number of power units by sharing them among a large number of actuators according to a switching schedule. Multiplexing introduces periodicity in the closed-loop plant, requiring the use of periodic linear systems theory to tune as optimal quadratic regulator. In this thesis a multiplexed implementation is developed to control the three actuators mounted on single smart beam, where the control inputs are updated sequentially and cyclically instead of simultaneously, thus allowing the use of a single power amplifier. Focus was placed on the application of multiplexed control theory to a smart structure, including experimental procedures to obtain a plant model using system identification tools, and tuning of a discrete-time periodic quadratic regulator. An observer was also implemented. Simulations and real-time implementation validate the approach.

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CHAPTER I

INTRODUCTION

This chapter gives brief idea about basis of the study undertaken and justifies the work conducted in this thesis. It also talks about the literature review pertinent to thesis work and at the end brief over the organization of the thesis.

1.1   Overview of Multiplexing on Smart Structure

The term multiplexing describes the basic functional requirement when it comes to selection and processing of more number of signals passing from one section of a circuit to another.

Multiplexer: Multiplexing indicates managing signals flow with the help of switch called a *multiplexer*, which selects the subset of signal to be passed to corresponding circuit sequentially and cyclically. *Multiplexer* schematic is shown in Figure 1.1.

Smart Structure: Smart structure is a structure which has ability to respond adaptively in pre-designed useful and efficient manner to changes in environmental conditions, including any changes in its own conditions.

In our experiment to demonstrate the functioning of multiplexing controller, beam with piezoelectric actuators mounted on it works as smart beam. Use of *multiplexing* on smart structure for control implementation denotes an arrangement in which a
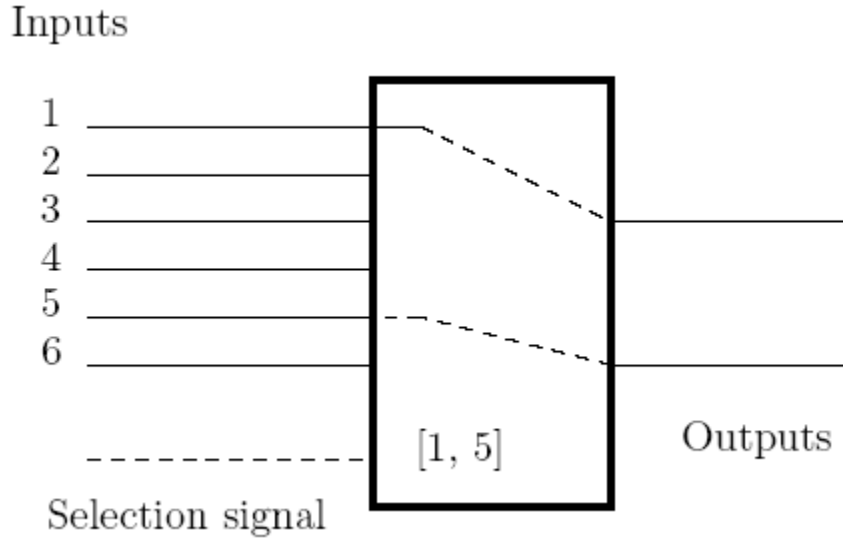
Figure 1.1: Multiplexer Switch

group of actuators are updated sequentially and cyclically instead of simultaneously as it has been done in traditional controllers. In this technique only a group of actuators are updated in every sampling instant keeping all other actuators held at their previous values. This technique of multiplexing can be used in software, hardware or in combination of both, using available resources and it also reduces the requirement of resources by sharing it over a sampling instant of time. In this thesis, focus is on the practical implementation of the multiplexing control theory on the smart structure, which is simple beam with three actuators mounted on it, which makes it follow the desired output. With multiplex controller theory, each actuator gets the control incremental signal cyclically and sequentially, saving on computational requirements of the software and hardware requirements, such as number of amplifiers.

1.2 Motivation

The motivation for implementation of multiplexing technique is to demonstrate and prove the practical implementation of theory in real-time and demonstrate the be-

havior of smart structure in control environment. For the complex plants, control strategy comes up with large number of states for the plant, which demands high computational requirements. Also from controlling different variables like sensors and actuators it demands for huge hardware resources. For real-time implementation these resource demands might fall short in supply. As in computation point of view, system may not be able to complete all control law computations for all channels simultaneously. Also from hardware point of view, there may not be enough signal conditioners and power units for all actuators and sensors.

Software limitations may arise due to number of control signals updated simultaneously for the complex structure having different actuators and sensors. It requires high computational resources on real-time basis to update all the control channels simultaneously. This makes it difficult and costly to provide software backup. At the same time, hardware limitations arise due to the number of resources needed which adds to the cost, weight and space limitations. For smart structure like one which we used in this thesis, requires large number of expensive piezoelectric actuators which demands for expensive high voltage output drivers, one per channel. So complex structure requires costly electronic resources, add on to the weight and space limitations. These limitations of software and hardware limit the usage in the control system to achieve the required control objective. So to overcome this limitation multiplexing technique is implemented, that allows the time sharing of reduced number of identical electronic drive units among actuators.

The other motivation for this thesis is to verify the modeling tool on the smart structure and compare the theoretical simulated behavior of the system with the real-time system implemented. The basic property of the smart structure is that, it has ability to respond adaptively to changes in environmental conditions. So to

form the smart structure, beam is built with three actuators, which demands for high software resources for control algorithm and hardware resources for control signal conditioning. Thus by using multiplexing control strategy, smart structure has been tested for basic implementation strategy, verifying smart structure behavior too.

1.3   Literature Review

The following paragraph lists several researchers' work which is related to this thesis. It explains the extent of the work done by individual and focuses on accomplishment in the field of smart structures and multiplexing control. Research works to reduce hardware where the cost limitations permit the use of reduced number of electronic drive units have also been provided.

For the smart structure development and control many efforts has been taken in past, as it helps to generate system which response to the changing external conditions. V. K. Wadhawan [7] describes the simple way of understanding the function of smart structure and focus on different types of smart structures with its usage in different fields of science. Multiplexing technique is tried by many researchers, and profound work has been done in this field. Thesis done by A. Singaraju, [5], talks about the multiplexing technique and its benefits. It also talks about mathematical modeling of multiplexed plant and multiplexing control theory. Based on the work done for control theory development, efforts have been put in our thesis to implement this multiplexing controller on smart structure. For simplification of the complicated structure and building the mathematical model, many efforts have been put and different techniques have been developed. Paper by Morari, Borrelli and Bemporad [1] develops a performance criterion based on sum of either the ∞ norm or the 1-norm

over time. For solving bigger equations with higher order states, computational time is one of the major factors to consider. As the system we are dealing with is complex in nature, and state space representation for the same will have higher order of state. For this, study done by Varga [2] for solving Riccati equation has been considered.

Apart from various research works conducted to overcome computational limitations in complex control algorithms like smart structure control with multiplexing technique, research has also be conducted in addressing the hardware limitations in the various control systems. A. N. Simonov, S. Hong, and G. Vdovin [6] describe a simple and efficient implementation of adaptive multiplexing control for high-order piezoelectric deformable mirrors. Piezoelectric actuators are important elements of adaptive optics control systems. A number of scientific, medical, and industrial applications use the piezoelectric actuators. Especially in the space shuttle, deformable mirrors have quite large number of piezoelectric actuators. So from the weight point of view in space application it is very critical to consider usage of minimum number. So in this work multiplexing technique has been addressed, which helps in reducing the usage of hardware.

An attempt to simplify the DM electronics by sequential addressing of actuators was made by Kibble White [4]. They built a 59-channel piezoelectric faceplate mirror that was driven by several high-voltage amplifiers (HVA) by the way of 16 high-voltage switches assembled from discrete components. Due to the implementation and control complexity, this approach has not won general acceptance.

1.4 Objective

This thesis is aimed at practical implementation of derived theoretical multiplexing controller, on smart structure, there by understanding and proving behavior of smart structure under control strategy. It basically talks about theoretical development of the multiplexing controller and then understands each component needed to implement the theory. It also accounts for the different variants, such as surrounding conditions, basic property of each components, non-ideal conditions, which we assumed while deriving the theory.

Following methodology is used to achieve this objective: First mathematical model is derived for smart structure with 3 actuators, using system identification technique. Then using mathematical model, control strategy is derived and verified using simulation tool. System behavior is verified with analyzer tool output. Also mathematical model is simulated for closed loop performance, with an observer.

Once mathematical model is verified for multiplex controller in simulation, practical implementation was big task. For this experiment, setup is built with smart structure, sensors, amplifier, spectrum analyzer, data acquisition system, power unit etc. Multiplexer is built physically using registers, capacitors, rectifiers etc. which connects between controller and amplifier. With this experimental setup, output is verified with theoretical output performance of the mathematical model, for controlled input to the system.

1.5 Composition of Thesis

The organization of the thesis is in the following manner. The experimental setup is introduced in chapter 2. Fundamentals of the system identification, control design,

system hardware and software details have been dealt in this chapter. In chapter 3 multiplexed control studies are explained in detail. It gives theoretical background and different design basics. It also explains simulated outcome of the system. In chapter 4 it explains the detail real time implementation with observer. It also talks about minute details about what went wrong and new inputs from the experiment. Chapter 5 briefs over the conclusion of the work and scope for future work.

CHAPTER II

EXPERIMENTAL SETUP

This Chapter introduces the experimental setup for the smart structure controlled operation using multiplex control strategy. It briefs over the arrangement of different basic components and their interconnections. In this chapter, detailed description of different hardware used to perform the experiment is given. Along with experimental setup, this chapter also gives over view of system identification theory and process.

2.1 Hardware Description

2.1.1 Smart Structure

The smart structure is a beam, made of stainless steel, with three piezoelectric actuators mounted on it as shown in Figure 2.1 As shown in schematic 2.2, one actuator is mounted on one side and two actuators are mounted on other side. These piezoelectric actuators receive voltage signals from the amplifier and in turn actuators bend to control the moment of the beam. Thus the moment of the beam, or any sudden change in the surroundings causes the beam to react and will be sensed by the optical sensor, which is mounted next to the tip of beam, will send signals to controller, generating control signals to operate the beam moment in desired manner.

This smart structure is mounted on the vibration table which helps to reduce the surrounding vibrations.
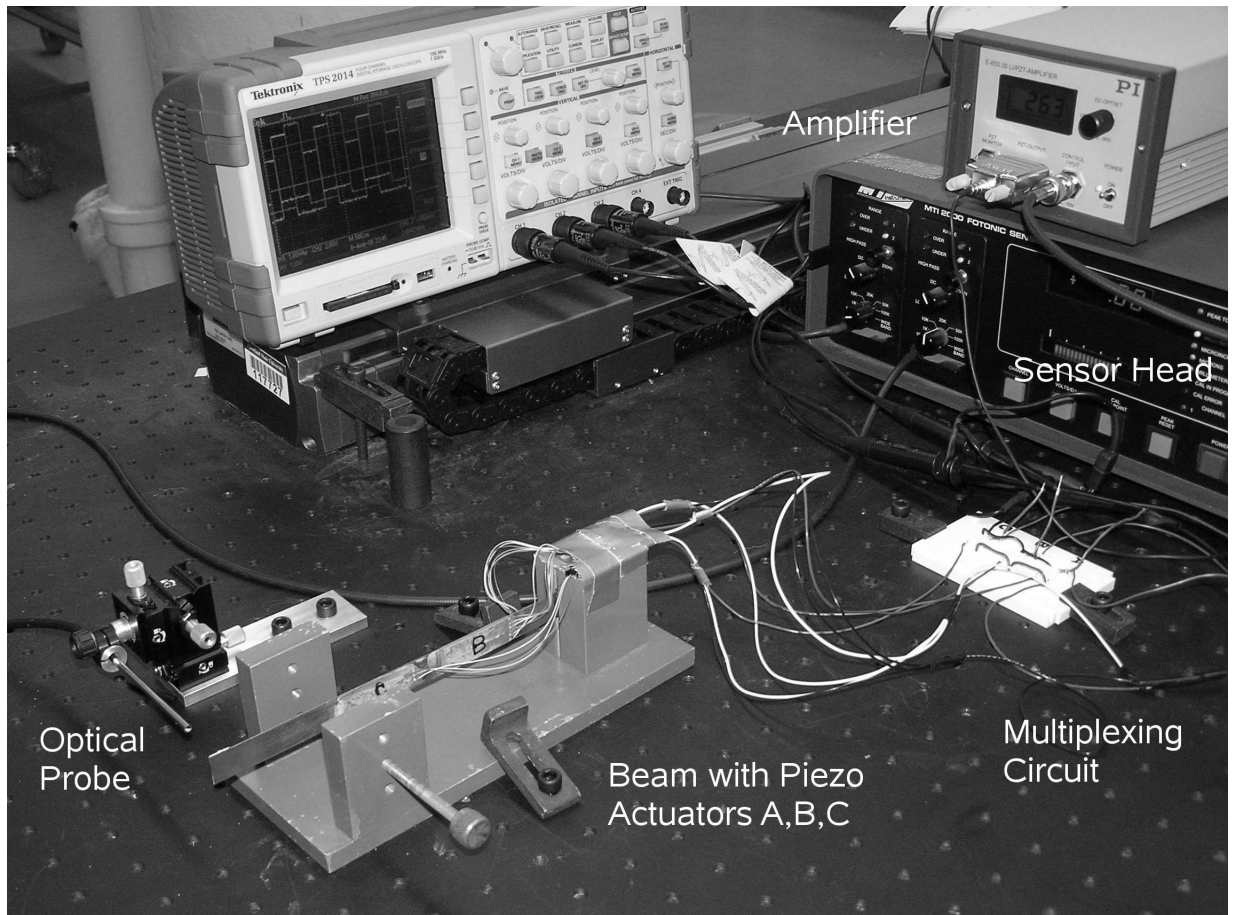
Figure 2.1: Experimental Setup

2.1.2 Piezoelectric Actuator

Piezoelectric actuators are made of piezoelectric materials. Piezoelectric materials have characteristics of generating electric voltage, when applied mechanical stress on it. Also it has characteristics of generating mechanical stresses and deformation when an electric field is applied to it. If an AC voltage is applied to the piezoelectric material, it will start vibrating at the frequency of the AC voltage.
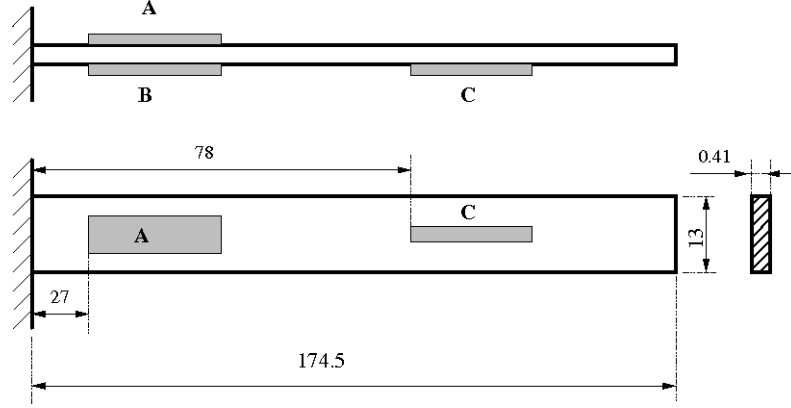
Figure 2.2: Experimental Setup Schematic

When piezoelectric materials are used for the purpose, where electric voltage is provided proportional to the control signal, and accordingly piezoelectric material makes moment to control the moment of the beam, then its called as actuator. For our experiment purpose, we used multilayered low voltage PZT (LVPZT) bender actuators. These actuators have been bought from Physik Instrumente (P.I.), a German company, leader in the market of piezoelectric transducers. These actuators are like classical bimorph actuators, but they provide several advantages over bimorph actuators. Bimorph actuators are composed of two ceramic plates glued together and connected in series or parallel. Such design offers higher stiffness and force than a single ceramic plate would. The LVPZT offers as even larger stiffness and response time. It requires low voltage to operate, which is very important to consider since it reduces the size of the power supply needed. Figure 2.3 shows piezoelectric actuators from P.I. and from the data sheet, the following information are extracted for PL128.255:

- Deflection (as a cantilever with a free length of 28mm): $\pm 450\mu$m

- Force generated (as a cantilever with a free length of 28mm): 0.5N

- Operating temperature range: -20 to +125°C

- Control signal range: 0 to +60V (through wire #1)

- Piezoelectric constant, d_{31}: -180E-12 C/N

- Relative dielectric constant, K^T: 1750

- Young's Modulus, e_{33}^{σ}: 111GPa

Figure 2.3: Picture of PL128.255 Piezoelectric Actuator

2.1.3 Amplifier

LVPZT actuator requires a 60V voltage to run. So to operate the actuator E-650 Piezo Driver amplifier is used. Figure 2.4. This amplifier can be connected to an analog control input ranging from -5V to 5V. It has built-in gain factor of 6, which multiplies the control input to be sent to the actuator. This amplifier has capability to drive two piezo actuators requiring the same control signals at the same time.

2.1.4 Optical Sensor

The Optical sensor we used for measurement of displacement of the tip of the smart structure is a MTI-2000. This optical sensor provides added advantages over traditional sensors. The MTI-2000 is a non-contact sensor which helps avoiding risk of interference in the motion of the smart structure. It measures the position of the beam with the help of light waves, in which light beam is focusing on the target surface, on which reflecting surface is created. Light signal reflected from the target
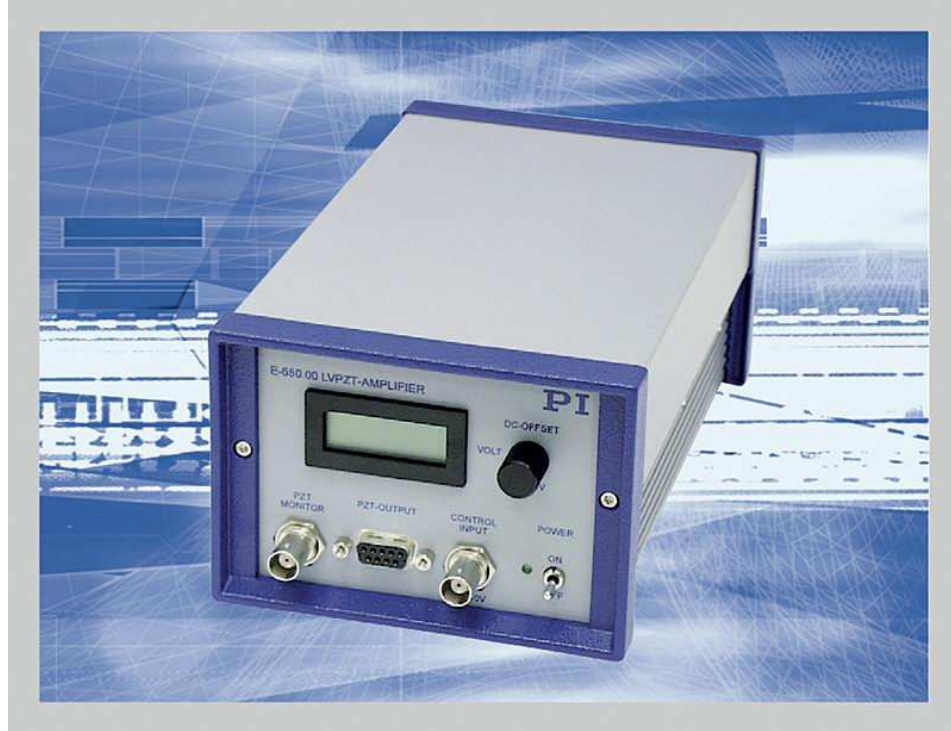
Figure 2.4: Picture of E-650

surface is collected by the sensor and based on the signal received, position of the object is defined. The MTI-2000 has two channels, which enable the measurement of the displacement of two target surfaces simultaneously. Also it has low-pass and high-pass filters for better accuracy. This unit is equipped with automatic calibration, which makes it easy to use for accurate measurement. It can also connects to different data acquisition systems, where data can be captured and analyzed for further analysis. The optical sensor was preferred over piezoelectric sensor, as optical sensor is very easy to use, less subject to noise and signals, and can be used directly for further analysis without any conditioning. Figure 2.5 shows the optical sensor MTI-2000. Following are specifications of the MTI-2000 optical sensor:

- Resolution of range 1: $0.03\mu m$

- Resolution of range 2: $0.25\mu m$

- Sensitivity of range 1: $0.03mm/mV$

- Sensitivity of range 2: $0.38mm/mV$

- Linear range of range 1: $0.14mm$

- Linear range of range 2: $1.32mm$

Figure 2.5: MTI-2000 Optical Fotonic Sensor

2.1.5   Spectrum Analyzer

In our experiment we used the Hewlett-Packard HP 3562A analyzer. It is a dynamic signal analyzer, which is capable of testing and analyzing of electronics and electromechanical control systems in the time and frequency domain. It has Fast Fourier Transform (FFT) capability, which gives a Bode plot of the analyzed system dynamics. For analysis, spectrum analyzer sends linear/logarithmic swept sine waves. The bandwidth of the device ranges from 6.4μ Hz to 100kHz. Feedback of the input swept from the system is measured by analyzer and give dynamics of the entire system in terms of Bode plot. This experimental Bode plot then compared to the Bode plot generated by system identification tool. Data transfer from the HP 3562A to a computer using an Agilent 82357B USB/GPIB interface. Figure 2.6 shows spectrum analyzer used for the experiment.

Figure 2.6: Spectrum Analyzer

2.1.6   Data Acquisition System

The data acquisition system is based on a combination of a PCI data acquisition card model 6259 and of a BNC 2110 block from National Instruments (NI). The BNC 2110 has 8 analog input channels and 8 analog output channels. It also has 9 digital input channels and 7 digital output channels. The BNC 2110 block connects to the data acquisition card mounted inside the computer on a PCI bus. The data acquisition card has a maximum sample rate of 1.25 MS/S coded on 16-bits. Figure 2.7 shows the BNC connector block.

2.1.7   Multiplexer

The multiplexer is a hardware switch, which helps to actuate a group of actuator, by sending the signals received from controller at particular instant of time. This multiplexer hardware switches the amplifier output to particular actuator depending

Figure 2.7: BNC 2110 Connector Block

on the response of multiplexing signal from the controller. multiplexer circuit was build on DAQ board, which has three digital output channels. The controlled signals are generated within the real-time Simulink interface. A custom multiplexer was built using three PVA13N series MOSFET photovoltaic relays. The input of a relay is readily connected to the multiplexing signal via a current-limiting resistor. The output of the relay is directly connected to the piezoelectric actuator. These relays were chosen on the basis of their output operating voltage range (0-100V) and their bounce-free operation with a 150 μs response time, significantly smaller than the control sampling period of 5×10^{-4}s. Figure 2.8 shows the multiplexing circuit.

2.2   Experimental Setup and Layout

Experimental setup is as shown in Figure 2.1. Smart structure has three actuators mounted on the stand, which is fixed on vibration table. On the beam, near
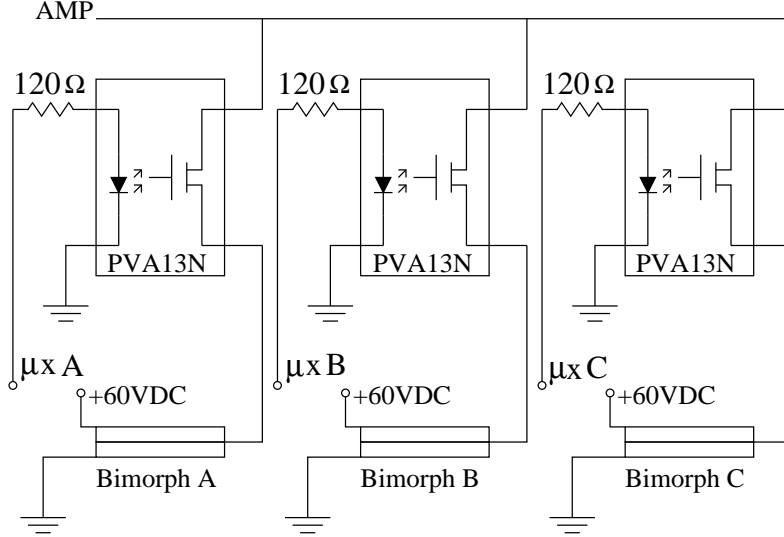
Figure 2.8: Multiplexer Switch Circuit

the tip, reflecting surface is glued for optical sensor operation. The optical sensor is mounted near the tip of the beam. Signals from the optical sensor are collected and sent to controller and observer based control strategy is developed. It calculates the required feed back control signal for each actuator, for controlling the desired moment of the smart structure. Smart structure is connected to data acquisition system and amplifier. Control signals from the controller are sent out to the amplifier, which amplifies the signal to the required signal needed to actuate the actuator. This amplified signal feeds to multiplexer unit built physically. Multiplexer sends signals to each actuator cyclically and sequentially. From actuators, signals are also sent to oscilloscope, where feedback signal from actuator are compared to input signals and also compared to theoretical output performance. Figure 2.9 shows the layout of the experimental setup and also gives idea about the connecting interface. It explains the direction of the flow of signals between important elements of the experiments. First spectrum analyzer is used to study the dynamics of the system. Study is done with each actuator individually, so that the system can be represented in state space form, using system identification technique. Using this individual state space form

and applying control strategy, whole system is defined and based on whole system state space form, observer based LQR controller is developed. This system is simulated to confirm the controller performance. After confirmation of the system and controller working, multiplexer circuit is developed and experimental setup is tested for performance.
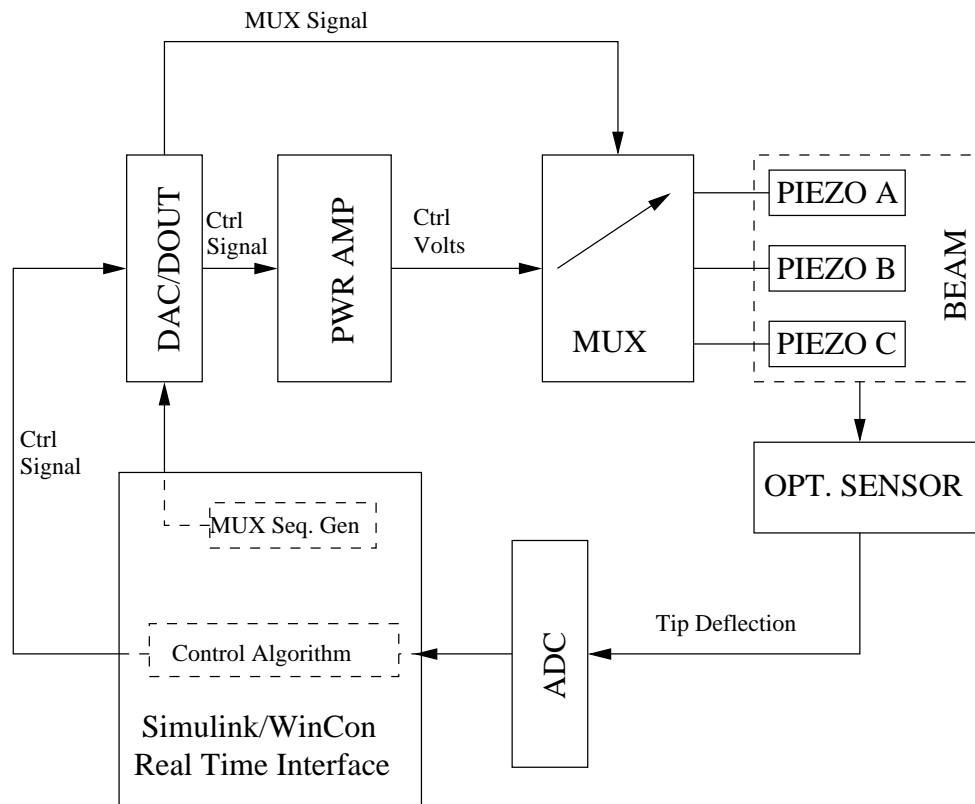
Figure 2.9: Experiment Setup Layout

CHAPTER III

System Identification And Multiplexed Plant Model

This chapter talks about the mathematical modeling of dynamic system, with systematic approach of input/output data processing. This chapter further develops the concept of mathematical model of the multiplexed plant. It derives the strategy for the multiplex control.

3.1   System Identification

For simplicity of control strategy development for any plant, it is very important to understand the plant and derive the mathematical model of the plant. So that, model can be tested and simulated for different input output parameters. For development of mathematical representation of dynamic system, there are many approaches to derive the model. Simple and traditional ways to build the model are based on the first principle. But for complex system, developing the model traditionally will be very time consuming and tedious. It may not even be possible to obtain the reasonable mathematical representation for complex system. So a common approach to develop the mathematical dynamic model is based on the study of system behavior using

measurements of system for known inputs, and then developing the relation between input and output to develop mathematical model. In this process there is no need to go into details of what actually is happening inside the system. This approach of development of dynamic mathematical model is called as system identification.

System identification in our experiment is done using System Identification toolbox. Two types of models are common for system identification

- Black box model: In this technique of mathematical modeling, no prior model of the system is available. It fits the linear and nonlinear models to data. We use this technique of mathematical modeling in our experiment.

- Grey box model: In this case, we have certain model available even if we do not know entirely system dynamics. This model may have certain number of unknown parameters which can be estimated using system identification.

3.1.1   System Identification of Smart Structure

For our experiment purpose we have smart structure of steel beam, with three actuators mounted on it, as shown in Figure 2.2. For simplicity and capturing the behavior of each actuator, operating of each actuator was done separately and input/output measures are captured for each actuator. For this a spectrum analyzer is used. Using HP Spectrum Analyzer, frequency domain response of the plant is measured. In this experiment, analog input was fed to each actuator at a time, with 1 V chirp signal with frequency range of 1Hz to 200Hz (Swept 20 s) and response was recorded at sampling rate of 5kHz. Same step is repeated for all actuators. Figure 3.1 shows the frequency reply plot for one of the actuator. This response data was then used for time-domain analysis and parametric estimation. This response data imported into system identification toolbox. Using this, parametric model was esti-

mated which closely resembles with actual plant as can be seen in figure 3.1. Now using output error method in system identification toolbox, 14th order discrete time transfer function was derived for the system. Same step is repeated for all actuators frequency response data, and three transfer functions $G_A(z)$, $G_B(z)$ and $G_C(z)$ were developed for smart beam, with each actuator acting at a time.

Figure 3.1, 3.2 and 3.3 show the frequency response for all three actuators, and comparison with parametric model.
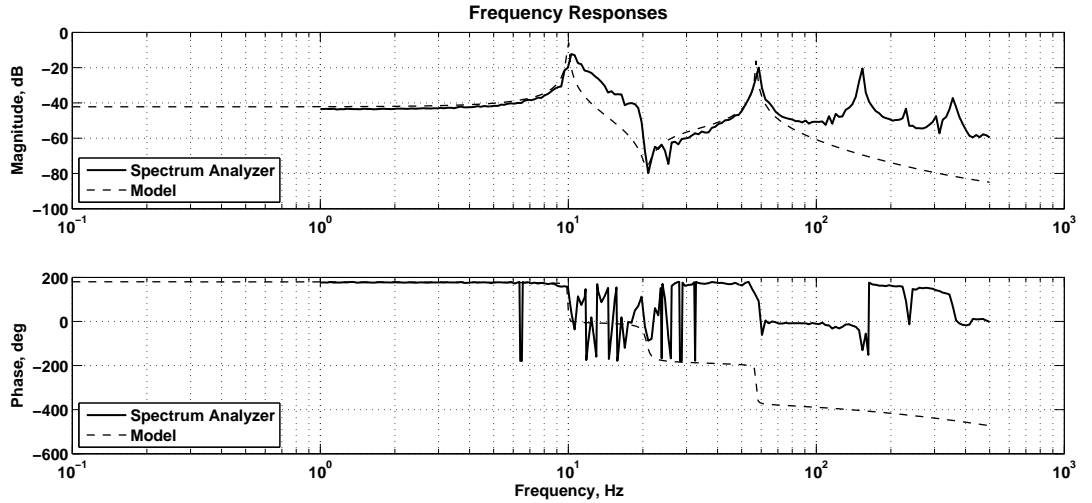
Figure 3.1: Frequency Response for Actuator 1

As now we have three transfer functions for same system, with different actuators acting at a time, these transfer function superimposed together to form the one transfer function, which represents the whole smart structure with three actuators acting together (three inputs and three outputs).

$$Y(z) = G_A(z)U_A(z) + G_B(z)U_B(z) + G_C(z)U_C(z) \qquad (3.1)$$

Where the identified transfer functions are as follows:

$$G_A(z) = \frac{4.366 \times 10^{-5}z^3 - 0.0001891z^2 + 0.0002473z - 0.0001021}{z^4 - 3.965z^3 + 5.927z^2 - 3.96z + 0.9979} \qquad (3.2)$$

20
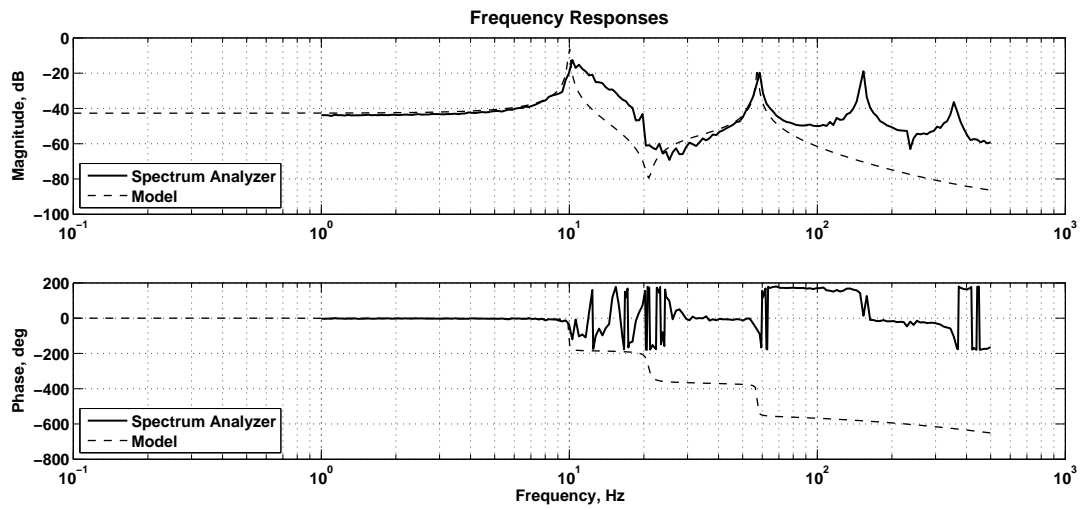
Figure 3.2: Frequency Response for Actuator 2
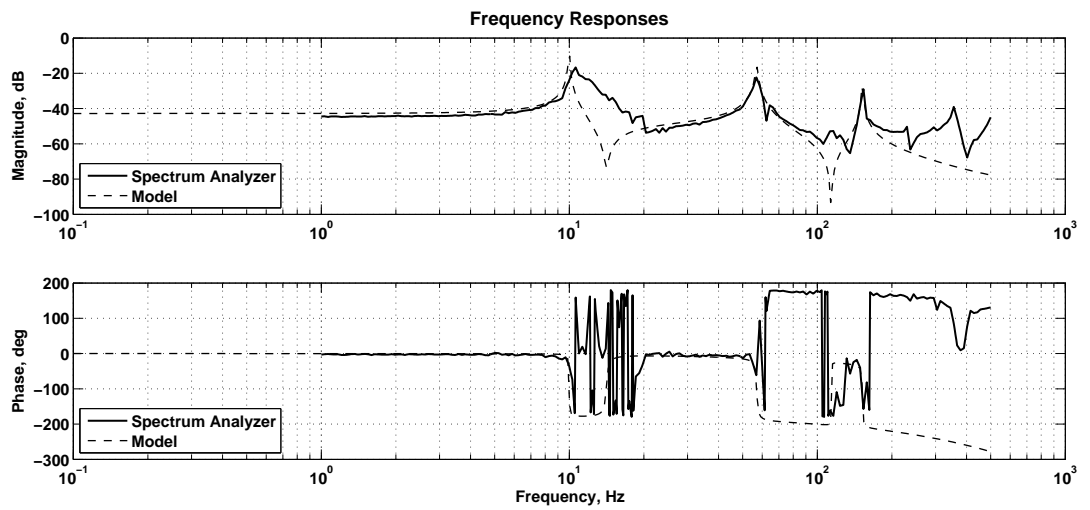
Figure 3.3: Frequency Response for Actuator 3

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$$G_B(z) = \frac{-3.413 \times 10^{-5}z^3 + 0.0001563z^2 - 0.0002103z + 8.834 \times 10^{-05}}{z^4 - 3.965z^3 + 5.927z^2 - 3.96z + 0.9978} \tag{3.3}$$

$$G_C(z) = \frac{-3.513 \times 10^{-5}z^5 + 0.000376z^4 - 0.001131z^3 + 0.001513z^2 - 0.0009631z + 0.0002396}{z^6 - 5.736z^5 + 13.95z^4 - 18.4z^3 + 13.91z^2 - 5.706z + 0.9923} \tag{3.4}$$

Using this complete system representation in transfer function form, we transform system in state space representation. State space form for the complete dynamic system is given as follows: A is block-diagonal of the form:

$$A = \begin{pmatrix} A_{11} & 0 & 0 \\ 0 & A_{22} & 0 \\ 0 & 0 & A_{33} \end{pmatrix} \tag{3.5}$$

Where $A_{11}, A_{22} and A_{33}$ are as given:

$$A_{11} = \begin{pmatrix} 3.965 & -1.482 & 0.495 & -0.2495 \\ 4 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \end{pmatrix} \tag{3.6}$$

$$A_{22} = \begin{pmatrix} 3.965 & -1.482 & 0.495 & -0.2495 \\ 4 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \end{pmatrix} \tag{3.7}$$

$$A_{33} = \begin{pmatrix} 5.736 & -3.486 & 1.15 & -0.4347 & 0.1783 & -0.124 \\ 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.25 & 0 \end{pmatrix} \tag{3.8}$$

$$B = \begin{pmatrix} 0.01563 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0.007813 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.01563 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \Bigg], \tag{3.9}$$

,

$$C^T = \begin{pmatrix} 0.002794 \\ -0.003025 \\ 0.001978 \\ -0.001634 \\ -0.004368 \\ 0.005002 \\ -0.003365 \\ 0.002827 \\ -0.002248 \\ 0.006016 \\ -0.004522 \\ 0.003026 \\ -0.001926 \\ 0.001917 \end{pmatrix} \qquad (3.10)$$

,

$$D = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \qquad (3.11)$$

3.2 Multiplexed Plant

By simplifying the plant in three different actuator modes, we would be able to develop the state space form and transfer function by system identification method as we discussed. After configuring the system representation it was needed to transform the plant in multiplexed plant so that multiplexing strategy could be applied to the

system. The state-space representation of the plant is as shown below,

$$x(k+1) = Ax(k) + Bu(k) \tag{3.12}$$

$$y(k) = Cx(k) \tag{3.13}$$

where A and B have dimensions n-by-n and n-by-m, respectively; $x(k) \in \mathbb{R}^n$ and $u(k) \in \mathbb{R}^m$. To facilitate modeling of the multiplexed implementation, the control input $u(k)$ is assumed to have the form

$$u(k) = u(k-1) + \Delta u(k) \tag{3.14}$$

Now for simplicity $\Delta u(k)$ is used as new input vector. The state vector is augmented with the m components of $u(k-1)$, resulting in the description

$$\left[\begin{array}{c} x(k+1) \\ \hline u(k) \end{array} \right] = \left[\begin{array}{c|c} A & B \\ \hline 0 & I_m \end{array} \right] \left[\begin{array}{c} x(k) \\ \hline u(k-1) \end{array} \right] + \left[\begin{array}{c} B \\ \hline I_m \end{array} \right] \Delta u(k) \tag{3.15}$$

Denoting the extended state vector as $\tilde{x}(k) = [x(k)^T \mid u(k-1)^T]^T$ and the new system matrices as Ag and Bg, the augmented system becomes

$$\tilde{x}(k+1) = A_g \tilde{x}(k) + B_g \Delta u(k) \tag{3.16}$$

3.2.1 Control Objective

Basic objective for this experiment is to implement the multiplexing technique to help realize benefit of multiplexing in reducing the hardware requirements. As by implementing multiplexing technique we will be reducing the requirement of three amplifier to one amplifier. And to make it realize multiplexing control theory is develop.

By developing the multiplex controller, we wanted to verify the system, first in simulation by giving known input to the system and plotting the output for closed-loop system. Once simulation confirms the controller working, it can be implemented in

real-time. In real-time experiment, controller objective is to control the motion of the beam and keep the beam to the mean position. This objective is confirm in real-time by giving the impulse disturbance to the beam and monitoring the motion of the beam in open-loop and closed-loop system. This confirms the objective of the controller and at the same time confirms the effectivity of the controller.

3.2.2 Multiplexed control

After development of the multiplexed plant, next thing to consider was to develop multiplexing technique, which takes care of control signal updating, sequentially and cyclically. For this LQR control strategy is used along with multiplexed control strategy. Now for multiplexed control, we assume that there are r groups of control inputs, which are to be simultaneously updated, where, for simplicity, $q = m/r \in \mathbb{Z}^+$ and $r < m \le n$. Also we assume that the q update groups are contiguous in Δu and that the update sequence coincides with the order in which the groups are stacked in Δu. Now to update required signals at desired time, selector matrix is developed. Define the *p-dimensional selector matrix at time k* as the q-periodic matrix

$$E_p(k) = \left[{}_p e^T_{r(k \bmod q)+1} \middle| {}_p e^T_{r(k \bmod q)+2} \middle| \cdots \middle| {}_p e^T_{r(k \bmod q)+r} \right]$$

with ${}_p e_j$ being the j-th canonical basis row vector of $\mathbb{R}^p$. The multiplexed plant can now be described as

$$
\begin{aligned}
\tilde{x}(k+1) &= A_g \tilde{x}(k) + b_g(k) w(k), \text{ where:} \qquad (3.17)\\
b_g(k) &= B_g E_{(m+n)}(k)
\end{aligned}
$$

Clearly, $b_g(k)$ is periodic with period q. $w(k)$ is an r-vector re-parameterizing the control inputs according to

$$w(k) = E^T_m(k) \Delta u(k) \qquad (3.18)$$

Note that the selector matrices are orthogonal, implying that

$$\Delta u(k) = E_m(k) w(k)$$

Multiplexing is equivalent to setting all columns of B to zero except for a block of columns corresponding to the actuators being updated at a particular instant. Therefore the plant has a periodically varying B matrix. So while designing the controller it needs to account for time invariant Eigenvalue condition for stability. Consider the *monodromy matrix* of $A(k)$ at time j is defined as $\Phi_A(T + j, j) \triangleq A(T + j - 1)A(T + j - 2)...A(j)$, that is product of all instances of $A(k)$ over one period.

Characteristic multipliers:

The Eigenvalues of Φ_A are known as characteristic multipliers. These Eigenvalues are independent of j.

Stability:

A periodic system associated with $(A(k), B(k))$ is said to be stabilizable if a T-periodic feedback matrix $F(k)$ can be found such that $A(k) - B(k)F(k)$ is asymptotically stable.

Asymptotic stability:

The periodic system associated with $(A(k), B(k))$ is asymptotically stable if and only if all characteristic multipliers lie in the open disc $\{z \in \mathbb{C} : |z| < 1\}$.

Now after defining the stability requirements for the multiplexed plant with varying B matrix, we need to chose control feedback F(k) which keeps the Eigen values of the monodromy matrix within the unit circle.

3.2.3 Control Law

Using a state feedback control law, we define:

$$\Delta u = -F(k)x(k) \tag{3.19}$$

The computation of $F(k)$ is done by solving the system of discrete-time periodic Riccati equations:

$$X_j = Q(j) + A_g^T(j)X_{j+1}A_g(j) \tag{3.20}$$

$$- A_g^T(j)X_{j+1}B_g(j)\left(R(j) + B_g^T(j)X_{j+1}B_g(j)\right)^{-1}B_g^T(j)X_{j+1}A_g(j) \tag{3.21}$$

for $j = 0, 1, 2...(q-1)$. Under stabilizability, a unique, symmetric and positive-semi definite sequence X_j can be found. The desired periodic feedback gain is then computed from

$$F_j = -(R(j) + B_g^T(j)X_{j+1}B_g(j))^{-1}B_g^T(j)X_{j+1}A_g(j) \tag{3.22}$$

Note that the q equations cannot be solved separately, and that the solution method can be described as numerically intensive. This provides the motivation for the order-reducing control scheme, where the optimal control problem is solved for system matrices of reduced dimensions, representing a computational advantage against calculating an LQR optimal controller for the multiplexed plant directly. Several methods exist to solve the above system of Riccati equations [2]. In one of them, an initial symmetric, positive-semi definite and stabilizing X_0 is calculated from a forward-time discrete periodic Lyapunov equation (FTDPLE). In turn, several methods are available for solving the FTDPLE. One of them reduces to the solution of a standard discrete Lyapunov equation. Once X_0 is available, a Newton step having the form of a reverse-time discrete periodic Lyapunov equation (RTDPLE) is repeated until convergence of X_j to a periodic sequence. More recent, computationally efficient methods involving the periodic Schur decomposition are also described in [2, 3]. The

reader is referred to these works and references therein for a detailed exposition of numerical methods.

To facilitate modeling of the multiplexed implementation, the control input $u(k)$ is assumed to have the form

$$u(k) = u(k-1) + \Delta u(k) \tag{3.23}$$

and Δu from section 3.2.2 is given by

$$\Delta u(k) = E_m(k)w(k) \tag{3.24}$$

So, we have

$$u(k) = u(k-1) - F(k)\tilde{x}(k) \tag{3.25}$$

Substituting the value of $u(k)$ (3.25) in equation 3.12, we obtain the closed loop dynamics of the system, given as

$$x(k+1) = Ax(k) + B\left(u(k-1) - F(k)\tilde{x}(k)\right) \tag{3.26}$$

where, $\tilde{x}(k) = \begin{bmatrix} x(k) \\ \hline u(k-1) \end{bmatrix}$

3.3 Observer

This section introduces the complete working setup and explains the observer based simulation. The Main emphasis is placed on working of an observer with multiplexed plant. It gives overview of theoretical base of observer and compares simulation results with real time working of the system. For feedback control system, it is desirable to know the different components of the system as a measurable state vector, so that these components can be processed by control strategy and required control signal

can be sent back to the system. This cyclic process of signal conditioning is called as closed loop feedback system. In the full state feedback design procedure, it is assumed that all the state vectors are measurable and available for feedback, based on which feedback controller works. However, ideally speaking all the states of the system are difficult to measure at all times, and eventually it will become costly as to measure each state vector we need complete setup of the sensor combination.

In this regards, if system is completely observable with given set of outputs, it will be possible to determine the states which are not measurable or observable, with the help of observer technique. A state observer is designed to estimate the signals that cannot be measured.

The plant to be observed and controlled is given by

$$x(k+1) = Ax(k) + Bu(k) \tag{3.27}$$

$$y(k) = Cx(k) \tag{3.28}$$

where A and B have dimensions n-by-n and n-by-m, respectively; $x(k) \in \mathbb{R}^n$ and $u(k) \in \mathbb{R}^m$.

The Luenberger observer is given as

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + H(y(k) - \hat{y}(k))$$

$$\hat{x}(k+1) = (A - HC)\hat{x}(k) + Bu(k) + Hy(k) \tag{3.29}$$

The equation for the error dynamics of the system is obtained by taking the difference between the estimated values of state $\hat{x}(k)$ and the actual state $x(k)$. The equation

is found as

$$
\begin{aligned}
e(k) &= \hat{x}(k) - x(k) \\
e(k+1) &= \hat{x}(k+1) - x(k+1)
\end{aligned}
\tag{3.30}
$$

The Figure 3.4 shows the structure of a state observer as described by the mathematical equations. Once the states are estimated through the observer they are fed back to the system to achieve the desired control objective.

3.4   Observer for Multiplexed plant

For the multiplex plant we have, we used reduced order observer, which estimates the states of non-measurable states and uses the raw measurement data for those that are measurable. In the multiplexed plant, state vector is actually an augmented vector working with states and previous control inputs. The states are augmented with previous control inputs, so as to work with control input updates to achieve multiplexing. Using the reduced order observer, augmented state vector fed to the observer consists of the states and the calculated previous control inputs. The states estimated by the observer are fed back to the plant.

As shown in Section 3.2, introduction of multiplexing to a plant results in a periodic system. Even though the system being observed is periodic in nature, the observer designed in this case is a linear time invariant system. This system is stabilized independently using a feedback gain which drives the error between the states and the estimates to zero.
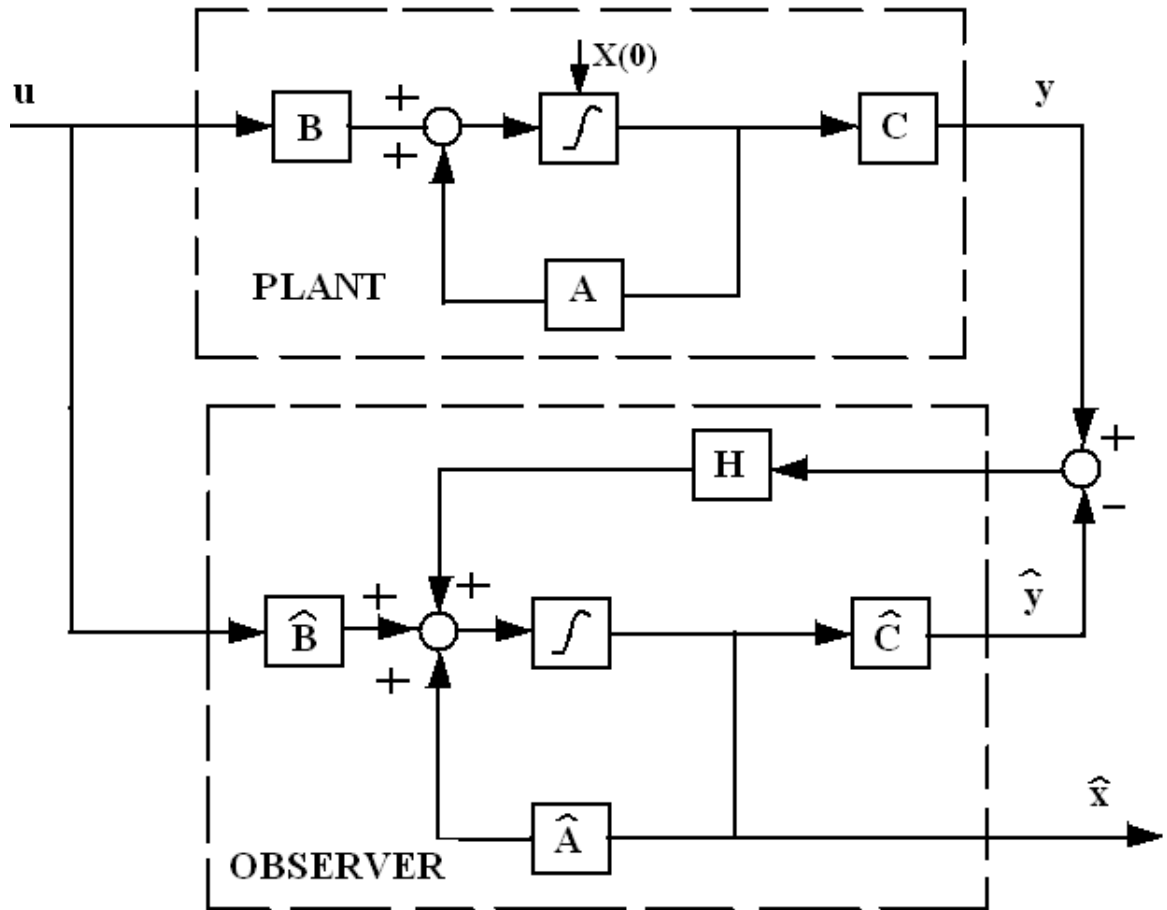
Figure 3.4: Observer

3.4.1 Reduced-order observer

Plant:

The underlying plant to be controlled has the standard state-space form

$$x(k+1) = Ax(k) + Bu(k) \tag{3.31}$$

$$y(k) = Cx(k) \tag{3.32}$$

where A and B have dimensions n-by-n and n-by-m, respectively; $x(k) \in \mathbb{R}^n$ and $u(k) \in \mathbb{R}^m$.

Estimator:

A reduced-order estimator equation which only estimates the values of the states is given as

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + H(y(k) - \hat{y}(k)) \tag{3.33}$$

$$\hat{x}(k+1) = (A - HC)\hat{x}(k) + Bu(k) + Hy(k) \tag{3.34}$$

To facilitate modeling of multiplexing implementation, the control input $u(k)$ is assumed to have the form:

$$u(k) = u(k-1) + \Delta u(k) \tag{3.35}$$

$$\Delta u(k) = E_m(k)w(k) \tag{3.36}$$

$$w(k) = -F(k)\tilde{x}(k) \tag{3.37}$$

$$\text{where, } \tilde{x}(k) = \left[\frac{\hat{x}(k)}{u(k-1)} \right] \tag{3.38}$$

Estimator error dynamics:

The equation for the error dynamics of the system is obtained by taking the difference between the estimated values of state $\hat{x}(k)$ and the actual state $x(k)$. The equation is given as

$$
\begin{align}
e(k) &= \hat{x}(k) - x(k) \tag{3.39} \\
e(k+1) &= \hat{x}(k+1) - x(k+1) \tag{3.40}
\end{align}
$$

From equation 3.31 and 3.34

$$
\begin{align}
e(k+1) &= (A - HC)\hat{x}(k) + Bu(k) + Hy(k) - Ax(k) - Bu(k) \tag{3.41} \\
e(k+1) &= (A - HC)\hat{x}(k) - (A - HC)x(k) \tag{3.42} \\
e(k+1) &= (A - HC)e(k) \tag{3.43}
\end{align}
$$

Closed loop dynamics

As shown in Anil's thesis [5], the closed loop dynamics of the plant using the estimated values of state from the observer is changed accordingly:

$$
x(k+1) = Ax(k) + Bu(k) \tag{3.44}
$$

Combining the closed loop plant dynamics and the error dynamics, we have

$$
\begin{bmatrix} \acute{x}(k+1) \\ \hline e(k+1) \end{bmatrix} = \left[\begin{array}{c|c} A - b_g F(k) & b_g(k)F(k) \\ \hline 0 & A - HC \end{array} \right] \begin{bmatrix} \acute{x}(k) \\ \hline e(k) \end{bmatrix} \tag{3.45}
$$

Since the plant dynamics only depend on the Eigenvalues of $A - b_g F(k)$ and the error dynamics only depend on the Eigenvalues of $A - HC$, so if we find feedback gains $F(k)$ for the plant and H for the observer which stabilize the corresponding systems individually, then the whole closed loop system would be stabilized.

3.5   Simulation

After discussing development of the multiplexed plant, multiplexed controller, LQR controller and observer, its time to confirm the working of derived plant in simulation, so that controller can be implemented in real time. For simulation verification refer the simulation model 3.5 as shown in figure. Matlab 'm' files described in appendix gives the step by step formulation of system simulation. It develops the multiplexed plant from the measured data from analyzer tool. This plant is augmented for multiplexing purpose. By applying control theory for this augmented plant, periodic feedback gain is calculated for control signal development. Optimized observer based LQR controller is developed for simulation.

To simulate the plant as closely as to real condition, plant is discretized, to make it continuous state space model. The plant is multiplexed by setting all columns of B to zero except for a block of columns corresponding to the actuators being updated at a particular instant as shown in Section 3.4.1. In this case only one control input is being updated at a sampling instant holding the other three control inputs at their previous values, thus the control algorithm calculates the each control update after every three sampling instants, as we have 3 input states for three actuators. This multiplexed plant is connected to an observer which is a linear time invariant plant as compared to the multiplexing plant which is periodic. The observer used here is a reduced-order observer which only estimates the states of the system but uses the calculated previous control inputs in the augmented state vector fed to it for calculations. The simulations show the working of the multiplexed plant with the reduced-order observer. The program used for simulations is attached in Appendix A.
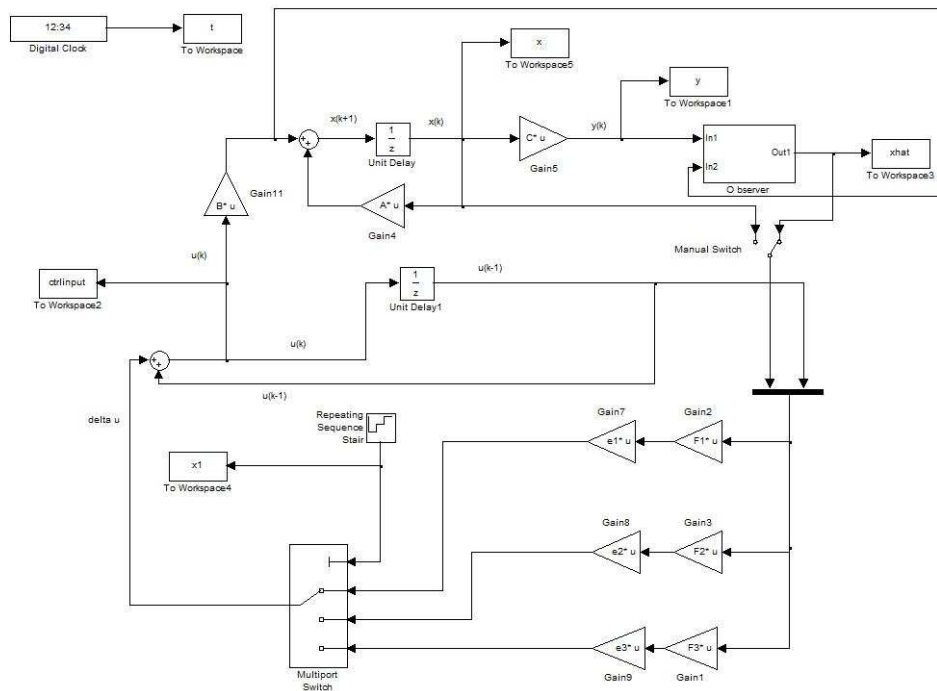
Figure 3.5: Simulink Model
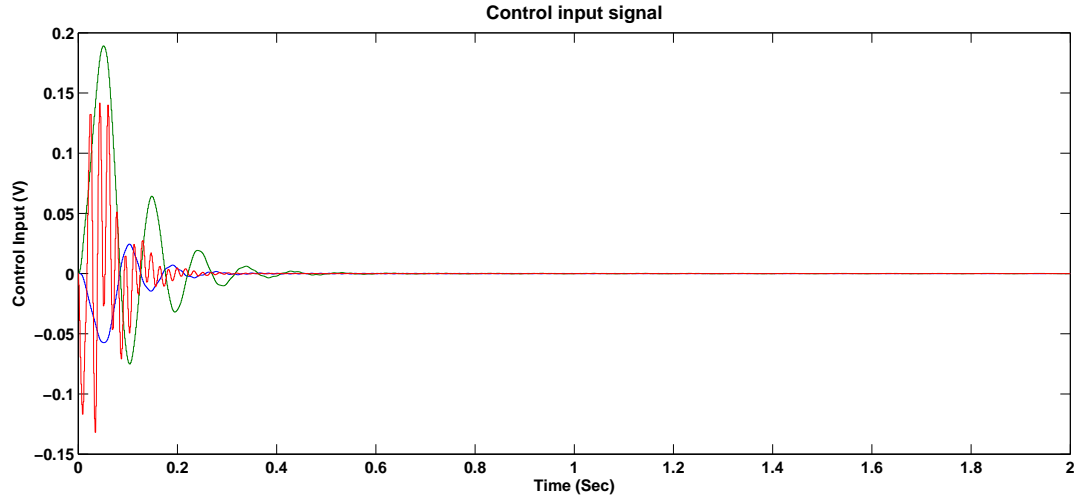
Figure 3.6: Control Input Signal

3.5.1 Simulation results

The simulation plot shows the working of the multiplexed plant with the reduced-order observer. The states estimated by the observer are used in the control to achieve the required control objective. Figure 3.6 show the control input signal to the plant. It shows the three control input signal, with initially higher actuating control signal, which eventually reduces as position of the beam comes to center position after initial deflection. Figure 3.7 show the regulation of state trajectory to zero using an observer. For simulation purpose it is consider that initial position of the beam is at 0.01mm deflected from mean position. So output plot shows the closed loop control of the beam position, which eventually comes to mean position due to control actuation.

Figure 3.7: Output Signal

CHAPTER IV

Real Time Implementation

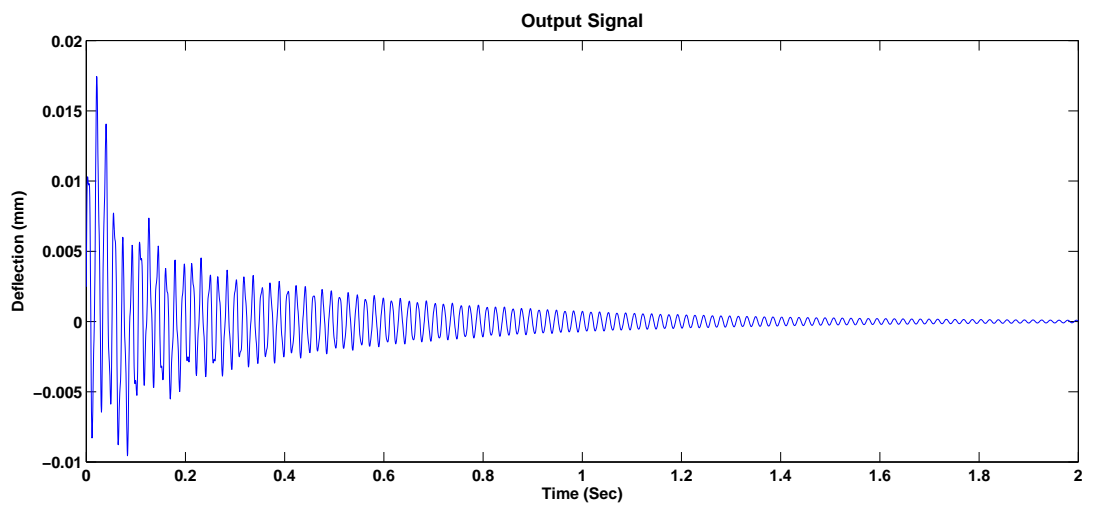This chapter talks about the overview of real time implementation of the multiplexing controller for multiplexed plant, which is the smart structure with three actuators. We focus mainly on the performance of the controller with smart structure and how effectively it controls the moment of the smart structure. At the end we compared the real time results with simulation results and confirms the effective utilization of the theory.

4.1 Experiment Setup Details

We have already discussed about the different components of the experimental setup and their arrangement. Schematic shows the arrangement of the experimental setup. For shown experimental setup 2.9, we will discuss here about different operating parameters and specifications.

For performing practical test of multiplexed controller, connections of the system is made as shown. WINCON software interface is used collecting signal data from data acquisition board. This data is processed and conditioned for controlling of moment of the smart structure. Calculated controlled signals from the controller sent

out through same data acquisition board and passed through amplifier. Amplifier amplification factor is 5, which amplifies the controlled signal and further send it to multiplexer, where signal is cyclically and sequentially sent to individual actuators. Movement of the smart beam is sense by optical sensors and data is sent to controller through data acquisition board. This is closed loop feedback system.

During preparation of the smart structure, with three actuators mounted on the beam, it was critical due to very delicate nature of the piezoelectric actuator samples we have. Also to make actuators integral part of beam, it was having delicate and stringent conditions of assembly.

When updating the actuator signals sequentially and cyclically, it needs to hold the remaining actuators to previous value of signal. For this there should be a mechanism to hold the signal to its value. As such in our configuration of multiplexer, we don't have any zero order hold for signal, but still working of the controller is as expected. This is because of the fact that, actuators act as capacitors, so while there is signal being sent to one actuator, remaining actuators stays at previous value of signal. There is little drop in the signal, but it is very low, as actuator signal shows hysteresis effect on the current. This effect is observer and recorded in oscilloscope. Figure 4.1shows the actuator terminal voltage and switching signals. From the figure we can see that, when one actuator signal is active, remaining actuator signals are at constant voltage. At this period of time there is little drop in the voltage, but due to hysteresis effect of actuator voltage this drop in signal voltage is very negligible. Also as sampling time is very small, there is no much effect of switching of signals.
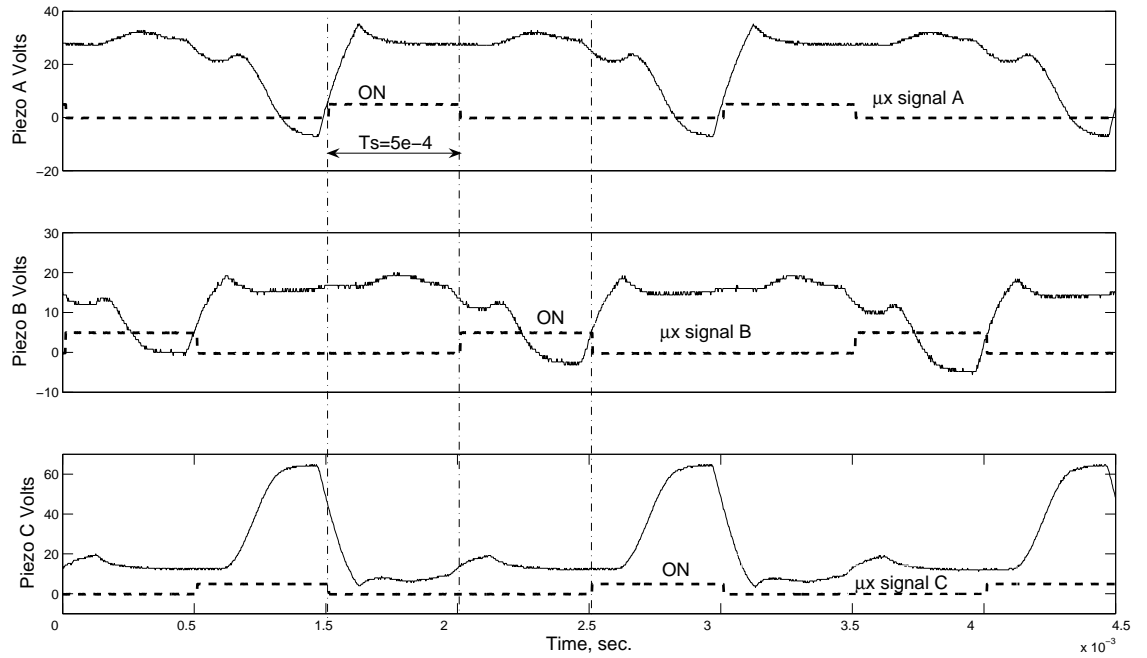
Figure 4.1: Actuator terminal voltage and switching signals

4.2 Real time operation verification

Controller is tested for abrupt moment of the smart structure and controlling efforts by controller. The WINCON software is used to build a real-time Simulink control interface. Figure 4.2 shows a comparison between the free vibration response (uncontrolled) and the multiplexed control response. In each case, a small mechanical impulse was applied to the beam which resulted in a tip displacement of about 2 mm. Note that the impulse had to be stronger in the closed-loop case to attain the same peak displacement as in the open-loop case. Figure shows, the multiplexed controller effectively reduces the time constant of vibrations by an order of magnitude.

Figure 4.2: Open and closed-loop response

CHAPTER V

Conclusion and Future work

This chapter describes the different outcomes and inferences obtained from the research work we did in this thesis. All the inferences are based on the simulation study and on the real time implementation.
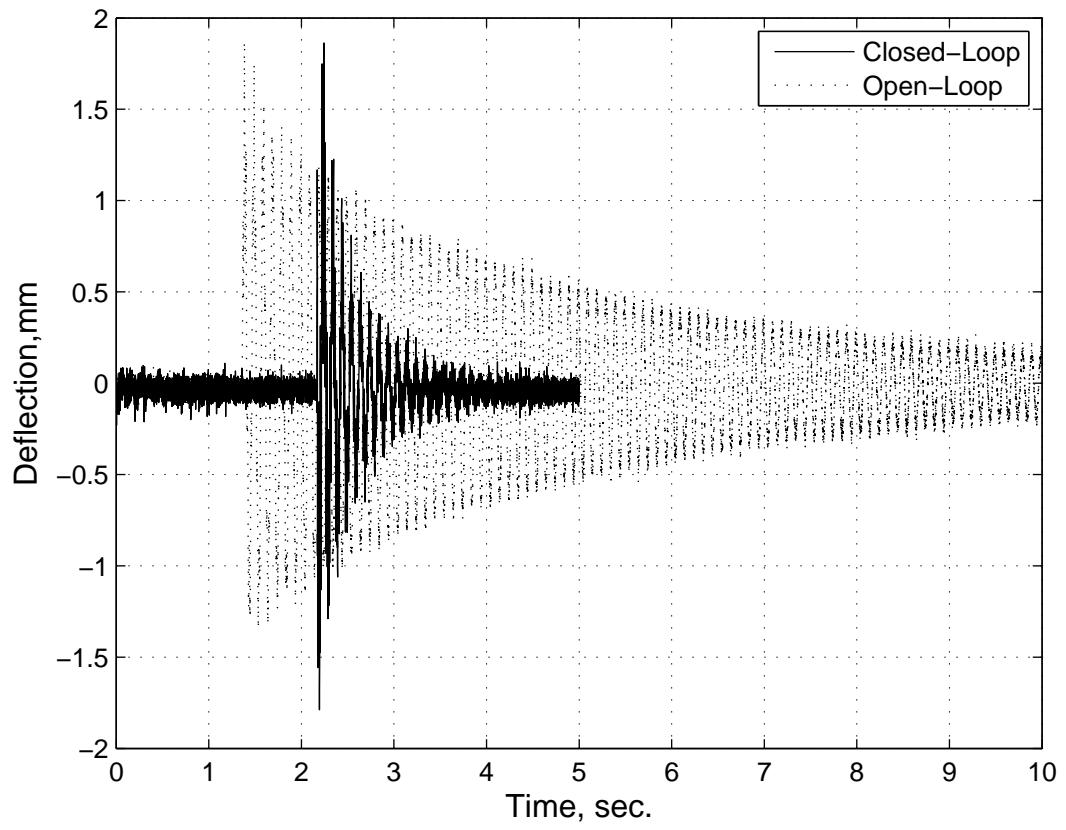
5.1   Conclusion

Main focus of this thesis was to prove the multiplexing control theory, and implement it in real time on smart structure. So first basic understanding of the theory is done and then eventually experimental setup is developed to accommodate theoretical proof. This thesis investigates multiplexing control as an alternative to solving hardware and computational constraint problems in systems with large number of interconnected actuators. In this work multiplexing of linear discrete time system was demonstrated using the standard state feedback control law. An observer model was developed to work with the multiplexed plant so that the estimates of the states can be used to achieve the required control objective. Once required control strategy is achieved, real-time implementation of controlled strategy was done on smart structure.

Based on the Simulation results and real-time inference, following conclusions are drawn:

- Use of multiplexing for operating system with large number of actuators and sensors, reduces the computational time required for calculating the optimal control law by reducing the dimensionality of the complex dynamic system without adversely affecting performance of the closed loop system.

- The computational advantage of the multiplexed implementation lies in that all Quadratic Programming routines are now performed over just one degree of freedom. It is a well-established fact that the time required to solve a QP problem grows with the cube of the number of inputs. The sample rate reduction is only linear in the number of inputs. Therefore, the time savings earned by reduced order multiplexed plant consideration may even allow increasing the original sample rate to help recover any lost performance due to slower sampling.

- Multiplexing control provides a way to maximize the hardware resources. In systems with cost and space constraints, multiplexing can be used to time share the electronic drive units among actuators.

- Reduction in the number of actuators being updated in a group although reduces the hardware required but it increases the period and the dimensionality of the matrices involved, thus increases the computational effort required. So multiplexing introduces a hardware-computational tradeoff.

- The use of observer with the multiplexed plant avoids the need to measure all of the state variables. It is shown that the multiplexed plant can be observed by a linear time invariant observer system.

- One of the key issue surfaced out in practical implementation is that the multiplexer circuit periodically leaves the terminals of the actuator open, constituting

a deviation from the zero-order-hold assumption commonly used in digital control design. The capacitance of the actuators and the converse piezoelectric effect prevent the terminal voltage to drop to zero upon disconnection, limiting the impact of this deviation.

5.2   Future work

A number of avenues are available for future research, in both theory and application of this technique. The following is a list of areas where future work can be carried out regarding the multiplexed implementation:

- Control systems in the real world are infused with disturbances which affect the system performance in significant manner. So research can be carried out to test the performance of multiplexing control when applied to a disturbance plant.

- Research on developing a multiplexed observer to work with a multiplexed plant to reduce the computation time in calculating the estimates when observer uses a complex computation intensive algorithm.

- Develop a systematic procedure for tuning the system for MMPC to satisfy all performance requirements.

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APPENDICES

APPENDIX A

MATLAB PROGRAMS AND

SIMULATION MODELS

1.1   Multiplexing with observer

```
%buildmodel3new

%Builds the 3-input mathematical model for the 3-actuator beam.


load rawfreq_3beam_new        %contains raw frequency traces from HP3562A


start_f=1;

end_f=500;

data=Adata;

process_dump;

Ts=5e-4;

load modelfromAnew               %in output error form

ss441=ss(oe441);

[Ad,Bd,Cd,Dd]=ssdata(ss441);

Bd=Bd(:,1);

Dd=Dd(1);                        %eliminate noise input components
```

```
full_dss_A=ss(Ad,Bd,Cd,Dd,Ts);


%Plotting section: plot HP data and model


w=logspace(log10(0.1*2*pi),log10(500*2*pi),500);

[magmod,phmod]=dbode(Ad,Bd,Cd,Dd,Ts,1,w);


fsize=12;                          %font size for all labels

lw1=1.5;                           %line width

lw2=1;                             %line width

subplot(2,1,1)

semilogx(f,mag+offset,'k','Linewidth',lw1);

hold on

semilogx(w/2/pi,20*log10(magmod),'k--','Linewidth',lw2)

ylabel('Magnitude, dB','Fontsize',fsize)

title('Frequency Responses','Fontsize',fsize)

legend('Spectrum Analyzer','Model','Location','SW')

grid

%axis([0.1 5000 -80 20])

subplot(2,1,2)

semilogx(f,phas,'k','Linewidth',lw1)

hold on

semilogx(w/2/pi,phmod,'k--','Linewidth',lw2);

grid

xlabel('Frequency, Hz','Fontsize',fsize)

ylabel('Phase, deg','Fontsize',fsize)

legend('Spectrum Analyzer','Model','Location','SW')

%axis([0.1 5000 -200 100])
```

```
%Repeat for actuator B

data=Bdata;

process_dump;

load modelfromBnew    %in output error form

ss441=ss(oe441);

[Ad,Bd,Cd,Dd]=ssdata(ss441);

Bd=Bd(:,1);

Dd=Dd(1);                %eliminate noise input components

full_dss_B=ss(Ad,Bd,Cd,Dd,Ts);


w=logspace(log10(0.1*2*pi),log10(500*2*pi),500);

[magmod,phmod]=dbode(Ad,Bd,Cd,Dd,Ts,1,w);


figure(2)

subplot(2,1,1)

semilogx(f,mag+offset,'k','Linewidth',lw1);

hold on

semilogx(w/2/pi,20*log10(magmod),'k--','Linewidth',lw2)

ylabel('Magnitude, dB','Fontsize',fsize)

title('Frequency Responses','Fontsize',fsize)

legend('Spectrum Analyzer','Model','Location','SW')

grid

%axis([0.1 5000 -80 20])

subplot(2,1,2)

semilogx(f,phas,'k','Linewidth',lw1)

hold on

semilogx(w/2/pi,phmod,'k--','Linewidth',lw2);

grid

xlabel('Frequency, Hz','Fontsize',fsize)
```

```matlab
ylabel('Phase, deg','Fontsize',fsize)

legend('Spectrum Analyzer','Model','Location','SW')

%axis([0.1 5000 -200 100])


%Repeat for actuator C

data=Cdata;

process_dump;

load modelfromCnew                    %in output error form

ss661=ss(oe661);

[Ad,Bd,Cd,Dd]=ssdata(ss661);

Bd=Bd(:,1);

Dd=Dd(1);                             %eliminate noise input components

full_dss_C=ss(Ad,Bd,Cd,Dd,Ts);


w=logspace(log10(0.1*2*pi),log10(500*2*pi),500);

[magmod,phmod]=dbode(Ad,Bd,Cd,Dd,Ts,1,w);


figure(3)

subplot(2,1,1)

semilogx(f,mag+offset,'k','Linewidth',lw1);

hold on

semilogx(w/2/pi,20*log10(magmod),'k--','Linewidth',lw2)

ylabel('Magnitude, dB','Fontsize',fsize)

title('Frequency Responses','Fontsize',fsize)

legend('Spectrum Analyzer','Model','Location','SW')

grid

%axis([0.1 5000 -80 20])

subplot(2,1,2)

semilogx(f,phas,'k','Linewidth',lw1)
```

```
hold on

semilogx(w/2/pi,phmod,'k--','Linewidth',lw2);

grid

xlabel('Frequency, Hz','Fontsize',fsize)

ylabel('Phase, deg','Fontsize',fsize)

legend('Spectrum Analyzer','Model','Location','SW')

%axis([0.1 5000 -200 100])



%Assemble MIMO model for multiplexed design

[numA,denA]=tfdata(full_dss_A);

[numB,denB]=tfdata(full_dss_B);

[numC,denC]=tfdata(full_dss_C);

mimo_model=tf({numA{1},numB{1},numC{1}},{denA{1},denB{1},denC{1}},Ts);



%Obtain state-space realization


model=ss(mimo_model);

[A,B,C,D]=ssdata(model);



%Model is order 14, fully controllable, not fully observable

%Need to check periodic reachability:


[n,m]=size(B);


%Augmented discrete model

Ag=[A B;zeros(m,n) eye(m)];

Bg=[B;eye(m)];

r=1;  %this is the size of the update block
```

```
%Computation of periodic regular form and transformation

q=m/r;  %period

Bset={Bg(:,1),Bg(:,2),Bg(:,3)};

R1=[Bset{1} Ag*Bset{3} Ag^2*Bset{2} Ag^3*Bset{1} Ag^4*Bset{3}

    Ag^5*Bset{2} Ag^6*Bset{1} Ag^7*Bset{3} Ag^8*Bset{2}]; %at time =1

R2=[Bset{2} Ag*Bset{1} Ag^2*Bset{3} Ag^3*Bset{2} Ag^4*Bset{1}

    Ag^5*Bset{3} Ag^6*Bset{2} Ag^7*Bset{1} Ag^8*Bset{3}]; %at time =2

R3=[Bset{3} Ag*Bset{2} Ag^2*Bset{1} Ag^3*Bset{3} Ag^4*Bset{2}

    Ag^5*Bset{1} Ag^6*Bset{3} Ag^7*Bset{2} Ag^8*Bset{1}]; %at time =3

rank(R1)

rank(R2)

rank(R3)

%Periodic syst is completely reachable if the above matrices are full-rank.

%Periodic reachability demonstrated.


%Design/tune feedback gains for regulation using theory.


%Begin calculation of periodic generator Xo

%include margin bb

bb=2;

AA=Ag*Ag*Ag/bb;                         %monodromy matrix

V1=Bg(:,1)*Bg(:,1)'/sqrt(bb);

V2=Bg(:,2)*Bg(:,2)'/sqrt(bb);

V3=Bg(:,3)*Bg(:,3)'/sqrt(bb);           %changed signs to plus (typo in Varga?)

QQ=Ag*Ag*V1*(Ag*Ag)'/bb+Ag*V2*Ag'/sqrt(bb)+V3;

X1prev=dlyap(AA,QQ);


%Recursion to find remaining matrices

X2prev=Ag*X1prev*Ag'+V1;
```

```
X3prev=Ag*X2prev*Ag'+V2;


for i=1:100,
 X1=Ag*X3prev*Ag'+V3;

 X2=Ag*X1*Ag'+V1;

 X3=Ag*X2*Ag'+V2;

 percent1=norm(X1-X1prev)/norm(X1prev)

 percent2=norm(X2-X2prev)/norm(X2prev)

 percent3=norm(X3-X3prev)/norm(X3prev)

 X3prev=X3;

 X2prev=X2;

 X1prev=X1;
end


%Calculate a periodic feedback gain:


F1=-Bset{1}'*inv(X2+Bset{1}*Bset{1}')*Ag;

F2=-Bset{2}'*inv(X3+Bset{2}*Bset{2}')*Ag;

F3=-Bset{3}'*inv(X1+Bset{3}*Bset{3}')*Ag;


%check monodromy matrix for A+BF
Acl={Ag+Bset{1}*F1, Ag+Bset{2}*F2, Ag+Bset{3}*F3};


MONO=Acl{3}*Acl{2}*Acl{1};
eig(MONO)


%Simulate from arbitrary IC
%x=rand(1,17);
```

```
%for i=1:1000,
  %  x(i+1,:)=Acl{mod(i+2,3)+1}*x(i,:)';
%end


clc
%We can find an optimized (LQR) solution as well using F1,F2,F3 as a seed


%Q=10*eye(17); R=1e5; %Works well in simulation,
    excites higher modes in realtime experiment


Q=eye(17); R=1e10;


%Q=[1e5*C'*C zeros(14,3);zeros(3,14) eye(3)];
%R=1;
Fprevcell={F1,F2,F3};
for i=1:20,  %loop the controlling number of newton steps
   AAcell={Ag+Bset{1}*Fprevcell{1},Ag+Bset{2}*Fprevcell{2},
           Ag+Bset{3}*Fprevcell{3}};
   Qcell={Q+Fprevcell{1}'*R*Fprevcell{1},
           Q+Fprevcell{2}'*R*Fprevcell{2},Q+Fprevcell{3}'*R*Fprevcell{3}};
   %Now solve the RTDPLE:
   %Begin calculation of periodic generator Xo
   AA=AAcell{3}*AAcell{2}*AAcell{1};  %monodromy matrix
   QQ=(AAcell{2}*AAcell{1})'*Qcell{3}*AAcell{2}*AAcell{1}
        +AAcell{1}'*Qcell{2}*AAcell{1}+Qcell{1};
   %X1prev=dlyap(AA',QQ);


   %Use dlyapchol for improved numerics:
   BBB=sqrtm(QQ);     % do not understand why???
```

```
XXX=dlyapchol(AA',BBB);

X1prev=XXX'*XXX;


Xgensol=X1prev;

%Recursion to find remaining matrices

X3prev=AAcell{3}'*X1prev*AAcell{3}+Qcell{3};

X2prev=AAcell{2}'*X3prev*AAcell{2}+Qcell{2};


for i=1:20,% intially was 5

    X1=AAcell{1}'*X2prev*AAcell{1}+Qcell{1};

    X2=AAcell{2}'*X3prev*AAcell{2}+Qcell{2};

    X3=AAcell{3}'*X1prev*AAcell{3}+Qcell{3};

    percent1=norm(X1-X1prev)/norm(X1prev)

    percent2=norm(X2-X2prev)/norm(X2prev)

    percent3=norm(X3-X3prev)/norm(X3prev)

    X3prev=X3;

    X2prev=X2;

    X1prev=X1;

end    %for convergence of each RTDPLE

%Note: In theory, one cycle of recursion should suffice. However, the

%solution to dlyap may not be exact enough to guarantee that the

%periodic generator indeed is an element of the periodic sequence of

%matrices. For this reason, more cycles are iterated.

%Problem can also be circumvented by using Cholesky factorization form


 %Calculate a periodic feedback gain:


  F1=-inv(R+Bset{1}'*X2*Bset{1})*Bset{1}'*X2*Ag;

  F2=-inv(R+Bset{2}'*X3*Bset{2})*Bset{2}'*X3*Ag;
```

```
        F3=-inv(R+Bset{3}'*X1*Bset{3})*Bset{3}'*X1*Ag;

        norm(F1-Fprevcell{1})

        norm(F2-Fprevcell{2})

        norm(F3-Fprevcell{3})

        Fprevcell={F1,F2,F3};

    end


%Check closed-loop monodromy matrix


Acl={Ag+Bset{1}*F1, Ag+Bset{2}*F2, Ag+Bset{3}*F3};

MONO=Acl{3}*Acl{2}*Acl{1};

eig(MONO)


%Simulate from arbitrary IC

x=rand(1,17);


for i=1:1000,

    x(i+1,:)=Acl{mod(i+2,3)+1}*x(i,:)';

end


%Design observer

Q=eye(14);

R=1;

h=dlqr(A',C',Q,R)';


e1=[1 0 0 ]';

e3=[0 0 1 ]';

e2=[0 1 0 ]';
```