THE BATTLE BETWEEN IMPECCABLE INTONATION AND COMPLETE CHROMATICISM

Thesis

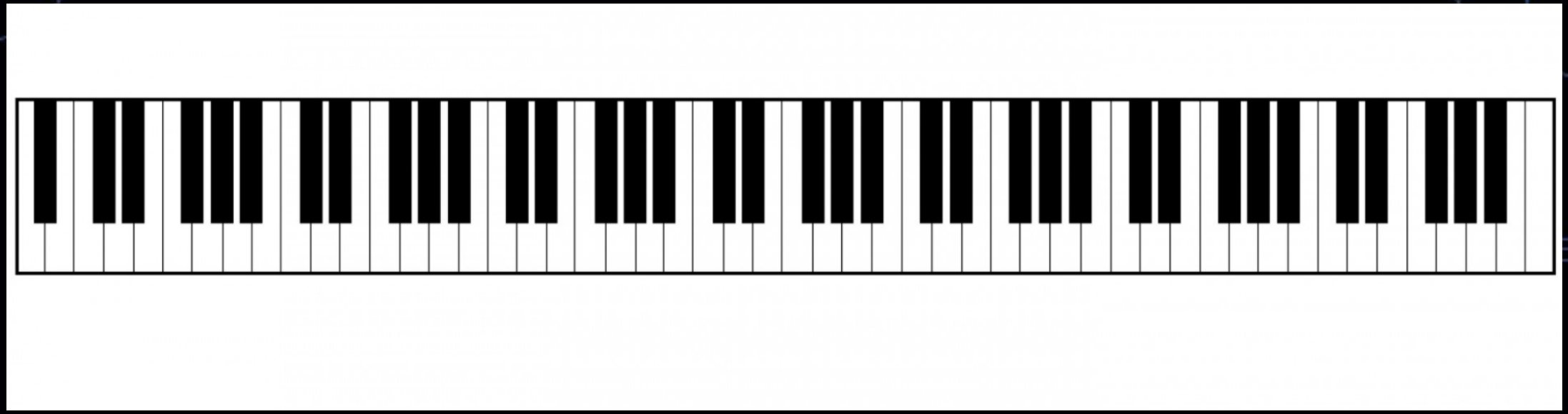
- While equal temperament is now universally hailed as the standard tuning system, it is not perfect.
- Rather, it represents a compromise designed to best accommodate the needs of tonal music since the Baroque Era.

Basic Intervals

Interval	Freq. Ratio	Decimal	Cents
Perfect Octave	2:1	2.00	1200
Perfect Fifth	3:2	1.50	702
Perfect Fourth	4:3	1.33…	498
Major Third	5:4	1.25	386
Minor Third	6:5	1.20	316
Major Sixth	5:3	1.66…	884

Cent value = 3986 × log (frequency ratio)

Closing the Circle



Defined via fifths: Frequency Ratio $= \frac{3}{2} \times \frac{3}{2} \times \cdots \times \frac{3}{2} \times \frac{3}{2} = \left(\frac{3}{2}\right)^{12}$

Defined via octaves: Frequency Ratio $\frac{2}{1} \times \frac{2}{1} \times \cdots \times \frac{2}{1} \times \frac{2}{1} = 2^7$

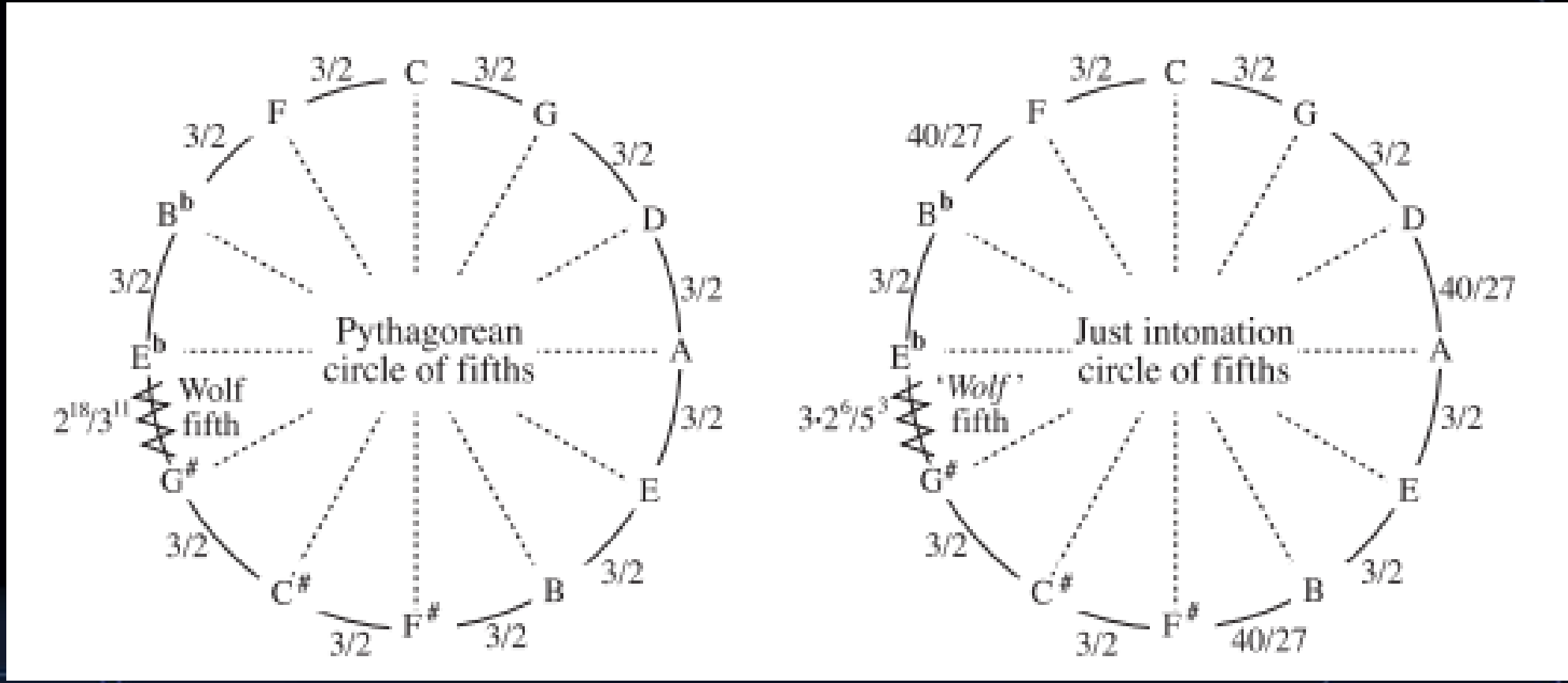
The Problem

- $\left(\frac{3}{2}\right)^{12} = \frac{531441}{4096} \approx 129.75$
- $2^7 = 128$
- This difference is equivalent to 24 cents
  - Pythagorean Comma
- “In order for the twelve pitches generated through the proportion 3:2 to complete a path from *do* to *do,* the circle has somehow to be adjusted, ‘rounded off.’”

Various Solutions

- Intonation
  - Just Intonation
  - Pythagorean Intonation
- Temperament
  - Mean-tone Temperament
  - Equal Temperament

Pythagorean and Just Intonation



Equal Temperament

- Divide up the Pythagorean comma equally
- Define octave to be 2:1
  - Break up octave into 12 equal semitones (100 cents each)
- $2^{\frac{1}{12}} \approx 1.059463$
  - One half step (A 440 to A# 466 Hz)
- $2^{\frac{7}{12}} \approx 1.498$
  - Seven half steps (fifth)

Meantone Temperament

- Flatten the fifth
  - 1/4 comma
  - Works very well in closely related keys
  - Breaks apart in keys that are further off

C^{0}	$C\#^{-7/4}$	$D^{-1/2}$	$Eb^{-3/4}$	E^{-1}	$F^{+1/4}$	$F\#^{-3/2}$	$G^{-1/4}$	$G\#^{-2}$	$A^{-3/4}$	$Bb^{+1/2}$	$B^{-5/4}$	C^{0}
0	76	193	310	386	503	579	697	773	890	1007	1083	1200

Listening Examples

- Tone Generator
  - Perfect fifth (660 Hz)
  - Wolf-fifth (651 Hz)
  - Meantone fifth (658 Hz)
  - Equal-tempered fifth (659.26 Hz)
  - Equal-tempered third (554 Hz)

History of Temperament

- Pythagoras – 550 B.C.
- Francisco Salinas – 1513-1590
- Rene Descartes – 1596-1650
  - Trade-off between simple ratios and interesting complexities

History of Temperament

- J.S. Bach – 1685-1750
  - Complicated
  - Equal or unequal?
    - Different styles for different keys?
  - Scholars remain unsure
  - Regardless of exactly which temperament Bach used, the point of the WTC clearly remains.
    - It is possible to perform in all 24 musical keys on a keyboard instrument, and to sound good doing it.

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Questions??