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4-2-2011

The Establishment of Equal Temperament

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The Establishment of Equal Temperament

Equal temperament is the foundation for modern music, yet most musicians have no concept of its meaning. In a modern culture that treats music solely as a means of entertainment, there is no need to understand the complex mathematical foundation on which music was created. However, hundreds of years ago, there existed an entire culture that integrated these numbers into their philosophies and way of living. Unfortunately, there were flaws in the relationships between the numbers that were kept secret for several centuries. These imperfections could not be hidden forever, and as the years wore on, people began to fight the old traditions. Many intellectual and religious leaders created solutions to compensate for those flaws and thus old systems of tuning were challenged by these new methods. Humans have always struggled with change and implementation of modern tuning and temperament methods were no exception. Thus, the road to the establishment of equal temperament tuning became a long, and difficult battle against the laws of the universe.

The recorded history of tunings and temperament began about 500 B.C. Ancient Greece was flourishing and many great philosophers were pushing the limits of understanding. People in this culture lived by what we now call the “Great Tradition.” This system of belief held that musical harmony was a micro-image of the cosmic order of the universe. Those who study Greek influence such as Pope Benedict XVI describes the Great Tradition as, “The universe of the revolving planets are like melodies, the numerical order is the rhythm, and the concurrence of the individual courses is their harmony. The music made by man must be taken from the inner music and order of the universe. The beauty of music depends on the conformity to the rhythmic and harmonic laws of the universe. The more that human music adapts itself to the musical laws of the universe, the more beautiful it will be" (Begbie, 78-79).

Great philosophical thinkers of that day concluded that the laws of the universe were expressed in simple mathematical ratios. The Greeks of that time were fascinated with math and how they could use numbers to describe relationships between objects. In the realm of music, Pythagoras is credited with the discovery of the relationships between numbers and harmony. Legend holds that he discovered these relationships while listening to a blacksmith working in a shop pounding metal with hammers of different sizes. The most beautiful harmonies were made when the weights of the metal were in simple proportion (2:1, 3:2, 4:3). Although the story cannot prove to be historically true, there is evidence that he replicated this phenomenon using strings. This experiment reproduced the same results.

The string experiment allowed Pythagoras to discover the first three concordances (tones that are joined in simple proportions). The first concordance was what we call the octave. Its ratio is 2:1, meaning the higher sound will vibrate twice as fast as the lower one. He then discovered the perfect fifth (3:2) and the perfect fourth (4:3). These ratios further developed the idea that the universe could be expressed in a mathematical language.

The Pythagorean method of tuning is based on the use of the simple ratios. To tune an instrument using this method, one simply sets a series of fifths (circle of fifths) so they sound pure (no beats). Pythagoras must have been ecstatic to think that he had discovered this perfect and pure method of using numbers to tune an instrument and thus reflect the music of the spheres. But what horror and dread he must have felt to realize there was a flaw. If one were to go around the circle of fifths, starting on, for example, a "C," it would take twelve fifths to complete the circle and end on a "C" that is seven octaves higher. The presumption is that the two "C's" would end with the same ratios. But upon completing the math, Pythagoras realized that $(3:2)^{12}$ equals 129.746 and $(2:1)^{7}$ equals 128.0. It is a mathematical fact that the, "powers of different prime numbers can never be equal" (Isacoff, 41). The ratio becomes 1.014:1, approximately about a quarter of a semitone higher than pure. This difference is called a "comma" (Duffin, 25). There was only one way to hide this discrepancy, and that was to temper, or alter one of the perfect fifths. Any one of these intervals had to be lowered by twenty-four cents (there are 100 cents between semitones in equal temperament) in order to complete the circle of fifths (Donahue, 21). The interval that is out of tune is called the "wolf" interval and it is this interval that made it impossible to play in all twenty-four keys. Greeks called this imperfection between the ratios an *alogon*, meaning "the unutterable" (Isacoff, 38). Today, it is simply called an irrational number.

This flaw was kept hidden from many people for hundreds of years. There was no need to address it because harmony was not introduced until the medieval era. Even as the shift from monophonic to homophonic music occurred, only the perfect consonances (perfect fourths and fifths) that Pythagoras had discovered were used. Plainchant and organa were performed without tempering and the "pureness" of these harmonies freely rang in the ears of the people. Even in melodic terms, the Pythagorean tuning system worked beautifully because, "the large major 2nds are handsome and the incisiveness of the small minor seconds is of potential expressive value. Hence Pythagorean intonation is well suited not only to parallel organum but also to late Gothic polyphonic compositions in which the role of harmonic major 6ths is somewhat analogous to that of dominant 7^{th}

chords in later triadic music, while the use of double leading-note cadences, places a premium on the incisive melodic quality of the small semitones," (Lindley, 643).

Pythagoras's tuning system was used well into the fifteenth century. It was not until the Renaissance period that it first began to bear scrutiny. Thirds became a new defining harmonic characteristic of this period. Under Pythagoras's method, the major third had a ratio of 81:64, resulting in what is known today as the Pythagorean third. It is slightly higher than what modern ears are used to hearing (21.51 cents higher, also called the syntonic comma). This easily explains why thirds were so dissonant; they were about ¾ of the way from being another semitone (100 cents between each semitone). Although musicians and composers tolerated these harsher harmonies for some time, they eventually began to experiment and alter the thirds to make them sound pure. The result was what is referred to today as meantone temperament.

Some credit Prosdocimo de' Beldomandi's treatise, *Brevis Summula Proportionum Quantum Ad Musicam Perytinet* (1409), as the first work to depart from the Pythagorean methods of dividing the monochord for tuning purposes (Herlinger, 12). Along with him and other theorists, meantone temperament developed as an alteration of Pythagoras's tuning system. This method of tuning was supposedly used from the late fifteenth century to the mid-seventeenth century (Donahue, 110). Although Spanish theorist Bartolomeo Ramos de Pareja describes temperament similar to meantone in 1482, actual instructions on how to implement meantone temperament were not found until published in Pietro Aaron's *Toscanello in musica* in 1523. Unlike Pythagoras's method, meantone temperament tuned major thirds pure instead of fifths.

Aaron specifically referred to the quarter-comma meantone temperament in his work. The process of tuning in this temperament is completed in three steps. Step one requires the C and the E to be tuned pure with no beats and then the C and the G to be tuned slightly flat. From there, the D above the G is lowered in the same fashion as well as the A and then the E. This way, all four of these fifths are equally lowered (hence the name, quarter-comma; the comma is divided evenly amongst these four fifths). Step two tunes the F, Bb, and Eb below the C as diatonic fifths. Step three tunes the C# to the A and the F# to the D. Although there is no mention of G#, it is assumed that it would be tuned pure in relationship to the E (Barbour, 26).

When keyboard instruments are tuned this way, it results in the distances between C-D and D-E to be split evenly. In other words, it is the distance of the pure third cut in half, also known as the "mean" (Isacoff, 105). This is thus the origin of the term, "meantone." Although the thirds are pure and the fifths are still consonant, the flaw in meantone temperament is that the semitones sound harsh and dissonant. Because of this, meantone temperament will only sound good and pleasing in six keys at a time. For example, if the C is tuned as the nominal tonic, then only the major keys of Bb, F, C, G, D, and A and the minor keys of G, D, and A will work. Changing the initial tuning note will result in a new set of key signatures.

Quarter-comma was not the only type of meantone temperament used for keyboard instruments during the Renaissance period. Other examples of meantone temperaments include 1/5-, 2/7-, 2/9-, and 1/9-comma. The difference between these is simply how much of the comma is dispersed to the fifths. There is no proof that any of these divisions were favored over the others. Thomas Donahue argues that the reason for this may be that, “some of these variants are difficult to tune accurately or the fact that tuners may have been merely seeking the ‘spirit’ of meantone: pure or near-pure thirds and sixths, and fifths that did not beat too badly” (111). Despite the subtle difference among the different variations among meantone temperament, “the real cost of this system was a fifth that was really not very pleasant but that became tolerable when heard with the perfectly euphonious third,” (Duffin, 33).

Meantone temperament would dominate the western world of music well throughout the Renaissance period. Hundreds of compositions would be written, highlighting the sweet sounds of thirds and sixths. The emphasis on humanism evolved into a desire to illustrate and portray the emotions. This attempt to convey emotions and feelings was a defining characteristic of the Baroque period. Sculpture, painting, architecture, literature, theatre and music were all media used to feature the affections.

In the music world, this need to portray the emotions began to demand more from musical harmony. Composers increasingly began to implement the Doctrine of Affections in their music. Remote keys were an unexplored territory for most composers and incorporating them into their compositions became a challenge. The Pythagorean tuning and meantone temperaments made it impossible to utilize these remote keys. Lesser intervals became less consonant as the greater intervals became more pure. The end result was limited use of simple keys and harsh dissonance between certain intervals. Composers wanted to expand their musical palette of colors and thus the world of tuning would be forced to compromise.

The threat of compromise had been looming for centuries. A piece written by Adrian Willaert (supposedly written before 1520) titled, *Quid non ebrietas* brought the battle to center stage where it would have to be addressed. Willaert's vocal composition (Wibberly) brings out the flaws of Pythagorean tuning and meantone temperament (See Appendix A):

> The piece opens in an ordinary fashion, using harmonies built from the tones of the *do* scale. In the beginning, *do* serves as the work's central tone-the hub around which all the musical lines will orbit. The musical scale that begins and ends on *do* thus serves as the music's framework. But Willaert's tenor part, twisting and turning in unexpected directions, snakes its way toward another tonal center, with a different constellation of scale tones; then it moves on to yet another. Every time the music seems to fix itself around one particular point, it swerves abruptly, winding this way and that, sleekly traveling through unexpected pathways, alighting for a brief moment on every one of the twelve distinct tones Pythagoras found in his chain of perfect fifths. Finally, it concludes its journey with an octave leap (Isacoff, 113-114).

Reconciling the commas with either tuning system was impossible. Harmonic instability and wolf tones were inevitable. Musicians knew that a compromise was needed in order to bridge the gap between the laws of the universe and harmonic freedom. For centuries, theorists, philosophers, and religious leaders had set the standard that music was to abide by strict rules in order that the music of the spheres could be adequately reflected on earth. The next milestone in the history of tuning and temperament would shatter the chains that so tightly held back the unexplored possibilities of musical expression.

The Baroque era ushered in new developments and ways of thinking in regard to not only music, but also different philosophies. Rulers began to use the arts as a way to assert their authority among their people. The rest of the people used the arts to express the emotions. Although the Doctrine of the Affects as developed by the Greeks was influential in the development of the arts, it became a domineering force in their creation. In musical terms, the Doctrine of Affections manifested itself in the various associations between certain keys and their characteristics. Although there were some variations between some beliefs, there seemed to be a set that emerged as most common. This set is found in Christian Schubart's *Ideen zu einer Aesthetik der Tonkinst* (1806). The keys and their characteristics correspond with the method of tunings. For example, the key of C is described as, "Completely pure. Its character is: innocence, simplicity, naïvety, children's talk (Steblin). Unlike piano tuners today who use "A" as the pitch from which they tune the rest of the instrument, people in the Baroque period and long before began tuning on "C." Because of this, the key of C was the closest to being tuned perfectly. In contrast, the key of Db is associated with gloom and harshness because in the Pythagoras tuning system and meantone temperaments, it was very dissonant and never worked harmonically.

One way that composers could convey the different characteristics while utilizing all of the different keys was to create keyboard instruments with more than the standard twelve keys. A man named Marin Mersenne was one of the many innovators who developed different keyboards with different divisions of the octave. He helped develop some with nineteen, twenty-seven, and even thirty-two keys. Although the idea sounded like a solution on paper, the actual act of playing them was simply too difficult.

The other solution developed into what we call well temperament. This next stage in the development of tuning was the result of a discovery by the famous philosopher, Descartes. He discovered the fact that certain strings will sympathetically vibrate along with others. William Noble and Thomas Pigot later confirmed his conclusion that, within a single string exists all of the smaller strings that create the harmonies we now call "overtones." Physicists describe them as, "a collection of exponentially decaying sine waves," (Deutsch, 10) (See Ex. 2).

Ex. 2 Frank, The Overtone Series

These overtones are what cause the "beating" that is heard when two notes sound that are not tuned according the perfect ratios. When the overtones clash very harshly, the result is the wolf intervals that were found in the Pythagorean tuning system and meantone temperaments. Well temperament sought to eliminate these wolf tones by tempering both the thirds and the fifths. In general, the major thirds surrounding C were made narrow while the rest were tuned no more than 21.51 cents, which if gone beyond that, were considered to be unbearably out of tune (Kanter).

Although many contributed to the development of the well temperaments, two names stand out among the rest. The first is Andreas Werckmeister. "He took a middle-of-the-road position in the ancient argument as to whether Ratio (reason) or Sensus (the senses) should rule music and preferred to believe in a rational interplay of the two forces," (Buelow, 286). His stance in the controversy was reflected in the temperament he created. In his method, eight perfect fifths were tuned pure, resulting in some Pythagorean thirds, while the others were tempered by a quarter comma (Ibid). The second is Johann Kirnberger. As a student of Johann Sebastian Bach, he is credited with creating a temperament that "contains some of the purest harmony acoustically possible," (Kanter). His tuning is not considered equal temperament because not all of the keys were equally in tune. In fact, some keys still sounded better than others. Both of these temperaments allowed for the freedom to modulate between all twelve keys while upholding the character of the keys.

One of the greatest treasures that came from the Baroque era amidst the tuning and temperament controversies was *The Well Tempered Clavier* by Johann Sebastian Bach. It is comprised of two solo keyboard books written in 1722 and 1724 that include different pieces using all twenty-four major and minor keys. There is a debate that still rages on even today among scholars as to which temperament Bach desired *The Well Tempered Clavier* to be performed. Jack Greenfield argues that, "The fact that many of the compositions discussed do not exceed twelve scale degrees suggests the possibility that claviers could have been tuned in regular meantone with several accidentals retuned to give the specific range required for the piece being performed," (Greenfield, 21). Either way, the need for a temperament that allowed free use among all of the keys was here to stay.

Although the ability to use all twenty-four keys was now available, composers wanted more. They wanted to be able to create music that could be played in all of the keys *equally*. Thus, equal temperament was established. Lloyd describes it as an, "acoustical compromise, tolerated by many ears on the piano, and designed to satisfy as completely as possible three incompatible requirements—true intonation, complete freedom of modulation and convenience in practical use in keyed instruments—and that it sacrifices the first of these to the second and third," (66). Using the logarithmic function, the ability to divide the octave into twelve equal parts finally became a mathematical reality. Mathematically speaking, the flaw, known as the "wolf tone," that had haunted theorists and musicians or centuries, "became exactly the same size as its genuine fifth, (Frosch, 188). Keyboard instruments could now enjoy the freedom of equal temperament just as stringed instruments had for centuries. There is a lot of debate about who discovered it, but one interesting theory states that, “while the finding was the nearly the same and even the timing almost coincidental, the East (Zhu Zaiyu) and West (Simon Stevin) had in fact pursued equal temperament through entirely different passages and for quite different reasons,” (Cho, 7). Without equal temperament, future Impressionist composers could not compose music that seemed to have no tonal center and future twelve-tone row composers could not create music with enharmonic spellings.

Isacoff best describes the relationship between tunings and temperaments by saying that, ”just intonation is like a pure child that lives inside of every equal-tempered adult,” (231). The road to equal temperament was full of “growing pains.” Many of these pains were controversies argued amongst the intellectual and philosophical elites of the Medieval, Renaissance, and Baroque eras. These lesser-known debates not only affected the music of their time, but also made a long-lasting impact on the music that is heard today. Charles Butler simply states that, “The history of temperament is a history of changing philosophies,” (Jorgensen, 111). The future of equal temperament is unsure, but many are revisiting the ancient tunings and temperaments today. Musicians such as Michael Harrison are exploring just intonation with his own modified harmonic piano. As musical digitization advances, the compromises between just intonation and the compromises of temperaments may possibly be reconciled, opening up a whole new realm of music to be explored by future musicians.

Appendix A

QUID NON EBRIETAS
Adrian Willaert

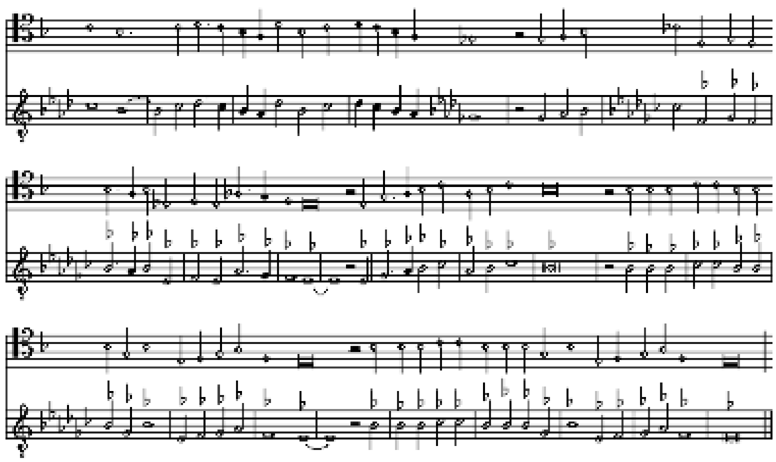
Quid non ebrietas dissignat? Operta recludit,
Spes iubet esse ratas, ad proelia trudit inertem,
Sollicitis animis onus eximit, addocet artes,
Fecundi calices quem non fecere disertum?
—Horace, Epistles I, V, 16-19

[What cannot be accomplished through drinking? It reveals secrets,
Fulfils hopes, encourages the unarmed into battle,
Removes the burden from worried minds, teaches new skills,
Whom has the wine-cup not made more skilful?]

Quid non ebrietas dissignat (Tenor)

Transcription by Roger Wilbberley

Quid non ebrietas dissignat (with double ficta)



Ficta realization by Roger Wibberley

Quid non ebrietas dissignat

Willaert

Edition by Roger Wibberley

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