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On the utility of i = √-1

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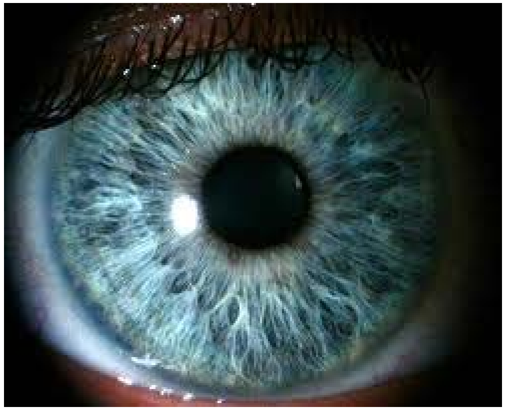

i

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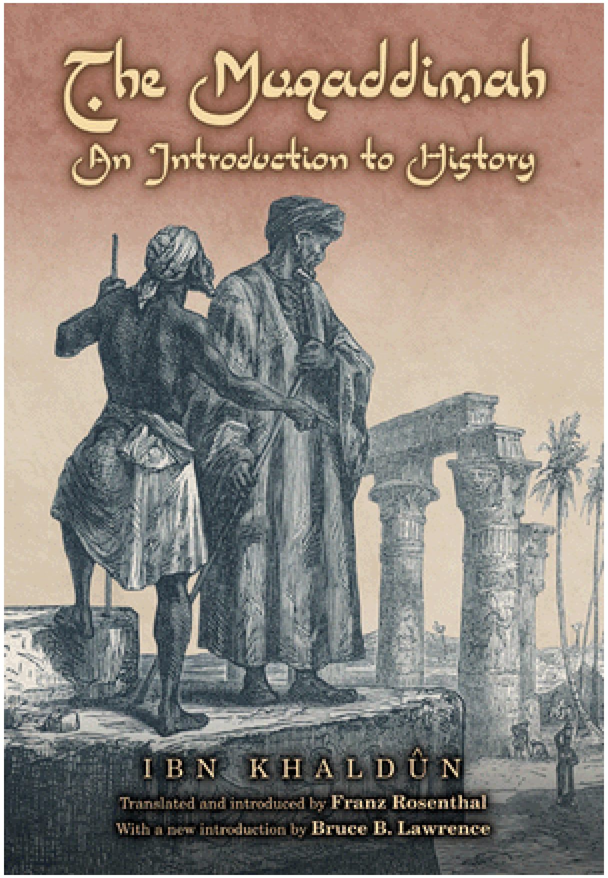
What even consider $i = \sqrt{-1}$ as a legitimate thing?

1. Solving ***polynomial equations*** (like $x^2 - 4x + 3 = 0$ for x) is at the core of issues such as taxation, maximizing profit, minimizing cost and a host of other economic factors critical to the advancement of society. i shows up quickly when one investigates solutions of polynomial equations rigorously.
2. Incorporation of i enables us to fully understand models for ***physical phenomena*** such as stress on beams, resonance, fluid flow, electrical currents, transmittance of radio waves and population dynamics among other things. Clearly, each of these contributes mightily to the betterment of society.

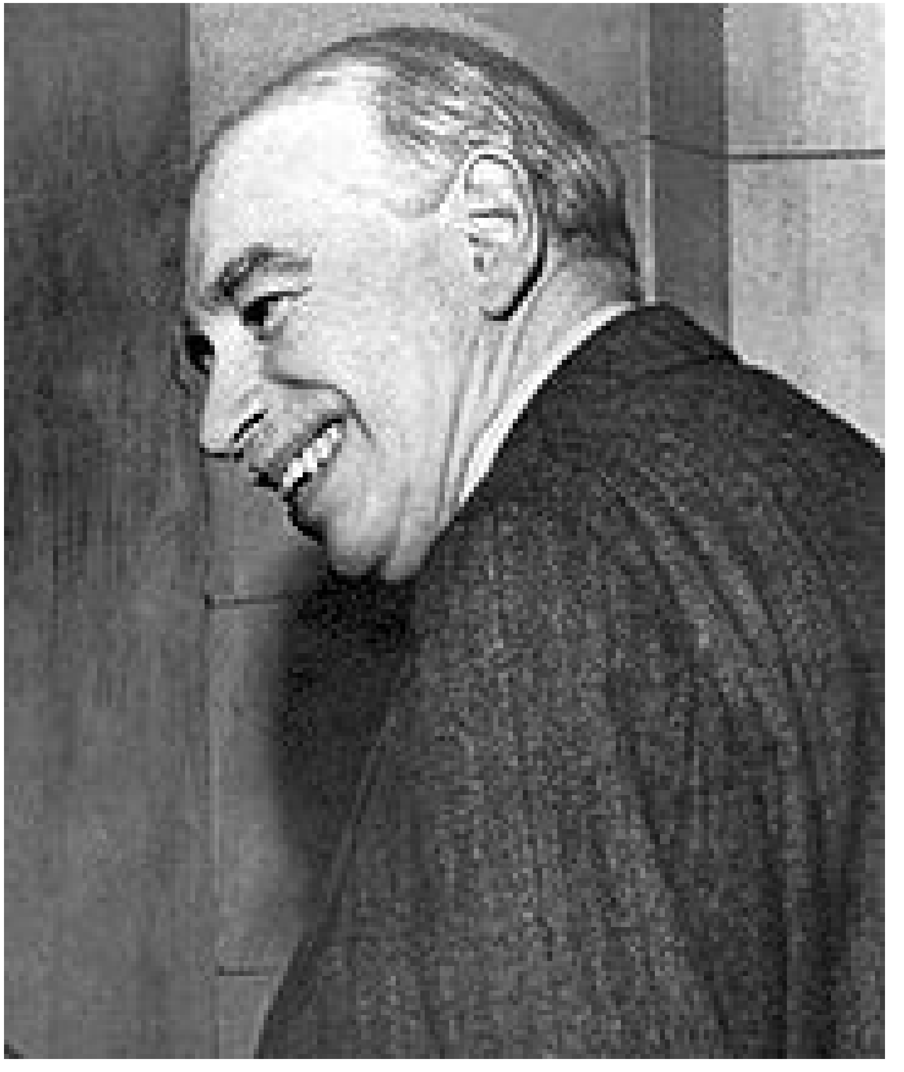


Ibn Khaldun (1332-1406), Muslim philosopher

"It should be known that at the beginning of the dynasty, taxation yields a large revenue from small assessments. At the end of the dynasty, taxation yields a small revenue from large assessments."

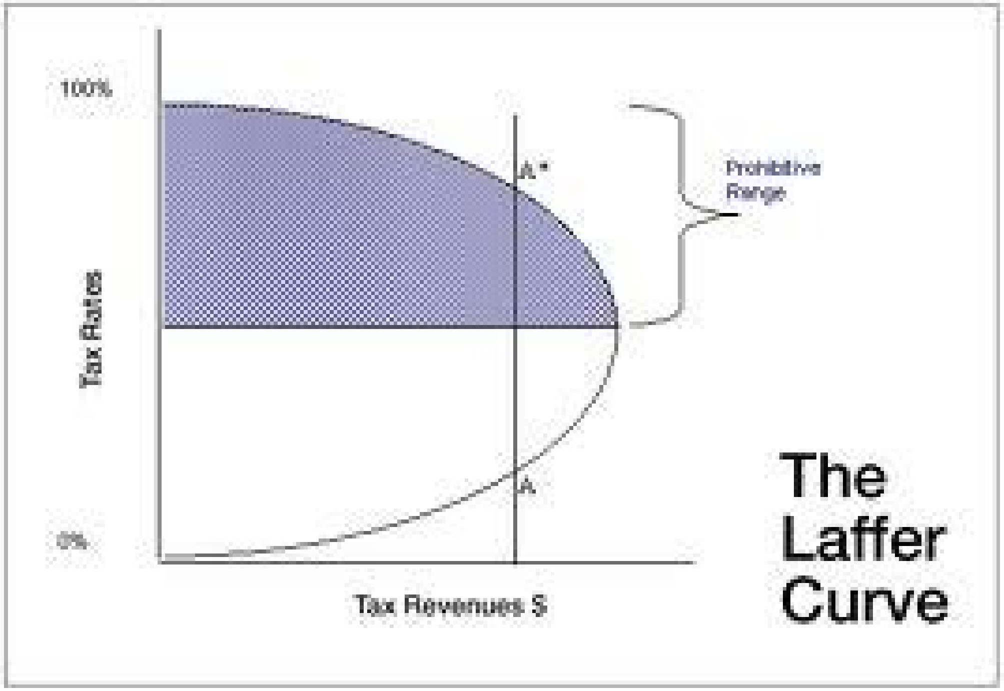
John Maynard Keynes (1883-1946), British Economist

“..Nor should the argument seem strange that taxation may be so high as to defeat its object, and that, given sufficient time to gather the fruits, a reduction of taxation will run a better chance than an increase of balancing the budget. For to take the opposite view today is to resemble a manufacturer who, running at a loss, decides to raise his price, and when his declining sales increase the loss, wrapping himself in the rectitude of plain arithmetic, decides that prudence requires him to raise the price still more--and who, when at last his account is balanced with nought on both sides, is still found righteously declaring that it would have been the act of a gambler to reduce the price when you were already making a loss.”



Arthur B. Laffer, Professor of Economics, University of Chicago; Reagan Economic Advisory Council

"As recounted by Wanniski (associate editor of The Wall Street Journal at the time), in December 1974, he had dinner with me (then professor at the University of Chicago), Donald Rumsfeld (Chief of Staff to President Gerald Ford), and Dick Cheney (Rumsfeld's deputy and my former classmate at Yale) at the Two Continents Restaurant at the Washington Hotel in Washington, D.C. While discussing President Ford's 'WIN' (Whip Inflation Now) proposal for tax increases, I supposedly grabbed my napkin and a pen and sketched a curve on the napkin illustrating the trade-off between tax rates and tax revenues. Wanniski named the trade-off 'The Laffer Curve.'"



Luca Pacioli (1445-1509): "The Father of Accounting"

- *Summa de Arithmetica* (1494)
- Presented a solution to the ***general quadratic:***
- $ax^2 + bx + c = 0$
- Rediscovered that $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ solves this equation (the Egyptians discovered it first about 4,000 years earlier)
- Ex: $x^2 - 4x + 3 = 0$
- Pacioli had no clue how to deal with the ***general cubic:***
- $ax^3 + bx^2 + cx + d = 0$

- **Scipione del Ferro** (1465-1526)
- Solved the ***depressed cubic:*** $x^3 + mx = n$
- Kept it "secret," and passed it to his student, **Antonio Fior**

- Fior challenged the great scholar of the day, **Niccolo Fontana** (1499-1557; known as **Tartaglia** "the Stammerer")
- Tartaglia prevailed!

Gerolamo Cardano (1501-1576)

- Completely bizarre character!
- Pestered Tartaglia relentlessly
- Tartaglia revealed the secret!
- With his student, Cardano was able to extend Tartaglia’s result on the *depressed* cubic to solve the *general* cubic.
- Despite being sworn to secrecy by Tartaglia, Cardano revealed all in his *Ars Magna* (1545)

Cardano/Tartaglia/del Ferro solution of the ***depressed cubic:***

- $x^3 + mx = n$ is solved by

$$x = \sqrt[3]{\frac{n}{2} + \sqrt{\frac{n^2}{4} + \frac{m^3}{27}}} - \sqrt[3]{-\frac{n}{2} + \sqrt{\frac{n^2}{4} + \frac{m^3}{27}}}$$

BUT WHERE'S i?!

- In *Ars Magna*, Cardano played around with alternative ways of finding maximum values for $x(10-x)$, ultimately leading him to solve the "impossible" equation $x(10-x)=40$.
- Despite his misgivings and referring to the techniques he used as "mental tortures," Cardano found that $x=5+\sqrt{-15}$ "solved" the equation. He dismissed this, though, as a "useless" purely abstract exercise.

Then, the bombshell…

- Rafael Bombelli (1526-1572) in his *l'Algebra* used the crazy *depressed cubic* formula to solve $x^3 - 15x = 4$, finding

$$x = \sqrt[3]{2 + \sqrt{-121}} + \sqrt[3]{2 - \sqrt{-121}}$$

Bombelli continued…

- It turns out that

$$x = \sqrt[3]{2 + \sqrt{-121}} + \sqrt[3]{2 - \sqrt{-121}}$$

is really just the number 4 in disguise! This was a bombshell to the mathematical community, as Bombelli's willingness to wade through the unfamiliar and uncomfortable territory of imaginary numbers had yielded, in the end, a REAL result! This opened the floodgates, and established $i = \sqrt{-1}$ as an intellectual reality worthy of rigorous study.

A bit more history…

- The word “imaginary” to describe negative square roots was first used by Rene Descartes (1596-1650)

- Leonard Euler (1707-1783) first used the notation $i = \sqrt{-1}$

Primary References

- *Journey Through Genius,* William Dunham, Chapter 6
- *The Laffer Curve: Past, Present, and Future*, Arthur B. Laffer,

http://www.heritage.org/research/reports/2004/06/the-laffer-curve-past-present-and-future

- *A Short History of Complex Numbers*, Orlando Merino,

http://www.math.uri.edu/~merino/spring06/mth562/ShortHistoryComplexNumbers2006.pdf

Questions?