



Spectral Mixture Modeling using Principle Component Analysis

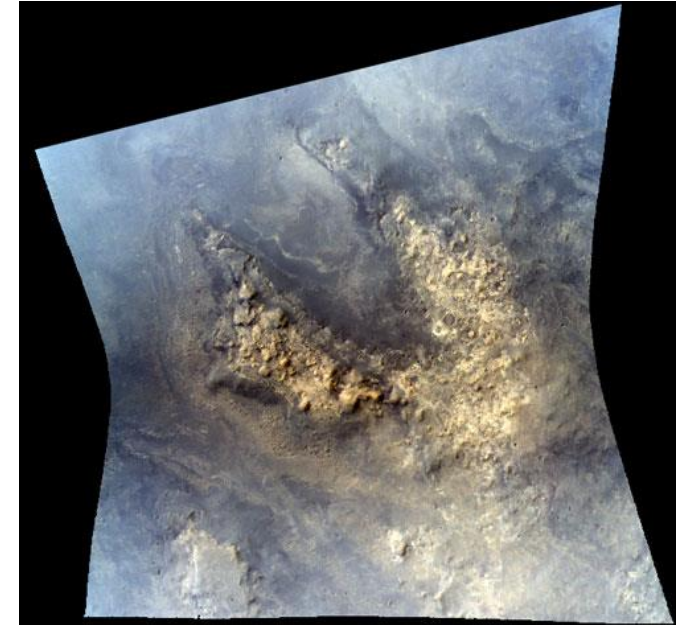
April 18, 2018

Joseph S. Makarewicz

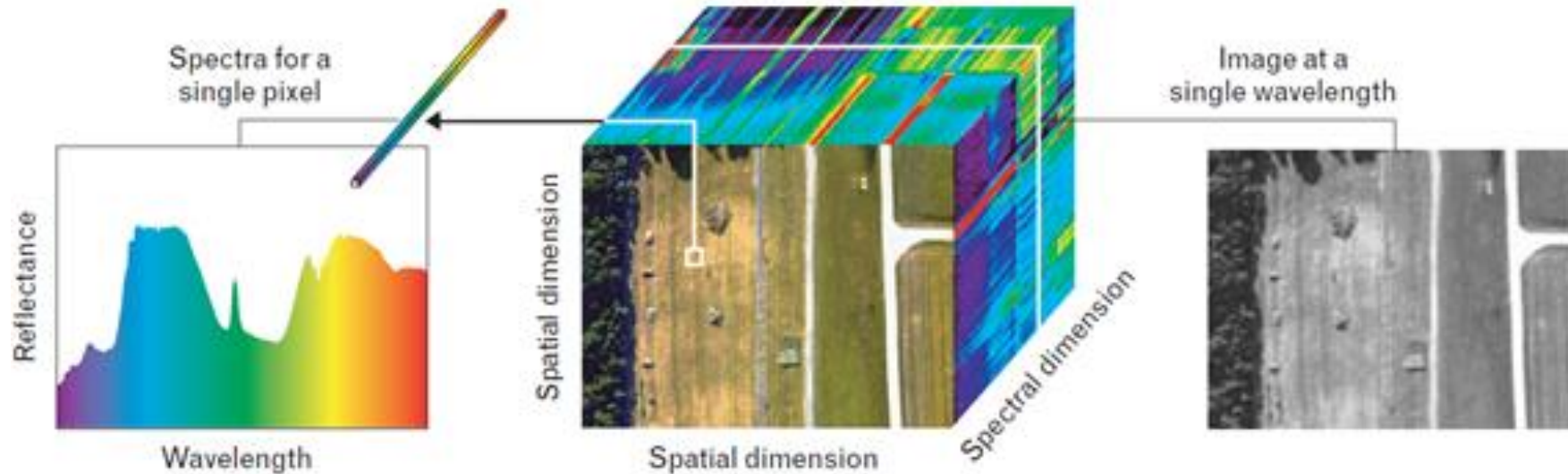
Heather D. Makarewicz

- Background
- Spectra Pre-processing
- Principle Component Analysis
- Principle Component Correlation
- Classification
- Binary Mixture Models
- Discussion
- Future Work

- Mars Reconnaissance Orbiter (MRO)
 - Launched in 2005
 - Capturing images of Mars using several different instruments
 - Still sending back images
- CRISM
 - One of several imaging instruments onboard MRO
 - A pushbroom imaging VNIR spectrometer that produces hyperspectral images



www.nasa.gov



www.tankonyvtar.hu

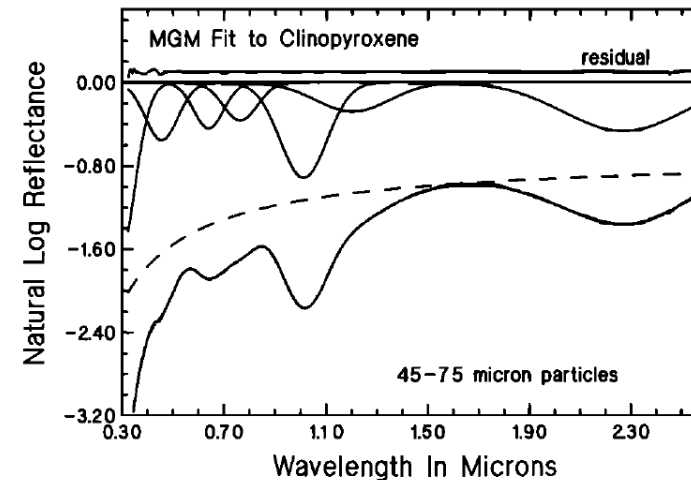
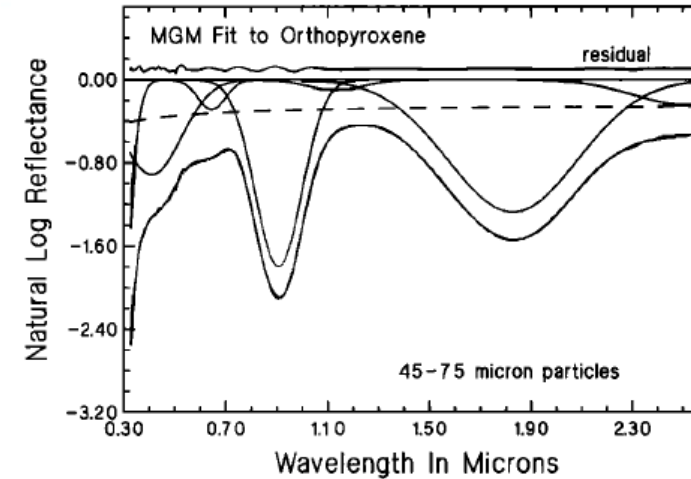
- **Hyperspectral Cube**

- A stack of images of the same scene at many different wavelengths
- Has 2 spatial dimensions and 1 spectral dimension
- Acquired with a spectral imager (grid of sensors)
- Large hyperspectral libraries available (USGS, NASA, etc.)

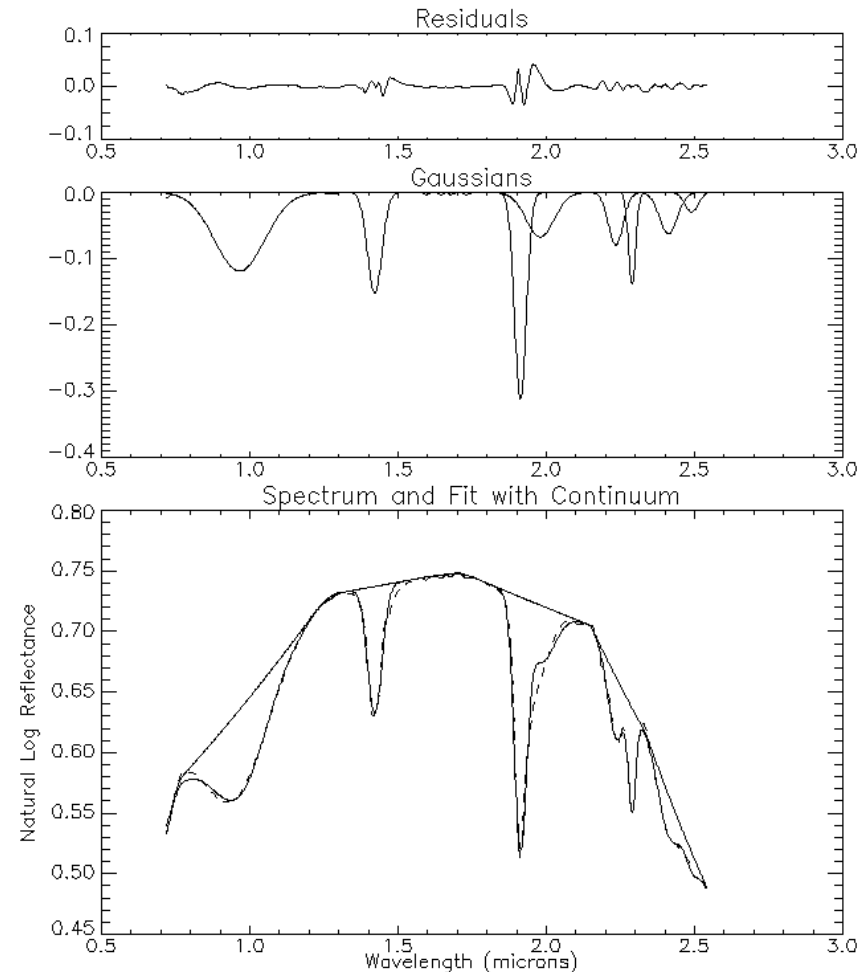
- **Spectra**

- A single pixel in a hyperspectral image
- A plot of reflective Intensity versus wavelength (or frequency)
- Can also be acquired with a spectrometer (single sensor)
- Large spectral libraries available (USGS, RELAB)

- MGM Laboratory Studies
 - Use modified-Gaussian modeling (MGM) to model spectral features
 - Use MGM parameters to determine mineral composition
 - Band Center
 - Amplitude
 - Full Width at Half Max (FWHM)
- Has been performed on:
 - Pyroxenes (Enstatite, Clinopyroxene)
 - Olivine Binary Mixtures (Forsterite, Fayalite)
 - Phyllosilicate Binary Mixtures (Kaolinite, Montmorillonite)

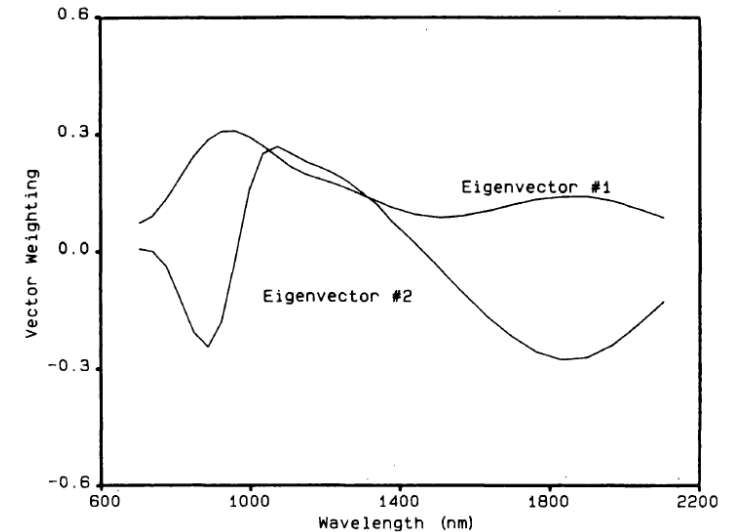
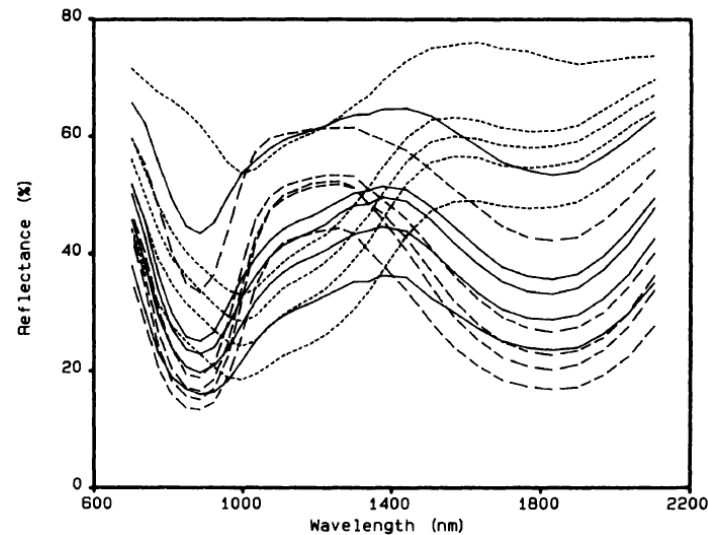


- Modified Gaussian Modeling
 - Uses Optimization
 - Levenberg-Marquardt
 - Tarantola and Valette
 - Automated parameter initialization
 - Estimate number of Gaussians and their parameters
 - Applied to Lunar and Martian hyperspectral images



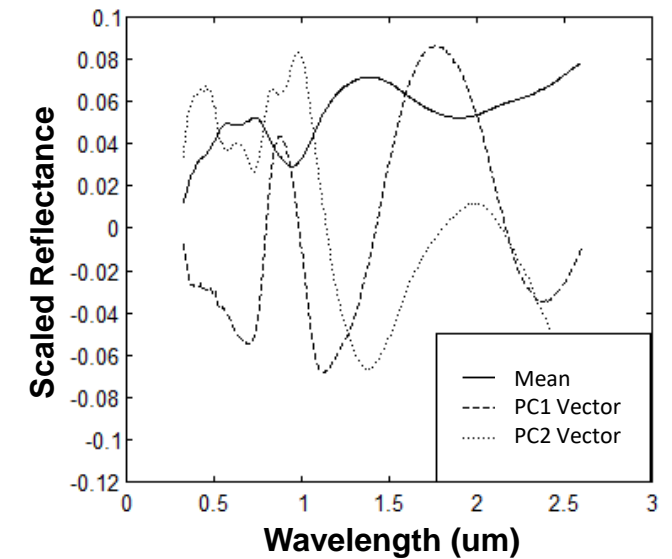
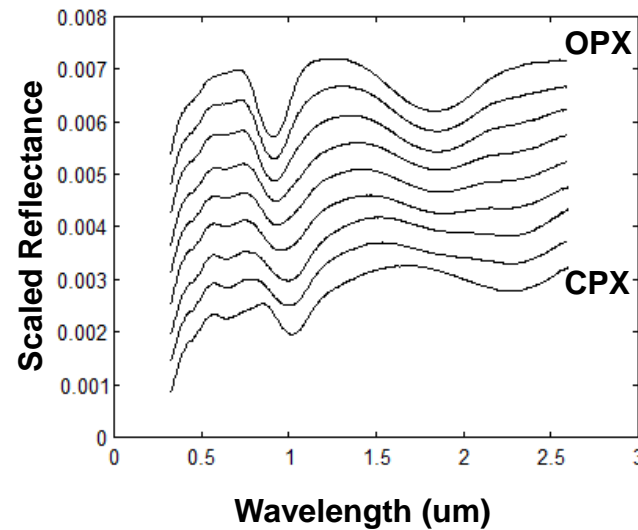
H.D. Makarewicz, M. Parente, and J.L. Bishop

- PCA Laboratory Studies
 - Use Principle Component Analysis (PCA) to derive a set of PC vectors and PC values
 - Retain a subset of PC values as principle components
 - Use principle components to determine mineral composition
 - No optimization
 - Has been performed on:
 - Tertiary Mixtures (Enstatite, Olivine, and Calcite)

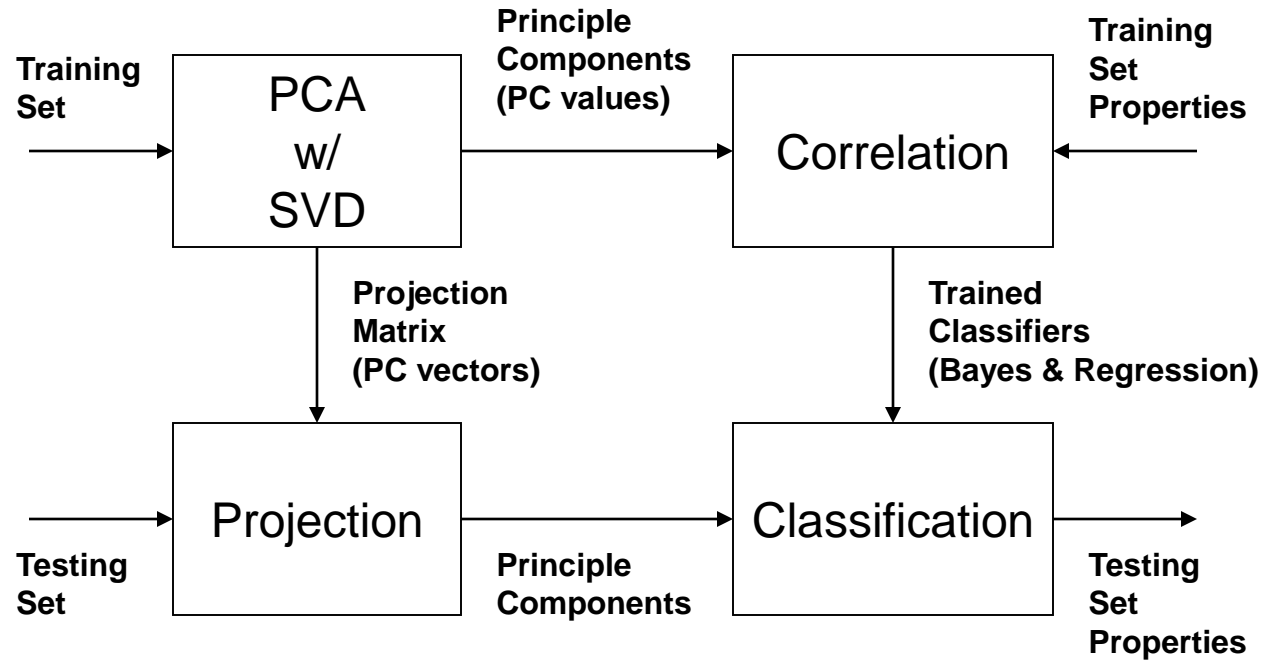


M. O. Smith , P. E. Johnson and J. B. Adams

- Perform principle component analysis to determine mineral composition
 - Use M. O. Smith's methodology
 - Apply to a variety of mixture data sets

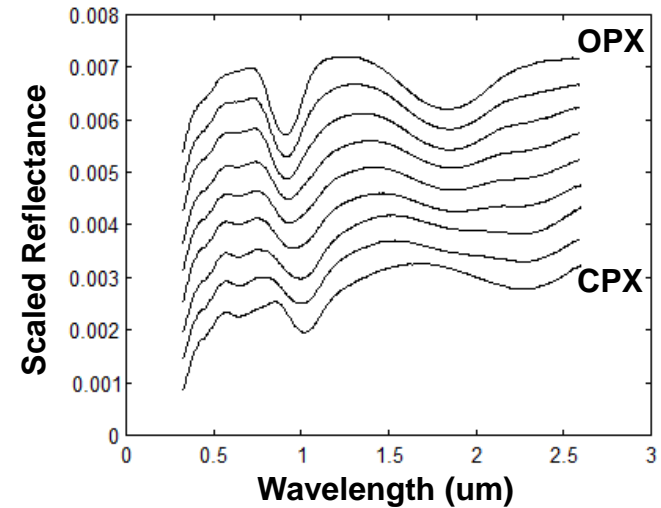


Methods



Spectra Pre-processing

- Crop
 - All spectra must have the same range of wavelengths.
 - 0.325 μm to 2.6 μm
- Resample
 - Fine enough to resolve spectral features
 - 0.05 μm
- Normalize
 - Area under spectrum equal 1



Principle Component Analysis

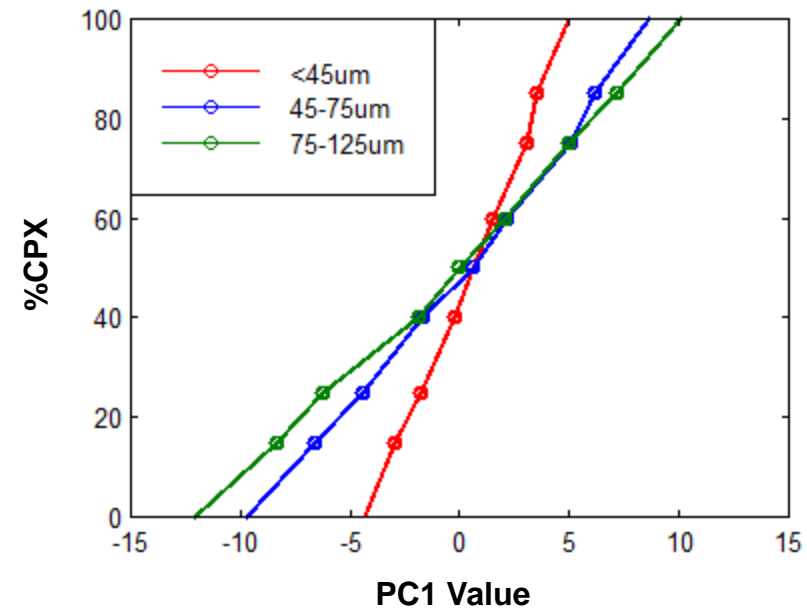
- Subtract the Mean Normalized Spectrum
 - Similar to 50% CPX
 - Features that are present in all sample spectra
- Computed Using Singular Value Decomposition
 - A set of PC vectors
 - Sets of PC values

Table 1 – Training Set Principal Components

| Name | Ratio (%CPX) | Grain Size (um) | PC1 Value | PC2 Value |
|--------|--------------|-----------------|-----------|-----------|
| C1PP21 | 100% | <45 | 4.9834 | 14.6796 |
| C1XP16 | 85% | <45 | 3.5224 | 14.1942 |
| C1XP14 | 75% | <45 | 3.0545 | 13.1861 |
| C1XP12 | 60% | <45 | 1.4616 | 17.5569 |
| C1XP10 | 50% | <45 | 0.5630 | 18.3265 |
| C1XP11 | 40% | <45 | -0.1848 | 18.5786 |
| C1XP13 | 25% | <45 | -1.7919 | 19.8681 |
| C1XP15 | 15% | <45 | -2.9467 | 21.0338 |
| C1PE30 | 0% | <45 | -4.3693 | 22.3367 |
| C1PP22 | 100% | 45-75 | 8.6066 | -5.8343 |
| C1XP26 | 85% | 45-75 | 6.1334 | -7.1253 |
| C1XP24 | 75% | 45-75 | 5.0372 | -9.2222 |
| C1XP22 | 60% | 45-75 | 2.1198 | -6.2064 |
| C1XP20 | 50% | 45-75 | 0.6355 | -6.2429 |
| C1XP21 | 40% | 45-75 | -1.7180 | -3.9304 |
| C1XP23 | 25% | 45-75 | -4.4424 | -4.2354 |
| C1XP25 | 15% | 45-75 | -6.5749 | -4.8977 |
| C1PE31 | 0% | 45-75 | -9.7554 | -4.4636 |
| C1PP23 | 100% | 75-125 | 10.091 | -11.9483 |
| C1XP07 | 85% | 75-125 | 7.1403 | -10.5483 |
| C1XP05 | 75% | 75-125 | 4.9318 | -10.7730 |
| C1XP03 | 60% | 75-125 | 2.0399 | -14.4355 |
| C1XP01 | 50% | 75-125 | -0.0357 | -14.2804 |
| C1XP02 | 40% | 75-125 | -1.8646 | -12.0175 |
| C1XP04 | 25% | 75-125 | -6.2256 | -11.0001 |
| C1XP06 | 15% | 75-125 | -8.3312 | -8.1490 |
| C1PE32 | 0% | 75-125 | -12.0808 | -14.4500 |

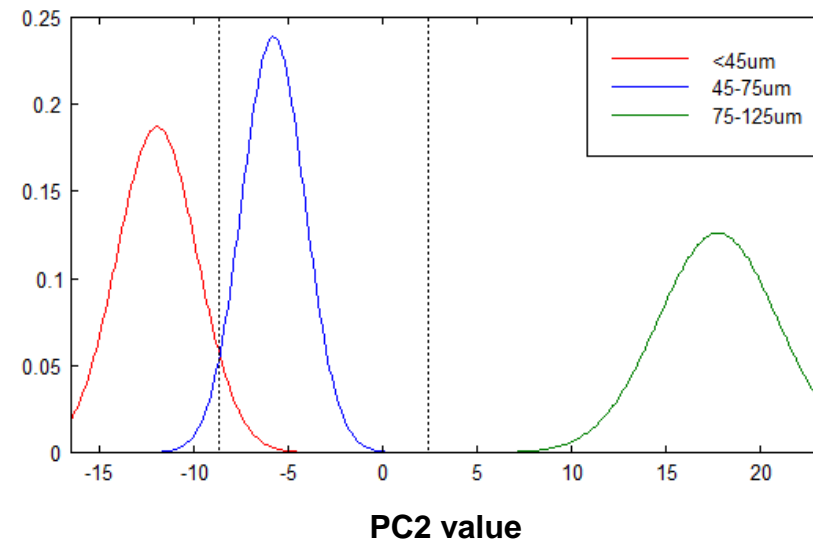
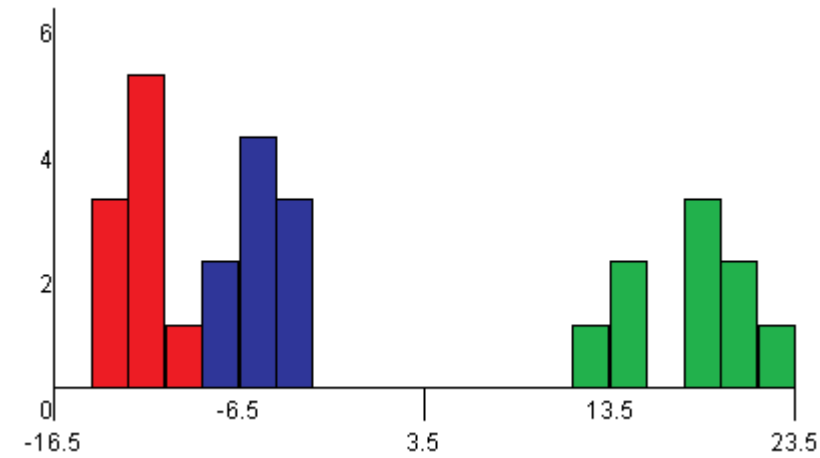
Principle Component Correlation

- First Principle Component was correlated with Percent Clinopyroxene
- Linear Regressions
 - <45um
 $\%CPX = 50 + 10 \times PC1 \text{ value}$
 - >45um
 $\%CPX = 50 + 5 \times PC1 \text{ value}$



Principle Component Correlation

- Second Principle Component was correlated with Grain Size
- Bayesian Classifier
 - <45um
 $\mu=17.75$, $\sigma=3.16$
 - 45-75um
 $\mu=-5.79$, $\sigma=1.67$
 - 74-125um
 $\mu=-11.95$, $\sigma=2.13$



- Process
 - Pre-process spectrum.
 - Compute the first two principle component values using the first two principle component vectors.
 - Determine the grain size using the second principle component value.
 - Determine the percent composition using linear regression and the first principle component value.

Results

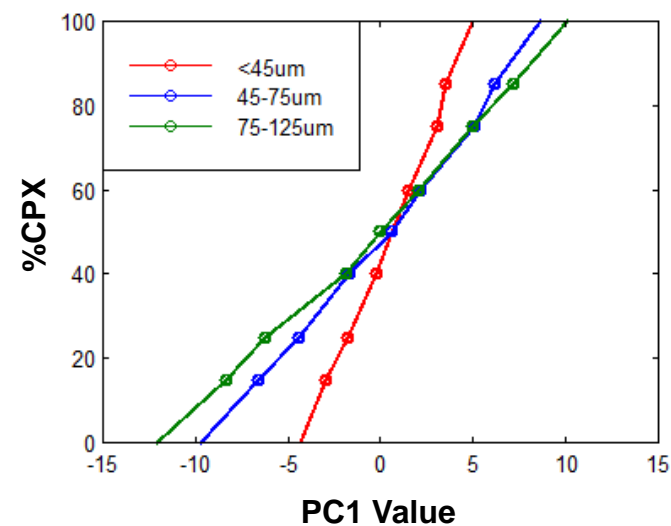
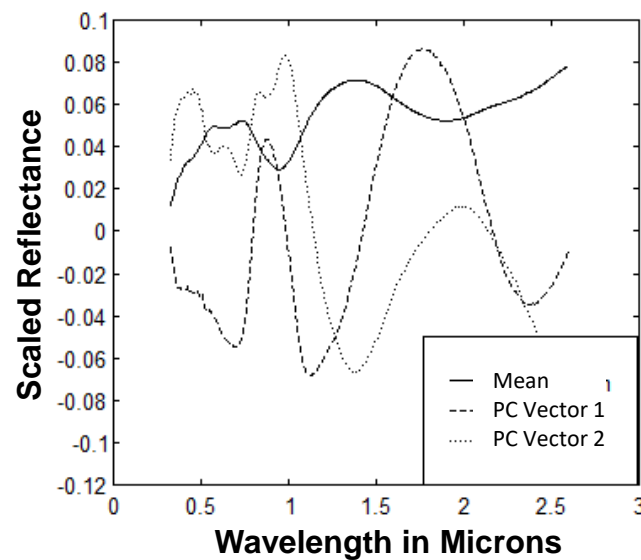
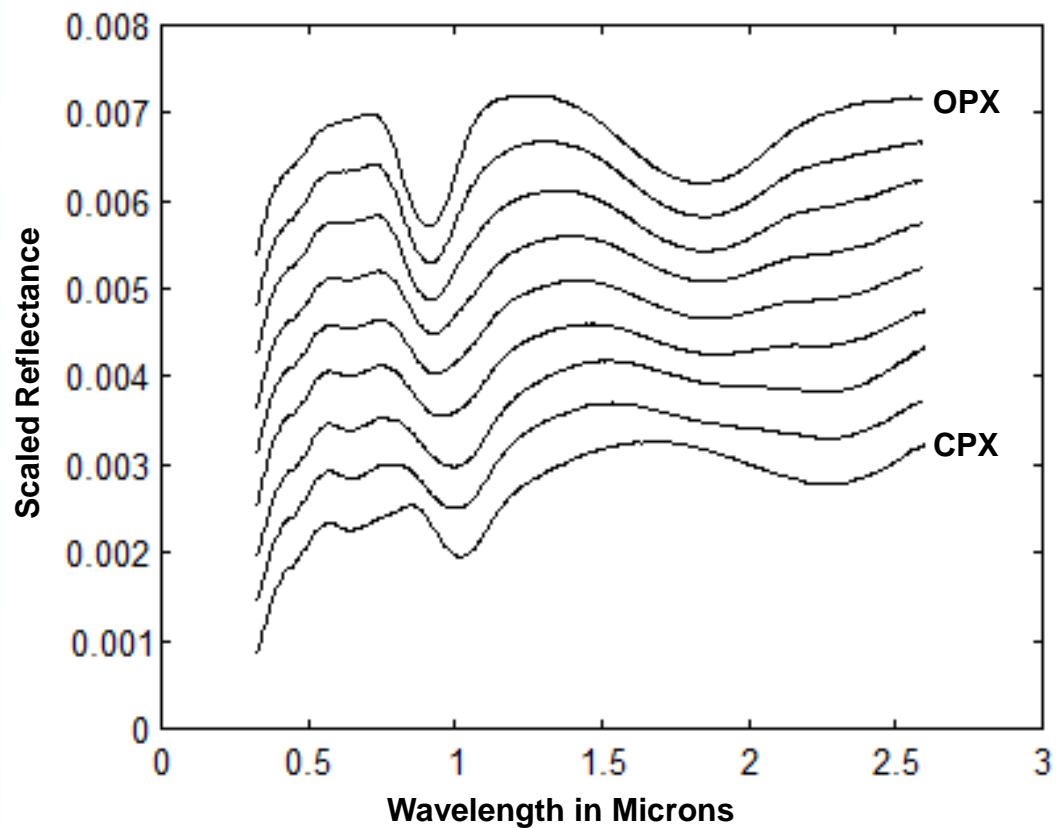
Table 2 – Test Set Grain Size

| Name | Actual Ratio (%CPX) | Actual Grain Size (um) | PC2 Value | Predicted Grain Size (um) |
|--------|---------------------|------------------------|-----------|---------------------------|
| C1PE30 | 0% | <45 | 22.3367 | <45 |
| C2PE30 | 0% | <45 | 22.3361 | <45 |
| C4PE30 | 0% | <45 | 24.9357 | <45 |
| C5PP21 | 0% | <45 | 13.4838 | <45 |
| C2XP15 | 15% | <45 | 21.0338 | <45 |
| C5PE30 | 50% | <45 | 25.3391 | <45 |
| C4XP14 | 75% | <45 | 13.3947 | <45 |
| C1XP17 | 75% | <45 | 15.1154 | <45 |
| C2XP16 | 85% | <45 | 14.1942 | <45 |
| C4XP16 | 85% | <45 | 13.7501 | <45 |
| C1PE31 | 0% | 45-75 | -4.4636 | 45-75 |
| C2PE31 | 0% | 45-75 | -4.4653 | 45-75 |
| C3PE31 | 0% | 45-75 | -4.5904 | 45-75 |
| C3PP22 | 100% | 45-75 | -7.9079 | 45-75 |
| C1PP22 | 100% | 45-75 | -5.8343 | 45-75 |
| C5PE32 | 0% | 75-125 | -10.9787 | 75-125 |
| C1PP23 | 100% | 75-125 | -11.9483 | 75-125 |
| C2PP22 | 100% | 75-125 | -5.8349 | 45-75 |
| C2PP23 | 100% | 75-125 | -11.9487 | 75-125 |
| C5PP23 | 100% | 75-125 | -11.8181 | 75-125 |

Table 3 – Test Set Percent Composition

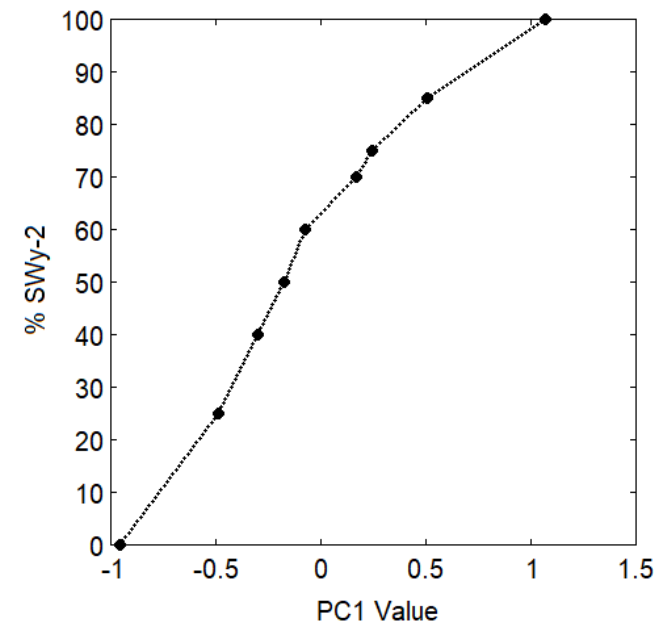
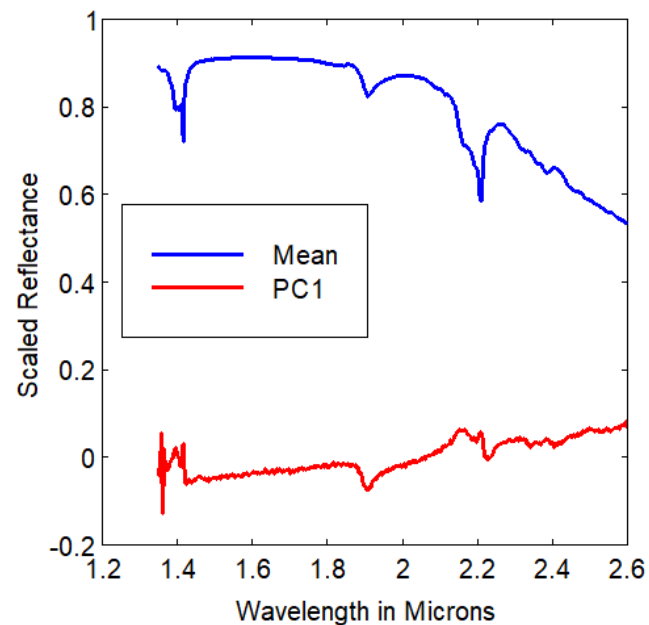
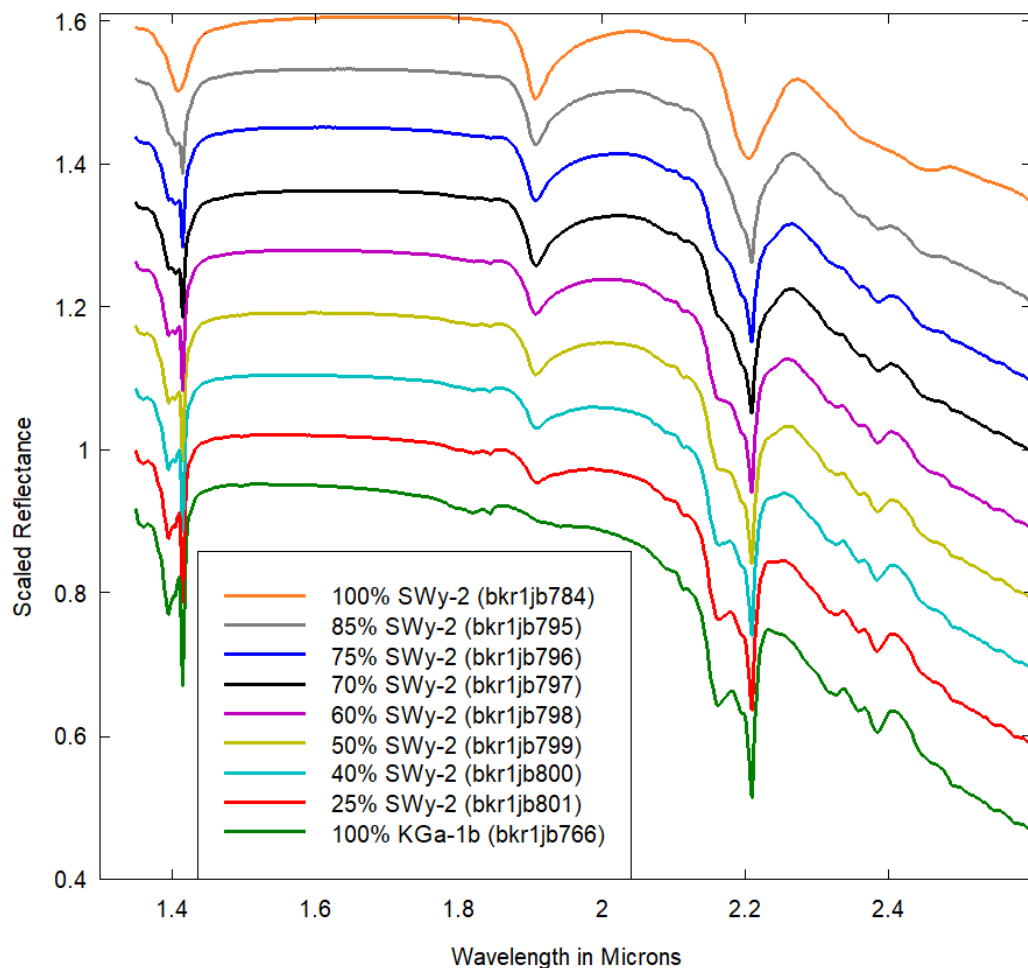
| Name | Actual Ratio (%CPX) | Actual Grain Size (um) | PC1 Value | Predicted Ratio (%CPX) |
|--------|---------------------|------------------------|-----------|------------------------|
| C1PE30 | 0% | <45 | -4.3693 | 6.3067 |
| C2PE30 | 0% | <45 | -4.3693 | 6.3074 |
| C4PE30 | 0% | <45 | -4.1384 | 8.6156 |
| C5PP21 | 0% | <45 | 5.3783 | 103.783 |
| C2XP15 | 15% | <45 | -2.9467 | 20.533 |
| C5PE30 | 50% | <45 | -4.1980 | 8.0199 |
| C4XP14 | 75% | <45 | 3.1054 | 81.0539 |
| C1XP17 | 75% | <45 | 2.9716 | 79.7157 |
| C2XP16 | 85% | <45 | 3.5224 | 85.2244 |
| C4XP16 | 85% | <45 | 3.475 | 84.75 |
| C1PE31 | 0% | 45-75 | -9.7554 | 1.2229 |
| C2PE31 | 0% | 45-75 | -9.7553 | 1.2236 |
| C3PE31 | 0% | 45-75 | -10.2877 | -1.4383 |
| C3PP22 | 100% | 45-75 | 9.0428 | 95.2139 |
| C1PP22 | 100% | 45-75 | 8.6066 | 93.033 |
| C5PE32 | 0% | 75-125 | -12.2364 | -11.1822 |
| C1PP23 | 100% | 75-125 | 10.0919 | 100.4594 |
| C2PP22 | 100% | 75-125 | 8.6066 | 93.0332 |
| C2PP23 | 100% | 75-125 | 10.0919 | 100.4596 |
| C5PP23 | 100% | 75-125 | 10.3279 | 101.6394 |

Binary Mixture Models – Clinopyroxene-Orthopyroxene

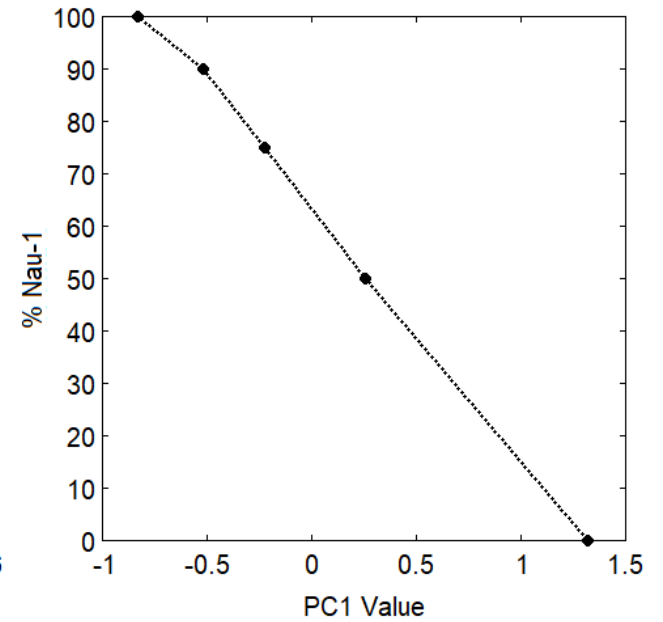
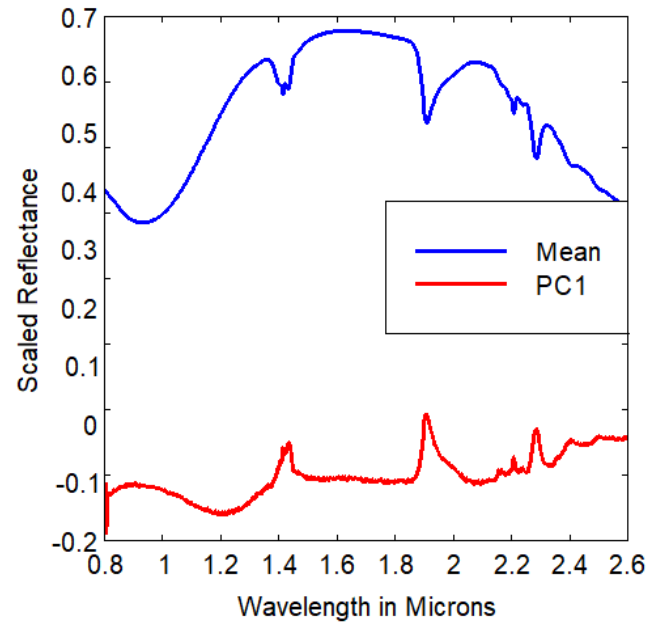
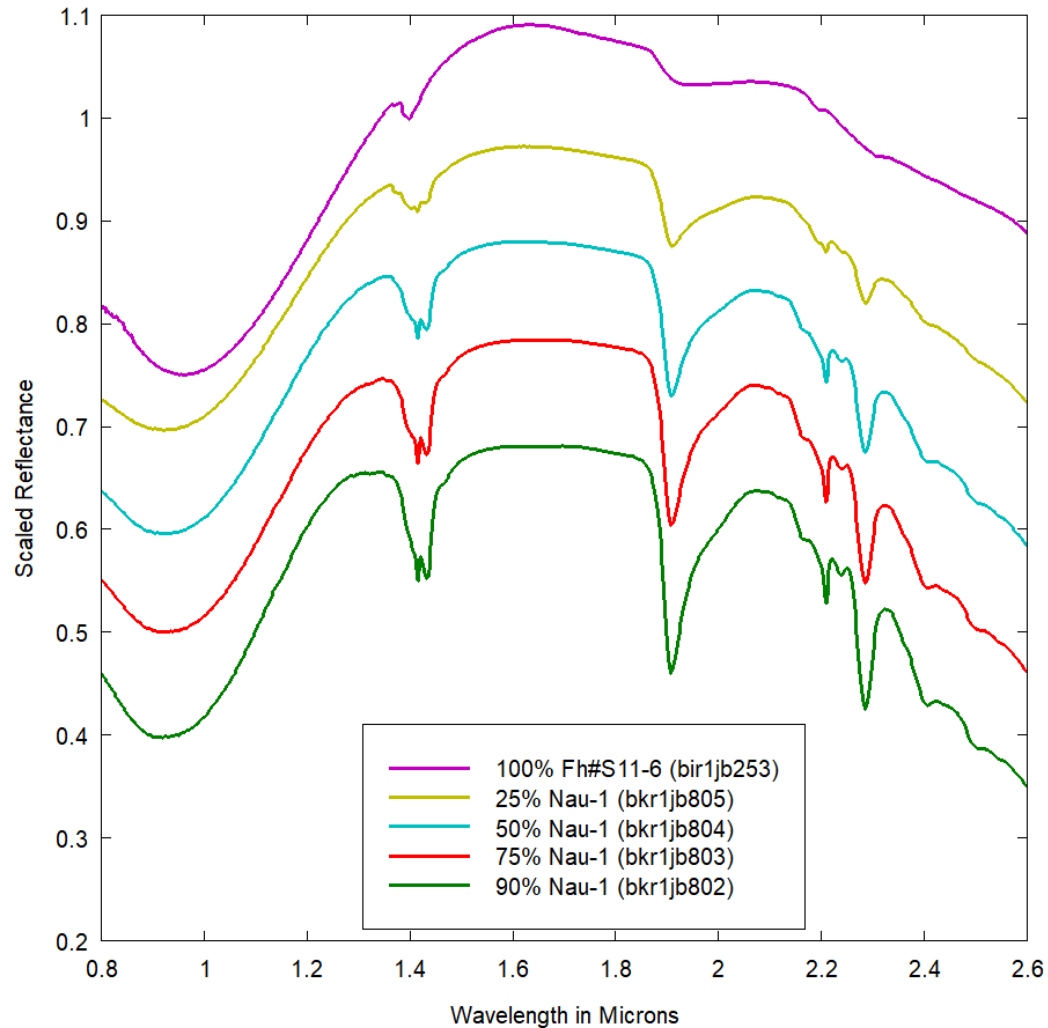


$\text{<45um: \%CPX} = 50 + 10 \times \text{PC1 value}$
 $\text{>45um: \%CPX} = 50 + 5 \times \text{PC1 value}$

Binary Mixture Models – Kaolinite-Montmorillonite



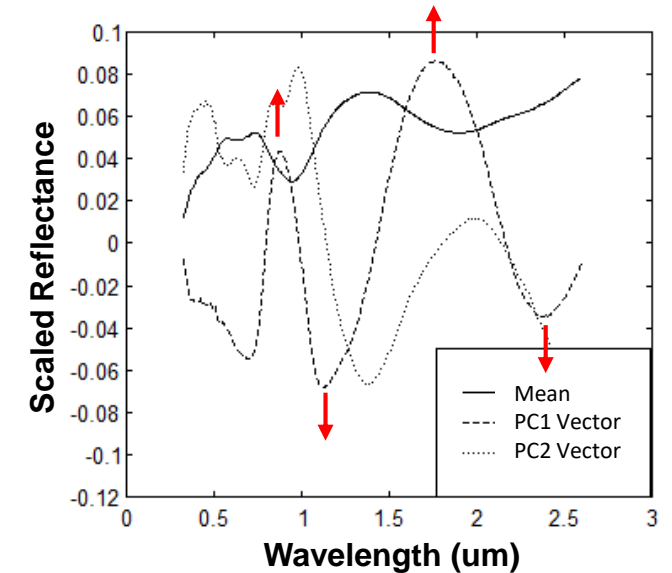
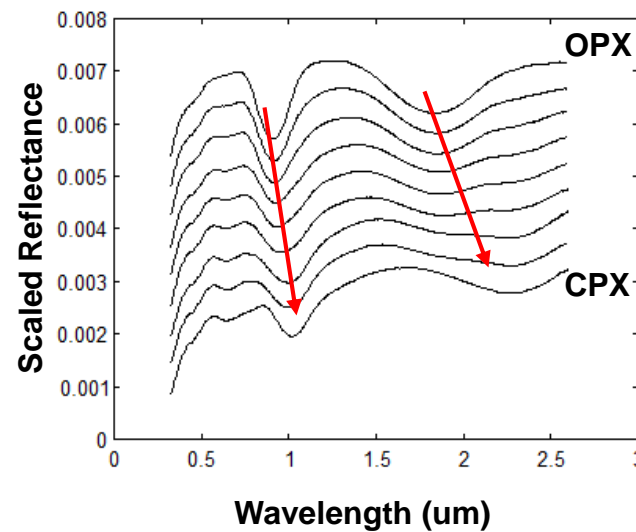
Binary Mixture Models – Nontronite-Ferrihydrite



$$\% \text{Nau-1} = 62 - 48 \times \text{PC1 value}$$

- Normalization
 - Lose information about albedo
 - Allows analysis to be invariant to light source intensity
 - Important for generalizing algorithm to remotely sensed data
- Grain Size
 - Strongly correlated with albedo
 - May not always be detectable
 - If spectral features change with grain size, then it will still be detectable after normalization.

- PC vectors
 - Used to form the projection matrix
 - Contain reflection and absorption band information
- Example
 - As PC1 value increases, %CPX increases, and PC1 vector is weighted more and absorption bands at 1.02 μm and 2.29 μm become stronger, but bands at 0.91 μm and 1.83 μm become weaker.



As %CPX increases

Future Work

- Apply algorithm to small sets of binary and tertiary mixtures for mass percent composition models.
- Apply algorithm to larger sets of various minerals for coarse identification of mineral presence.
 - Variance will increase amongst sample spectra
 - More principle components vectors and values will need to be retained
 - PC vectors will represent bands that are unique to different mineral groups
- Use models to identify minerals and estimate mineral abundance in lunar and Martian hyper spectral images.
 - Investigate residuals from model to identify trace minerals.