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ATTITUDES AND PERCEPTIONS OF HIGH SCHOOL MATHEMATICS

TEACHERS REGARDING STUDENTS' COGNITIVE-

METACOGNITIVE SKILLS

by

Peter A. Babich

Dissertation

Submitted to the Faculty of

Olivet Nazarene University

School of Graduate and Continuing Studies

in Partial Fulfillment of the Requirements for

the Degree of

Doctor of Education

in

Ethical Leadership
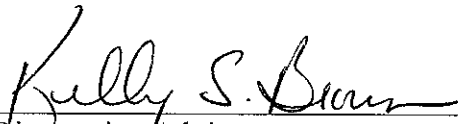
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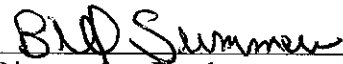
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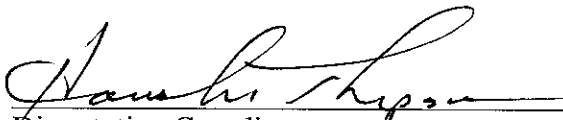
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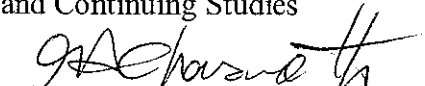
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Program Director

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Date

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Dean of the School of Graduate
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Date

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Vice-President for Academic Affairs

5-5-2010

Date

ACKNOWLEDGEMENTS

DEDICATION

This dissertation is dedicated to my parents and brother. Without your love and support, I would not have found the time, patience, or courage to complete this study.

ABSTRACT

by
Peter A. Babich, Ed.D.
Olivet Nazarene University
May 2010

Major Area: Ethical Leadership   Number of Words: 119

The purpose of the study was to identify the attitudes and perceptions held by select

teachers in a Midwest high school regarding teaching strategies related to students

solving mathematics problems from a cognitive-metacognitive approach. The case study

utilized a questionnaire regarding instructional practices and teacher beliefs and opinions

as well as semi-structured interviews. Teachers commented on definitions and beliefs

regarding thinking about thinking, thinking mathematically, and conceptual and

procedural understanding. Furthermore, teachers discussed teaching strategies utilized to

teach thinking about thinking in mathematics, effects of school-wide metacognitive

training efforts, and usage of student reflection activities. The specificity and

sophistication of responses related to perceptions held and strategies utilized seemed to

increase with the years of experience teaching for participants.

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CHAPTER I

INTRODUCTION

The debate regarding the content in mathematics education in the United States was decided, in large part, over 50 years ago by individuals living halfway around the world. The launch of the Soviet Union's *Sputnik 1* satellite on October 4, 1957 (Lindee, 2007) ignited a flurry of activity in the United States to transform the educational system regarding mathematics and science. *Sputnik's* launch has had far-reaching consequences in mathematics and science classrooms even to this day. Shortly after the Soviet Union's evident superiority, schools began implementing more rigorous mathematics and science curricula in an attempt to close the gap between the U.S. and Soviet Union in the new "Space Race." Americans felt as if the United States had catching up to do because the Soviet Union had already experienced success in their first and subsequent satellite attempts. The Soviet Union's salient event provided the catalyst for a great turning point in the education of the United States (Launius, 1994).

Since the late 1950s, numerous reports and studies have focused on the achievement of students in the United States in relation to those of other countries. The *A Nation at Risk* report (National Commission on Excellence in Education [NCEE], 1983) portrayed America as lagging behind other countries in mathematics and science achievement. The 1983 *A Nation at Risk* report succinctly stated the dismal state of education in the United States:

If an unfriendly foreign power had attempted to impose on America the mediocre educational performance that exists today, we might well have viewed it as an act of war. As it stands, we have allowed this to happen to ourselves. We have even squandered the gains in student achievement made in the wake of the Sputnik challenge. … We have, in effect, been committing an act of unthinking, unilateral educational disarmament. (NCEE, p. 1)

The *A Nation at Risk* report's strong words served as yet another catalyst for educational reform. As with the Sputnik challenge, legislators and educators began brainstorming and integrating new ideas and curricular changes for the sake of bridging the gap for students in America in relation to those internationally.

The *Third International Mathematics and Science Study* (*TIMSS*) (Gonzales et al., 2004) reported similar findings, although with more data comparing the achievement of American students to those abroad. Although not as strong of a mechanism of change as *A Nation at Risk* (NCEE, 1983), the *TIMSS* data were still troubling to the mathematics and science education community. Again, students in the United States were behind their international counterparts.

More recently, mathematics education has again taken center stage in the public's eye due to the No Child Left Behind (NCLB) Act (2002). The two main foci of the law are increased reading and mathematics achievement of students. In addition, by 2014, all students must meet state standards for these two academic areas (NCLB). If schools have subgroups that do not attain the needed state standards, they do not meet the required adequate yearly progress (AYP). Schools not making AYP for two years in a row can be branded as failing schools. With the need for all students to reach high standards,

teachers, parents, and administrators are looking at what schools have done in the past, what schools are doing presently, and what schools need to do in the future to improve student achievement. As such, many schools have greatly increased their focus on mathematics and reading skills in their curriculum (Dillon, 2006; Hoerandner & Lemke, 2006).

Teachers bear most of the responsibility in conveying the updated, intended curriculum to students. Even though schools are ultimately held accountable for undesirable results and students take the tests, teachers are the individuals who implement various strategies to reach students and teach them the necessary skills to succeed on these tests (Schmidt et al., 1996).

Statement of the Problem

The purpose of this study was to identify the attitudes and perceptions held by select teachers in a Midwest high school regarding teaching strategies related to students solving mathematics problems from a cognitive-metacognitive approach.

Current pedagogical practices in mathematics education are not sufficient in preparing students to think mathematically. Schoenfeld (1992) explained this notion as follows:

> Learning to think mathematically means (a) developing a mathematical point of view — valuing the processes of mathematization and abstraction and having the predilection to apply them, and (b) developing competence with the tools of the trade, and using those tools in the service of the goal of understanding structure — mathematical sense-making. (p. 3)

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The first part of Schoenfeld's definition describes how students must realize and accept the process and purpose of mathematics as an academic area of study. In the second part, students must actually learn to do mathematics by solving problems and using the processes that have been created within the field. The two parts also covertly imply that students should learn to evaluate both their solutions and methods of solution when doing mathematics (Schoenfeld). Without a propensity for self-reflective mathematical thinking, students generally employ an algorithm or enlist the help of a procedure without thought to its effectiveness or efficiency.

While a lack of mathematical thinking does not seem disastrous, its effects are actually quite far-reaching. Mathematics itself is grounded in the domain of logic. Investigating cause-effect relationships, examining patterns, and hypothesizing about relationships are all mathematical exercises. Students who are not educated to think about mathematics are simply limited in their ability to think (Schoenfeld, 1992).

Schools are charged with the task of instilling and conveying the intrinsic value of mathematical thought in today's youth. Additionally, with ever increasing accountability for schools and teachers, along with constant pressure for student achievement from a myriad of external and internal sources, it is vitally important to implement successful strategies in teaching mathematics. National and state standards will soon require all students to meet benchmarks in mathematics (NCLB, 2002). Although passing a standardized test does not guarantee that a student can think mathematically, students who can think mathematically are more apt to showed increased achievement on these high stakes assessments.

The problem arises when the attained curriculum falls short of the intended curriculum set forth by school districts. The gap either stems from a breakdown in learning by the student or teaching by the teacher. If the teacher's implemented curriculum does not meet the intended curriculum, the student suffers and fails. There are multiple reasons for student failure. One major reason students struggle is that teacher perception influences what he or she teaches and how that content is taught. In order for schools to maximize student potential for success on standardized tests, teachers need to incorporate the intended curriculum via strategies providing opportunities that foster mathematical thinking. Strategies focusing on authentic mathematical thinking are vital to improve students' critical thinking and problem solving abilities (Schmidt et al., 1996).

Background

Cognition and metacognition have undergone a long and arduous journey in finding a place within the field of education. Intrinsically linked, yet nebulously different, the two terms have caused great controversy, not only because their definitions are debated to this day, but also because they are synonymous with numerous other educational expressions (Garofalo & Lester, 1985). Garofalo and Lester helped to differentiate the two by defining cognition as being "involved in doing" (p. 164) and, as cited in Garofalo and Lester, Brown and Palincsar's (1982) definition of metacognition as "knowledge about cognition and regulation of cognition" (p. 2). The distinction was important, yet Schoenfeld (1992) better qualified the notion of cognition as "the knowledge base", "problem solving strategies", "monitoring and control", "beliefs and affects", and "practices" (p. 42). The third aspect of cognition, monitoring and control, seems very similar to the ideas behind metacognition. As such the definitions of

cognition and metacognition are cyclical, with each representing a subset of the other. Although paradoxical, this describes the very nature of cognition and metacognition.

Because of the duality of the terms, frameworks involving thinking, as well as thinking about thinking, generally refer to the cognitive-metacognitive domain rather than attempting to separate the two (Artzt & Armour-Thomas, 1992). Artzt and Armour-Thomas delineated some specific activities based upon their predominant level. For example, Artzt and Armour-Thomas's framework classifies reading as generally cognitive whereas understanding, analyzing, and planning are metacognitive. Additionally, exploring, implementing, and verifying are all dual because they require both cognitive and metacognitive processes (Artzt & Armour-Thomas). The activities listed explicate the slight nuances and clarify the major differences between the two levels. Artzt and Armour-Thomas's framework serves as the basis that will be utilized within this research.

Numerous researchers have studied the effect of metacognitive strategies on students' mathematics achievement (Dahl, 2004; Marge, 2001; Mevarech & Kramarski, 2003). Results vary in degree, but students achieve at higher levels when utilizing metacognitive problem solving strategies (Maqsud, 1998; Schurter, 2002; Teong, 2003). Therefore, teachers must include activities that foster students' problem-solving skills and encourage students to use their own cognitive and metacognitive resources (Stillman & Galbraith, 1998). Unfortunately, the extent to which teachers are prepared to assist students in utilizing these methods varies. Teachers of mathematics generally teach a new concept using illustrative examples and then have students practice solving similar exercises. Unfortunately, completing exercises is far different from problem solving that

requires thinking and the utilization of metacognitive skills. Schoenfeld (1992) explained that most mathematics classrooms segregate problem solving from the normal mathematics that is taught day to day. In order to maximize student potential, teachers must move problem solving from the sidelines to the forefront.

A teacher's attitudes, beliefs, and perceptions greatly influence not only how, but what, he or she teaches (Schoenfeld, 1992). Teachers who believed that mathematics was a fixed, unchanging entity expected students to absorb the content rather than explore and conjecture. Even past experiences in mathematics influence current teaching styles. Pittman (2002) found that there was great influence to the way in which mathematics was taught, at least at the elementary level, based upon the beliefs of newer mathematics teachers. As a result of Pittman's finding, it is important to study the beliefs of teachers when analyzing the way they teach.

Finally, teachers often forego problem solving from their classrooms in favor of teaching content contained in high-stakes tests. The pressure for their students to achieve is immense. Results are highly scrutinized by administration, the community, and governmental entities. In this new era of increased accountability, problem solving is not highly valued. Few high-stakes assessments measure problem solving ability, so teachers focus on those concepts that will be tested (Schoenfeld, 1992). Although unfortunate, this is often the reality in American classrooms. Despite the widespread lack of problem solving on high-stakes assessment coupled with increased accountability, teachers need to continue to implement problem solving in mathematics classrooms to produce students who can think mathematically (Schurter, 2002).

Research Questions

This study was guided by the following research question: What strategies are perceived by mathematics teachers in a Midwest high school as best at fostering students' cognitive-metacognitive skills in solving select mathematics problems?

The research question was divided into two sub-questions:

1. What attitudes and perceptions do these teachers have regarding students' cognitive-metacognitive abilities?

2. How do these attitudes and perceptions influence the strategies utilized by these teachers to teach cognitive-metacognitive skills?

Description of Terms

A Nation at Risk. A report by the National Commission on Excellence in Education (1983) that outlined the academic achievement of students in the United States. The dismal results of American students prompted the report and an outline for improvements in the educational arena for the future.

Attained curriculum. What "students have learned, and what they think about these subjects" (Mullis et al., 2005, p. 4).

Attitude. "State of mind, behavior, or conduct regarding some matter, as indicating opinion or purpose" (Read et al., 1993, p. 94).

AYP. Adequate yearly progress.

Cognition. The five aspects of cognition are: "the knowledge base", "problem solving strategies", "monitoring and control", "beliefs and affects", and "practices" (Schoenfeld, 1992, p. 42).

Implemented curriculum. The mathematics that "is actually taught in classrooms, who teaches it, and how it is taught" (Mullis et al., 2005, p. 4).

Intended curriculum. The mathematics "that society intends for students to learn" (Mullis et al., 2005, p. 4).

Metacognition. "Knowledge about cognition and regulation of cognition" (Brown & Palincsar, 1982, p. 2).

No Child Left Behind (NCLB). The "reauthorization of the Elementary and Secondary Education Act of 1965" (Cunningham & Cordeiro, 2006, p. 47). Cunningham and Cordeiro continued by explaining that testing in reading and mathematics is mandated for grades three through eight and eleven. Proficiency by all students is required by 2014. Increased accountability measures toward meeting AYP are included. Additionally, schools failing to meet the proficiency goals set for each year may be required to undergo changes toward increasing student achievement.

Perception. "Any insight or intuitive judgment that implies unusual discernment of fact or truth" (Read et al., 1993, p. 936).

Sputnik 1. A Soviet Union satellite launched on October 4, 1957 (Lindee, 2007).

Trends in International Mathematics and Science Study (*TIMSS*). A study in which mathematics and science achievement of students is compared. Over 40 countries, including the United States, have participated. The 2004 data represent the third installment of this landmark study conducted by the International Association for the Evaluation of Educational Achievement (Gonzales et al., 2004).

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Significance of the Study

The most significant part of this study was that it focused on the attitudes and perceptions of mathematics educators. Most research has focused specifically on the attitudes and perceptions of students, whereas the role of the teacher in the learning process has been controlled. The purpose of this study was to learn about teachers' perceptions and to what extent they may influence what they teach, how they teach, and how their students learn. If a possible relationship is discovered, the results could lead to further research into the relationship between student learning and teacher perception.

Mathematics educators, especially those at the high school level, will find this study to be of interest. Also, researchers in the field of mathematics education may find the results worthy of further research. The results could shed light on the learning process from a unique and unconventional perspective.

Process to Accomplish

An ethnographic case study was the best method to complete this research. The mathematics teachers in this school were studied to find best practices for strategy use and to learn about their attitudes, beliefs, and perceptions (Gay, Mills, & Airasian, 2006; Robson, 2002). The use of an ethnographic case study model was most appropriate due to the need to study "participants' perspectives and behaviors," as well as "what people actually do and their reasons for doing it" (Gay et al., p. 445). The study itself was completed using a mixed-methods approach. Twenty six teachers, working in high school mathematics and special education mathematics classrooms, and two student teachers were invited to participate in the first phase of the research. The researcher was an active participant observer in this phase. Gay et al. explained that, as a teacher in the school

district of the study, the researcher was immersed in the culture of the school while conducting the study.

In the first phase, participants were asked to complete a questionnaire consisting of three sections. The questionnaire was adapted from an instrument developed by the Center for Research in Mathematics and Science Education at Michigan State University. As part of a National Science Foundation (NSF) supported evaluation study of a Mathematics and Science Partnership (MSP) Institute, the instrument was developed to measure attributes of teacher leaders. The instrument was previously used by Kher and Burrill (2005) with teachers from the "Park City Mathematics Institute (PCMI)'s Math-Science Partnership Project" (p. 1). The sections consisted of Instructional Practices of Teachers, Beliefs and Opinions, and Teacher Background Characteristics. The second of these sections consisted of two subsections: Beliefs Regarding Mathematics and Mathematics Teaching and Learning. No criteria validity was cited.

The Beliefs Regarding Mathematics instrument was adapted by Kher and Burrill (2005) from the Mathematics Beliefs Scales of Fennema, Carpenter, and Peterson (1987) and Capraro (2001), who later reduced the size of the instrument. Both the initial researchers and Kher and Burrill calculated the coefficient-alpha reliability of the instruments. The original 48-item versions of the Mathematics Beliefs Scales instrument used by Fennema et al. and Capraro had coefficient-alpha reliabilities of .93 and .78, respectively. The abbreviated version utilized by Kher and Burrill had an overall coefficient-alpha reliability of .80. Additionally, Kher and Burrill tested validity of differences between PCMI and non-PCMI teachers on responses from the Beliefs Regarding Mathematics instrument via MANOVA finding an overall statistical

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significance with $p < .0009$. Although utilized, no previous data were reported for the results associated with the Instructional Practices of Teachers section.

The Mathematics Teaching and Learning subsection was adapted from the Mathematics Teaching Efficacy Belief Instrument (MTEBI) created by Enochs, Smith, and Huinker (2000). The original instrument was subdivided into the Mathematics Teaching Outcome Expectancy subscale (MTOE) and Personal Mathematics Outcome Expectancy subscale (PMTE). Enochs et al. found coefficient-alpha reliability of .77 for the MTOE and .88 for the PMTE. No overall value was calculated. Kher and Burrill (2005) found alpha coefficients of .72 for the MTOE and .63 for the PMTE with an overall value of .66 for the instrument in its entirety. A further factor analysis by Kher and Burrill led to a possible explanation in the differences between the two groups of participants. It appeared that "teachers who self-selected participation have different beliefs regarding mathematics than teachers who are non-participants" (p. 9). This was evident in PCMI participating teachers having "statistically significantly higher responses for the total score" (p. 9) of the instrument. Kher and Burrill found a statistical significance of $p < .025$ using MANOVA when comparing differences between PCMI and non-PCMI teachers on responses from the PMTE subsection of the Mathematics Teaching and Learning instrument. Difference between PCMI and non-PCMI teachers on responses from the MTOE and overall Mathematics Teaching and Learning instruments using MANOVA, however, showed no statistical significance.

The first sub-question of the study was:

1. What attitudes and perceptions do these teachers have regarding students' cognitive-metacognitive abilities?

This sub-question was addressed through the use of the questionnaire and follow-up interviews. The questionnaire phase of the study was conducted from April to June of 2009. As respondents completed the questionnaires, data were collected and organized by the researcher. The questionnaire phase allowed the researcher to gain background knowledge about the teachers in the school, as well as information regarding beliefs, opinions, and interests. The responses from the Beliefs and Opinions section of the questionnaire, consisting of Beliefs Regarding Mathematics and Mathematics Teaching and Learning, were analyzed via chi-square tests to find differences between expected and actual responses, and coefficient-alpha reliability was calculated to be compared with results obtained by other researchers. Responses were given along the continuum Strongly Agree, Agree, Disagree, and Strongly Disagree. The alpha level was set at the .05 level for the chi-square analysis. Percentages and frequencies of responses were also calculated. The results served as a basis for studying the attitudes and perceptions held by teachers in the school. In phase two, eight initial respondents were invited to participate in semi-structured interviews. Initial respondents with unique perspectives, backgrounds, and questionnaire responses were invited to the interview phase. The interviews were completed from April to October of 2009. The questions for the interview related to knowledge of cognitive-metacognitive techniques and beliefs about mathematics instruction. Follow-up interviews were conducted when necessary. Interview responses were then reviewed to establish a coding system (Leedy & Ormrod, 2005; Robson, 2002). The researcher utilized the coding strategy to determine if any patterns or relationships existed.

The second sub-question of the study was:

2. How do these attitudes and perceptions influence the strategies utilized by these teachers to teach cognitive-metacognitive skills?

This sub-question was also address through the use of the questionnaire and follow-up interviews. The questionnaire phase allowed the researcher to identify the instructional practices utilized by teachers in the school. The responses from the Instructional Practices section of the questionnaire were analyzed via chi-square tests to find differences between expected and actual responses. The probabilities of responses mapped along a variety of continua were set at $p < .05$. Percentages and frequencies of responses were also calculated. The results served as a basis for studying the practices and strategies used to teach cognitive-metacognitive skills. In the interview phase, the questions related to teaching strategies used in class. The researcher utilized the previously cited coding strategy to determine if any patterns or relationships existed.

CHAPTER II

REVIEW OF THE LITERATURE

Introduction

The purpose of this chapter was to review the literature regarding the topics of

cognition and metacognition in relation to student learning and thinking in mathematics

classrooms. Additionally, the relationships between the attitudes and perceptions of

teachers and student cognitive-metacognitive skills were discussed. The chapter begins

with a review of the evolution of both cognition and metacognition, specifically within

the educational arena. Then, the relationship between the two is explored, leading to a

discussion of frameworks created to delineate cognitive-metacognitive skills, abilities,

and processes. After the theoretical research has been exhausted, the effects of cognition

and metacognition in relation to learning, teaching, and strategy instruction are discussed.

Following the role of cognitive-metacognitive skills in schools, the focus shifts to the

roles of attitudes and perceptions on thinking and learning. Next, an examination of

external influences that play a role in the teaching and learning process reveals its effects.

Finally, a discussion of various studies from more specific examples of the topics

presented is included.

Cognition

The notion of education is often related to the concepts of thinking and learning.

Although the terms thinking and learning have definitive denotations, the connotations

generally associated with them can vary greatly. Similarly, since its inception, the term cognition has been both prevalent in its usage and elusive in its meaning.

According to Schoenfeld (1992), cognition is comprised of five generally agreed upon tenets: "the knowledge base", "problem solving strategies", "monitoring and control", "beliefs and affects", and "practices" (p. 42). Together, Schoenfeld's five aspects of cognition form the basis for thinking. Of course, not all researchers subscribe to Schoenfeld's definition (Galosy, 2006; Henningsen & Stein, 1997; Leron & Hazzan, 2006). Henningsen and Stein explained how the demand level on cognition is another factor to be considered. Similarly, Galosy identified three levels of cognitive demand: "recognition, memorization (of facts, concepts, and principles), [and] understanding and/or application" (p. 10). In relation to mathematics education, cognition involves intuition and analytical thinking (Leron & Hazzan). Leron and Hazzan continued by explaining how the field of cognitive psychology tends to view cognition as primarily unconscious thoughts.

Artzt and Armour-Thomas (1992) approached the concept of cognition from a different perspective. Instead of explaining the concept of cognition, Artzt and Armour-Thomas exemplified cognition via its observable actions, such as reading. In relation to mathematical problem solving, Mayer (1998) identified cognitive skills as "instructional objectives, learning hierarchies, and componential analysis" (p. 51). The approaches suggested by Artzt and Armour-Thomas, as well as Mayer, echoed previous research by Garofalo and Lester (1985) in which cognition was defined as "involved in doing" (p. 164). In combination, the different approaches give a more accurate picture of the varying interpretations of the term and how it can be applied to educational research.

Another view by Hutchins (1995) suggested that cognition is situational and relies greatly upon both the environment and people interacting within the environment. Therefore, knowledge, or the application of knowledge, is shared among a group of people. To an extent, Hutchins' view contradicts the cognitive psychology assertion that cognition involves unconscious thoughts. Norman (1993, as cited in Razo, 2001) explained that "distributed cognition" (p. 6) allows objects to retain knowledge and work in tandem with individuals. Concerning distributed cognition, a person's ability to demonstrate cognitive skill relies greatly upon knowledge of the situation. Unfamiliarity with a situation or set of available tools poses a barrier to successful cognition. Such an obstacle can be directly related to student thinking in education (Megowan, 2007). Megowan found that distributed cognition took the form of students working off of each others' comments toward a common goal. The methods utilized and solutions found, however, were not restricted by a particular algorithm. In other words, the class as a whole exemplified distributed cognition through shared knowledge and problem-solving strategies.

<center>Metacognition</center>

Similar to cognition, the use of the term metacognition in education has been nebulous. One of the earliest researchers in the area of metacognition was Flavell (1979). Flavell's theory attempted to explain the main aspects of metacognition. Flavell explained that metacognition consists of both knowledge and experiences. Metacognitive knowledge is comprised of three categories: "person, task, and strategy" (p. 907). The person category refers to how an individual comes to understand people. The task category refers to how an individual perceives completion of a task or goal. Finally, the

strategy category refers to how an individual analyzes and chooses a strategy in a situation. Flavell continued by citing that metacognitive knowledge usually encompasses at least two of the categories in tandem. On the other hand, metacognitive experiences are usually specific experiences that create a change in metacognitive knowledge. For example, a new experience with another person might cause an individual to change perceptions, therefore adjusting the previous metacognitive knowledge in the person category. Similarly, an individual might revise a previously used strategy based upon feedback. Metacognitive knowledge and experiences are truly intertwined.

Although Flavell clarified numerous aspects of metacognition, the overall approach does not easily translate into practice. Brown and Palincsar (1982) took a different, simpler approach. According to Brown and Palincsar, metacognition refers to "knowledge about cognition and regulation of cognition" (p. 2). Brown and Palincsar's succinct definition has generally served as a foundation of subsequent research on metacognition. Building upon Brown and Palincsar's work, numerous researchers have added or adjusted theories related to the notion of metacognition (Artzt & Armour-Thomas, 1992; Holton & Clarke, 2006; Kayashima, Inaba, & Mizoguchi, 2004; Livingston, 1997).

Livingston (1997) gave an overview of much of the changes to metacognitive theory since both Flavell's (1979) and Brown and Palincsar's (1982) work. Livingston explained that metacognition was defined in a myriad of ways because different disciplines used similar yet distinct terms to elucidate it. To illustrate, terms as varied as self-regulation, executive control, and meta-memory were all used in relation to metacognition. Despite the differences, a common thread did exist. Livingston found that

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terms equating to metacognition "all emphasize[d] the role of executive processes in the overseeing and regulation of cognitive processes" (p. 2). The commonality was nearly identical to the denotation proposed by Brown and Palincsar some 15 years earlier. Both Brown (1987) and Flavell (1987) continued refining metacognitive theories in relation to other research. Despite the continued research, a relative mystique still remained, and metacognition could still be accurately described as "fuzzy" and causing "confusion" (Brown, p. 66). As Brown and Flavell's distinct but related theories developed, both researchers cited how the application of metacognition could impact educational systems. Conversely, educational systems could affect metacognition in a reciprocal manner.

Just as with cognition, Artzt and Armour-Thomas (1992) identified behaviors that mapped primarily to metacognitive activities. The behaviors included understanding, analyzing, and planning as predominantly metacognitive activities. Additionally, the behaviors of implementing, exploring, and verifying were identified as activities that could be classified as either cognitive or metacognitive. Kayashima et al. (2004) found Flavell's (1979) taxonomy to be too simplistic to encompass the complexity that surrounds metacognition. Kayashima et al. first redefined cognition as the ability to "produce the mental representations of outside world…through perceptions" and cognitive activity as the ability to "achieve the goals that we have such as problem solving, reasoning and judgment" and use "a pre-compiled action such as computing" (p. 2661). From that point, metacognition was subdivided similarly to cognition. Therefore, Kayashima et al. classified metacognition as cognition of cognition and cognition of cognitive activity, whereas metacognitive activity was classified as cognitive activity with cognition of cognition and cognitive activity for cognitive activity. Although

confusingly enigmatic, Kayashima et al.'s definition does not oversimplify the meaning of metacognitive skill.

From an entirely different perspective, Holton and Clarke (2006) likened metacognition to scaffolding, a term dating back to Vygotsky (1934, as cited in the translation by Cole, John-Steiner, Scribner, & Souberman, 1978). In fact, even more explicitly, Holton and Clarke suggested that metacognition is a form of self-scaffolding. In other words, self-scaffolding "mediates between the learner and their cognition" (p. 132). Holton and Clarke were quick to point out that a self-scaffolder will not necessarily possess the knowledge necessary to overcome the problem posed or concept encountered successfully; however, the self-scaffolder benefits from knowing the limitations of his or her own knowledge. Outside sources providing scaffolding, such as teachers, might lack the ability to pinpoint a knowledge deficiency.

Once metacognition had been initially researched and defined, new researchers began theorizing as to its applicability to more specific areas of study, namely mathematics education (Garofalo & Lester, 1985; Leron & Hazzan, 2006; Mayer, 1998; Schoenfeld, 1987, 1992; Wilson & Clarke, 2002). Mayer approached the concept of metacognition as a necessary component for successful problem solving. Mayer's theory was comprised of three parts: "skill, metaskill, and will" (p. 51). In relation to problem solving in mathematics, Mayer's three components could be immediately identified. Skill refers to the knowledge students possess. Metaskill refers to the ability of students to identify, use, and monitor the knowledge applied to a problem. Finally, will refers to the motivation or interest level the student has to solve the problem. A student's will is an interesting addition to metacognitive theory that few researchers have explored.

Wilson and Clarke (2002) identified actions related to metacognition. The actions represented "awareness", "evaluation", and "regulation" (p. 9), and all were encompassed by cognition. The three actions related to metacognition interacted with and mediated cognition. Wilson and Clarke worked with students and asked them to identify the type of action taken as problems were solved. Students quickly moved among the different types of actions, with cognition interspersed among them. The students demonstrated that cognition and metacognitive actions are closely related within thinking systems.

Garofalo and Lester (1985), as well as Schoenfeld (1987, 1992), built upon Flavell's (1987) theory to relate metacognition to mathematics. Flavell's categories of person, task, and strategy can be easily translated into the mathematical arena. Garofalo and Lester explained each of the categories in relation to mathematics. In mathematics, person refers to an individual's "own capabilities and limitations with respect to mathematics in general and also with respect to particular mathematical topics or tasks" (p. 167). Task knowledge refers to an individual's "beliefs about the subject of mathematics as well as beliefs about the nature of mathematical tasks" (p. 167). Strategy refers to "knowledge of algorithms and heuristics, but it also includes a person's awareness of strategies to aid in comprehending problem statements, organizing information or data, planning solution attempts, executing plans, and checking results" (p. 168). In addition, just as Flavell cited that each of the categories often worked in tandem, Garofalo and Lester mentioned how the three categories could interact when influencing mathematical problem solving. Schoenfeld continued by explaining that metacognition actually could be taught in mathematics classrooms. Indeed, part of

successful problem solving involves monitoring of cognition through attention to strategy choice and evaluation of effectiveness and appropriateness.

Leron and Hazzan (2006) cited Schoenfeld's (1987) work regarding an additional aspect of metacognition: students' beliefs and intuitions. When students construct mathematical knowledge, prior experiences and individual identities affect the way in which learning can take place. Actually, Schoenfeld (1992) later refined the application of students' beliefs to include the beliefs of both students and teachers in the mathematics classroom. Therefore, mathematical knowledge is constructed by both teachers and students, and each can be influential in the knowledge development of the other.

<center>Cognition versus Metacognition</center>

As was evident in the discussion of cognition and metacognition, the terms can easily be misidentified, and are often used interchangeably. Due to the ambiguity of both cognition and metacognition individually, there is some debate as to the distinction. As Schoenfeld (1992) described, cognition involves "monitoring and control" (p. 42) as one of its five aspects. Schoenfeld's description closely resembles the definition of metacognition proposed by Brown and Palincsar (1982) of "knowledge about…and regulation of cognition" (p. 2), however. Therefore, according to Schoenfeld, metacognition is a subset of cognition. Though at the same time, Brown and Palincsar's definition places cognition within the scope of metacognition. Hence, the paradox that is the relationship between cognition and metacognition is established.

Because of the continued confusion between cognition and metacognition, a number of researchers refer to skills and abilities relating to thinking from the jointly defined cognitive-metacognitive domain (Artzt & Armour-Thomas, 1992; Garofalo &

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Lester, 1985; Hutchinson, 1992). As previously stated, Artzt and Armour-Thomas identified specific behaviors related separately to either cognition or metacognition. Additionally, certain behaviors related both to cognition and metacognition. Together, the specified behaviors were collectively termed within a cognitive-metacognitive model. Garofalo and Lester's theory was comprised of four categories: "orientation, organization, execution, and verification" (p. 171). Garofalo and Lester's four thinking tasks were jointly referred to as cognitive-metacognitive. Additionally, Hutchinson explained how mathematics instruction should address students' cognitive-metacognitive skills and abilities. Although Hutchinson cited other researchers who attempted to study one or the other, Montague, as cited in Hutchinson, focused on the processes related to both at once.

Livingston (1997) explained the difference as "cognitive strategies are used to help an individual achieve a particular goal…while metacognitive strategies are used to ensure that the goal has been reached" (p. 3), echoing Flavell's (1987) sentiment a decade earlier. Livingston also concurred with previous research in stating that strategies often overlap between the two parts of the domain. Brown (1987) defended the role of metacognition in educational research by further explaining its difference from cognition and relation to reading. Brown clarified that metacognition refers to deeper understanding and analyzing of test with respect to reading strategy. Criticism of metacognition as a reading strategy surfaced in great part due to the emergence of the term before a more clearly defined difference was proposed between metacognition and the generally less complex cognition. In other words, critics felt that, on the surface, reading itself is not metacognitive unless it incorporates some form of additional scrutiny.

Other researchers, such as Leron and Hazzan (2006), Kayashima and Inaba (2003), Kayashima et al. (2004), Holton and Clarke (2006), and Lim (2006), proposed distinctions between cognition and metacognition more tangential to the mainstream approaches. Leron and Hazzan explained how the differences in definitions between cognition and metacognition are extremely crucial in the field of mathematics education. Whereas, regulation and monitoring of thinking are generally sufficient in defining metacognition outside the realm of mathematics, the characteristics of beliefs, focus, and time management become increasingly important when expending effort in problem solving. The characteristics described by Leron and Hazzan are glaringly absent from most definitions of cognition or metacognition proposed from fields outside mathematics education. Contrastingly, Kayashima and Inaba suggested that metacognitive skills are not affected by subject area. Instead, one can apply metacognitive skills across any subject once mastered. Additionally, Kayashima and Inaba described metacognitive skills as an additional layer above cognitive skills. Kayashima et al. further delineated the barrier between cognition and metacognition by concluding that cognition served as a subset, although in numerous forms, of metacognitive skills and activities. Therefore, metacognition is placed at a strictly higher level of mental demand than cognition.

Holton and Clarke (2006) continued the research of differences between cognition and metacognition. The distinction, however, emerged in explanation of the context of metacognition in the real world. Holton and Clarke suggested that while metacognition allows one to regulate, monitor, and control cognition, cognition itself allows one to regulate, monitor, and control the real world. In reality, there is no direct link between metacognition and the real world except through cognition. From another perspective,

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Lim (2006) linked cognition and metacognition through the concept of anticipation. Lim qualified anticipation as the ability to "foresee" and "predict" (p. 5). The intuitiveness of cognition and metacognition, as well as beliefs, parallels the two aspects of anticipation. The acceptance of anticipation within the study of metacognition has yet to be seen on a larger scale.

<center>Cognitive-Metacognitive Frameworks</center>

Numerous researchers have created frameworks for interpreting student thinking in relation to the cognitive-metacognitive domain (Artzt & Armour-Thomas, 1992; Flavell, 1979; Garofalo & Lester, 1985). Garofalo and Lester's distinctions between cognition and metacognition were most evident in the cognitive-metacognitive framework created. Orientation concerned "strategic behavior to assess and understand a problem" (p. 171). Organization referred to "planning of behavior and choice of actions" (p. 171). Execution involved "regulation of behavior to conform to plans" (p. 171). Finally, verification required "evaluation of decisions made and of outcomes of executed plans" (p. 171). Within each of the four categories, additional strategies and descriptions were provided. The ingenuity of the framework was that it extensively accounted for numerous metacognitive activities. The limitations included lack of easily identifiable actions and generally limited only to students. Additionally, the framework was fairly general and lacked certain specificities that would make it more easily implementable.

Wilson and Clarke's (2002) framework included the actions of "awareness", "evaluation", and "regulation" (p. 9), each interrelated to cognition. Although similar to Garofalo and Lester's (1985) framework, Wilson and Clarke created a more compelling model by explaining each aspect through observable actions. The small distinction made

a huge difference in applicability. Conversely, the lack of distinction between types of regulation, namely organization and execution from Garofalo and Lester, reduced some of its potency. Regardless, both approached metacognition from a slightly unique perspective, helping further to delineate the intricacies of cognition and metacognition in education.

Flavell (1979) developed a model of monitoring cognition that was even more limited. Flavell's model consisted of "(a) metacognitive knowledge, (b) metacognitive experiences, (c) goals (or tasks), and (d) actions (or strategies)" (p. 906). Flavell's model is less refined than that of Garofalo and Lester (1985), and it lacks verbiage that would improve its accessibility to additional research. On the other hand, Schoenfeld's (1992) definition of cognition, which, in fact, represented a framework of thinking, fell somewhere between Flavell and Garofalo and Lester. The advantage of Schoenfeld's framework over both of the others is that it could relate to the relationship between teachers and students. Additionally, it was created in such a way that mathematics educators could interpret it as readily as mathematics education researchers.

Cognitive-metacognitive frameworks are generally concerned with the thinking that occurs in students. Artzt and Armour-Thomas (1992) created a framework that listed specific activities that related to either cognitive behavior or metacognitive behavior. Cognitive behaviors included reading, implementing, exploring, and verifying. Metacognitive behaviors included understanding, planning, analyzing, implementing, exploring, and verifying. Obviously, the last three behaviors of both types could fluctuate between the cognitive or the metacognitive depending upon the task or specific point in the problem-solving process. The strength of Artzt and Armour-Thomas' framework is

that it is easily accessible to researchers and novices alike. Moreover, it does not lack the rigor or theoretical background of other, deeper frameworks. Later, Artzt and Armour-Thomas (1998) researched the use of metacognitive approaches by mathematics teachers. The scope of metacognition was expanded outside the individuals involved in learning to encompass the individuals involved in providing or creating the learning environment. Artzt and Armour-Thomas believed that the learning tasks utilized to further the development of metacognition in students were influenced by teachers. The second model incorporated an entirely different scope. The importance of teacher lesson planning outside the classroom, monitoring and regulating in the classroom, and assessing and revising after the lesson all played a part in the teaching process. Being sensitive to students' metacognitive needs was an important aspect of this research, and Artzt and Armour-Thomas addressed students' needs through attention to how teacher beliefs eventually influence instructional practice. In conjunction with Schoenfeld's (1992) definition-style framework of cognition, Artzt and Armour-Thomas' frameworks (1992, 1998) most closely resembled the purposes and goals of this study.

Besides the frameworks that guided this study and the other major frameworks previously discussed, there were many other theories and frameworks proposed in the literature. Kayashima and Inaba (2003) proposed a double-loop model that placed metacognitive behaviors above behaviors solely dealing with cognition. Leron and Hazzan (2006) explained how the dual-process theory of cognitive psychologists was closely related to much of the recent literature regarding cognitive-metacognitive domains in mathematics education. In fact, Schoenfeld's (1987) earlier work on metacognition nearly matched the theory. Borkowski (1992) cited themes somewhat

similar, except including the ever increasingly important role of models in teaching. Metacognition in conjunction with scaffolding was another model proposed (Holton & Clarke, 2006; Marge, 2001).

In other cases, researchers narrowed in on very specific frameworks outside the mainstream. Fogarty and McTighe (1993), based upon a quotation by Oliver Wendell Holmes, analogized intellect to a three-story building. The first-story represents the acquisition of skill. The second, making meaning of the skills acquired. Finally, the third floor represents the transfer and application of the skills learned. In another study, a model incorporating "Consciousness, Unconsciousness, Language, Tacit, Individual, [and] Social" (CULTIS) (Dahl, 2004, p. 129) was created. The model attempted to incorporate many of the current theories to be more all-encompassing. Finally, McSweeney (2005) cited the taxonomy called Let Me Learn. The uniqueness of McSweeney's taxonomy was its basis on social cognitive theory. Of particular note was the inclusion of both human agency and conation in social cognitive theory; two aspects not explicitly discussed in most other frameworks.

Effects of Cognition and Metacognition on Learning Mathematics

The impacts of the theoretical research on more empirical research related to learning mathematics are as varied as the frameworks for metacognition. The National Council of Teachers of Mathematics (NCTM, 1989) established societal and student goals related to learning mathematics. The societal goals were "mathematically literate workers", "lifelong learning", "opportunity for all", and an "informed electorate" (pp. 3-4). The student goals were "learning to value mathematics", "becoming confident in one's own ability", "becoming a mathematical problem solver", "learning to

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communicate mathematically", and "learning to reason mathematically" (p. 5). The
NCTM's goals served as the purpose and foundation of mathematics education. Romberg
(1994) explained how the NCTM's goals viewed students as learners constructing
knowledge and impacting our world, rather than simply receptacles to be filled with
knowledge. On a somewhat similar note, Schoenfeld (1992) purported the purpose of
learning mathematics to be the need for "mental discipline" (p. 35).

Flavell (1976, 1979) suggested similar findings years before the NCTM created
its goals. With respect to metacognitive strategies being implemented in schools, Flavell
(1979) suggested that metacognitive strategies would help to make better citizens by
instilling an ability to think critically in students. Flavell's (1976) earlier work also cited
the ability to solve problems as a necessary component of mathematics curricula. The
relationship between mathematical problem solving and cognitive-metacognitive training
was further researched in small-group settings (Artzt & Armour-Thomas, 1992). The
research showed promising results in teaching problem solving through cognitive and
metacognitive strategies. The pervasiveness of success, however, may not be as
expansive as hoped. For example, special education students not only struggled to select
correct algorithms for solving problems, but also had difficulty carrying out calculations
using the procedures (Palincsar & Brown, 1987). Extra effort is needed to address
students' special needs, or implementing different strategies would better suit the learning
process for special needs students.

Carpenter and Fennema (1991) found that much previous research focused on
how teachers teach. In fact, the characteristics of "effective teaching" (p. 2) were defined
in great detail. "Process-product" authors (p. 2), however, largely ignored two important

parts of the equation: children and the thoughts of teachers. The characteristics identified only represented the surface behaviors observable in the classroom. The underlying beliefs and thinking involved were not considered. Carpenter and Fennema acknowledged that teachers' thinking in and out of the classroom is deeply cognitive, and has "a profound effect on the way they teach as well as on students' learning in their classroom" (p. 3). Of course, the thoughts and beliefs of students are just as important, if not more so, than the teachers' thoughts. The background knowledge and experiences that a student possesses can greatly influence the learning process. The teachers' knowledge can also impact the extent to which students can learn (Fennema et al., 1996). As a result of the dual influence of students and teachers on each other, as well as the importance of knowledge, thinking, and beliefs, Carpenter, Fennema, Franke, Levi, and Empson (2000) proposed a program called Cognitively Guided Instruction (CGI). In the CGI program, teachers are trained to understand how children think to better address the learning styles of students.

Metacognitive strategies should be incorporated in the classroom to assist students in achieving higher levels (Maqsud, 1998; Stillman & Galbraith, 1998). Carr, Alexander, and Folds-Bennett (1994) found that metacognitive instruction would prove beneficial to student learning. Similarly, Vygotsky (1978) investigated the "zone of proximal development" (ZPD) (p. 86) as it related to student learning. Vygotsky's research on the ZPD, which is closely related to scaffolding, deals with the point in learning or problem solving at which an individual can do better with help from an external source. Scaffolding, similarly, refers to the assistance given to an individual when in the ZPD to achieve greater learning or more successfully solve the problem. In either case, the

assumption is that assistance, as that given by a teacher, can greatly improve a student's

learning. Of course, it is necessary for the assistance to be given at a specific time, and

the assistance must still allow the learner to progress without oversimplifying the task.

Providing scaffolding within the ZPD requires teachers first to hone cognitive skills

themselves and then view learning from the student's perspective to learn how to assist

appropriately.

The missing piece of the puzzle is the student. Teachers can be perfectly attuned

to the needs of students, however unwilling students cannot benefit from assistance

(Basta, 1998; Mayer, 1998). Basta explained that disposition is an important factor in the

learning process. It was found that a positive correlation existed between a student's

mathematics disposition and need for cognition. Positive correlation is immensely

important to motivational research as it relates to student learning of mathematics. Mayer

defined three different motivating factors for students to want to achieve. Some students

are motivated because of an interest in a given topic. Other students are motivated

because of a need for self-efficacy. Still others will be motivated, or unmotivated, due to

attributions, such as blaming a poor grade on another student or the difficulty of the task.

Regardless, motivating factors can be hugely influential on a student's willingness to

learn.

The experience of the learner is another aspect of learning to consider. Novices

and experts possess different levels of background knowledge and access to

metacognitive strategy (Adelson, 1984; Blessing & Ross, 1996). Blessing and Ross

referenced studies in which novices relied upon surface content of problems to determine

the type of knowledge to be applied. Conversely, experts combine initial categorization

of problem content with an understanding of the deeper structure of the problem. As experts have a greater schema, or underlying organization of thought, the ability to make connections between the current problem and previously encountered situations surpasses that of novices. Interestingly, Adelson found that experts actually can be at a disadvantage with respect to novices. Whereas experts rely upon conceptual understanding of the abstract, novices think in generally concrete terms. When problems focus more on concrete ideas and procedures, experts can have a tendency to over analyze the problem by connecting to previous experiences. On the other hand, novices, possessing a more limited schema, have fewer options and, therefore, apply it to the few concrete ideas they understand. The avoidance of multiple connections benefits novices for straightforward, concrete problems. When problems become more abstract in nature, however, experts again have the advantage.

On a different note, Lim (2006) explained how the act of anticipating is an important part of learning. Students who exhibited a stronger ability to predict fared better when solving problems, which also related to metacognitive abilities. Henningsen and Stein (1997) found that solving mathematical problems required students to be cognitively aware. The warning was that learning mathematics could be derailed unintentionally. Superbly crafted tasks requiring a great deal of deep, mathematical thought could devolve into simple procedural algorithms depending on the method of implementation. Teachers must remain vigilant in requiring students to justify explanations and preserve the intent and scope of genuinely challenging problems. Kahveci and Imamoglu (2007) and Kayashima et al. (2004) echoed the sentiment that teachers must foster a classroom of collaborative problem solving, lending itself to the

promotion of metacognitive skills. Furthermore, student motivation and supportive classroom environments are important for students to be able to think deeply about mathematics (Kahveci & Imamoglu).

Kramarski and Mevarech (2003) cited that engaged learning is most effective for learning mathematics. In fact, students teaching material to other students is an extremely effective strategy for content retention. The reciprocal learning strategy requires a great deal of cognitive-metacognitive skill. Unfortunately, engaged learning, through reciprocal teaching or by other methods, is not always feasible or even possible. When learning occurs via E-learning situations, there is little interaction, if any, among the members of a class (Kramarski & Gutman, 2006). Despite the lack of interaction, metacognitive skills and strategies are important for the students to expand and utilize. The teacher could help facilitate metacognition through the use of student "self-metacognitive questioning" (p. 26). The teacher asked students to reflect upon the strategies and techniques used in solving mathematical problems. Although not ideal, student reflection did aid students in fostering at least minimal metacognitive skill. The implications of Kramarski and Gutman's study include the possible applicability to situations outside the E-learning environment, such as normal mathematics classrooms.

Effects of Cognition and Metacognition on Teaching Mathematics

Cognition and metacognition can positively affect student learning. Therefore, because sophisticated cognitive-metacognitive skills rarely occur spontaneously, the responsibility of preparing students to hone cognitive-metacognitive skills lies with teachers. Although intrinsically linked, student thinking is a separate research topic from teaching (Carpenter & Fennema, 1991). Carpenter and Fennema expanded upon the

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thought process of teachers. Because "thinking plays an important part in teaching" (p. 3), it is not surprising that "teachers have theories and belief systems that influence their perceptions, plans, and actions in the classroom" (p. 3). After instruction in the CGI program, Carpenter et al. (2000) found that teachers shifted teaching styles to accommodate increased problem solving rather than simply procedures and computations. Additionally, teachers were more sensitive to the needs of students, especially needs involving cognition and metacognition.

The CGI model combines the research of student thinking with teacher instruction, belief structure, and the impact of instruction and belief structure on students (Carpenter & Fennema, 1991; Carpenter et al., 2000; Clarke, 1997; Fennema et al., 1996). Created by Carpenter et al., the CGI program aims to inform and instruct teachers in practices that will help them better teach students how to think. Clarke's research of teachers using the CGI program revealed seven teacher roles directly related to the implementation of cognitive-metacognitive programs, such as CGI. First, one of the most important teacher roles involved using "nonroutine problems as the starting point and focus of instruction, without the provision of procedures for their solution" (p. 286). The second involved adapting teaching to the specific students taught. Next, group dynamics should be incorporated to maximize learning individually and in groups of varying sizes. Additionally, the classroom should foster group discussion and include the teacher as another view rather than the authority. Furthermore, teachers should spend more time on overarching concepts than specific, insignificant ones. Changes to instruction should be based upon continuous assessment from a myriad of sources. Finally, reflection should be incorporated into the thinking and problem-solving process to evaluate methods both

considered and actually implemented. In total, the teacher roles discussed comprised

Clarke's vision of "reconceptualized roles" (p. 303) for teachers in successfully

integrated problem-solving classrooms. Unfortunately, Clarke's research also referenced

the difficulties that threatened to impede full adoption of the changes. Not only did many

of the teachers possess a willingness to change, but were also supported through

"innovative materials, supportive colleagues, [and] time for reflection and planning" (p.

303).

> Kramarski and Mevarech (2003) explored similar teacher roles to Clarke (1997)

and articulated the findings as follows:

> Finally, beyond context and content, there is the role of the teacher. The study
>
> illustrates several important facets of teachers' roles, pertaining to metacognitive
>
> guidance, organization of the classroom, and the selection and use of worthwhile
>
> mathematical tasks that allow significant mathematical discourse to occur. Such
>
> tasks should include complex situations that present quantitative information in
>
> different contexts, allow multiple representations, or afford students opportunities
>
> to resolve mathematical conflicts. (p. 302)

The emphasis on the importance of teaching metacognitive skills, especially as it pertains

to mathematics, is clear. Garofalo and Lester (1985) suggested that many teachers

address the teaching of procedures in relation to problem solving yet ignore any type of

metacognitive training. Without metacognitive training, however, students often lack

knowledge of both the purpose and suitability of procedures for specific problems.

Instead, as procedures are taught, teachers also need to guide students in understanding

when certain procedures are applicable, thereby increasing student conceptual knowledge of mathematical problem solving.

Palincsar and Brown (1987) agreed that teaching would be enhanced when teachers explicitly teach metacognitive strategies, especially when working with special education students. Despite Palincsar and Brown's focus on special education students, instruction at all levels would likely improve based upon the following suggestions:

> The features of successful metacognitive instruction which have been identified throughout this review include (1) careful analysis of the task at hand, (2) the identification of strategies which will promote successful task completion, (3) the explicit instruction of these strategies accompanied by metacognitive information regarding their application, (4) the provision of feedback regarding the usefulness of the strategies and the success with which they are being acquired, and (5) instruction regarding the generalized use of the strategies. (p. 73)

Of course, suggestions for improvement are much more likely to be successful when teachers and students are motivated in the endeavor (Kahveci & Imamoglu, 2007). Moreover, plentiful support, especially in the form of time, is required to accompany the implementation process.

Lim (2006) suggested that starting to train students in metacognitive skills will likely be more successful if "teachers can be more sensitive to their students' ways of thinking and ways of understanding" (p. 39). Simply asking students for feedback by means of daily written reflections or questioning can help inform teachers as to instructional successes or needed improvements. The strategy can also aid teachers in confronting common misconceptions and errors. As teachers begin to understand

students' perspectives, the classroom discourse could change. Teachers could better predict the students' responses and thoughts. An overall change in teaching method would follow, but changes can only occur if teachers allow themselves to become aware of the cognitive-metacognitive needs of students (McSweeney, 2005; Megowan, 2007).

Unfortunately, just as students do not miraculously start utilizing refined metacognitive skills, nor do teachers suddenly flip a switch and understand students' ways of thinking. Instead, teachers need training in how to first understand student thinking and then how to train students to monitor thinking via metacognition (Murphy, 2004; Patton, Fry, & Klages, 2008). Another reason for the importance of training teachers is that students often imitate the help given by teachers (Webb, Nemer, & Ing, 2006). Furthermore, Borkowski (1992) cited that teachers influenced students through the use of mental models. In other words, if a teacher believed that a student was capable of a given task, it was more likely that that student would be successful in completing it. Unfortunately, the opposite was also true. In a sense, teachers would subconsciously promote self-fulfilling prophecies related to student performance; but the role of teachers went even beyond that. The beliefs and attitudes held by teachers regarding mathematics itself could be transferred into the teachers' means of instruction and, therefore, affect students' learning (Ernest, 1988; Munby, 1982; Patton et al.).

Schurter (2002) studied how teaching students certain techniques affected achievement. Using both metacognition and comprehension monitoring, students tended to score slightly higher on the test instruments. Schurter continued by citing the need for teachers to model problem-solving techniques for students. Modeling was, and is, an important tool in a teacher's arsenal. Stillman and Galbraith (1998) advocated that

teachers utilize activities aimed at fostering the problem-solving abilities of students. In addition, teachers must encourage students first to realize and then apply the cognitive and metacognitive resources they possess. One method for helping students to realize previously obtained knowledge and achieve at higher levels is for teachers to provide scaffolding in the ZPD (Henningsen & Stein, 1997; Holton & Clarke, 2006; Vygotsky, 1978). Henningsen and Stein proposed that teachers should emphasize the need for students to adjust schema and make connections. The role of the teacher, therefore, is to maintain constant mathematical activity through the use of effective classroom management, supportive collaboration, and robust discourse. At the same time, the teacher must prepare students to become self-scaffolders who can reason without an external influence (Holton & Clarke).

Wilson and Clarke (2002) proposed that schools move toward the utilization of a metacognitive curriculum, especially in mathematics. The *Third International Mathematics and Science Study* (*TIMSS*) model of curriculum focuses on the intended, implemented, and attained curricula (Mullis et al., 2005). Teaching of metacognitive skills must first be conceptualized as the intended curriculum. Next, teachers must support the change by teaching in such a way as to implement the change and foster metacognition. In that way, the attained curriculum of the students will most closely resemble the original intended curriculum. Schmidt et al. (1996), in studying numerous nations and curricula, found that mathematics classrooms are diverse in different countries. Not only are the curriculum plans driving countries different, but the format of classes are unique. In short, "instruction differs qualitatively among countries" (p. 132).

Students learn differently because teachers teach differently, and cultural differences likely play a role.

An interesting aside to the discussion of teaching is the role of calculators in the teaching of cognitive-metacognitive strategies (Wilkins, 1995). Few studies have addressed the issue, but technology is becoming increasingly important to the learning process, as well as the opportunities afforded to students. The use of graphing calculators can help students solve certain problems more quickly by allowing faster computations. Wilkins also cited the fact that students require higher levels of metacognition simply to use the calculators because accurate input is needed. Additionally, students exhibit higher levels of attention to determining appropriateness of solutions obtained through calculator usage. On the other hand, some students rely heavily on the calculator as a crutch to avoid almost all mental computation, and the usefulness dissipates when analysis of the results is ignored. Ideally, teachers would instruct students not only in metacognitive strategies, but also include strategies in calculator usage, too.

Dahl (2004) expressed one of the most important aspects of incorporating cognitive-metacognitive strategies into schools (and one that is most often ignored) as the "importance of giving time in the teaching for (further) developing the pupils' metacognition" (p. 153). Teachers need time to understand students' perspectives, instruct students in the methods and strategies of metacognition, and, perhaps most importantly, give students time to practice and experiment with the new techniques learned. Metacognition is not a fact to be memorized, but a skill to be developed and honed. Instruction in a traditional fashion is both inappropriate and highly unlikely to

succeed. Instead, problem solving must be desegregated from its current position as an enrichment topic and be at the forefront of mathematics teachers' minds.

Teaching Strategies for Developing Mathematical Understanding

Knowing that cognition and metacognition affect teaching and learning in mathematics, strategies that foster student skills related to cognitive-metacognitive thinking processes need to be implemented. Numerous researchers have proposed strategies to improve the cognitive-metacognitive development of students with respect to mathematical literacy (Carpenter et al., 2000; Costa, 1984; Dahl, 2004; Kramarski & Mevarech, 2003; Mevarech, 1999; Polya, 1945; Schoenfeld, 1987). Schoenfeld, one of the most published authors on metacognition and mathematics, explained a number of approaches that he had developed in his own classroom. The approaches were videotaping, teacher as role model, whole-class discussion, and small group exploration. Each method has a place in the classroom, and each can aid students in developing more of an awareness of how the problem-solving method is employed. For example, the teacher as role model technique allows students to observe the methods by which the teacher demonstrates metacognitive skills while solving mathematics problems.

Many strategies related to improving metacognitive skills trace back to one of the earliest problem-solving methods. Polya (1945), as cited in Kahveci and Imamoglu (2007), outlined steps used in successful problem solving. The genius of Polya's plan, however, was in how it suggested differentiating given information. The steps included analyzing the problem, considering a method of solution, implementing the method, and evaluating both the success and efficiency of the solution method. Another aspect of problem solving advised by Polya was that only those data applicable to solving the

problem should be considered. In other words, Polya encouraged metacognition before

the term really existed in the field. Schoenfeld (1992) cited Polya's problem-solving

model when describing his own problem-solving strategies, but Schoenfeld defined the

strategies as heuristics rather than problem solving. In fact, the term heuristics more

closely resembles the intent of investigation and discovery inherent to learning

mathematics with an emphasis on metacognitive skill. The phrase problem solving has

become as ambiguous and vague as cognition and metacognition due to overuse and

generally divergent connotations. Heuristics, on the other hand, has retained its potency

and robustness by remaining innocuously inconspicuous.

Livingston (1997) described several methods of instruction in metacognitive

processes:

While there are several approaches to metacognitive instruction, the most

effective involve providing the learner with both knowledge of cognitive

processes and strategies (to be used as metacognitive knowledge), and experience

or practice in using both cognitive and metacognitive strategies and evaluating the

outcomes of their efforts (develops metacognitive regulation). Simply providing

knowledge without experience or vice versa does not seem to be sufficient for the

development of metacognitive control. (p. 5).

More specifically, Livingston described the "Cognitive Strategy Instruction" (CSI) (p. 5)

approach as one in which students are encouraged to reflect upon mathematical thinking.

Students then learn how to hone skills related to thinking via metacognitive methods.

Flavell (1979) suggested that instruction in metacognitive strategies allows students to

interpret new experiences metacognitively. Interestingly, once students begin to

metacognate, metacognition breeds more metacognition. For example, if a student is self-monitoring while reading and realizes that his mind is wandering, he might begin to ask clarifying questions to check for understanding as he continues. The initial reading is a generally cognitive activity, the self-monitoring is metacognitive, and, due to the analysis through self-monitoring, the student incorporates a second metacognitive skill in response to the first. Of course, Flavell pointed out that students can only utilize cognitive and metacognitive skills that have been previously learned or experienced. Therefore, students need to learn numerous strategies in order to be able to implement appropriate ones in unique situations.

Brown and Palincsar (1982), while working with students with learning disabilities, described in more detail some of the characteristics necessary in cognitive-metacognitive strategy instruction:

> Ideal cognitive skills training programs would include practice in the specific task appropriate strategies (skills training), direct instruction in the orchestration, overseeing and monitoring of these skills (self-regulation training) and information concerning the significance of those activities and their range of utility (awareness training). The level of intervention needed will depend critically on the pre-existing knowledge and experience of the learner and the complexity of the procedures being taught. (p. 31)

The description above applies to almost any successful training program, not just for students with learning disabilities. Kher and Burrill (2005) cited that an additional aspect of strategy instruction should come from the teacher's experiences. Experiences, ranging from classroom occurrences to professional development opportunities, influence a

teacher's perspective and can give teachers insight into different teaching approaches to be used in the classroom. From another study, Dahl's (2004) CULTIS model is a general approach to encompass multiple theories and apply to a wide variety of teachers' instructional methods and students' learning styles. Therefore, Dahl's intent was for CULTIS to be applied to a multitude of arenas in the hope to further the cause of cognitive development.

Along similar lines to the CSI method described previously, CGI incorporates student and teacher thinking and belief structures in relation to mathematics (Carpenter & Fennema, 1991; Carpenter et al., 2000; Clarke, 1997; Fennema et al., 1996; Loeber, 2008). While teachers possess generally accurate interpretations regarding the skills necessary to solve particular mathematics problems and many of the strategies that students utilize, Carpenter et al. explained that:

> Most teachers' understanding of problems and strategies is not well connected and most do not appreciate the critical role that Modeling and Counting strategies play in children's thinking or understand that more than a few students are capable of using more sophisticated strategies. (p. 4)

As such, the CGI program aims to improve student problem solving by preparing teachers to be more sensitive to students' thinking needs. Loeber explained that the intent of the program is to train teachers in the skill of adjusting lessons, specifically lessons involving problem solving through exploration, to maximize student learning. Because students have different experiences and background knowledge related to problems, there is not one model or tailor made procedure for accomplishing an authentic problem.

Kramarski and Mevarech (2003) developed a program that includes "*I*ntroducing the new concepts, *M*etacognitive questioning, *P*racticing, *R*eviewing and reducing difficulties, *O*btaining mastery, *V*erification, and *E*nrichment" (IMPROVE) (p. 283). The IMPROVE program aimed at enhancing student metacognitive ability (Kramarski & Gutman, 2006; Mevarech, 1999). Mevarech reported that the IMPROVE program, in most cases, led to better student performance in mathematics, as well as an improvement in student justification. In conjunction with cooperative learning, the IMPROVE program enhanced student understanding because the small group interactions were guided by metacognitive monitoring. Kramarski and Gutman found that the IMPROVE methods were also beneficial in an E-learning environment. Students utilized the strategy of self-questioning both to stay on task and gain a deeper understanding of the concepts taught. In addition to the IMPROVE program, Kramarski and Mevarech expanded the scope to four different adaptations. The groups in the study included individualized learning with no metacognitive training, cooperative learning with no metacognitive training, individualized learning with metacognitive training (IND+META), and cooperative learning with metacognitive training (COOP+META). Results focused, at least in part, on students' fluency and flexibility in explaining solutions. Students in the COOP+META condition far surpassed students participating in the other conditions by demonstrating much greater fluency and flexibility in mathematical explanations.

Kayashima et al. (2004) and Fogarty and McTighe (1993) echoed the sentiment that collaboration is an appropriate complement to metacognitive training. In actuality, Kayashima et al. suggested that, as one student explains a solution for solving a problem, the remaining members of the group should monitor the solver. Therefore, the group can

recommend ways to improve the solution method while also honing personal

metacognitive skills for future situations that are completed individually. Fogarty and

McTighe added reflection as an important aspect in the metacognitive approach. Costa

(1984) described 12 different strategies for teaching students to become more

metacognitively aware. Such strategies included having students create questions while

reading or labeling students' metacognitive behaviors when observed. Costa also

emphasized the importance of modeling thinking for students to become aware of how

thoughts are created and how thinking can be monitored through metacognition.

Ironically, in younger children just learning metacognitive skills, knowledge of

multiplication can be superseded by previous experiences. Siegler (1988) explained that

even when "children possess extensive metacognitive knowledge, many of their strategy

choices fall out from efforts to retrieve answers, rather than through reference to that

metacognitive knowledge" (p. 272). Similarly, Carroll (1996) found that students would

sometimes give answers that were wholly unreasonable. When unreasonable answers

were given, students ignored metacognitive processes such as evaluation of solutions or

reflection on the solving method. Even when asked to clarify how a method was used,

students sometimes chose another strategy while explaining. In short, the ability to

evaluate appropriate strategies and the solutions the strategies yield is a complex process,

and there is no singular method or procedure that simplifies the process for teachers to

instruct or students to learn.

Besides the previously described strategies, other researchers identified methods

to improve students' cognitive-metacognitive abilities (Mevarech & Kramarski, 2003;

Patry, 2004; Pinon, 2000; Razo, 2001). Both Pinon and Razo explained how

representation was a strategy that could aid in improving the problem-solving abilities of students. Razo found that models, either physical or virtual, aided students in better understanding the problem. Virtual models were especially useful when representing objects "that would otherwise be costly, inaccessible, unsafe or simply inconvenient" (p. 68). Pinon explained that representations allow students to connect tangible, real life models of situations to previously limited conceptual knowledge. In short, representations helped students make mathematical connections. Patry identified concept mapping as a strategy that allowed students to delve deeper into understanding content knowledge while also honing metacognitive skills related to connections within previously attained knowledge. Moreover, students needed to utilize metacognitive skills in the form of understanding the new concepts as the map was planned and evaluating the map after its creation. Finally, Mevarech and Kramarski researched the effectiveness of teachers using worked-out problems versus metacognitive training. When teachers utilized worked-out problems, students were instructed to solve problems following the method shown in the examples whereas in metacognitive training, students learned a variety of solution methods from which to choose. The major limitation to the worked-out example method was that students really only learned the method shown in the worked-out example, so any innovative problem type would generally pose too great a challenge to students.

<div align="center">Attitudes and Perceptions</div>

As previously mentioned, attitudes and perceptions of both students and teachers influence the learning process. Attitudes and perceptions are directly influenced by beliefs. As such, Schoenfeld (1992) delved deeply into the interactions that occur between teachers and students, as well as studying the underlying belief structures that

influenced teacher-student interactions. Schoenfeld explained that students' beliefs about mathematics are strongly impacted by classroom experiences, and many negative experiences leave them with a skewed view of the nature of mathematics. While teachers use the terms problem solving and critical thinking, most of the work and assignments given are simply exercises requiring direct application of algorithms or procedures. A conceptual understanding of mathematics is generally lacking. The problem with most classrooms, however, can stem from different problems. First, less cognitively challenging curricula are often imposed on teachers due to the pressures of achieving certain benchmarks in schools. Even though strategies aimed at improving students' thought processes and metacognitive abilities would likely improve standardized test scores, thereby achieving the benchmarks required, the cost of change is greater than maintaining the status quo. In some cases, the reason for a lack of genuine problem solving is the teacher. A teacher's beliefs regarding mathematics and problem solving greatly influences how the topics are addressed in class. If teachers view problem solving as an enrichment activity, it will be banished to the bottoms of worksheets or utilized when the regular lesson has not occupied an entire class period. On the other hand, if a teacher sees problem solving as integral to the process of learning mathematics, it is more likely that nonroutine, authentic problems will lead class discussion and encourage learning. Interestingly, the beliefs that teachers have regarding mathematics have been traced to past learning experiences in school. The self-perpetuating cycle is hard to break.

Peterson, Fennema, Carpenter, and Loef (1989) studied mathematics teachers and the impact of teachers' beliefs. Peterson et al. outlined four assumptions regarding the learning of mathematics. First, students construct knowledge by making connections.

Next, teachers should instruct in a way that accounts for the construction of new knowledge and connections to previous knowledge. Furthermore, the learning of mathematics should be planned so as to be incremental and to maximize understanding. Finally, mathematics should not be isolated from problem solving. When teachers' scores on a belief questionnaire were compared to students' ability to solve problems, a positive correlation was found. Peterson et al. suggested that, despite the fact that the study concerned only first-grade teachers, the methods used and theories serving as a basis would be transferable to future research. The importance of the study was that "teachers' pedagogical content beliefs and their pedagogical content knowledge seem to be interrelated" (p. 38).

As teacher beliefs are so vital to the learning that takes place in the classroom, numerous studies have focused on studying teacher beliefs (Capraro, 2001; Kher & Burrill, 2005; Webb et al., 2006). Webb et al. explained that changing instruction first requires a change in what teachers believe about how students learn and effective teaching strategies. Kher and Burrill went further by adapting previously created questionnaires in an attempt to quantify teachers' beliefs better. The adapted instruments created sought to uncover constructivist tendencies of teachers, beliefs about the ways in which students learn, and comfort levels with mathematical content. Kher and Burrill agreed with Webb et al. by stating that "teacher beliefs play a role in how teachers present material to students, how willing and/or able teachers are in implementing new content or pedagogy, and in how effective teachers perceive themselves in helping students of different ability levels" (p. 9). Kher and Burrill adapted one of Capraro's instruments. Capraro reduced a previously created scale created by Fennema et al.

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(1987). The purpose of the reduction aided researchers "by (a) shortening the time it takes to administer the scale, (b) removing seemingly redundant items, and (c) focusing on specific constructs contained within the instrument" (p. 13).

Researchers have also begun to study the beliefs held by pre-service mathematics teachers (Enochs et al., 2000; Minor, Onwuegbuzie, Witcher, & James, 2002; Patton et al., 2008). As Capraro (2001) above, Enochs et al. adapted a previous scale while creating the Mathematics Teaching Efficacy Belief Instrument (MTEBI) focusing specifically on the beliefs of pre-service teachers. The purpose of the instrument relied upon the need to understand the beliefs of teachers and teacher candidates in order to implement classroom changes more effectively. Patton et al. cited that teacher beliefs varied between teacher candidates and expert teachers. Additionally, teacher candidates demonstrated a lower level of metacognitive understanding and ability. Patton et al. surmised from the teacher candidate responses and explanations that pre-service teachers held naive beliefs regarding the nature of mathematics, focusing on rote memory and procedural knowledge rather than deep conceptual understanding with an ability to apply what has been learned. Misconceptions must be corrected, and nurturing a deeper awareness of metacognition as it relates to solving mathematics problems along with conveying the importance of explicit instruction of metacognitive strategies should be an integral component of teacher candidate training. Minor et al. quantified some of the characteristics that pre-service teachers deemed as important for success in the classroom. Teacher candidates identified seven important themes associated with effective teaching: "student centered, effective classroom and behavior manager, competent instructor, ethical, enthusiastic about teaching, knowledgeable about subject, and professional" (p.

119). An ancillary benefit of the study was having the teacher candidates reflect upon teaching characteristics, a necessary yet often underdeveloped skill in newer teachers.

The specific impacts of teacher belief on student achievement have been of increasing interest to researchers (Battista, 1999; Beckman, 1970; Ernest, 1988; Loeber, 2008; Pittman, 2002). Beckman stated that, not only do a teacher's beliefs about learning and teaching mathematics affect efficacy, but also a teacher's expectations of individual students. When teachers viewed students as having low motivation or lower achievement capabilities, the teachers' actions in the classroom mirrored the belief, regardless of its accuracy. The same was true for teachers who viewed students as high achieving. Therefore, teachers must be careful not to form negative impressions of students to avoid inadvertent self-fulfilling beliefs. Battista, as cited in Loeber, explained how teachers with misconceptions or incorrect beliefs regarding mathematics can be damaging to student learning. The issue with misconceptions is that teachers can sabotage the learning process by attempting to ease difficulties for the students. Ironically, struggling with authentic and challenging problems is precisely how students learn. By removing any obstacles, teachers reduce worthwhile problems to repetitious drill and straightforward use of algorithms. In another study, Pittman found that teachers who taught in a primarily traditional fashion generally held traditional beliefs about mathematics. Interestingly, some of the teachers studied "had teaching practices that were more nontraditional than their beliefs about the nature of mathematics and mathematics pedagogy" (p. 182). In other words, teachers can change teaching methods prior to or without regard for mathematical beliefs.

In another instance, Ernest (1988) described how a teacher's beliefs are deeply

rooted philosophies, and beliefs are often not created consciously but formed

unconsciously throughout years of learning in school. Other restrictions on a teacher's

beliefs include the assumed teacher role of instructor, explainer, or facilitator and the role

of provided curriculum by the school. Teachers in the instructor role emphasize skill

acquisition and performance, whereas facilitators provide exploratory problem-solving

opportunities. With respect to curriculum, some teachers view textbooks as the singular

source of material, others focus on the text with supplemental materials when needed,

and others view the text only as a tool to be used when the contents match the individual

curricular goals of the classroom or school. Both the teacher type and curricular view

held by teachers can dramatically impact the type of learning that occurs in the

classroom.

Although similar to belief structures, the perceptions and conceptions held by

teachers also influence teaching methods (Clarke, 1997; Munby, 1982; Thompson, 1985).

Thompson explained that perceptions are impacted by conceptions, and conceptions

represent individual schema for interpreting experiences. Thompson mentioned how

more research needs to be done in determining the stability of conceptual patterns over

time. Programs designed to influence teachers' perceptions can only be successful if

perceptions cannot only be changed but if changes to perception remain long after the

intervention. When teachers teach, actions are often influenced by underlying

conceptions. Interestingly, though, teachers' actions can be based upon a mixture of

conscious and unconscious thoughts. The difficulty in determining teachers' conceptions

and perceptions lies in the fact that conceptions and perceptions are psychologically

based and can only be accessed by observation, which can be influenced by other factors, and subject response, which can be affected by bias, untruths, or inability to verbalize thoughts and beliefs. Regardless, teachers' psychological beliefs and constructs can impact instruction. Munby created a cycle of issues influencing teachers and the instructional methods used. At the heart of the cycle were the characteristics and cognitive processes of teachers. Similar to Thompson's findings, beliefs, conceptions, judgments, and expectations all impacted the teaching cycle. Although Munby did not research the impact of teacher characteristics on students, Clarke studied the effect of teachers' perceptions on students, at least with teachers who were reconceptualizing roles by changing from instructors to facilitators. Clarke cited that changes could occur, but teachers needed support in the form of professional development.

In addition to teachers' attitudes and perceptions, students' attitudes and perceptions impact the mathematics classroom (Mullis et al., 2005; Schoenfeld & Herrmann, 1982). Mullis et al. explained that student attitudes, often influenced by value of education at home, can "contribute heavily to student learning and achievement" (p. 81). The TIMSS program included research of attitudes of both teachers and students nationally and abroad. Mullis et al. stated that student attitudes are influenced by enjoyment of mathematics, self-confidence, and motivation. In turn, the factors, and attitude itself, impact student achievement. Schoenfeld and Herrmann examined the perceptions of mathematics held by students in a similar vein to Battista (1999) and Ernest (1988). Schoenfeld and Herrmann explained that students with correct mathematical perceptions and problem interpretation were more likely to be able to solve problems relying upon previous knowledge. Students lacking mathematical perception

were often guessing at solution methods without discretion for nuances of the problem.

Results of the study suggested that as students gain additional knowledge about

mathematics and mathematical solving strategies, the perceptions held became more

accurate. As Schoenfeld and Herrmann summarized, "students' problem perceptions

change as the students acquire problem-solving expertise. Not only their performance,

but their perceptions, become more like experts'" (p. 491).

Unique People, Unique Perspectives

In relation to the literature regarding cognition and metacognition, the strategies

related to them, and the role of attitudes and perceptions, there are numerous articles

tangential to the eminent and seminal works in the field. Starting from the beginning,

cognition is not a concept strictly confined to the field of education. Hutchins (1995)

likened the notion of distributed cognition to the cockpit of a plane and the interworkings

of the flight crew. Individually, no one can fly the plane or even possesses the knowledge

to instruct an entire crew simultaneously. Together, however, the many parts incorporate

numerous specialties into a singular outcome, the successful flight of an airplane. In

relation to education, the classroom is a place where teacher and students alike should

share individual expertise to enlighten everyone and further the cause of learning.

Stevens (1999) approached the idea of cognition from a very different perspective. Real

problem solving rarely has a tidy answer or takes place in the scope of a classroom, rather

Stevens preferred to think of "cognition in the wild" (p. 274). True learning through the

use of thinking through a problem simply emerges under authentic conditions.

As stated earlier, Fogarty and McTighe (1993) proposed a model of thinking, the

three-story model, where each story added a layer of thought process and understanding.

Maqsud (1998) and Hart and Martin (2008) both approached the thinking and learning process from unique perspectives. Maqsud studied the effects of metacognitive training on low achieving mathematics students in South Africa. Results showed significantly higher scores, and Maqsud cited improved student attitudes, likely due to encouragement during the training. Hart and Martin, on the other hand, studied the need for and content of high school mathematics standards. An interesting discovery of Hart and Martin's study was the content of standards from Singapore's Ministry of Education (2006). Singapore's mathematics framework not only keeps problem solving literally at its core, but also includes metacognition as one of the five surrounding components. Not surprisingly, another of the five components is attitudes, including beliefs, interest, and confidence.

Although this study focused on the use of metacognition of high school students and teachers, a number of studies focused on elementary and middle school students of diverse backgrounds (Hoard, 2005; Muniz-Swicegood, 1994; Teong, 2003). Hoard studied gifted children in the first, third, and fifth grades. On tasks requiring skills such as rapid automatized naming, gifted children demonstrated a superior working memory and greater mathematical cognitive ability. Additionally, gifted children were more adept at selecting strategies applicable to the tasks given. In another study, third grade bilingual students learned metacognitive strategies related to reading skills (Muniz-Swicegood). Students started to exhibit metacognitive skills based upon the responses given to questions and even self-directed questioning that occurred. Finally, Teong studied the effects of learning metacognitive skills on low achieving Singapore students. In the computer-based class, students learned a number of strategies that encouraged

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metacognition. Even after metacognitive training, students in the study continued to make mistakes, sometimes even after demonstrating metacognitive awareness in the form of self-monitoring. Therefore, the goal of metacognitive training should be to teach students a cadre of strategies for various situations but not expect immediate or complete success in all instances.

Teaching and learning from the cognitive-metacognitive perspective has additional factors in urban settings (Espinosa & Laffey, 2003; Galosy, 2006). Galosy cited the fact that urban schools can have difficulty obtaining science teachers who are adequately prepared to teach science. Galosy's study reviewed the curricular goals of science teachers in an urban high school district, including the level of "cognitive demand" (p. 13) required by the teacher. In most cases, because the teachers were inexperienced, the decisions depended greatly upon the resources available, such as colleagues, textbooks, or standards. In turn, the teachers sought resources based upon comfort with the concepts to be taught and personal knowledge levels. The teachers' self-perceptions impacted instruction. Espinosa and Laffey also studied perceptions in urban schools. Teachers' perceptions of students, rather than of themselves, influenced instructional methods utilized. In the study, teachers rated many students as problematic, and students labeled as such were also considered to be of a lower academic ability, which was not supported by the data. The problem was exacerbated by the negative feedback given to the students. Additionally, cultural differences could have played a part in the study, with none of the teachers having received any diversity training.

Minor et al. (2002) cited the discrepancies between beliefs of pre-service teachers from different cultural backgrounds. Minor et al. found that minority candidates were

more likely to rank enthusiasm over content knowledge as an important teaching skill. The rationale for the difference was that more of the minority teachers would likely teach in urban schools "characterized by large proportions of minority students, high student failure rates, and low academic motivation and self-esteem" (p. 123). In that situation, enthusiasm is important to raise student morale. Another factor in learning was that of gender. Women were much more likely to rate enthusiasm higher than content knowledge. In fact, Minor et al. explained that schools are still more male-centered and not proactive in establishing a more balanced environment for males and females. Smith (2000) agreed that gender differences in schools persist and also explained the importance of confidence, both in relation to the ability to complete mental computations.

Motivation, similar to confidence, can greatly influence students' willingness and ability to do mathematics (Mayer, 1998; McDonald & Hannafin, 2003). As cited previously, Mayer described the three aspects of student motivation in mathematics. Without a motivation to learn, metacognition and wonderful instruction have no effect. Because motivation plays such a large part in learning, McDonald and Hannafin utilized web-based computer games while studying students reviewing social studies concepts. The use of web-based computer games greatly increased student motivation because that type of review more closely mirrored the types of activities to which children played at home for enjoyment. Because the games used asked progressively more difficult questions, and students spent time conversing about the games, the games served as a cognitive tool sparking in-depth conversations about the concepts included. The role of motivation cannot be discounted in the learning process.

Finally, Mevarech and Kramarski (2003) focused on the use of metacognitive training versus the use of worked out mathematics examples. The long-term effects of the two strategies were also studied. The students participating were tested over two academic years via a pretest, a posttest, and a delayed posttest. The results showed that students performed better having had metacognitive training in addition to the ability to explain mathematical solutions better. All of the included students were working in collaborative settings. The use of worked out examples is particularly important to this study in which numerous participant teachers utilize worked out examples as a primary instructional method.

<center>The Impact of External Forces</center>

Education does not take place within a vacuum. Regardless of research findings and previous literature, there are numerous external forces that impact the lives of students and teachers. As far back as the 1950s, with the launching of *Sputnik I*, education, specifically in the field of mathematics, has undergone transformations as responses to external stimuli in an attempt to improve American students (Launius, 1994; Lindee, 2007). Later, additional international comparisons shed light on the lack of achievement of youth in the U.S. (Gonzales et al., 2004; National Commission on Excellence in Education, 1983). In more recent years, education became a more mainstream topic due to the implementation of the No Child Left Behind (NCLB) Act (2002). A renewed emphasis on reading and mathematics are hallmarks of recent educational reform policy, with serious consequences if the pre-specified goals are not met.

The short historical account above paints a hugely negative picture of the role of external forces on mathematics education; however, many external forces have positively influenced mathematics teaching and, therefore, student learning. The NCTM (1989), in an attempt to unify the field of mathematics education, proposed the *Curriculum and Evaluation Standards for School Mathematics*. The NCTM's 1989 standards represented a collective vision for mathematics educators and other policymakers, as well as suggestions for a common mathematics curriculum. The NCTM (2000) later published the *Principles and Standards for School Mathematics* as an update to the previous document. The second set of standards and principles added additional focus to the need for technology and student mathematical literacy. Together, the NCTM's two landmark documents greatly influenced mathematics education in the United States. De Leon (2003) cited the report response to *A Nation at Risk* by a Carnegie Corporation task force, *A Nation Prepared: Teachers for the 21st Century* (1986). The report, rather than chastising our educational system much like the original, pointed out the need for educational reform instead. Because America's ability to compete in a global market relies upon our youth, teachers stand at the front lines in preparing students for the future. The authors of the 1986 report also recommended that teachers earn bachelor's degrees with "a broad base of knowledge as well as a specialty knowledge of the subject they teach" (p. 5). Another proposal included the formation of what would later become the National Board for Professional Teaching Standards. In short, both the NCTM and the Carnegie Corporation task force influenced the advancement of mathematics education in a much more positive manner than previous incidents.

More recently, the enactment of the NCLB Act (2002) has had major impacts on

U.S. education. With lofty goals, all students are to "make the grade on state-defined

education standards by the end of the 2013-14 school year" (United States Department of

Education, 2004). As the deadline approaches, schools are required to meet certain

benchmarks, or adequate yearly progress (AYP). Schools not achieving AYP for two

consecutive years are labeled as needing improvement, and assistance is given in the

hopes of changing student achievement. The accountability measures used ensure that no

schools, and therefore, no students, are left behind. Hoerandner and Lemke (2006)

explained that, due to the testing associated with NCLB, schools have greatly increased

the focus on mathematics and reading skills in the curriculum. Although focusing on

reading and mathematics in and of itself is not detrimental to students, it has often been at

the expense of art, music, and other subjects not included in state testing. Therefore,

students are not learning as broad a curriculum as in the past. Dillon (2006) cited similar

findings, also stating that students become bored when instruction is almost solely

mathematics and reading. At one California school, students have mathematics, reading,

and gym for five out of six class periods. In such situations, it is difficult to fault schools

for focusing on the exact areas in which students, teachers, and schools will ultimately be

judged through standardized tests.

As a nation, NCLB has had a dramatic effect on education. Looking

internationally, for years America has felt the pressure to compete with other developed

nations (Gonzales et al., 2004; Mullis et al., 2005). Since the launch of *Sputnik I*,

Americans have felt inferior and in need of catching up with other nations. The *TIMSS*

reported mathematics and science achievement of students on an extensive, international

scale at specified intervals (Gonzales et al., Mullis et al.). Information as diverse as test scores, curriculum models, and teacher attitude responses were collected to make comparisons. The results of the numerous *TIMSS* events, especially the most recent installment, will aid in guiding nations while improving education in mathematics and science.

Other researchers have reviewed the differences in the mathematics standards of foreign nations, as well as the variety that exists within the United States (Hart & Martin, 2008; Massell, 2000; Mervis, 2006). Hart and Martin cited the mathematics framework of Singapore as including both problem solving and metacognition as integral components. Although Singapore is a much smaller nation than the U.S., a nationally identified set of standards, much like that proposed by the NCTM, might have a unifying effect on the education of mathematics. Similarly, Mervis explained how China's national curriculum focuses on very few topics, yet the depth of learning is greater than here. Teachers are also trained to implement the national curriculum. Mervis also explained how, in the United States, there are numerous competing forces with stakes in mathematics standards. The NCTM, graphing calculator companies such as Texas Instruments, and industries relying upon graduates with mathematical backgrounds all have competing interests in what and how mathematics is taught. Massell added that politicians can often impact the adoption of standards. Mathematics content standards documents that are created are often either too vague or too specific to be of use to most educators. Additionally, adoption of standards can pose difficulties based upon ideology. For example, California standards were heavily influenced by innovation whereas the NCTM's standards, without governmental support, were developed through consensus.

Finally, the implementation of metacognitive strategies is threatened by other challenges (Henningsen & Stein, 1997; Kher & Burrill, 2005; Wilson & Clarke, 2002). Kher and Burrill explained that:

Schools function differently from firms and businesses, specifically when dealing with innovations of technology and teaching practices. Reforms and changes in schools require more than simply voting to adopt new techniques and practices. The success of the innovation's implementation depends upon the existing resources of the schools and the perceptions of efficacy of the changes. Social pressures, experience or knowledge of the innovation, and how teachers perceive his or her affiliation with the school all help determine the ultimate success of any innovation in a school. (p. 12)

As such, schools must plan ahead and garner support for any initiatives involving teachers and incorporating new teaching strategies. Specific to the incorporation of metacognition is the need for the concept of metacognition to become legitimized (Wilson & Clarke). Policymakers, including teachers and administrators, must value the use of genuine problem solving in the classroom and provide resources for its successful execution. Additionally, Henningsen and Stein cited that schools must be vigilant in preventing such programs from devolving into lower cognitive activities. The main responsibility lies with the teacher to avoid oversimplifying rich, multi-faceted problems and allow students to struggle through to experience authentic learning of mathematics.

Conclusion

The major research cited in this chapter supported the inclusion of cognitive and metacognitive strategies in mathematics classrooms. As metacognition is practically in its

infancy, there continue to be numerous studies relating to various facets of its meaning. Additionally, researchers have proposed a variety of frameworks in an attempt to clarify the roles of cognition and metacognition in the teaching and learning process. Although debates still continue as to the specific definitions of both cognition and metacognition, there is growing agreement as to the need for students to learn about thinking and mathematics teachers to teach cognitive-metacognitive strategies in school. The influence of external factors, such as international comparisons, national organization's recommendations, and governmental policy, have yet to be seen as detrimental or beneficial to the educational system.

From another perspective, researchers have focused on the impacts of attitudes and perceptions on student learning. Relatively little research, however, has focused on both teachers' attitudes and perceptions in conjunction with teaching cognitive-metacognitive strategies. Therefore, this study sought to reveal the attitudes and perceptions teachers have regarding students' cognitive-metacognitive abilities and the impact of the strategies used to teach cognitive-metacognitive skills in the classroom. In the next chapter, the researcher will explain the methodology utilized in the study.

CHAPTER III

METHODOLOGY

Introduction

The purpose of the study was to identify the attitudes and perceptions held by select teachers in a Midwest high school regarding teaching strategies related to students solving mathematics problems from a cognitive-metacognitive approach. The key difference between the present study and previous research was examining the role teachers' attitudes and perceptions play in the teaching of cognitive-metacognitive mathematics strategies. The unique difference greatly influenced both the research design and analytical methods performed. This chapter begins with a description of the research design and population. Then, the procedures utilized for data collection are outlined. Finally, the analytical methods and limitations are delineated. The study was guided by the following research question: What strategies are perceived by mathematics teachers in a Midwest high school as best at fostering students' cognitive-metacognitive skills in solving select mathematics problems?

The research question was divided into two sub-questions:

1. What attitudes and perceptions do these teachers have regarding students' cognitive-metacognitive abilities?

2. How do these attitudes and perceptions influence the strategies utilized by these teachers to teach cognitive-metacognitive skills?

Research Design

Due to the emphasis on teachers' attitudes and perceptions, as well as the strategies and approaches teachers used, a two-pronged approach best suited the research. A mixed-methods ethnographic case study was the utilized methodology. The rationale for the case study format was because, as Merriam (1998) described it, "a case study design is employed to gain an in-depth understanding of the situation and meaning for those involved" (p. 19). The case study was ethnographic in nature based on the need to learn about the culture, perspectives, attitudes, and behaviors of the participants (Gay et al., 2006; Merriam; Robson, 2002). As a teacher at the school, the researcher was an active participant observer immersed in the school's culture (Gay et al.). By using a mixed-methods approach, the researcher was able to gain greater insight through the use of quantitative and qualitative approaches. The first phase consisted of distributing a questionnaire greatly influenced by the work previously done by Kher and Burrill (2005), and the second phase consisted of semi-structured interviews to gain a deeper understanding of the interconnectedness of teachers' attitudes, perceptions, and mathematics teaching strategies.

The first phase of the study involved the completion of a questionnaire consisting of three sections. An instrument created by the Center for Research in Mathematics and Science Education at Michigan State University was adapted in this questionnaire's development. Because the researcher adapted the questionnaire from copyrighted material with permission from Neelam Kher at Michigan State University (see Appendix A), the sections are not included in the appendix. The original instrument was created to measure teacher leader attributes as part of an NSF-supported evaluation study of a

Mathematics and Science Partnership (MSP) Institute. Teachers from the "Park City Mathematics Institute (PCMI)'s Math-Science Partnership Project" (Kher & Burrill, 2005, p. 1) were participants with the original instrument. The sections consisted of Instructional Practices of Teachers, Beliefs and Opinions, and Teacher Background Characteristics. The second of these sections consisted of two subsections: Beliefs Regarding Mathematics and Mathematics Teaching and Learning. No criteria validity was cited.

The Mathematics Beliefs Scales of Fennema et al. (1987) and Capraro (2001), who later reduced the size of the instrument, were adapted by Kher and Burrill (2005) in the creation of the Beliefs Regarding Mathematics instrument. The original 48-item versions of the Mathematics Beliefs Scales instrument used by Fennema et al. and Capraro had coefficient-alpha reliabilities of .93 and .78, respectively. The abbreviated version utilized by Kher and Burrill had an overall coefficient-alpha reliability of .80. Further, differences between PCMI and non-PCMI teachers on responses from the Beliefs Regarding Mathematics instrument via MANOVA found an overall statistical significance in validity with $p < .0009$ (Kher & Burrill). No previous data were reported for the results associated with the Instructional Practices of Teachers section.

The Mathematics Teaching and Learning subsection was adapted from the Mathematics Teaching Efficacy Belief Instrument (MTEBI) created by Enochs et al. (2000). The original instrument was subdivided into the Mathematics Teaching Outcome Expectancy subscale (MTOE) and Personal Mathematics Outcome Expectancy subscale (PMTE). Coefficient-alpha reliability of .77 for the MTOE and .88 for the PMTE were found (Enochs et al.). No overall value was calculated. Kher and Burrill (2005) found

alpha coefficients of .72 for the MTOE and .63 for the PMTE with an overall value of .66

for the instrument in its entirety. Kher and Burrill completed factor analyses to explore

the possible differences between the two groups' results. Kher and Burrill's conclusion

pointed toward different beliefs between "teachers who self-selected participation" and

"teachers who are non-participants" (p. 9). On the instrument, PCMI teacher participants

had higher scores that were statistically significant. The PMTE subsection of the

Mathematics Teaching and Learning instrument yielded statistical significance at the $p <$

.025 level using MANOVA in comparing PCMI and non-PCMI teacher responses. No

statistical significance was found on the MTOE subsection, however.

The first sub-question of the study was:

1. What attitudes and perceptions do these teachers have regarding students'

cognitive-metacognitive abilities?

The first sub-question was addressed through the use of both the questionnaire and

follow-up interviews. The questionnaire phase was conducted from April to June of 2009.

As respondents completed questionnaires, the researcher collected and organized the

data. Information regarding beliefs and opinions was obtained through the use of the

questionnaire, as well as background and demographic information. The results from the

questionnaire formed a foundation for studying the attitudes and perceptions held by the

teachers. The interview phase was scheduled from April to June of 2009, but one

additional interview was conducted in October 2009 due to conflicting schedules.

Purposive sampling was utilized to select previous respondents with unique perspectives

or backgrounds as established by questionnaire responses for the semi-structured

interviews (Leedy & Ormrod, 2005; Merriam, 1998). The prepared questions in the

interviews concerned knowledge of cognitive-metacognitive techniques, beliefs about mathematics instruction, and perceptions regarding students and the school. Interviews were then reviewed to create a coding system of analysis (Leedy & Ormrod; Robson, 2002). The researcher reviewed the interview data for any emerging patterns or relationships.

The second sub-question of the study was:

2. How do these attitudes and perceptions influence the strategies utilized by these teachers to teach cognitive-metacognitive skills?

The second sub-question was addressed through both the questionnaire and interview phases. The questionnaire results allowed the researcher to review instructional practices favored by teachers in the school and the emphasis on those strategies fostering cognitive-metacognitive skills. The interviews further explored the thought-processes and strategies utilized by teachers. Again, the coding strategy cited previously was applied to identify recurring themes in responses.

Population

All twenty-six high school teachers working in mathematics and special education mathematics classrooms in the school, as well as two student teachers placed at the school, were invited to participate in the first, quantitative phase of the study. Of the 28 invitees, 18 elected to participate and returned the questionnaire. In the sample, six of the participants were male and 12 were female, with eight males and 20 females in the original population. The race/ethnicity of all participants in the sample was Caucasian/white. The years of teaching experience at their current school ($M = 8.67$, $SD = 7.24$) and years of teaching experience at previous and current schools ($M = 10.94$, SD

= 8.59) were calculated for the questionnaire participants. Eight of the respondents to the initial questionnaire were invited to participate in the second phase interviews. Respondents with unique perspectives or backgrounds as determine by the questionnaire were selected. The eight interviewees consisted of two males and six females. The years of teaching experience at their current school ($M = 9.38$, $SD = 8.26$) and years of teaching experience at previous and current schools ($M = 12$, $SD = 10.36$) were calculated for the interview participants.

<div align="center">Data Collection</div>

With permission from the school principal (see Appendix B), all mathematics and special education mathematics teachers were given informed consent forms briefly describing the purpose and format of the study. Those wishing to participate were asked to sign and date the consent form (see Appendix C) and were then given a copy of the questionnaire. The questionnaire contained a unique alpha or numeric code for the researcher to identify respondents needed for the interview phase. Participants were asked to complete the questionnaire, which could take up to 30 minutes to complete, and return the questionnaire to the researcher. As participants returned the questionnaires, the data were collected and organized.

The questionnaire consisted of three sections: Instructional Practices, Beliefs and Opinions, and Background. The Instruction Practices section was comprised of four multipart, Likert-style questions. The Beliefs and Opinions part was comprised of two sections with Likert-style questions, Beliefs Regarding Mathematics with 18 questions and Mathematics Teaching and Learning with 14 questions. Finally, the Background section asked basic demographic information such as gender and ethnicity, as well as

years of experience teaching at their current school, total years of experience, attained

degrees and institutions, courses taught this year, and courses taught in previous years.

Participants could also provide written comments at the end of the questionnaire although

no formal prompt was present.

Because of the timing of the study with respect to the school year, some interview

participants were selected prior to the completion of the questionnaire phase. Participants

taking part in the interview phase were asked to sign a second consent form (see

Appendix D). Participants were informed that the interview process could take up to 40

minutes to complete. Interviews were conducted in a school classroom after school hours

and audio recorded. Participants were given a copy of the prepared interview questions

(see Appendix E) during the interview process so that questions could be referred to more

easily. All interviews were conducted before the end of the 2008-2009 school year except

one due to conflicting schedules. The final interview occurred at the beginning of the

following school year. The recordings were transcribed from July to October of 2009.

After all data had been collected and transcribed, participants were given copies of the

signed consent forms.

<center>Analytical Methods</center>

The purpose of this study was to delve more deeply into the relationship existing

between cognitive-metacognitive ability and student learning from the teachers'

perspective. The key components of the study were to learn about the role of the teacher

in this process. As evidenced by the literature review, little research had been done in this

area. Therefore, studying the attitudes and perceptions of teachers required innovative

methods of data analysis. Participants completed the questionnaire in the first phase, and

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some respondents were asked to participate in the interview process of the second phase. The data collected from the questionnaire were analyzed differently by section.

Data from the Instructional Practices section were analyzed via chi-square tests to find differences between actual responses and theoretical ones. The probabilities for responses in the first section were measured for significance at $p < .05$. Percentages and frequencies for the responses given were also calculated. The results from the analysis guided the researcher in grasping the instructional practices commonly utilized by mathematics teachers in the school.

The two parts of the second section were also analyzed via chi-square tests to find differences between actual and expected responses. Additionally, coefficient-alpha reliability was computed to be compared with values obtained by previous research (Capraro, 2001; Enochs et al., 2000; Fennema et al., 1987; Kher & Burrill, 2005). The possible responses to the questions in the Beliefs Regarding Mathematics and Mathematics Teaching and Learning sections were all Likert-style. The response choices were Strongly Agree, Agree, Disagree, and Strongly Disagree. The alpha level for the chi-square analysis was set at the .05 level. Percentages and frequencies for responses were also computed.

The final section, Background, was analyzed by obtaining frequencies for gender and race/ethnicity. Means and standard deviations for years in teaching and years in present school were calculated. Other demographic information was reviewed but not analyzed quantitatively. The information under Background did greatly influence the purposive sample selection for the interview phase. Teachers were selected with varying

years of teaching experience, courses taught, degrees attained, and gender. The selected interviewees represented a diverse subsection of the original population.

The interview data were analyzed qualitatively by creating a coding system. The data were reviewed to identify emerging themes or recurring patterns that could represent common categories (Merriam, 1998; Robson, 2002). The quantitative questionnaire data were also revisited as the interview responses were coded. The interviewees' questionnaire responses were compared to verbal responses for internal reliability. Also, patterns or inconsistencies were identified. The analyses utilized directly related to the research sub-questions of the study. The questionnaire analysis aided in providing both basic demographic information for comparison as well as initial information regarding teachers' opinions, beliefs, and instructional methods related to students' thinking in mathematics. Then, analysis of the interview responses provided a deeper, richer explanation of the underlying attitudes and perceptions of the selected teachers. Those beliefs were also analyzed in studying responses to interview questions related to mathematics teaching strategies related to cognitive-metacognitive reasoning.

Limitations

The nature of the study did present some limitations that could have affected the results obtained and extrapolated. Some of the limitations were unavoidable due to the nature of the study. As a participant observer and fellow teacher within the same school setting, it was possible that the researcher had unintentional bias. Additionally, as a member of the unique school culture, the researcher may not have been conscious of certain qualities of the data unique to the singular setting. Conversely, the researcher may have been able to obtain data from participants that would have otherwise been

unattainable. The researcher had an established rapport with the teachers in the school as a colleague. Perhaps the previously established relationship also led to participants viewing the researcher as more trustworthy than an outside investigator. Another concern that arose during the data collection process was that participants commented on whether the responses given were what the researcher was looking for. The participants may have attempted to please the researcher by providing particular data in questionnaire or interview responses.

A possible limitation of the study could have been directly tied to the nature of the study. Because teachers were asked to verbalize personal attitudes and perceptions, there existed a minimal risk to the participants. Although the researcher ensured that responses would be kept anonymous and confidential, readers of the study could possibly identify participants ultimately posing a job security risk. The possible risks were clearly outlined in the consent forms. Due to the possible risks, it is possible that some invited teachers declined to participate. Teachers having already attained tenure status, teachers leaving the school at the end of the school year, and student teachers were more apt to participate because of job security or no lasting ties to the school. Non-tenured teachers volunteered to participate at a much lower frequency. Additionally, of those non-tenured teachers who did participate, it was possible that responses were skewed or inaccurate because of possible risks associated with certain responses. Interview participants who were non-tenured may have been hesitant to speak openly about attitudes and perceptions with the possible risks that existed.

Some actual data collection procedures could have limited the data collected. The format of the questionnaire could have affected the data collection. One portion of the

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questionnaire has two part response questions that could have confused some participants. There were two participants who responded to only one part of the two part response questions. The omission could have been voluntary or accidental, in which case the overall results would be somewhat affected. Additionally, the timing within the school year for the questionnaire distribution could have been improved. The questionnaires were distributed and collected from April to June of 2009. It might have been more beneficial to have participants complete the questionnaires during the first semester when there are not distractions such as spring break and state testing days. The interviews occurred in the same months as the questionnaire distribution, and participating in interviews near the end of the academic year could have been distracting. If nothing else, the participants may have been preoccupied with school-related thoughts in completing the school year while being interviewed.

Another possible limitation affecting the data was the delay for the final interview. Because the school year ended before all the interviews could be conducted, the researcher conducted the final interview in October 2009. The mindset at the beginning of the school year in comparison to that at the end of the school year could very likely have been different. Additionally, the familiarity with the teaching strategies implemented throughout the previous school year for that participant had probably diminished over the summer break. The participant's beliefs about students and thinking could have also been distorted by not having the constant daily contact as the other participants had previous to the interview process. For all the participants, the time elapsed between the completion of questionnaires and participation in the interviews

could also have been reduced. Some participants in both phases did not recall responses from the questionnaires when participating in the interviews.

Other interview concerns included the presence of the interview questions and the taping of the interviews. Because interview participants were given a copy of the prepared questions used in the interview, some participants were distracted while answering. Some participants were looking ahead to future questions while on the current question. The advantage to giving the participants the questions, however, was that it was easier to refer to the current question and maintain focus on that question. Taping the interviews also concerned some of the participants. The participants were not hesitant to participate, but it is possible that the responses given were more guarded because of the taping procedure.

On the other hand, some of procedures followed could have been adjusted to maximize the quality of the responses obtained. In retrospect, although participants were asked how many years they had taught at their current school and how many years they had taught at their current and previous schools, participants were not asked about their age. Age could have been an interesting addendum to the currently collected information to gain insight into possible differences not only between teaching experience and perception but also between age and perception. Additionally, there were no open-ended questions in the questionnaire used. Although the questionnaire used was very close to the original version, the addition of qualitative components may have added richness to the data for all participants, especially those not participating in the interview process. The researcher did not include open-ended questions because the questionnaire was shorter and easier for participants to complete in a shorter time span and the interview

contained open-ended questions. Despite this, the data collected via the interviews did accomplish that which was established in the research question and sub-questions, namely, to gather in-depth beliefs and thoughts of classroom teachers.

CHAPTER IV

FINDINGS & CONCLUSIONS

Introduction

The study attempted to ascertain the attitudes and perceptions held by select

teachers in a Midwest high school regarding teaching strategies related to students

solving mathematics problems from a cognitive-metacognitive approach. The chapter

begins with a summary of the research question and methodology utilized. Then, the

findings of both the quantitative and qualitative aspects of the study are presented.

Additionally, conclusions related to the research question and based upon the findings are

drawn. Finally, implications of the results are explained and recommendations for future

study are proposed. The study was guided by the following research question: What

strategies are perceived by mathematics teachers in a Midwest high school as best at

fostering students' cognitive-metacognitive skills in solving select mathematics

problems?

The research question was divided into two sub-questions:

1. What attitudes and perceptions do these teachers have regarding students'

cognitive-metacognitive abilities?

2. How do these attitudes and perceptions influence the strategies utilized by these

teachers to teach cognitive-metacognitive skills?

As established in the literature review, cognitive-metacognitive teaching

strategies can and should be an integral aspect of mathematics education. Unfortunately,

there are numerous frameworks and approaches to implementing thinking strategies. Even the definitions of cognition and metacognition are ardently debated. Despite the contention, agreement exists that emphasis on student thinking can be beneficial. Teachers' attitudes and beliefs about students can directly influence instruction and therefore student cognitive-metacognitive abilities. Therefore, the study sought to reveal the attitudes and perceptions teachers have regarding students' cognitive-metacognitive abilities and the impact of the strategies used to teach cognitive-metacognitive skills in the classroom.

The study was completed through the utilization of a mixed-methods case study approach with the researcher acting as an active participant observer. The initial phase consisted of the distribution and completion of a questionnaire adapted from the Center for Research in Mathematics and Science Education at Michigan State University. The questionnaire consisted of sections related to Instructional Practices of Teachers, Beliefs and Opinions, and Teacher Background Characteristics. Responses from the questionnaire guided the researcher in selecting participants for the second phase. The second phase consisted of semi-structured interviews aimed at gaining deeper information regarding teachers' attitudes, perceptions, and mathematics teaching strategies. Both the questionnaire results and interview responses attempted to answer the research question and sub-questions.

<div align="center">Findings</div>

The findings for the study are presented according to the corresponding research sub-question. The first sub-question related to the attitudes and perceptions held by teachers regarding the cognitive-metacognitive abilities of students. The second sub-

question related to how teachers' attitudes and perceptions influence strategy selection and utilization with respect to teaching students pertinent cognitive-metacognitive skills. The sample participating in the questionnaire phase consisted of 18 of the 26 high school teachers working in mathematics and special education at the school plus two student teachers who were invited. The invited population consisted of eight males and 20 females, with six males and 12 females completing the questionnaire. Eight of the respondents to the initial questionnaire were invited to participate in the second phase interviews. The eight interviewees consisted of two males and six females.

Attitudes and Perceptions Held

The quantitative data related to teachers' attitudes and perceptions are presented in Table 1 and Table 2. Table 1 indicates the frequency and percentages of responses from the questionnaire section Beliefs and Opinions: Beliefs Regarding Mathematics. Additionally, chi-square values were calculated for each item assuming equal expected frequencies. Because the items were Likert-style with response choices Strongly Agree, Agree, Disagree, and Strongly Disagree, the degrees of freedom were set at three. The number of participants responding varied among the items because some items were omitted by participants. Coefficient-alpha reliability was calculated to compare to values obtained in previous studies. The Beliefs Regarding Mathematics instrument was adapted and shortened by Kher and Burrill (2005) from the Mathematics Beliefs Scales of Fennema et al. (1987) and Capraro (2001). Both the initial researchers and Kher and Burrill calculated the coefficient-alpha reliability of the instruments. The original 48-item versions of the Mathematics Beliefs Scales instrument used by Fennema et al. and Capraro had coefficient-alpha reliabilities of .93 and .78, respectively. The abbreviated

Table 1

Responses to Section II Part A: Beliefs and Opinions: Beliefs Regarding Mathematics

Question	Strongly Agree		Agree		Disagree		Strongly Disagree		X^2
	f	%	f	%	f	%	f	%	
1[a]	2	11.8	13	76.5	2	11.8	0	0.0	24.65***
2[b]	3	16.7	7	38.9	7	38.9	1	5.6	6.00
3[a]	2	11.8	9	52.9	5	29.4	1	5.9	9.12*
4[b]	0	0.0	7	38.9	9	50.0	2	11.1	11.78**
5[b]	3	16.7	6	33.3	8	44.4	1	5.6	6.44
6[b]	2	11.1	12	66.7	4	22.2	0	0.0	18.44***
7[a]	2	11.8	11	64.7	4	23.5	0	0.0	16.18**
8[b]	1	5.6	4	22.2	12	66.7	1	5.6	18.00***
9[b]	1	5.6	9	50.0	8	44.4	0	0.0	14.44**
10[b]	2	11.1	6	33.3	10	55.6	0	0.0	13.11**
11[b]	0	0.0	12	66.7	5	27.8	1	5.6	19.78***
12[b]	3	16.7	7	38.9	8	44.4	0	0.0	9.11*
13[b]	1	5.6	6	33.3	10	55.6	1	5.6	12.67**
14[b]	2	11.1	16	88.9	0	0.0	0	0.0	39.78***
15[a]	0	0.0	13	76.5	4	23.5	0	0.0	26.53***
16[b]	3	16.7	12	66.7	3	16.7	0	0.0	18.00***
17[b]	2	11.1	6	33.3	10	55.6	0	0.0	13.11**
18[b]	2	11.1	13	72.2	3	16.7	0	0.0	22.44***

[a]$n = 17$.   [b]$n = 18$.

*$p < .05$.   **$p < .01$.   ***$p < .001$.

version utilized by Kher and Burrill had an overall coefficient-alpha reliability of .80. The shortened, 18-item version was utilized in the present study. The reliability finding was .65.

Table 2 indicates the frequency and percentages of responses from the questionnaire section Beliefs and Opinions: Mathematics Teaching and Learning. Chi-square values were also calculated assuming equal expected frequencies for each item. The degrees of freedom were set at three because the response choices consisted of Strongly Agree, Agree, Disagree, and Strongly Disagree. The sample size varied among items. Most items had a response rate of 17 participants because one respondent did not complete the subsection. Another participant omitted a question, so only 16 responses were recorded. The Mathematics Teaching and Learning subsection was created based upon the original Mathematics Teaching Efficacy Belief Instrument (MTEBI) utilized by Enochs et al. (2000). The original instrument was subdivided into the Mathematics Teaching Outcome Expectancy subscale (MTOE) and Personal Mathematics Outcome Expectancy subscale (PMTE). The subscales were not utilized in the present study but are cited here for purposes of establishing previous reliability values for the subsection of the questionnaire. Enochs et al. found coefficient-alpha reliability of .77 for the MTOE and .88 for the PMTE. No overall value was calculated for the instrument. Kher and Burrill (2005) found alpha coefficients of .72 for the MTOE and .63 for the PMTE with an overall value of .66 for the instrument as a whole. Factor analysis posed a possible rationale for the intergroup differences in responses. Possibilities exist that responses were different between the groups due to self-selection to be participants in the program offered and the related "statistically significant

Table 2

Responses to Section II Part B: Beliefs and Opinions: Mathematics Teaching and

Learning

Question	Strongly Agree		Agree		Disagree		Strongly Disagree		X^2
	f	$\%$	f	$\%$	f	$\%$	f	$\%$	
1[b]	0	0.0	2	11.8	15	88.2	0	0.0	36.88***
2[b]	0	0.0	5	29.4	11	64.7	1	5.9	17.59**
3[a]	0	0.0	0	0.0	11	68.8	5	31.3	20.50***
4[b]	1	5.9	9	52.9	7	41.2	0	0.0	13.82**
5[b]	1	5.9	4	23.5	11	64.7	1	5.9	15.71**
6[b]	11	64.7	6	35.3	0	0.0	0	0.0	19.94***
7[b]	1	5.9	3	17.6	11	64.7	2	11.8	14.77**
8[b]	0	0.0	5	29.4	12	70.6	0	0.0	22.77***
9[b]	0	0.0	10	58.8	7	41.2	0	0.0	18.06***
10[b]	0	0.0	7	41.2	10	58.8	0	0.0	18.06***
11[b]	1	5.9	1	5.9	7	41.2	8	47.1	10.06*
12[b]	15	88.2	2	11.8	0	0.0	0	0.0	36.88***
13[b]	17	100.0	0	0.0	0	0.0	0	0.0	51.00***
14[b]	0	0.0	2	11.8	11	64.7	4	23.5	16.18**

[a]$n = 16$.   [b]$n = 17$.

*$p < .05$.   **$p < .01$.   ***$p < .001$.

higher responses for the total score" (Kher & Burrill, p. 9) in the study. In the present

study, coefficient-alpha reliability was calculated to compare with values obtained in

previous research. Three possible differences existed in the present study as compared to

previous studies. The present study utilized a 14-item subsection related to mathematics

teaching and learning, whereas previous studies utilized 15-item subsections. Again, one respondent did not complete the subsection which affected the calculation. Additionally, for one item, all participants responding selected the same choice, so the question was omitted from the calculation having no variance in answer selections. The reliability value was .65. The value obtained for the present study was close to the value obtained by Kher and Burrill.

The coefficient-alpha reliability was also calculated for the overall Beliefs and Opinions section of the questionnaire. The reliability coefficient obtained for the overall section was .75. No overall reliability was reported for the combined subsections in previous studies. The same possible differences in comparison from above apply to the overall calculation. The overall number of items differed, one item received a univariate response, and respondents omitted certain question or an entire subsection.

With respect to responses on the Beliefs and Opinions: Beliefs Regarding Mathematics subsection as presented in Table 1, the items with the highest level of statistical significance, $p < .001$, were 1, 6, 8, 11, 14, 15, 16, 18. Over two-thirds of the participants selected one particular response choice for each of the items. The items concerned students finding individual methods of solution through self-discovery, persevering in problem solving, learning computational procedures, and listening to explanations given by teachers. The items with the greatest variance in responses were items two and five having no one response represented by more than 45% of the sample. The items concerned the need for explicit instruction in mathematics.

On the Beliefs and Opinions: Mathematics Teaching and Learning subsection as presented in Table 2, responses to items 1, 3, 6, 8, 9, 10, 12 and 13 were statistically

significant, $p < .001$. At least 10 participants selected one particular response choice for each of the items. The items regarded the impact of increased teacher effort, teacher efficacy, teacher mathematical knowledge, the relationship between student interest and teacher performance, the use of manipulatives, and student questions in mathematics classes. None of the items in the subsection showed as great of variance as in the previous subsection, however. Item 11 showed the greatest variance with $n = 17$, X^2 (3) = 10.06, $p < .05$. Item 11 concerned evaluations by the principal. Item 13 showed no variance because all 17 respondents chose the same response, X^2 (3) = 51.00, $p < .001$. Item 13 regarded how participants welcome student questions.

Definitions of mathematics and metacognition. Deeper, more specific responses were gathered through the use of the interview. The first questions asked in the interview included "What is your definition of 'Thinking mathematically'?", "What is your definition of 'thinking about thinking'?", "To what extent, if any, do you teach students to think mathematically?", and "Is there a difference between thinking mathematically and doing mathematics?" Although quite extensive, the answers to the baseline questions served as a foundational lens through which to evaluate responses to the questions that followed.

Responses to the question "What is your definition of 'Thinking mathematically'?" were similar for many participants. Logical thinking and problem solving were common themes in participants' responses. The responses from each participant are listed in Table 3.

Table 3

Responses to Interview Question 1 Part 1: Definitions of Thinking Mathematically

Participant	Description
1	Having a clear cut, like, answer and, like, looking at everything logically in a series of steps not just looking at it and trying to solve it right away. Trying at the whole process behind it and trying to figure out the steps for anything in general. Any problem or anything that would arise.
2	Thinking mathematically to me would be for a student to, um, in a situation, think through the process but thinking through using math concepts, math vocab, um, so whatever problem they come across they can look at it from a math standpoint using whatever the concept is we're talking about in class.
3	My definition of thinking mathematically is being able to look at a problem, any problem, and figure out ways to solve it.
4	Thinking mathematically would be thinking logically, sequentially, more abstract, about how you're going to think logically, and mathematically.
5	I think it's kinda thinking about thinking and metacognition. And thinking about reasoning and utilizing skills to apply them.
6	Thinking mathematically would be not rote memorization, but thinking through the problem – how to set up an equation and solving from there, not, um, not just rote memorization.

(*table continues*)

Table 3 (continued)

Participant	Description
7	Any time that you are thinking in a problem solving mode, it doesn't necessarily have to be with numbers. Um, but basically any time you're thinking through a problem, a situation, you're checking out your alternatives, you're choosing the best strategy, anything like that.
8	Thinking logically, um, in an organized fashion to solve problems.

When participants were asked "What is your definition of 'thinking about thinking'?", however, initial responses were mixed (see Table 4). Participant 5 did not give a separate definition for thinking about thinking because that was encompassed by the definition for thinking mathematically. The participant did not believe that there is a difference between thinking mathematically and thinking about thinking.

Table 4

Responses to Interview Question 1 Part 2: Definitions of Thinking About Thinking

Participant	Description
1	I think that's just kinda just thinking just for the sake of doing so and … see to me that would be like the opposite of thinking and processing. You would just thinking about stuff in general. I don't know that is kind of a weird question.

(*table continues*)

Table 4 (continued)

Participant	Description
2	You'd have to, you have to think about, you know, you sometimes have to take time to think about what you are thinking? Um, where you are basically kind of, um, what's the word, I can't think of the word, um, evaluating, like, your thinking process. Or going though what was I thinking, why do I do this, um, what's the reasoning for the steps here. Or as you kinda go through, think about what it is that you're actually doing or thinking about.
3	I don't really have a definition for that.
4	What do I do to begin the problem is thinking about thinking, I guess, and that I think is the essential first step for all of math.
6	Having the kids think, maybe even sh- think with a partner. So they can actually talk about and discuss out loud what they're thinking because I've found many a times that students perform better when they are talking out loud cause I think they're actually listening to what they're saying and actually rethinking about it.
7	Analyzing how you as a person think through problems. Are, are you a person that has to write things out, are you a person that can think about things and, and look at situations and figure them out, do you have to talk through them, um, and knowing what your strengths and your weaknesses are about how you think as a person.
8	Being able to analyze one's thinking process and make adjustments.

After a few more questions, the investigator gave Participant 3 a definition of thinking about thinking as metacognition or self-monitoring to prompt the participant to ponder the use of that in the classroom. The participant responded that:

> I think that I do. In, I never really thought of it that way. But I think I do in going over, okay, here's what we're going to do today, or here's what we did do today. "Do you understand what I'm saying when I say that? What's an example of a procedure that you do when you do that?" Like, I'm just thinking of factoring now cause you said that. Is that when we would do factoring or we talk about it in words instead of like writing stuff down so they can evaluate. "Do you understand what I'm saying? Do you understand what you, I want you to do?"

After the majority of the interview, participants were asked to clarify any responses given. In particular, Participant 3 continued by explaining what thinking about thinking meant after having responded to the other interview questions. The participant explained how the wording of the question was confusing yet acknowledged the difficulty in addressing or asking the question. The investigator also explained the intent of the item to the participant:

> P: Thinking about thinking, like people could take that so many different ways. … I don't know. Like give the definition like you used the word metacognition. Obviously I've heard that a million times but I don't know like the specific definition of what that is. I'm not sure how you could change it without, without edging somebody towards an answer, you know?
>
> I: Right.

P: So, or maybe you could use, like, instead of thinking about thinking…, like what you're, are you saying like you want, you wanna know how teachers would have students evaluate the process that they use in their brain to determine what it is they want to do? Is that what you're saying?

I: It depends. Um, it could mean just thinking about thinking. There's many different definitions of metacognition. So, um, I pretty much want to know what each teacher's definition is and then pretty much how that influences their other answers, if it does.

P: Oh. Okay. Because like when I think about that then, when I just said like my own little definition I thought about like I always try to make sure that my students like don't, uh, feel like they cannot do things, because I think that blocks their thinking. So if they have low confidence about it, if they go into it they're already, they're already blocked. Because now I see with my lower classes that that really does happen.

The investigator then asked questions related to the extent to which teachers teach students to think mathematically. Most participants replied how it is a continuous and daily goal. Responses often incorporated some discussion of strategies utilized in teaching students to think mathematically. Participant 7 explained how:

I try to get them to ask themselves, "What is the problem asking for? And therefore what, what way can I solve this, what plan of attack do I have, what tools do I have in my toolbox that I can use." Um, all different kinds of strategies to get them to think, about first what the problem's asking about, ways they can

solve it, and then once they do solve it does their answer make sense, do they go

back through the problem and check it.

Similar ideas were expressed by Participant 2 who explained the following:

 I try to make sure that I get in there how you would think through this. How we

should do this. Why do we do this? And I think those kind of things help them

understand this is why we're doing it. Um, cause a lot of times they're like why

am I gonna use this? So you try to bring in those real life things to say, so well

this is a real life situation. There is math involved. So let's think about it. What do

we have to do? How are we gonna solve this?

The notion of using real world or application problems was also mentioned by Participant

3 in the following:

 I think when we teach students like application problems or real world problems

that that is giving them, um, the ability to see it, to think mathematically … if we

do real world or application problems we're giving them like a problem instead of

just like two plus two …. But if they're like reading something and there is a

situation with it, then I feel like that is teaching them how to think that way where

they're looking at like a broader problem not just, not just numbers. And they're

having to come up with different ways. Oh I could solve it this way or I could

solve it that way. And that's where like the math part will actually come in. But

like using all their logic skills and everything before that.

Participant 5 focused on students in special education classes in saying that "it's a matter

of getting them to even think and use their thinking processes and it's more of even

teaching them that process and modeling it for them." And finally, Participant 4

described the notion of teaching students to think mathematically with respect to Geometry:

> One of the main things that I am focusing on my kids in Geometry is write down what you know. … so I stress that heavily. I would contend that that's what Geometry is as a class. Is thinking and teaching them to think mathematically and thinking about thinking.

The last aspect of defining mathematics and metacognition related to teachers' interpretations of the possible differences between thinking mathematically and doing mathematics. Some responses from the previous question and the current question overlapped. A common theme that appeared was the idea that doing mathematics can simply be completing calculations using arithmetic or an algorithm, whereas thinking mathematically cannot be reduced to calculations. Participant 4 discussed the importance of thinking in Geometry by saying "That's why I enjoy teaching geometry as whole. Is because I think you can take math out of it. They've never seen it before, so therefore you're able to work on that structure." Participant 6 had an extremely concise and lucid response, "I think the kids that don't like math are the ones who do math and I think the ones that do like math think math." Similarly, Participant 8 succinctly stated that "doing mathematics can be just applying an algorithm, a series of steps. Thinking mathematically is coming up with those steps on your own."

Beliefs and attitudes about school. After establishing an understanding of participants' views regarding mathematics and metacognition, the investigator asked questions regarding school-wide efforts to influence student thinking, especially in mathematics. The question asked was "What are your opinions regarding the effect of

school-wide initiatives related to student metacognition (the reading course and school problem solving goal)?" Additionally, participants often commented on the school-wide use of "I can" statements. In the school, "I can" statements are objectives worded so that students can immediately apply them to delineate daily learning targets. A great deal of participants' comments focused on both student and teacher buy-in to school-wide initiatives. From the perspective of the participants, the effectiveness of the initiatives is questionable. Participant 5 explained how it is important for teachers to transfer the responsibility of metacognition to students. On the other hand, the participant conceded that "it's good to develop the awareness in the staff how important it is. Um, but I think it comes down to, um, whether or not that teacher buys into that." Participant 1 had similar sentiments regarding the implementation of school-wide initiatives:

> I think they're great if the teachers are actually having them be a part of the lesson. I mean a lot of times schools can implement stuff, and if all the teachers aren't on board, it's not gonna work. I mean you have to have, I mean, obviously it's gonna be the teachers that are gonna be the ones, you know, drilling it into the kids in order for the kids to be able to think that way. But, I mean, a lot of times, it…it's hard to do it especially when they're in high school, cause they kind of already have a, I don't know, they've - kindergarten through eighth grade they've had a certain way of thinking so it's kind of hard to retrain them freshmen year, and you know. So unless you have all the teachers on board, I think it's very difficult to do something like that. I mean, even with the "I can" statements we're doing here, some teachers do 'em, some don't, you know. So then some kids love 'em and some absolutely hate 'em and are sick of them. It's just, it's kind of hit or

miss. … I mean with the freshmen kids, they love the "I can" statements. They know exactly what they are expected to know, and I think it helps them because then it's not we're just doing this for nothing. They know the objectives and what is required of them, opposed to older kids think it's a joke.

Participants 2 and 3 both mentioned how the strategies taught in the reading course would have been personally beneficial in high school. Participant 2 said how "I struggled hard core with reading and I probably would've benefitted from something like that," but "I know my kids get to the point sometimes where they're like, this is so overdone." The participant suggested that "if it's done in a decent amount or a moderate amount I think it's a little more beneficial than kind of like bombarded across, across the board."

Participant 6 described how "I think the reading goal is a nice goal for those students. … I think it's a lot to do with that self-assessing and the reflecting with the problem solving." Alternatively, Participants 4, 7, and 8 had more negative comments related to the reading program and problem solving goal. Participant 7 had the following to say:

I think it's important that we recognize the fact that we had kids graduating from here that were not reading at a high school level. Um, I think it's important that we recognize that kids are leaving here without the ability to think on their own. Just stating that we're gonna do the reading program just stating that we're gonna do problem solving as a goal doesn't fix the problem. I think the reading program has done a lot of good work with kids who were marginal and below average. And those kids, I think, have really come a long way. Even the average kids, I

think, have learned more about how to read appropriately, how to read effectively.

Um, I think in the math department we have always strived on problem solving,

I'm not sure that problem solving thing is getting the same attention throughout

the curriculums as the reading one was. I mean I think all of us in every subject

area does something with reading now quite often. I'm not sure everybody does

often enough problem solving.

Participant 8 suggested the following:

Um, I think the effect of the problem solving model goal, the, the model I don't

think is very effective. The problem solving goal, I think it's been effective in

math, but I don't think we've changed anything. I don't see it happening as a

school-wide initiative. … Um, the reading course, I think, is, has increased

metacognition. Um, the problem is the transfer of that metacognition to the other

subject areas. I think that's where the issue lies.

Finally, Participant 4 suggested a rationale for the possible problem with the school-wide

initiatives:

Reading course, school problem solving goals, I, I like them but I don't think that

they're, when you do a school-wide situation, they have a tendency to, uh, water it

down, it's kinda like fashion, okay. Once it hits the main market, it's not really

high end anymore, and now it's lost in transition. Um, reading across the

curriculum is a great idea but I think it's lost across the curriculum. … Um,

Geometry, I've said, write down what you know, answer the question, that

answering the question is part of reading. You have to go back and say what was

the question again, cause usually you lost it. You were just so used to solving the

equation and writing down the answers instead of answering what was it they asked. And I think that's where the reading course might be lost because they think it's reading and, and they're not really realizing that both reading and problem solving go together. And so it's a school-wide initiative of both those two problems and I think they're almost, the kids think they're separate but they're really the same thing. And um, I don't know if that's good or bad. I, I don't know whether I've seen that much of a difference by having that focus or not having the focus.

Despite some of the negative attitudes toward the school-wide initiatives, the participants still expressed the importance of both reading and problem solving in mathematics.

Beliefs and attitudes regarding students. In addition to discussing participants' opinions regarding school-wide initiatives related to metacognition, participants were asked to discuss the implications of students' conceptual understanding, procedural understanding, and silly mistakes. The specific questions were "To what extent is it necessary for students to have a conceptual understanding of a topic to be successful in thinking about or applying that concept?", "To what extent is that conceptual understanding necessary to be successful at incorporating related procedural processes (such as factoring or solving certain equations)?", and "In your opinion, how do "silly mistakes" factor into student thinking (related to causes, rationale, and or possible avoidance)?"

First, participants were asked if understanding a concept was a prerequisite for applying the concept. Regarding the notion of understanding a concept before applying it, Participant 7 said:

I don't think a student can fully understand what they are doing and how to apply it if they don't have a good grasp of the fundamentals. If you cannot add one third plus one half, you are never gonna to be able to add rational algebraic expressions. You, you need a good firm solid understanding of the basics in order to build upon those in my opinion.

Participant 8 agreed stating that "you can think about a concept without having a conceptual understanding of it, but applying the concept I think would require a pretty thorough conceptual understanding." Participant 2 held the same belief saying "I definitely think they do need to know the basics and the understanding of what it is you're asking before they can actually apply that to a more real-life situation."

Participant 1 gave further insight into the motivation of students:

Some students actually understand what's going on while others are just like okay you plug this number in here and that's the end of it, whereas others actually see the whole process behind it. I'm hoping most teachers are actually trying to show the whole process. But, unfortunately, many, many students just don't care enough.

Participant 4 had a different perspective citing how "the higher you get into math, you don't really understand the topic, you're just going through the motions of doing it. So if you did learn to think, then you'll be okay" and, referring to a specific student applying a conceptual understanding of mathematics, said "I'm sure she's able to do it because I think she learned how to think."

With respect to incorporating related procedures based on a concept, Participant 7 said:

Again, if they don't understand the basics when presented with a new problem,

they may not know how to approach it. If they understand, okay, this is the type

of approach I can use with this kind of a problem; this is the type of approach I

can use with this kind of a problem. Okay, now I'm presented a new problem.

Well, how can I use my previous strategies to approach this new problem? If they

understand those other ways, then they can use those to either create a new way,

or use one of them that they already know. If they don't have a firm

understanding, they can sit there and go around in circles with the new procedure

trying to figure out how to implement it, trying to figure out how to use it. And

they may get lucky and, and do it right or they may get frustrated and quit and

give up because they don't understand what they're doing and they don't see a

way out.

Dissimilar to the ideas related to apply concepts, Participant 8 disagreed with the ideas

described by Participant 7 in saying "Yes, you can do the process, because the processes

could just be a series of steps, or just do this and when you see this do this and when you

see this do that." Participant 5 clarified the idea by saying that "that's definitely

beneficial. Then that way it connects meaning to the math and they understand what

they're doing or why they're doing what they're doing."

Unfortunately, the possibility exists that some students do not see the connections.

Participant 1 explained the following:

They might not have any idea what's actually going on, the process behind it, but

as far as, you know, they just memorize the formulas and plug in the right

numbers and do whatever the teacher does and they never actually understand

why they're doing it or how it applies to real life, unfortunately. And, um, that is

something I definitely learned in college, was that they're not teaching the kids

enough how to apply it. I know they're trying to change it now, but I remember

when I was in school they never told you the reasoning behind it. It was like,

"here's the formula, this is what you're suppose to do, plug in the numbers, end of

story." I think it's important to have it but I, unfortunately, I don't think enough

kids know why. … I think they're just implementing a procedure. They're doing

what they have to do to get the right answer to get whatever grade they need.

Participant 4 addressed similar concerns in the interview:

You don't have to be good at it but you have to have the concept. I mean if you

were taught to think then I think you can understand the concept. Even the kids

that don't understand FOIL understand what FOIL is; they just don't do it well.

But they understand it. And there is the disconnect between doing it well and

understanding it. They can tell you what to do, but when they actually go to do it,

they make careless mistakes. And then their interpretation is they don't know

what to do. So I think that answers your question is, they know what to do but if

you ask them globally how do you do this problem and they shout out FOIL, they

shout out distribute, then they understand the concept, and then I do think most of

our kids get hung up on the actual manipulation of the, the fine tuning or whatever

you wanna call it. … I think our kids have a real rough time with the actual, um,

pencil to paper material. But they understand the global concept. They understand

how to think.

Participants were also asked about how silly mistakes play into student thinking and metacognition in general. Participant 3 focused on the positive aspects of student mistakes:

 I love mistakes! Mistakes are my favorite thing. Like when we put stuff on the board. … I don't know all the mistakes that they're gonna make cause I'm not thinking in their head. So I love seeing their mistakes and I point them out. And, um, like if after they take a quiz, all the mistakes that people made, if I think it's something unusual or if it's something everybody's doing; both of those situations I put them up and show them here's what I don't want you to do. I didn't think of this before, but now I see that you could do this. And now let's talk about this and let's not do that. … Yeah, I think that's how they learn. That's how I learn what they need to learn.

Other participants pointed out some of the possible causes for students' silly mistakes. Participant 5 suggested "that sometimes it's about them being focused and being able to attend to things that are happening at the same time. They get lost and trapped in where they're at and not being able to see the whole."

Participant 8 explained how devastating silly mistakes can be to students' confidence and ability to learn:

I think silly mistakes prevent students from believing they understand. I think that, uh, they avoid it because they make so many silly mistakes, and … if they make a silly mistake, they believe they don't understand, which most of the time is not true, um, so then they will avoid doing it. Um, the rationale is that they don't understand it so they will shut down. When in fact silly mistakes tend to be

reasonably minor errors and don't affect total understanding or comprehension of

the main concept. … I think they relate it to their thinking. I think they believe

that anything, anytime they miss points they don't get it. They don't understand

the problem if they miss points on it. And getting them to understand that silly

mistakes are just that, and can be avoided, uh, is difficult. Saying that you

understood the problem, you just missed this piece of it … is difficult.

Participant 1 held similar beliefs saying "they don't want to take the time to check their

answers … even though regardless of how many times you say you need to check your

answers" and "unfortunately, I think they're unavoidable." The participant continued

saying "when it comes down to it, it's up to the student." Participant 2 agreed citing how

often a silly mistake "happens cause they rush" and "they don't go back to check if

they've made a silly mistake." The students who are able to avoid silly mistakes are

"usually the ones who take it that next step and say, okay, I need to work on this; can I

ask you a question, can I stay after, they usually fix it a little bit better." Participant 7

echoed all the previous participants in describing how students make mistakes when

rushing and how it is the responsibility of the student to fix silly mistakes:

Um, the silly mistakes, I find, usually come into play when kids are questioning

themselves. For the most part, that or they're trying to hurry through something.

Um, silly mistakes are generally not made because the kid doesn't know what

they are doing. It's usually made in haste, or, like I said, when they aren't real

sure bout what they're doing because it's presented in a new way. Um, I, I refer to

the silly mistakes to the kids as things you have to fix those. You have to see

them, you have to slow down, you have to check your answers, you have to see

does this answer make sense. You know, you multiply three times four and get fifty, does that make sense in the context of the problem. Um, so I, as far as avoiding them, again I think that goes back to step back, take a second, think through the problem, think about what you have to do, is what you did on your paper what you thought you needed to do.

Participant 4 mentioned more about the pencil-paper methods utilized in mathematics, as well as the effect of silly mistakes on self-esteem in mathematics:

I think that silly mistakes goes to what I was describing as the pencil-paper. They understand the concept, but they pencil-paper it wrong and do things wrong mathematically or arithmetic wise. And I think the basic skills are what's lacking more than anything else, and because of that they have a lower self-esteem on math which then affects their ability to believe that they can solve problems as opposed to their actual ability. … And that's usually what you see in the higher level classes, that their basics are better than the lower end classes. Not that they're great, but they're better. They don't make as many silly mistakes therefore they can believe more positively about their abilities.

Influence on Strategy Utilization

The following delineates the data obtained via the questionnaire and interviews related to the influence of teachers' attitudes and opinions as related to strategy utilization in homework techniques, classroom practices, general cognitive-metacognitive strategies, and specific metacognitive strategies in the form of student reflection activities. Although utilized in previous research (Kher & Burrill, 2005), no data were reported for the results associated with the Instructional Practices of Teachers section of the questionnaire.

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Homework techniques. The results from the questionnaire related to the question concerning the types of homework teachers assign are found in Table 5. Frequency and percentage of responses are provided. The question had an additional response option of "Do not assign homework", but no participants selected that option. Therefore, the degrees of freedom for the chi-square analysis ($n = 18$) were set at three for the response choices Never, Rarely, Sometimes, and Often or always. The chi-square expected frequencies were treated as equal for the calculation.

Table 5

Responses to Section I Question 3: Instructional Practices: Types of Homework

Part	Never f	$\%$	Rarely f	$\%$	Sometimes f	$\%$	Often or always f	$\%$	X^2
A	1	5.6	0	0.0	12	66.7	5	27.8	19.78***
B	1	5.6	1	5.6	8	44.4	8	44.4	10.89*
C	2	11.1	10	55.6	6	33.3	0	0.0	13.11**
D	2	11.1	12	66.7	4	22.2	0	0.0	18.44***
E	8	44.4	10	55.6	0	0.0	0	0.0	18.44***
F	8	44.4	8	44.4	2	11.1	0	0.0	11.33*
G	9	50.0	7	38.9	1	5.6	1	5.6	11.33*
H	3	16.7	9	50.0	6	33.3	0	0.0	10.00*
I	10	55.6	3	16.7	3	16.7	2	11.1	9.11*

Note. "Do not assign homework" was omitted because the option was not selected.

p < .05.   **p* < .01.   ***p* < .001.

The type of homework garnering the most "Often or always" responses consisted of assignments in the form of worksheets or problems from a workbook or textbook. Two-

thirds of all participants responded with choices of "Never" or "Rarely" for the remainder of the types of homework listed in the question. Oral reports and journal assignments were the least often used with at least half of the participants indicating a selection of "Never."

Additionally, some interview participants referenced the use of homework in the classroom. Specifically, Participant 3 commented on how the questionnaire led to self-reflection regarding the definition and use of homework in class in discussion with the investigator:

P: Yeah, I guess it did. Because like I thought the one that you asked about like homework, you know, do I let students work on homework in class? So my answer to that was no, never. Because, but even though sometimes it is work that I want them to do on their own, like if I have time at the end, I'll pick out like a couple of the problems that would have, I would have assigned but I'm not going to assign it for homework anymore. I want them to do it right then and there. Because I think it is important for them to start in class so they know what to do when they get home, cause I think that's the biggest barrier to kids doing stuff at home.

I: Right.

P: Is that they don't even know where to start. So if they start in class, so, I said no to it, but then I'm like, well am I, am I doing that? Like, am I not using my class time wisely, you know? But, I, I think that's an important thing to do. And sometimes when I don't have time to do that I feel bad.

I: And I think some teachers might call that homework but give them class

        time to do it and you just identify it as class work.

P: Exactly.

I: So it's just your definition of homework.

P: But I don't have them do it in order.

I: Right.

P: Like if it was, you know, if there's one through ten that's one topic and

        then there's like, you know, eleven through fifteen, I would give them

        number two and number twelve, you know, or something to do. I wouldn't

        give them one, two, three, four in order.

In the dialogue, Participant 3 explained the realization reached regarding the personal

definition of homework versus class work. From the participant's last comment, textbook

problems are utilized. As another example of the use of homework in the classroom,

Participant 6 mentioned how a great deal of time is spent reviewing homework

assignments, upwards of 10 to 15 minutes, so students can ask questions regarding the

problems. The participant cited how reviewing homework gives students an opportunity

to reflect upon the homework. Participant 8 echoed the sentiment of Participant 6 stating

how homework is an avenue to allow students to self-assess understanding of the

concepts included in assignments.

    Classroom practices. The next aspect of strategy utilization relates to the

classroom practices teachers utilized as evidenced by specific questionnaire items and

through the interview process. Teachers first responded to the amount of classroom

control each is afforded in the classroom (see Table 6). The question had answer choices

of None, Little, Some, and A Lot. Frequencies of responses and percentages of

participants are listed. The degrees of freedom were set at three for the chi-square

analysis with equal expected frequencies utilized. All 18 participants in the questionnaire

phase of the study completed the question.

Table 6

Responses to Section I Question 1: Instructional Practices: Classroom Control

	None		Little		Some		A Lot		
Part	f	$\%$	f	$\%$	f	$\%$	f	$\%$	X^2
A	2	11.1	7	38.9	4	22.2	5	27.8	2.89
B	0	0.0	0	0.0	4	22.2	14	77.8	29.11***
C	0	0.0	1	5.6	1	5.6	16	88.9	39.33***
D	0	0.0	1	5.6	10	55.6	7	38.9	15.33**

**$p < .01$. ***$p < .001$.

In Part A, great disparity existed among the beliefs of the participants with respect to the

topics taught in the classroom, X^2 (3) = 2.89. A statistically significant amount of the

participants felt that a great deal of control is afforded in selecting instructional methods

and homework problems, each with $p < .001$, whereas participants perceived slightly less

control available over assessments.

Another question on the questionnaire related to the instructional practices

utilized by teachers with respect to student work. Table 7 lists the responses by frequency

and percentage. The possible responses included Never, Some lessons, Most lessons, and

All or almost all lessons. Assuming equal expected frequencies for the response choices,

the chi-square calculations utilized three degrees of freedom. All 18 participants

responded to the items in this question.

Table 7

Responses to Section I Question 2: Instructional Practices: Student Work

	Never		Some lessons		Most lessons		All or almost all lessons		
Part	f	$\%$	f	$\%$	f	$\%$	f	$\%$	X^2
A	7	38.9	6	33.3	5	27.8	0	0.0	6.44
B	2	11.1	10	55.6	4	22.2	2	11.1	9.56*
C	0	0.0	3	16.7	13	72.2	2	11.1	22.44***
D	0	0.0	14	77.8	3	16.7	1	5.6	27.78***
E	5	27.8	10	55.6	3	16.7	0	0.0	11.78**
F	0	0.0	14	77.8	3	16.7	1	5.6	27.78***

*$p < .05$.   **$p < .01$.   ***$p < .001$.

Parts C, D, and F were the most statistically significant, $p < .001$. These parts concerned whole class teacher-led instruction, whole class peer-to-peer dialogues, and assisted small group work. According to the responses provided, the majority of participants utilized the listed strategies in "Some lessons" or "Most lessons." On the other hand, student work without assistance received the most "Never" responses in Parts A and E.

Table 8 summarizes the first half of the data and Table 9 shows the second half of the data related to Classroom Activities as found via the questionnaire. Table 8 shows the Frequency of Use of selected activities by frequency and percentage ($n = 18$). The answer choices were Never, Seldom, Sometimes, and Often. Table 9 shows the Amount of Time Used. Frequency and percentages are provided for the response choices of <5 minutes, 5-10 minutes, 11-15 minutes, and >15 minutes. Chi-square calculations with three degrees of freedom and equal expected frequencies are provided for both tables. Some

participants omitted certain questions and one participant omitted all items related to

Amount of Time Used as reported in Table 9.

Table 8

Responses to Section I Question 4: Instructional Practices: Classroom

Activities: Frequency of Use

	Never		Seldom		Sometimes		Often		
Part	f	$\%$	f	$\%$	f	$\%$	f	$\%$	X^2
A	0	0.0	0	0.0	4	22.2	14	77.8	29.11***
B	0	0.0	0	0.0	3	16.7	15	83.3	34.00***
C	0	0.0	6	33.3	4	22.2	8	44.4	7.78
D	1	5.6	6	33.3	6	33.3	5	27.8	3.78
E	0	0.0	4	22.2	11	61.1	3	16.7	14.44**
F	0	0.0	0	0.0	3	16.7	15	83.3	34.00***
G	3	16.7	10	55.6	5	27.8	0	0.0	11.78**
H	0	0.0	6	33.3	7	38.9	5	27.8	6.44
I	0	0.0	0	0.0	5	27.8	13	72.2	25.11***
J	1	5.6	5	27.8	5	27.8	7	38.9	4.22
K	0	0.0	1	5.6	7	38.9	10	55.6	15.33**
L	0	0.0	10	55.6	7	38.9	1	5.6	15.33**
M	0	0.0	0	0.0	6	33.3	12	66.7	22.00***
N	0	0.0	2	11.1	12	66.7	4	22.2	18.44***

**p < .01. ***p < .001.

The activities gaining the most "Often" responses included review of topics and

homework, explanation of new topics, paper-and-pencil exercises, small group work, and

technology use. Items most frequently answered "Sometimes" concerned class discussion

and connecting mathematics to the real world. Finally, over half of participants selected

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"Never" or "Seldom" for both Parts G and L. These parts related to using manipulatives and using mathematics in exploration. Responses were most varied for Parts D and J which regarded oral recitation, oral drills, and using class time for homework.

Table 9

Responses to Section I Question 4: Instructional Practices: Classroom

Activities: Amount of Time Used

	<5 minutes		5-10 minutes		11-15 minutes		>15 minutes		
Part	f	%	f	%	f	%	f	%	X^2
A[b]	5	31.3	10	62.5	0	0.0	1	6.3	15.50**
B[b]	1	6.3	12	75.0	3	18.8	0	0.0	22.50***
C[a]	8	50.0	4	25.0	3	18.8	1	6.3	6.50
D[a]	9	60.0	4	26.7	2	13.3	0	0.0	11.93**
E[b]	8	50.0	6	37.5	1	6.3	1	6.3	9.50*
F[b]	0	0.0	1	6.3	3	18.8	12	75.0	22.50***
G[b]	5	31.3	4	25.0	6	37.5	1	6.3	3.50
H[b]	2	12.5	2	12.5	3	18.8	9	56.3	8.50*
I[b]	0	0.0	5	31.3	6	37.5	5	31.3	5.50
J[b]	6	37.5	8	50.0	1	6.3	1	6.3	9.50*
K[b]	0	0.0	2	12.5	9	56.3	5	31.3	11.50**
L[b]	1	6.3	9	56.3	5	31.3	1	6.3	11.00*
M[b]	0	0.0	2	12.5	6	37.5	8	50.0	10.00*
N[b]	0	0.0	8	50.0	6	37.5	2	12.5	10.00*

[a]$n = 15$.   [b]$n = 16$.

*p < .05.   **p < .01.   ***p < .001.

Seventy-five percent of respondents reported using ">15 minutes" on introducing new material in the classroom. The same percentage of respondents reported utilizing "5-10 minutes" reviewing homework. Both activities represented statistically significant responses via identical chi-square values, X^2 (3) = 22.50, p < .001.

Although the quantitative data regarding classroom practices were extensive, additional information was gleaned through the interview process. The interview question most closely related to addressing the use of specific classroom activities was "In what ways, if any, does the type of class (remedial, regular, honors, special education) affect your method(s) of instruction as related to thinking mathematically?" Participant 7 mentioned how, when teaching freshmen, classroom activities focus on "getting them to understand what the rules are, and how they affect other things as they progress through the different levels." Additionally, the participant explained how "I ask them higher level questioning, I start asking them to analyze more, to synthesize their own questions [because] basically at the freshman level, it's cognitive" and "most of them aren't ready to see how the big picture fits together." Older students in more advanced mathematics classes "have more mathematical background to them. They understand, they remember the topics better, they, they can kinda start putting things together, they can see how things are connected." For these reasons, the method of instruction is directly influenced in the participant's classroom.

Participant 6 mentioned similar ideas in classroom teaching methods in using instruction to deepen mathematical connections. The following are the participant's thoughts regarding instruction in varied class types:

Very different cause I teach a variety of different classes. So for … collaborative

… we do a lot of guided notes. Um, it is a lot of, of rote. It's a lot of this, if you

have this set up, this is what you're gonna do. So you have to do take baby steps.

Whereas my … honors class, it's more so let's, let's figure out here's, um, a

binomial x squared minus 16, what can we do with it. What connections are you

seeing? So I think it's more so self-discovery in those upper level classes, versus

these are the baby steps for the lower, um, classes.

Participant 4 focused more on specific activities in a variety of class rather than

connections, echoing the information from the questionnaire related to the prevalence of

pencil-paper activity usage:

Um, the higher the level of class the more you can think about thinking. The

lower level class, I think, you have to teach them the pencil-paper, actual

manipulation because that's where they struggle with, that's where they get it

wrong. When they're uh higher, like honors or regular, I think they can handle

manipulation better, and then you need to teach them how to think and how to

problem solve more so they can see a variety. The remedial … class, they need to

just hammer out the low end ones, but I'm told that's not true, uh, they need to see

the higher ones. I think they just want to be successful at the low end and move on

to the next success next success, whereas regular kinda gets bored with the pencil

paper. … The pencil paper writing as opposed to they would like to see different

problems in different ways to be challenged a little bit more, so that when will I

ever do something like that, and then you throw a little, a little bit different. You

see that in Geometry a lot.

Finally, Participant 3 realized a possible deficit in classroom instruction related to students in different level classes:

> Um, I think that's definitely something I need to work on for next year. It's because I feel like I've been, um, so much in the lower classes, worrying about this. "Do you understand what I'm saying? Do you understand what we did today? Do you understand how it relates to what we're going to do tomorrow?" Yesterday, now when it all comes together, I'm so worried about that for them, but I feel like in the higher level classes, I'm just assuming that they understand what I'm saying. I give them the I can [statement]s, I give them their study guides, I tell them the topics we're gonna be discussing, but I don't, I, I suppose I definitely do not do anywhere near as much discussion with them at all.

General cognitive-metacognitive strategies. In the interviews, participants were asked "To what extent, if any, do you teach students to think about thinking in mathematics classes?" rather than specifically asking about cognitive-metacognitive strategies. The responses were varied, but a common theme among most participants was the attempt to implement such strategies on a daily basis, just as with teaching students to think mathematically. Participant 1 cited the use of journals and "just having students put down their thoughts and journaling" as examples of activities to foster thinking about thinking, but also remarked that "I haven't done that much, with the type, time constraint I've had trying to get everything together." Participant 2 mirrored Participant 1 in reflecting upon the extent to which cognitive-metacognitive thinking activities were used in the classroom:

Probably not as much as I could or should. Um, cause it's probably good for the students to kinda reflect on their thinking and what they're doing and why they're doing it. Um, you know, sometimes you teach 'em and you kinda let 'em go and see where they're at. And some of 'em get stuck and you help them along. Um, but, as far as actually doing that, I guess I don't do it a whole lot.

Participant 5 expressed how thinking about thinking is a classroom focus by "continuously, um, trying to get them to tap into those parts of their brain and being able to utilize those, those types of processes." Finally, Participant 7 expressed the teaching method utilized to aid students in learning to think:

Again, I try to get them, I talk out loud the way I walk through a problem. I, I model to them how I go about, uh, different problems. And again, not just math-related problems. Not just out of a textbook. I, I ask them, "Well what if, um, you wanted to, uh, make a decision about buying a dress or something? What, what's your criteria, what, what do you want to get out of this, what's your point to doing this versus what's the best price, what's the best buy?" I mean there is numbers involved, but there's more to it than just the numbers. Think through the process.

Student reflection activities. Whereas the previous strategies mentioned by interview participants were more general in nature, the following examples of metacognitive strategies were explicitly addressed through the interview process. Participants were asked "In what ways do you have students reflect upon their thinking (such as on graded assessments or via I can statements)?" To follow-up the responses, interviewees were also asked "To what extent, if any, have these efforts been effective?" and "Are there ways in which they could have been or could be improved?" The

commonly cited student reflection activities included reflections after assessments and the use of learning objectives worded as "I can" statements. Notably, the use of "I can" statements was mentioned by teachers with the least amount of experience. Other activities mentioned less often included journaling, stressing vocabulary building, using study guides and sample problems, encouraging self-prompting, utilizing think-pair-share, incorporating self-assessment with homework, addressing common student errors, and opening and closing lesson activities.

Participant 1 explained the use of "I can" statements as a student reflection activity:

How "with the "I can" statements, I tried to always have them figure out whether or not they actually knew how to do it to reflect on their performance. … I think it's important that they do know what they know and what they don't know how to do. I can, you know, I can think they all know it and then the next day they come in and nobody has any idea. So I think it's important to always have assessments where they personally know and I also know too but then it's their responsibility especially as high school students to do something about it. Do extra practice or ask for help if they don't understand it, you know, whatever it takes. … Some students will actually take advantage of it. Others, they just don't care enough, unfortunately.

Participant 2 explained how "I can" statements allow students to self-assess and determine whether the topics of the daily lesson were achieved. Participant 3 expressed how objectives are presented, "not necessarily in the form of the words 'I can', but here

are the topics and the concepts that we've gone over. And I definitely think that really, really helps."

Other participants briefly explained how students would review individual understanding after assessments. Participant 8 had students complete "self-assessments that they do after quizzes and tests." Participant 6 cited how students reflect upon "their tests and quizzes" and how, along with other reflection activities, "it's a lot of self-assessing." Participant 2 expounded more regarding the use of reflection after assessments:

> I usually do this after, um, most often after a quiz or a test. … So they're going
>
> back and they're looking through it as I'm going through the problems …
>
> especially after a quiz, cause you want to get them ready for that next step, that
>
> test, and if there's things you can fix right after the quiz, I want to fix that before
>
> we move on.

Journaling was another student reflection activity cited by multiple participants. Interestingly, at least half the participants in the questionnaire phase of the study reported not utilizing journals in mathematics classes. Participant 5 succinctly explained that "We do journaling frequently or we have student feedback within think-pair-shares or classroom discussions about what points the students bring up about how they approach problems." Participant 7 described the use of journals more deeply:

> Um, I do journal statements. Journals once a week. Um, some of those are based
>
> upon, think over the last quarter. What are your strengths? What are your
>
> weaknesses? What are you still having difficulties with? Um, some of them are if
>
> you were given this situation, how would you go about approaching it, what

would you do with it. Um, some of them are, uh, be creative. Think of something new on their own. Um, how would, how would you, uh, what, what holiday would you create and why? Always a why component, always an explain your answer component so they have to think through what their decision is and why their decision is that.

Teachers participating in the interview phase then commented on the effectiveness of the student reflection activities utilized. For the most part, participants expressed how the efforts have been effective. Participant 8 explained how "I've seen some change, some effect, uh, it's, it's not a tremendous effect but … I do believe it has some effect." Participant 7 recounted the comments students made regarding the use of journals in mathematics class:

Most of them said yes you should continue to do them, some of course said because they're free points but some, a lot of the students responded that, you know, it got me to think about what I needed help with, what I was doing right, what I was doing wrong, it was good introspectively for me, that I don't normally take time to sit down and think about the way I look at things and the way I do things. So, I think it was effective in that regard.

Participant 4 explained how the effectiveness is not evident within the immediate school year:

You won't see it this year, you don't see it in their grades, but you will see it in their thought process the next year when they try to apply what you've learned. So, I don't see it, but I'm willing to bet if you ask the former students or teachers

that have my students, they would be the better one to judge whether it was

effective or not

As for ideas for improving the effectiveness of student reflection activities,

participants described utilizing a variety of methods to increase enthusiasm and attention,

analyzing the specific problems students are encountering, helping students to accept

responsibility and understand the importance of maturity, receiving student feedback, and

gaining teaching experience in general. Participant 7 explained how direct student

feedback will influence the use of journaling in the future in saying:

> I have a lot of positive responses in the journals because the last one I did I asked
>
> them what was your be-, your favorite entry, your least favorite entry and should I
>
> do them? … Well, this was my first year doing the journals, so, um, I, I'm taking
>
> a lot of their feedback from that last journal and different things I'm going to use
>
> as topics for next year.

Participant 5 focused on how increasing student responsibility for learning made a

difference in thinking:

> Um, I think for those students who are invested in their education and are now
>
> seeing that it really isn't about me being their teacher, it's about them being them
>
> and learning for them, those students have taken off and thrived this year. My
>
> students who aren't there yet, um, it's a process. And at least I feel like I've
>
> planted some seeds into getting them to think cause they mature. … It's more
>
> about them and their maturity and what they're willing to take responsibility for.

From a different perspective, Participant 6 expressed how teaching experience led to

improvement in helping students to reflect because "I think I have improved as a teacher

obviously over the … years." Additionally, Participant 1 explained how students could verbalize misunderstandings to the teacher:

> Actually have the students look at their work and, like as they're doing their problem, after they do it and stuff, rather than them just saying if they can or can't do the main topic, maybe actually break it down to what don't they understand. Like, where do they fall, or where did they go wrong. What are they not understanding? What part of the process are they not understanding? And I think that would probably be more helpful cause maybe that's something I could easily clear up with just talking with them for a minute or two.

> Finally, Participant 4 introduced the analogy of blockage of student thinking to

Star Trek:

> I think it's up to the kid more than anything else. Um, at that point to, to buy in to the philosophy of how to think. Some, um, I call it shields up, they stop you, you know, Star Trek idea of … shields up, they, they want nothing to do with that process. And I think you can see that in Geometry a lot. They, they say "Oh, it's Geometry, I don't get it, I don't get it, I don't get it." But sometimes, as I was told by an administrator, sometimes if you have a good teacher, their effects last years after that. And my contention is that you will see it next year whether you were successful or not.

<div align="center">Conclusions</div>

Based on the data and results presented, numerous conclusions can be drawn. The inferences are organized according to the attitudes and perceptions held by participants and the influence of attitudes and perceptions on strategy utilization.

Attitudes and Perceptions Held

First, addressing teachers' attitudes and beliefs regarding students' cognitive-metacognitive abilities relied heavily upon the teachers' definitions of cognition and metacognition. For the purposes of the study, no true distinction was made between the two despite the sometimes contentious relationship among seminal authors. Instead, at least for the interview phase of the study, participants were asked to define "thinking mathematically" and "thinking about thinking." The intent was to learn about teachers' definitions of metacognition without using terms that might be unfamiliar or too technical. Additionally, because even seminal authors regarding metacognition (Artzt & Armour-Thomas, 1992; Brown & Palincsar, 1982; Flavell, 1979; Holton & Clarke, 2006; Kayashima et al., 2004; Livingston, 1997; Schoenfeld, 1992) disagree about where cognition ends and metacognition begins. Therefore, the more generic and all-encompassing "thinking about thinking" allowed participants to express definitions and opinions without being possibly restricted by the term metacognition. A secondary purpose was to allow participants to access opinions related to the research topic more easily and comfortably without feeling intimidated by terminology.

Although less deep than the responses found via interviews, the questionnaire gave important information that allowed for choosing interview participants. Specifically, the Beliefs and Opinions section consisted of the Beliefs Regarding Mathematics subsection and the Mathematics Teaching and Learning subsection. The Beliefs Regarding Mathematics subsection had a reliability value of .65. The value obtained differed greatly from values obtained from previous research. Although the original 48-item version utilized by Fennema et al. (1987) and Capraro (2001) had reliability values

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of .93 and .78, respectively, the current questionnaire version consisted of only 18 items. The 18-item version was used by Kher and Burrill (2005), who found alpha to be .80. Possible reasons for the disparity in reliability values between the present study and previous incarnations include how some participants did not answer all items in the subsection and how the sample size of the present case study was much smaller than the 102 participants responding to the same 18-item instrument previously (Kher & Burrill).

The Mathematics Teaching and Learning subsection had a reliability value of .65 for the present study. The value for the current study is closer to the previous value of .72 obtained by Kher and Burrill (2005). Again, the value obtained for the present study could have differed from the previous study because one participant omitted the entire subsection and another did not complete all the items. Additionally, the previously utilized version was misnumbered to have 15 items, whereas the present study only utilized 14 items. Finally, one item received responses of zero variance because all participants selected the same response. Although no overall reliability value was obtained for the composite instrument, the present study had a reliability value of .75, much closer to the acceptable range. Obviously, the obtained value could be affected by the individual subsections' limitations.

Next, the verbal responses in the interview phase showed some glaring differences between participants, especially between less experienced, non-tenured teachers and more experienced, tenured teachers. Some of the dichotomies that seemed to emerge between the groups included the following: developing responses versus sharing views, lengthier responses versus concise insights, abstract versus concrete explanations, positive or optimistic versus negative views of the school, and prolific use of educational

buzz words versus the acknowledgment of the swinging educational pendulum. As bleak a picture this paints, both groups contributed greatly to a deeper understanding of the attitudes and perceptions of teachers regarding metacognition, education, and working with students.

In selecting interview participants, the researcher strategically chose teachers with different backgrounds. Three teachers possessed fewer than four years of experience, one just more than four years experience, and the remainder with more than four years teaching experience. The comparison revolved around the four year mark because of tenure status achievement after completing four years in the school. Specifically, in comparing the responses given, the first two participants were earlier in teaching careers than other participants. The responses given were almost discovered as the interview continued. The participants had not given much thought to the questions in the small number of years of teaching completed. For example, responses to the interview questions related to the "difference between thinking mathematically and doing mathematics" were much lengthier for the less experienced teachers than those with more experience. It seemed as though the less experienced teachers believed they needed to give more substantial responses than was necessarily necessary. Often, the added portions to the responses were wordy, unfocused, and detracted from the genuinely insightful comments provided.

Conversely, the more experienced teachers gave more concise responses that demonstrated that previous thought had been dedicated to considering the questions asked, even if only in passing. When the more experienced teachers did give lengthier responses, the length added more depth to the explanation rather than grasping for the

right words to express underdeveloped ideas, attitudes, or beliefs. Perhaps a deeper level of expertise stemming from years of teaching experience also aided in lucid, concise responses. Additionally, having taught various courses for numerous years likely afforded the more experienced teachers a deeper understanding of the connections not only existing within and throughout the curriculum, but also connecting the art of teaching to the inherent thinking involved in mathematics.

Despite the years of experience, the participating teachers grappled with the definitions of "thinking mathematically," "thinking about thinking," and the difference between them. In particular, Participant 3 struggled to give an initial definition of thinking about thinking or metacognition. After some prompting, the participant verbalized how "thinking about thinking, like people could take that so many different ways" and "the word metacognition. Obviously I've heard that a million times but I don't know like the specific definition of what that is. I'm not sure how you could change it without, without edging somebody towards an answer, you know?" Participants most often cited using logical steps to solve problems in defining "thinking mathematically." As for "thinking about thinking," most participants referenced student self-analysis of thought processes.

Obviously, the responses point toward a difference of belief regarding the terms. The distinction is important because it points to how teachers differentiate between teaching students to think mathematically and think about their thinking. Having the attitude or belief that a distinction exists can affect instruction and, therefore, student learning, understanding, and performance as in the Cognitively Guided Instruction (CGI) program (Carpenter & Fennema, 1991; Carpenter et al., 2000; Clarke, 1997; Fennema et

al., 1996). The underlying beliefs held by teachers that started to surface through the study are integral to the classroom learning environment (Capraro, 2001; Kher & Burrill, 2005; Webb et al., 2006). Interestingly, as evidenced by interview responses, Ernest (1988) explained how teachers' beliefs are formed in large part due to teaching experience rather than any formal though process to "decide" on a particular philosophy. While participants with more experience expressed more sophisticated responses, the underlying beliefs were likely formed based upon teaching experiences that shaped and formed the individuals.

Additionally, beyond the beliefs that began to be uncovered, teachers' responses gave insight into certain perceptions of teaching. Regardless of the validity, Participants 4, 7, and 8 held perceptions of a somewhat negative nature in relation to the school. Perhaps because of prior incidents or greater experience with administrators in the past, but these participants seemed to perceive the school-wide initiatives as less effective than other interviewees. Another possible explanation for the pessimistic perception held by the participants is how programs and initiatives have been implemented previously. Eventually, the programs dissipate or are abandoned in favor of a newer, more promising plan. The inevitable swinging pendulum of educational change can be disheartening and could point to the underlying reasoning behind the responses. As Participant 4 eloquently stated, "it's kinda like fashion." Likely for similar reasons, the less experienced teachers cited educational jargon more readily and eagerly than more experienced teachers. For example, the newly implemented "I can" statement versions of student learning objectives were only shared by the less experienced teachers, whereas more veteran teachers spoke more commonly about problem solving.

Along similar lines to the interview responses, one of the questionnaire items referred to teachers being evaluated directly by the principal. The responses were quite mixed. The results implied a possible sense of distrust by some teachers of the administration. The sentiments were echoed by select participants in the interview process regarding the effects of school-wide initiatives.

Participant 5 shared an interesting comment regarding the relationship between the study, the classroom, and the effect of the study on teaching methods:

> What I've appreciated is I've seen connection to the questions that you're asking and what I've been doing this year in the classroom and how interrelated that they are and it's got me thinking about even more, what I'm doing in my classroom.

Creating change in the thought process of the participating teachers and encouraging both teacher and student reflection has been rewarding. Participant 4, however, expressed the ever present difficulties of both teaching and learning mathematics, encompassing all the intricacies, cogently in the following:

> Isn't that the conundrum of math as a whole? Is that you need both basics and problem solving to be successful. It's the devil's in the detail. And we need to make sure that they understand that detail is extremely important and I think that's sometimes lost on the kids.

Influence on Strategy Utilization

Next, certain conclusions can be drawn with respect to the influence of teachers' attitudes and perceptions of students' cognitive-metacognitive abilities on selecting and utilizing instructional strategies. The questionnaire results evidenced numerous instructional activities that various teachers use. The extent to which each activity fosters

deeper thinking skills is debatable. In the interview process, participants were probed more deeply to discuss activities that encouraged student thinking in mathematics, specifically with a focus on reflection activities. The responses given were outlined in the findings section.

The study pointed toward the caring nature of the participating teachers. The questionnaire item related to the welcoming of student questions was overwhelmingly answered affirmatively by all participants. The responses demonstrated that the teachers in the school are dedicated to maximizing student potential and are open to aiding students. On the other hand, some of the results, specifically from the interviews, were unexpected. Only one participant referenced the use of bell ringers, or warm up activities, and entrance or exit slips as a student reflection activity during the interviews. I found this interesting because the use of the activities listed is quite pervasive in the mathematics department at the school. Additionally, the use of "I can" statement versions of objectives was only discussed by non-tenured, less experienced teachers. Again, "I can" statements are a seemingly constant classroom fixture in the department and the school as a whole. Even more striking, the use of "I can" statements was mentioned as a part of the interview question discussing the influence of school-wide initiatives, yet the less experienced interview participants referenced them more often than more veteran teachers. Instead, more experienced teachers explained the importance of problem solving.

Regardless of experience, the methods described by participants showed the extensive knowledge regarding problem solving, as well as the diversity of instructional methods utilized to reach students and truly teach mathematical thinking and

metacognition. Polya's (1945) problem solving method (in Kahveci & Imamoglu, 2007) included problem analysis, consideration of solutions, implementation, and evaluation of solution method. A similar model is present in the mathematics classroom in the form of the school problem solving model in conjunction with the school problem solving goal. Polya also explained the importance of discerning pertinent information from unnecessary or extraneous data. Parts of Polya's model were mentioned by numerous participants in the interview phase. The thinking process described predated metacognition, but definitely applies to the definitions suggested by interview participants. In fact, Schoenfeld (1992), a prominent proponent of mathematical thinking, cited Polya's problem solving model. Even though Schoenfeld redefined it from strategies to heuristics, the notion of problem solving still pervades the mathematics curriculum. Admittedly though, the term problem solving is just as ill-defined as metacognition, especially due to misuse and ambiguous implementation.

Garofalo and Lester (1985) related teaching procedures, problem solving, and metacognitive training. Again, the definition of both problem solving and metacognition can be elusive, so explicating the relationship may pose insurmountable. From the information obtained through the interviews, participants seemed to understand and acknowledge the intrinsic link between problem solving and thinking at the metacognitive level, however. Garofalo and Lester explained that, without metacognitive training, problem solving is a fruitless and often frustrating exercise in futility. Of course, the specifics of metacognitive training in contrast to problem solving were not addressed in the present study.

Finally, a commonality with the previous section was the disparity between responses of teachers with different years of experience. The specificity and sophistication of responses seemed to increase with the years experience teaching. Kher and Burrill (2005) explained that teachers' experiences, from professional development to daily classroom situations, inform teachers of a variety of instructional methods that can influence decision-making in the classroom. The insights gleaned from experience cannot be replaced nor artificially replicated in less experienced teachers. Truly, experience is the best teacher.

Implications and Recommendations

The present study has only scratched the surface of exploring the attitudes and beliefs of mathematics teachers with respect to students' cognitive-metacognitive abilities. Because of the small, case study nature of the study, the study needs to be expanded in numerous aspects. First, the study should be replicated in the current case study format to add more pertinent data. Additionally, an expanded version of the study is needed. On a larger scale, more participants and more data collected could give a more accurate picture of teachers' attitudes and perceptions than a small sampling from a single school. For that reason, the results and conclusions of the present study are probably not generalizable far beyond the school, the school district, or the local area.

Another idea to expand the scope of the study would be to create a longitudinal measurement to determine growth of teachers in attitudes and beliefs as well as strategy utilization. In that way, it may be possible to determine if an understanding of metacognition is tied to teaching experience or familiarity with the profession in general. Additionally, along with the questionnaire and interviews, future research could

incorporate classroom observations to observe teaching strategies currently in use that foster cognitive-metacognitive skills in students. Researchers could then perform follow-up interviews to debrief the teachers and learn about the thought process utilized in utilizing and selecting specific classroom techniques. Comparing interview responses with classroom observations could also evidence proof of interview comments and draw parallels between beliefs and practical application or uncover inconsistencies between beliefs and actions in the classroom.

Regardless, in any future study, utilizing the questionnaire would likely be important because it has been a constant part of numerous previous studies as well as the current one. Utilizing the questionnaire would allow for comparison and contrast with the previously obtained data. With respect to the interview process, future researchers might benefit from specifically asking for definitions of metacognition rather than the presently utilized term "thinking about thinking." A direct approach may be appropriate depending upon the population participating in the research. Additionally, rather than asking about student reflection activities or activities related to thinking about thinking, results might be more pertinent if participants were asked about the cognitive-metacognitive strategies utilized in the classroom. As done in the present study, if participants were unfamiliar with the term metacognition, the investigator gave a brief description as based upon the literature. If the study is to be replicated, however, the interview sample size seemed appropriately large to represent the population studied.

Future study would likely benefit from delving deeper into two particular aspects of the present study. First, the negative attitudes surfacing regarding the school seemed to run deep. Less experienced teachers did not demonstrate such passionate feelings,

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though. Future study into the juxtaposition between feelings toward school and teaching efficacy seems interesting. Second, the issue of students' silly mistakes was a unique topic with tangential importance to the present study. Obviously, increased metacognitive awareness could lead to reduced occurrence of silly mistakes. The underlying causes, rationale, and avoidance techniques seem like rich areas warranting further investigation, however.

The implications for the future based upon the present study are mixed. Because the study was restricted to a single school, the results cannot be generalized to other schools in other situations. Assuming that the data are accurate for the presently used school, however, the following recommendations can be made. First, the relationship between teachers' attitudes and perceptions and instruction exists. Therefore, attention must be paid to nurturing and encouraging teachers to hold high expectations of students and to improve any negative attitudes and perceptions related to students' abilities. Professional development opportunities and addition of a perception component to pre-service mathematics education programs could address the need. Ideally, teachers can learn to become self-aware of the impact attitudes and beliefs regarding students' abilities play in teaching mathematics. Additionally, because of the importance of cognitive-metacognitive skills to students to be successful in mathematics, teachers must be trained in methods to instill those skills in students. Again, pre-service teacher training and professional development activities could address teaching methods. Together, attitude and perception awareness in conjunction with training in teaching cognitive-metacognitive skills would lead to improved student abilities to reason mathematically through the teaching of metacognition.

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Appendix A

Permission for Questionnaire Adaptation

From: Neelam Kher
Sent: Mon 11/10/2008 9:27 AM
To: Peter Babich
Subject: Re: Request for PROM/SE Research Instruments

Peter,

Sorry for the delay. We encountered an issue that needed to be resolved with reference to the items your were requesting.

The issue did not have anything to do with whether you can use the items or not but how to provide appropriate credit. You have our permission to use the items/sections. The issues related to credit/citation are discussed below.

Some of the sections that you plan to use are sections that are adaptations of other scales so in your write-ups you this connection will need to be reflected.

I think the best way to for you to proceed with the use of the items is to read our writeup of the instrument development. It provides references for the original scales, outlines our changes and describes our attempt to determine the psychometric quality of our instrument. I'm attaching the writeup for your information. The paragraphs highlighted in blue are particularly germane to our discussion.

You may find that some of the sections identified in the write-up may have been rearranged and some sections may have been deleted from the version that was sent to you but the sections you plan to use are part of both versions. If this creates any confusion I can send you the original version.

What we would like you to do is to credit the center(identified in my address block) for the development of the instrument (identify it by name), the purpose for which it was developed (indicated in the write-up) and provide us with the results of an independent reliability and validity analysis of the instrument that you conduct as part of your study.

Additionally, you will need to identify the source (references) from which we obtained the original scales embedded in our instrument. If you send me a draft write-up of the credit/citation I can review it.

With reference to your question about analysis-the analysis strategies for reliability/validity are described in the writeup. If you need additional information about those let me know. We also have results based on the revised version of the instrument.

As far as other analysis is concerned, the choice will really depend on the research questions/hypotheses that you are trying to answer/substantiate. We considered the scales to be categorical data and provided a distribution of responses in terms of percentages. We also used Chi-Square analyses.

Let me know if you have any additional questions.

Thanks and have a good day.

Neelam

Neelam Kher, Ph. D.
Center for Research in Mathematics and Science Education
Center for the Study of Curriculum
240-C Erickson Hall - Michigan State University
East Lansing, MI 48824-1034

Appendix B

Site Letter

April 1, 2009

Chair, Institutional Review Board
Office of the Vice President of Academic Affairs
Olivet Nazarene University
One University Avenue
Bourbonnais, IL 60914

Olivet Nazarene University Institutional Review Board,

After reviewing the proposed study, Attitudes and Perceptions of High School
Mathematics Teachers Regarding Students' Cognitive-Metacognitive Skills, presented by
Mr. Peter Babich, a doctoral student at Olivet Nazarene University, I have granted
permission for the study to be conducted at █████████████████████.

The purpose of the study is guided by the following research question: What strategies
are perceived by mathematics teachers in a Midwest high school district as best at
fostering students' cognitive-metacognitive skills in solving select mathematics
problems? The primary portions of the research will involve teachers completing
questionnaires and conducting interviews.

I understand that the questionnaires will be completed by teachers during the second
semester and that interviews will be conducted within the school building. Only teachers
in the mathematics department or teachers in mathematics classrooms will participate. I
understand that this project will end no later than June 2009.

I understand that Mr. Babich will provide me with a copy of all IRB approved documents
related to his research. Any data collected by Mr. Babich will be kept confidential and
will be securely stored at the researcher's home.

Sincerely,

Dr. █████████, Principal
█████████████████████

Appendix C

Informed Consent Form - Questionnaire

INFORMED CONSENT FOR PARTICIPATION IN RESEARCH ACTIVITIES

Project Title: Attitudes and Perceptions of High School Mathematics Teachers
Regarding Students' Cognitive-Metacognitive Skills

Investigator: Peter Babich, Olivet Nazarene University Department of Graduate and
Continuing Studies, Investigator's Phone: ███████████ E-mail:
███████████

You are being asked to participate in a project conducted through Olivet Nazarene
University (and -- if applicable -- any other cooperating institution). The University
requires that you give your signed agreement to participate in this project.

The investigator will explain to you in detail the purpose of the project, the procedures to
be used, and the potential benefits and possible risks of participation. You may ask
him/her any questions you have to help you understand the project. A basic explanation
of the project is written below. Please read this explanation and discuss with the
researcher any questions you may have.

If you then decide to participate in the project, please sign on the last page of this form in
the presence of the person who explained the project to you. You will be given a copy of
this form to keep.

1. **Nature and Purpose of the Project:** You are being asked to participate in a
   research study to learn more about the attitudes and perceptions you hold
   regarding teaching strategies related to students solving mathematics problems
   from a cognitive-metacognitive approach. This first stage involves completion of
   a questionnaire regarding instructional practices, beliefs, and opinions, and a
   second stage will include interviews of approximately eight persons. Responses to
   the questionnaire will aid in selecting second stage participants.

2. **Explanation of Procedures:** Your participation in this first stage of the study will
   only consist of completing a questionnaire. As a participant in this study, you will
   be asked to complete the questionnaire and return it to the principal investigator.
   The questionnaire should take about 20-30 minutes to complete.

3. **Discomfort and Risks:** Opinions and attitudes you express will be conveyed
   anonymously in the published research. There is the slight possibility of your
   identification by potential readers of this study, but this will be minimized by
   keeping your responses anonymous in the published research. There is also a
   minimal possibility that based upon your responses, your identification could be a
   risk to your job. All your responses will be kept confidential, and those published
   in this study will be kept anonymous to safeguard your identity. Your
   participation is voluntary, and you may withdraw at any time. If you withdraw,
   your submitted questionnaire would be destroyed.

4.   **Benefits:** Your participation in this study may aid in our understanding of how and what teachers believe regarding students' cognitive and metacognitive abilities.

5.   **Confidentiality:** Only the investigator and members of the research team will have access to your questionnaire. If information learned from this study is published, you will not be identified by name. Any excerpts from your responses used for illustrative purposes will be kept anonymous.

6.   **Refusal/Withdrawal:** Refusal to participate in this study will have no effect on any future services you may be entitled to from the University. Anyone who agrees to participate in this study is free to withdraw from the study at any time with no penalty.

You understand also that it is not possible to identify all potential risks in an experimental procedure, and you believe that reasonable safeguards have been taken to minimize both the known and potential but unknown risks.

_____   _____
Signature of Participant   Date

THIS PROJECT HAS BEEN REVIEWED AND APPROVED BY
THE OLIVET NAZARENE UNIVERSITY INSTITUTIONAL REVIEW BOARD
(IRB@olivet.edu)

Appendix D

Informed Consent Form - Interview

INFORMED CONSENT FOR PARTICIPATION IN RESEARCH ACTIVITIES

Project Title: Attitudes and Perceptions of High School Mathematics Teachers
                       Regarding Students' Cognitive-Metacognitive Skills

Investigator: Peter Babich, Olivet Nazarene University Department of Graduate and
                  Continuing Studies, Investigator's Phone: ▮▮▮▮▮▮▮▮ E-mail:
                  ▮▮▮▮▮▮▮▮▮▮

You are being asked to participate in a project conducted through Olivet Nazarene University (and -- if applicable -- any other cooperating institution). The University requires that you give your signed agreement to participate in this project.

The investigator will explain to you in detail the purpose of the project, the procedures to be used, and the potential benefits and possible risks of participation. You may ask him/her any questions you have to help you understand the project. A basic explanation of the project is written below. Please read this explanation and discuss with the researcher any questions you may have.

If you then decide to participate in the project, please sign on the last page of this form in the presence of the person who explained the project to you. You will be given a copy of this form to keep.

1.   **Nature and Purpose of the Project:** You are being asked to participate in a research study to learn more about the attitudes and perceptions you hold regarding teaching strategies related to students solving mathematics problems from a cognitive-metacognitive approach. This first stage involved a questionnaire, and this second stage will include interviews of approximately eight persons. These individuals were chosen based upon responses to the first stage questionnaire.

2.   **Explanation of Procedures:** Your participation in this second stage of the study will consist of answer interview questions. As a participant in this study, you will be asked to come to my classroom to answer a number of questions regarding your initial questionnaire responses and your beliefs, attitudes, and perceptions about student cognitive-metacognitive abilities. I will audio tape this interview and take detailed notes afterward. The interview should take about 30-40 minutes to complete.

3.   **Discomfort and Risks:** Opinions and attitudes you express will be conveyed anonymously in the published research. There is the slight possibility of your identification by potential readers of this study, but this will be minimized by keeping your responses anonymous in the published research. There is also a minimal possibility that based upon your responses, your identification could be a risk to your job. All your responses will be kept confidential, and those published in this study will be kept anonymous to safeguard your identity. Your

participation is voluntary, and you may withdraw at any time. If you withdraw, your interview recording will be destroyed. The researcher will also ask if you would prefer that your questionnaire be destroyed.

4. **Benefits:** Your participation in this study may aid in our understanding of how and what teachers believe regarding students' cognitive and metacognitive abilities.

5. **Confidentiality:** Only the investigator and members of the research team will have access to your interview recording and transcript. If information learned from this study is published, you will not be identified by name. Any excerpts from your responses used for illustrative purposes will be kept anonymous.

6. **Refusal/Withdrawal:** Refusal to participate in this study will have no effect on any future services you may be entitled to from the University. Anyone who agrees to participate in this study is free to withdraw from the study at any time with no penalty.

You understand also that it is not possible to identify all potential risks in an experimental procedure, and you believe that reasonable safeguards have been taken to minimize both the known and potential but unknown risks.

_____   _____

Signature of Participant   Date

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Appendix E

Semi-Structured Interview Questions

Semi-Structured Interview Questions:

1. What is your definition of "Thinking mathematically"? Of "thinking about thinking"?

2A. To what extent, if any, do you teach students to think mathematically? Is there a difference between thinking mathematically and doing mathematics?

2B. To what extent, if any, do you teach students to think about thinking in mathematics classes?

3A. In what ways do you have students reflect upon their thinking (such as on graded assessments or via I can statements)?

3B. To what extent, if any, have these efforts been effective? Are there ways in which they could have been or could be improved?

4A. To what extent is it necessary for students to have a conceptual understanding of a topic to be successful in thinking about or applying that concept?

4B. To what extent is that conceptual understanding necessary to be successful at incorporating related procedural processes (such as factoring or solving certain equations)?

5. In what ways, if any, does the type of class (remedial, regular, honors, special education) affect your method(s) of instruction as related to thinking mathematically?

6. What are your opinions regarding the effect of school-wide initiatives related to student metacognition (the reading course and school problem solving goal)?

7. In your opinion, how do "silly mistakes" factor into student thinking (related to causes, rationale, and or possible avoidance)?