The Impact of Early Order Commitment on the Performance of a Simple Supply Chain

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Abstract

An example of supply chain coordination is early order commitment, wherein a retailer commits to purchase a fixed-order quantity and delivery time from a manufacturer before the real need takes place. In this paper, an analytical model is developed that quantifies the impact of early order commitment on the performance of a simple two-level supply chain consisting of a single manufacturer and a single retailer. The model reveals that the effect of early order commitment depends on a lot of factors such as the cost structure of the supply chain, the lengths of manufacturing and delivery lead times, and the correlation of the demand over time. This model can be used to evaluate the benefit of early order commitment, to determine the optimal early commitment periods of the supply chain, and to estimate the maximum incentives the manufacturer can provide to encourage the retailer to commit its orders in advance.

Keywords: Supply Chain Coordination, Early Order Commitment, Inventory Control

1. Introduction

Effective supply chain management requires coordination among the various members in a supply chain. Through coordinating activities across the boundaries of firms in a supply chain, it is believed that significant benefits can be achieved for the partners and the entire supply chain. Previous research has recognized different approaches for supply chain coordination, including information sharing -- sharing real-time demand data collected at the point-of-sales with upstream suppliers ([1],[6], [7]). A further-developed method concerning the effective use of such information suggests a centralized forecasting mechanism that accesses the final demand ([2], [3], [8]). These investigations reveal that information sharing can significantly enhance the performance of the supply chain by reducing the bullwhip effect, a negative phenomenon of demand variability amplification along a supply chain from downstream members to their suppliers.

Recently, as another alternative form of supply chain coordination, early order commitment has drawn attention from the researchers and practitioners. Early order commitment means that a retailer commits to purchase a fixed-order quantity and delivery time from a manufacturer before the real need takes place. [5] presented an analysis for a steel distribution supply chain, and quantified the benefits for the consumers who commit orders in advance. [9-11] conducted extensive simulation studies on the effect of early order commitment on supply chain performance under various operational conditions. These researches show that under some cases, practicing early order commitment can generate significant cost savings in the supply chain. [4] examined the tradeoff between strategic early order commitment and postponement. Generally speaking, the impact of early order commitment on the supply chain performance is intuitively clear. On one hand, early order commitment increases a retailer's risks of over-estimating the demand; On the other hand, it helps the manufacturer reduce planning and executing costs. In order to accept early order commitment to achieve best performance for the entire supply chain, the members at different levels of the supply chain should make optimal tradeoff based on careful evaluation of both the negative effect for the retailer and the positive effect for the manufacturer. Unfortunately, up to now, no general analytical model is available to quantify the integrated effect of early order commitment and to guide such kind of tradeoff decisions. Besides, a fundamental question about early order commitment is why should a retailer make commitment (with penalty charge) if information sharing (without cost penalty for order commitment) can provide equal or close enough benefits!

This paper aims at proposing an analytical model to quantify the impact of early order commitment on the performance of a simple two-level supply chain consisting of a single manufacturer and a single retailer. The model reveals the effect of early order commitment depends on a lot of factors such as the cost structure of the supply chain, the length of manufacturing and delivery lead time, and the correlation of the demand over time. Furthermore, under some cases, significant benefit can be achieved for the supply chain even under the environment with information sharing. This model can be used to evaluate the benefit of early order commitment, to determine the optimal early commitment periods of the supply chain, and to estimate

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the maximum incentives the manufacturer can provide to encourage the retailer to commit its orders in advance.

2. Supply Chain Model

2.1 Basic Assumptions

The basic assumptions underlying the model in this paper is similar to the one proposed by LST. The supply chain is assumed to be a simple one consists of a single manufacturer and a single retailer. Only the retailer faces external demand for a single product, and the demand is assumed to be a simple autocorrelated AR(1) process, i.e., the demand in period t is

$$D_t = d + \rho D_{t-1} + \varepsilon_t, \qquad (1)$$

where d>0, -1< ρ <1, and ε_t is *i.i.d.* normally distributed with mean zero and variance σ^2. We also assume that σ is significantly smaller than d, so that the probability of negative demand is negligible. Furthermore, the demand process and its characteristic parameters are common knowledge, i.e., both the retailer and the manufacturer know the demand distribution in (1) and the values of the parameters d, ρ, and σ. LST gave some evidences to show that the assumption of AR(1) demand process with known characteristic parameters is reasonable in real world supply chain management when demand information is shared from the retailer to the supplier.

The manufacturing leadtime for the manufacturer (including the leadtime for the replenishment of raw materials from the external suppliers) is a constant L, and the delivery leadtime from the manufacuturer to the retailer is a constant l. The most important feature of the current paper is that we incorporate an early order commitment period (A) to the system. Early order commitment period is the number of the time periods that the retailer places her order in advance.

Both the retailer and the manufacturer use the order-up-to policy, a periodic reviewing policy which is optimal for the stochastic inventory system without fixed ordering cost, to make their ordering (or manufacturing) decisions with the review interval being one period (i.e., daily review). The events occur in sequence as follows. Before the end of period t, after demand D_t has been realized, the retailer places her order of size O_t based on her inventory level. Please note that, because of the early order commitment, this order is scheduled for the period t+l+A+1, not for the period t+l+1. That's to say, because of the early order commitment period A and delivery leadtime l, this order will arrive the retailer at the beginning of period t+l+A+1. If the retailer does not hold enough stock to satisfy the demand, the excess demand is backordered. When the manufacturer receives this order from the retailer, she does not need to ship the order O_t to the retailer immediately. In fact, the quantity the manufacturer must ship to the retailer is the order placed by the retailer A periods ago, i.e. the order of size O_{t-A}. We assume that this order must always be completely filled by the manufacturer. This means that if the manufacturer does not hold enough stock to fill the order, she can obtain any quantity of the product from an external source with a penalty cost, where the same quantity of the product must be provided by the manufacturer to resupply the external source later. Finally, the manufacturer places his manufacturing order of size Q_t based on his inventory level. This order will arrive at the beginning of period t+L+1.

2.2 Retailer's Ordering Decision

Let S_t be the retailer's order-up-to level in period t, minimizing the total expected holding and shortage costs in period t+l+A+1. Comparing our case with the case in LST, we found that the only difference is that in LST's case, the retailer's order must cover the demand uncertainty up to t+l+1, while in our case the retailer's order must cover more periods' demand uncertainty, i.e., the uncertainty up to t+l+A+1. In fact, if we take the value l+A as the leadtime periods, then all the behavior of the retailer will be the same as in LST.

Denote X_t as the total demand during periods [t+1, t+l+A+1], then using (1) we have

$$\begin{aligned} X_t &= \sum_{j=1}^{l+A+1} D_{t+j} \\ &= \frac{1}{1-\rho}\left\{ d\sum_{j=1}^{l+A+1}(1-\rho^j) + \rho(1-\rho^{l+A+1})D_t \right\} \\ &+ \sum_{j=1}^{l+A+1}\sum_{i=0}^{l+A-j+1}\rho^i \varepsilon_{t+j} \end{aligned} \qquad (2)$$

From (2), X_t is a normal distributed variable with the mean value

$$\begin{aligned} m_t = E(X_t) &= \frac{d}{1-\rho}\left\{ (l+A+1) - \sum_{j=1}^{l+A+1}\rho^j \right\} \\ &+ \frac{\rho(1-\rho^{l+A+1})}{1-\rho} D_t \end{aligned} \qquad (3)$$

and the variance

$$v_t = Var(X_t) = \frac{\sigma^2}{(1-\rho)^2}\sum_{j=1}^{l+A+1}(1-\rho^j)^2. \qquad (4)$$

Thus the order-up-to level

$$S_t = m_t + k\sqrt{v_t}, \qquad (5)$$

where k is the safety stock factor depending on the unit holding cost h and unit shortage penalty cost p. In fact, if Φ is the standard normal distribution function, then

$$k = \Phi^{-1}\left(\tfrac{p}{h+p}\right). \qquad (6)$$

Therefore the retailer's order quantity at period t is

$$\begin{aligned} O_t &= D_t + (S_t - S_{t-1}) \\ &= D_t + \frac{\rho(1-\rho^{l+A+1})}{1-\rho}(D_t - D_{t-1}). \end{aligned} \qquad (7)$$

2.3 Manufacturer's Ordering Decision

Throughout this paper, we assume that the early order commitment period A is no more than the manufacturing leadtime L, i.e., $0 \le A \le L$, since committing the orders even earlier is harmful to the retailer and not beneficial to the manufacturer. Please note that A=0 corresponds to the case that the supplier does not commit orders to the manufacturer in advance. As we have pointed out in Section 2.1, the quantity the manufacturer must ship to the retailer in period t is the order placed by the retailer A periods ago, i.e. the order of size O_{t-A}. From (1) and (7), we have

$$O_{t+1} = d + \rho O_t + \frac{1-\rho^{l+A+2}}{1-\rho}\varepsilon_{t+1} - \frac{\rho(1-\rho^{l+A+1})}{1-\rho}\varepsilon_t. \quad (8)$$

Repeating to use (8) gives

$$O_{t+i} = \frac{1-\rho^i}{1-\rho}d + \rho^i O_t + \frac{1-\rho^{l+A+2}}{1-\rho}\varepsilon_{t+i} + \sum_{k=1}^{i-1}\rho^{l+A+1+k}\varepsilon_{t+i-k} - \frac{\rho^i(1-\rho^{l+A+1})}{1-\rho}\varepsilon_t. \quad (9)$$

Denote Y_t as the total orders that must ship by the manufacturer during periods [t+1, t+L+1], then

$$\begin{aligned} Y_t &= \sum_{j=1}^{L+1} O_{t-A+j} = \sum_{j=1}^{A} O_{t-A+j} + \sum_{j=1}^{L-A+1} O_{t+j} \\ &= \sum_{j=1}^{A} O_{t-A+j} + \frac{d}{1-\rho}\left\{(L-A+1) - \frac{\rho(1-\rho^{L-A+1})}{1-\rho}\right\} \\ &\quad + \frac{\rho(1-\rho^{L-A+1})}{1-\rho}O_t + \frac{1}{1-\rho}\sum_{j=1}^{L-A+1}(1-\rho^{L+l+3-j})\varepsilon_{t+j} \\ &\quad - \frac{\rho(1-\rho^{L-A+1})(1-\rho^{l+A+1})}{(1-\rho)^2}\varepsilon_t. \end{aligned} \quad (10)$$

Let T_t be the manufacturer's order-up-to level at the end of period t. We assume that the retailer shares her demand information to the manufacturer, thus the manufacturer knows both the retailer's order quantity O_t and demand error ε_t up to period t. Raghunathan (2001) showed that the cases with information sharing and without information sharing do not make a great difference.

From (10), Y_t is a normal distributed variable with the mean value

$$\begin{aligned} M_t = E(Y)_t &= \sum_{j=1}^{A} O_{t-A+j} + \frac{d}{1-\rho}\left\{(L-A+1) - \frac{\rho(1-\rho^{L-A+1})}{1-\rho}\right\} \\ &\quad + \frac{\rho(1-\rho^{L-A+1})}{1-\rho}O_t - \frac{\rho(1-\rho^{L-A+1})(1-\rho^{l+A+1})}{(1-\rho)^2}\varepsilon_t. \end{aligned} \quad (11)$$

and the variance

$$V_t = Var(Y_t) = \frac{\sigma^2}{(1-\rho)^2}\sum_{j=1}^{L-A+1}(1-\rho^{L+l+3-j})^2. \quad (12)$$

Thus the order-up-to level

$$T_t = M_t + K\sqrt{V_t}, \quad (13)$$

where K is the safety stock factor depending on the unit holding cost H and unit shortage penalty cost P. In fact, if Φ is the standard normal distribution function, then

$$K = \Phi^{-1}\left(\tfrac{P}{H+P}\right). \quad (14)$$

Therefore the manufacturer's order quantity at period t is

$$\begin{aligned} Q_t &= O_t + (T_t - T_{t-1}) \\ &= O_t + \frac{\rho(1-\rho^{L-A+1})}{1-\rho}(O_t - O_{t-1}) \\ &\quad - \frac{\rho(1-\rho^{L-A+1})(1-\rho^{l+A+1})}{(1-\rho)^2}(\varepsilon_t - \varepsilon_{t-1}). \end{aligned} \quad (15)$$

3. Supply Chain Performance

In this section we evaluate the performance of the supply chain in terms of inventory holding and shortage cost.

3.1 Supply Chain Cost

According to LST (2000), the retailer's and manufacturer's expected inventory holding and shortage costs in a period can be expressed, respectively, as

$$c = \sqrt{v_t}[(h+p)F(k) + hk], \quad (16)$$

$$C = \sqrt{V_t}[(H+P)F(K) + HK], \quad (17)$$

where $F(\bullet)$ is the right loss function for the standard normal distribution, i.e.,

$$F(x) = \int_x^{\infty}(z-x)d\Phi(z). \quad (18)$$

Thus the percentage of the retailer's cost increasing in a period due to early order commitment is

$$\begin{aligned} \Delta c &= (\sqrt{v_t} - \sqrt{v_t|_{A=0}})/\sqrt{v_t|_{A=0}} = \sqrt{v_t}/\sqrt{v_t|_{A=0}} - 1 \\ &= \sqrt{\sum_{j=1}^{l+A+1}(1-\rho^j)^2} \Big/ \sqrt{\sum_{j=1}^{l+1}(1-\rho^j)^2} - 1 \\ &= \sqrt{1 + \sum_{j=l+2}^{l+A+1}(1-\rho^j)^2 \Big/ \sum_{j=1}^{l+1}(1-\rho^j)^2} - 1 \end{aligned} \quad (19)$$

and the percentage of the manufacturer's cost saving in a period due to early order commitment is

$$\begin{aligned} \Delta C &= (\sqrt{V_t|_{A=0}} - \sqrt{V_t})/\sqrt{V_t|_{A=0}} = 1 - \sqrt{V_t}/\sqrt{V_t|_{A=0}} \\ &= 1 - \sqrt{\sum_{j=1}^{L-A+1}(1-\rho^{L+l+3-j})^2} \Big/ \sqrt{\sum_{j=1}^{L+1}(1-\rho^{L+l+3-j})^2} \\ &= 1 - \sqrt{1 - \sum_{j=L-A+2}^{L+1}(1-\rho^{L+l+3-j})^2 \Big/ \sum_{i=1}^{L+1}(1-\rho^{L+l+3-j})^2} \end{aligned} \quad (20)$$

According to (16) and (17), the total cost of the supply chain is

$$\begin{aligned} SC &= c + C \\ &= \sqrt{v_t}[(h+p)F(k) + hk] + \sqrt{V_t}[(H+P)F(K) + HK]. \end{aligned} \quad (21)$$

In order to simplify the expression, we introduce a cost structure ratio (r) to represent the cost structure of the supply chain. Specifically, we define

$$r = [(h+p)F(k) + hk]/[(H+P)F(K) + HK]. \quad (22)$$

Then the percentage of the entire supply chain's cost saving in a period due to early order commitment is

$$\Delta SC = [(c+C)|_{A=0} - (c+C)]/(c+C)|_{A=0}$$
$$= 1 - \frac{r\sqrt{v_t} + \sqrt{V_t}}{(r\sqrt{v_t} + \sqrt{V_t})|_{A=0}}$$
$$= 1 - \frac{r\sqrt{\sum_{j=1}^{l+A+1}(1-\rho^j)^2} + \sqrt{\sum_{j=1}^{L-A+1}(1-\rho^{L+l+3-j})^2}}{r\sqrt{\sum_{j=1}^{l+1}(1-\rho^j)^2} + \sqrt{\sum_{j=1}^{L+1}(1-\rho^{L+l+3-j})^2}}. \quad (23)$$

Obviously, $\Delta c > 0$ and $\Delta C > 0$ are both increasing function with respect to the early order commitment period A. Formula (19) shows that the retailer's relative cost increase due to early order commitment depends on the correlation of the demand process, the delivery leadtime from the manufacturer to the retailer, and the early order commitment period. Besides these three factors, formula (20) shows that another factor, the manufacturer's replenishment leadtime, affects the manufacturer's relative cost saving. Furthermore, formula (23) shows that the cost structure ratio also affects the integrated performance of the whole supply chain due to early order commitment. However, these results reveal that although the variance of the error item in the demand process (1) amplifies the absolute values for the costs, it does not have any impact on the relative cost increase or decrease for the retailer, the manufacturer, and the whole supply chain, unless there is no demand uncertainty (i.e., the variance of the error item is zero).

From (23), we can easily obtain the following condition where the early order commitment is beneficial to the whole supply chain: $\Delta SC > 0,$ or equivalently

$$r\left(\sqrt{\sum_{j=1}^{l+A+1}(1-\rho^j)^2} - \sqrt{\sum_{j=1}^{l+1}(1-\rho^j)^2}\right) < \sqrt{\sum_{j=1}^{L+1}(1-\rho^{L+l+3-j})^2} - \sqrt{\sum_{j=1}^{L-A+1}(1-\rho^{L+l+3-j})^2}. \quad (24)$$

This is the critical condition that guides us to determine whether we should use the early order commitment or not in a supply chain. The smaller the cost structure ratio r, the more that early order commitment can benefit the whole supply chain.

In order to get more insights about the interaction between early order commitment and the parameters of the supply chain, we conduct some numerical results in the following Subsections.

3.2 Performance of Early Order Commitment

In this Subsection, we fix the parameters r=1, L=l=10, ρ=0.5, and vary A from 0 to L and calculate the corresponding relative cost increase or decrease for the retailer, the manufacturer, and the whole supply chain according to (19), (20) and (23). The corresponding relative cost increase or decrease are plotted in Figure 1. From this example, we can see that the earlier the retailer commit orders with the manufacturer, the more benefits can be achieved for the total supply chain. When A reaches the largest value (10 periods), the retailer's cost is increased by 43.92%, the manufacturer's cost is reduced by 69.85%, and the whole supply chain's cost is reduced by 15.30%.

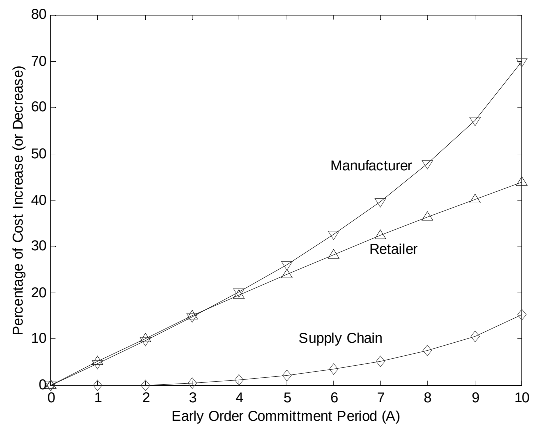

Figure 1. Performance of early order commitment period (r=1, L=l=10, ρ=0.5)

3.3 Impact of Demand Process Characteristics

In this Subsection, we first fix the parameters r=1, L=l=10 and vary ρ from –0.9 to +0.9 with increment 0.1. For each value of ρ, we vary A from 0 to L and calculate the corresponding relative cost increase or decrease for the retailer, the manufacturer, and the whole supply chain according to (19), (20) and (23). Then the value of A that results in the largest cost saving for the whole supply chain according to (23) is recorded as A^*, the optimal early order commitment period. We find that the optimal value of A^* =L=10 periods under all the values of ρ for this setting. The corresponding relative cost increase or decrease are plotted in Figure 2. From the figure, we can see that the retailer's cost increases quickly as ρ increases to 0.5 or above. However, it shows that the cost savings for the manufacturer and the whole supply chain is about 70% and 15% respectively, and the results are relatively stable under different values of ρ.

However, there are interactions between A, ρ and the leadtimes L, l. Figure 3 represents the results for fixed parameters r=1, L=10, l=3. Similarly, we vary ρ from –0.9 to +0.9 with increment 0.1, and for each value of ρ, we vary A from 0 to L to find A^*, the optimal early order commitment period. We find that the optimal value of A^* equals ten periods (A=L=10) when ρ is not over 0.7, and A^*

=0 periods when ρ is larger than 0.8. From the figure, we can see that the retailer's cost increase due to early order commitment climbs to extremely higher value as ρ increases, and finally exceeds the cost savings of the manufacturer. Under the cases where ρ is smaller than 0.5, the cost savings of the manufacturer and of the whole supply chain are about 70% and 10%, respectively.

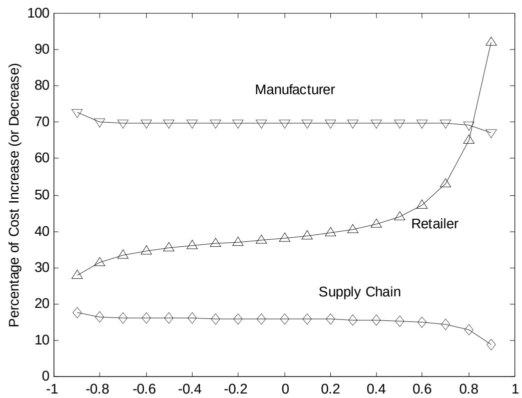

Figure 2. The impact of demand correlation (r=1, L=l=10)

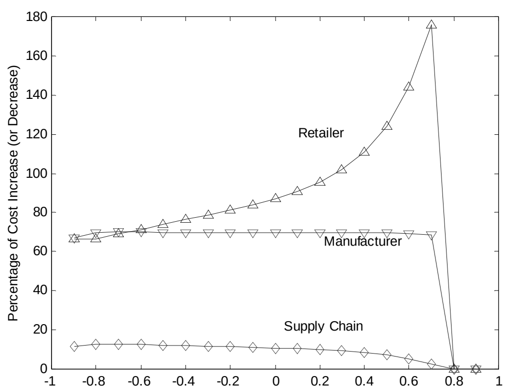

Figure 3. The impact of demand correlation (r=1, L=10, l=3)

3.4 Impact of Leadtimes

In this Subsection, we first fix the parameters r=1, ρ=0.5, L=10 and vary l from 0 to 20. For each value of l, we vary A from 0 to L to find A^*, the optimal early order commitment period. We find that A^* =L=10 periods when l is greater than or equals to 2, and A^* =0 when l is very small (0 or 1). The corresponding relative cost increase or cost decrease is plotted in Figure 4.

Now we fix the parameters r=1, ρ=0.5, l=10 and vary L from 0 to 20. For each value of L, we vary A from 0 to L to find A^*, the optimal early order commitment period. We find that A^* =L periods for all values of L. The corresponding relative cost increase or cost decrease is plotted in Figure 5.

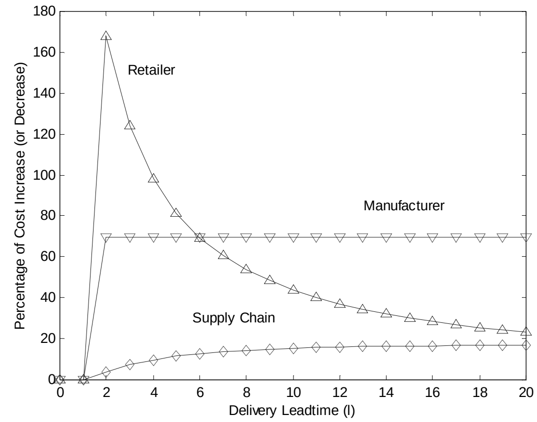

Figure 4. The impact of Delivery Leadtime (r=1, ρ=0.5, L=10)

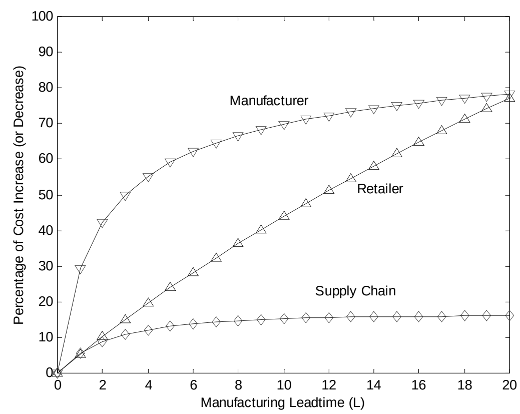

Figure 5. The impact of Manufacturing Leadtime (r=1, ρ=0.5, l=10)

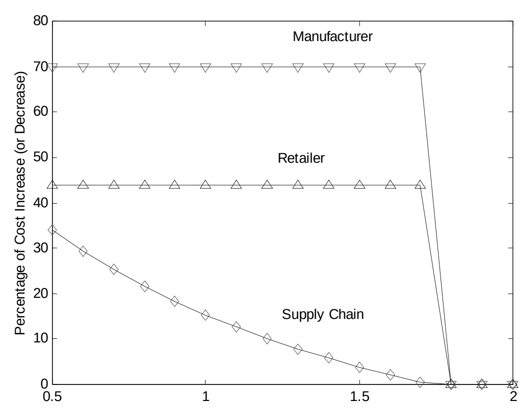

Figure 6. The impact of Cost Structure Ratio (ρ=0.5, L=l=10)

3.5 Impact of Cost Structures Ratio

In this Subsection, we fix the parameters ρ=0.5, L= l =10 and vary r from 0.5 to 2.0 with increment 0.1. For each value of r, we vary A from 0 to L to find A^*, the optimal early order commitment period. We find that A^* =L=10 periods when r is smaller than or equals to 1.7, and A^* =0 when r is relatively loarge (over 1.8). The corresponding relative cost increase or decrease are plotted in Figure 6. Obviously, the cost savings decrease as r increases.

4. Conclusions

In this paper, an analytical model is developed that quantifies the impact of early order commitment on the performance of a simple two-level supply chain consisting of a single manufacturer and a single retailer. The model reveals that the effect of early order commitment depends on some key factors such as the cost structure of the supply chain, the lengths of manufacturing and delivery lead times, and the correlation of the demand over time. This model can be used to evaluate the benefit of early order commitment, to determine the optimal early commitment periods of the supply chain, and to estimate the maximum incentives the manufacturer can provide to encourage the retailer to commit its orders in advance.

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