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Abstract

Errors or inaccuracy always occurs when we use the Analytic Hierarchy Process to aid decision making. This paper shows the errors can be divided into two parts. One is called System Error and another is called Judgment Error. System Error is caused by the judgment ratio of pairwise compare matrix which must be taken from set of {1/9, 1/8, ..., 1,2,..., 9}. The Judgment Error is caused by human wrong judgment. The computational results in this paper demonstrate that the System Error may cause the confusable priority of the alternatives and a proposed method which increase the ratio accuracy can clear the priority of the alternatives.

Key Word: Decision Support, Analytic Hierarchy Process (AHP), Reversal Rank, System Error

1.Introduction

The Analytic Hierarchy Process (AHP) is a powerful decision support tools which aid decision makers in solving complex decision problems. It decomposes the real complex decision problems into several levels of hierarchies. The first level of the hierarchy is the goal of decision problem, the second and the lower levels are placed by many relevant criteria, subcriteria, and/or alternatives. Usually, the alternatives occupy the last level of the hierarchy and represent the goal to be accomplished. The decision maker assesses a pairwise comparison judgment matrix (PCJM(n)) for each level of hierarchy to establish local vectors of priorities for n criteria, subcriteria, and alternatives. The composite vector of priorities for decision alternatives is obtained by combining the local priority vectors associated with the criteria or subcriteria of all levels of the hierarchy.

How to get accurant priority of alternatives from the judgments given by decision maker is a important problem in decision making science [3]. In the Analytic Hierarchy Process (AHP), this problem is solved by getting the information of redundant judgments of a decision maker. According to the AHP theory, the decision maker is required to give $n(n-1)/2$ pairwise judgments ratio for an n alternatives decision problem and only $(n-1)$ judgments is necessary. The AHP method modifies the accuracy of judgments by using the $n(n-1)/2$ - $(n-1)$ redundant judgments [1]. But using the AHP makes at least two kinds of errors. One is called System Error (SE) and another is called Judgment Error (JE). SE is caused by the judgments ratio must be taken from the set = {1/9, 1/8, ..., 1,2,..., 9}. The JE is caused by human wrong judgment. Now we focus on the SE problems and propose how to clear the confusable priority which caused by SE.

When we use the AHP to aid in our decision making, the System Error is always caused since the judgment ratio must be taken from the set S1= {1/9, 1/8, ..., 1, 2, ..., 9}. Therefore, the ratio is approximate to the real value. Here the real value of weight of alternatives is continuous, which is true in more application problems. The priority we obtain by using the AHP is also an approximation of the true priority. In our research, we try to find the difference between the computational priority and the real priority.

Triantaphyllou and Mann gave some relative research in this problem. He concluded that it is possible for some alternatives to have the same rank while in reality they are distinct [4]. In additionally, he demonstrated that it is possible alternatives which in reality are less important than others to appear to be more important after the AHP are used. In our present simulation, we confirm the Trantaphyllou and Mann's results and suggest a method to avoid reversal problem caused by SE.

2.Our Simulation Analysis

As assumption that the real weights of alternatives are continuity, the $w_1, w_2, ..., w_n$ are the real weight values of the n alternatives. We suppose that the decision maker knew the above real values and he would be able to construct a matrix with the real pairwise comparisons. This matrix is called Real Continuous Pairwise matrix (RCP). In RCP, each element, a_{ij} is obtained by the ratio w_i/w_j. Also, the judge would be able to determine the entry a_{ij} as close as possible w_i/w_j value taken from the set S1 = {1/9, 1/8, ..., 1, 2, ..., 9} when the AHP is used. Notice that this determination has a litter bit difference with Trantaphyllou and Mann's. It is because the matrix is a reciprocal matrix. The minimal of $|a_{ij} - w_i/w_j|$ is not same as that of $|1/a_{ij} - w_j/w_i|$ when the $w_i/w_j < 1$. This matrix is called Closed Discrete Pairwise matrix (CDP).

Our simulation includes that randomly generate the real vector of the alternatives, construct the RCP and CDP, calculate the eigenvectors of RCP and CDP, then compare the eigenvectors with the real vector. In generating the real vector, the ratio of largest against smallest value is less than 9 since the comparable axiom limited [1]. The eigenvectors of RCP and CDP are calculated by Power Method because it is suit for the simulation [2]. The comparison of the vectors just compare the rank of the vectors. That is, if the eigenvector of the CDP has the same rank with real vector, the Correct Sign (CS) is given. If the eigenvector of the CDP has rank changed for real vector rank, the Rank changed Sign (RS) is given. If some elements of eigenvector of the CDP have the same rank while in real vector they are distinct, the causing Equal elements Sign (ES) is given. Here is a number example which given in the Trantaphyllou and Mann's paper, but the result is not exact same as theirs because their calculation has some mistake.

Real vector : (0.1325 0.0890 0.5251 0.2533)

$$RCP = \begin{bmatrix} 1.0000 & 1.4888 & 0.2523 & 0.5231 \\ 0.6717 & 1.0000 & 0.1695 & 0.3514 \\ 3.9603 & 5.9000 & 1.0000 & 2.0730 \\ 1.9117 & 2.8461 & 0.4824 & 1.0000 \end{bmatrix}$$

Eigenvector of RCP : (0.1325 0.0890 0.5252 0.2533)

$$CDP = \begin{bmatrix} 1.0000 & 1.0000 & 0.2500 & 0.5000 \\ 1.0000 & 1.0000 & 0.1667 & 0.3333 \\ 4.0000 & 6.0000 & 1.0000 & 2.0000 \\ 2.0000 & 3.0000 & 0.5000 & 1.0000 \end{bmatrix}$$

Eigenvector of CDP: (0.1187 0.0970 0.5229 0.2614)

In this example, for CDP, the CS = 1, RS = 0 and ES =0.

Let us show another example:

Real vector : (0.0486 0.3551 0.2558 0.3406)

$$RCP = \begin{bmatrix} 1.0000 & 0.1368 & 0.1899 & 0.1426 \\ 7.3111 & 1.0000 & 1.3881 & 1.0426 \\ 5.2671 & 0.7204 & 1.0000 & 0.7511 \\ 7.0126 & 0.9592 & 1.3314 & 1.0000 \end{bmatrix}$$

Eigenvector of RCP : (0.0486 0.3551 0.2558 0.3406)

$$CDP = \begin{bmatrix} 1.0000 & 0.1429 & 0.2000 & 0.1429 \\ 7.0000 & 1.0000 & 1.0000 & 1.0000 \\ 5.0000 & 1.0000 & 1.0000 & 1.0000 \\ 7.0000 & 1.0000 & 1.0000 & 1.0000 \end{bmatrix}$$

Eigenvector of CDP : (0.0507 0.3249 0.2996 0.3249)

In this example, for CDP, the CS = 0, RS = 0 and ES =1.

From later example, we find the ES is caused by the approximation of ratio especially the ratio which equal to 1. That is, these alternatives have very close weight values. If we increase the accuracy of ratio of these weight values to 0.1, the eigenvector of CDP should present the correct rank. The following is the eigenvector of revised CDP:

$$CDP1 = \begin{bmatrix} 1.0000 & 0.1370 & 0.1887 & 0.1429 \\ 7.3000 & 1.0000 & 1.4000 & 1.0000 \\ 5.3000 & 0.7143 & 1.0000 & 0.7692 \\ 7.0000 & 1.0000 & 1.3000 & 1.0000 \end{bmatrix}$$

Eigenvector of CDP : (0.0485 0.3521 0.2573 0.3421)

In this example, for CDP1, the CS1 = 1, RS1 = 0 and ES1 =0. This revision which increase the accuracy is reasonable because we must increase the accuracy of measurement to determine the priority of the alternatives when present accuracy cannot identify difference between some alternatives.

Table 1 shows the results of our simulation for 1000 times and comparison results between real vector rank and eigenvector rank of CDP, CDP1, CDP2, CDP3 and CDP4 while eigenvector rank of RCP always same as the real vector rank.

Table 1. The Comparison Between Real Vector and eigenvector of CDP

Dimension	RS	ES	ES1	ES2	ES3	ES4
15	0	928	287	23	2	0
14	0	920	271	20	2	0
13	0	922	243	16	1	0
12	0	903	223	10	0	0
11	0	891	231	12	1	0
10	0	856	191	10	0	0
9	0	819	175	12	1	1
8	0	775	161	16	1	0
7	0	740	130	10	1	1
6	0	679	121	7	0	0
5	0	624	113	0	1	0
4	0	517	89	5	0	0
3	0	486	65	2	0	0

3.Conclusions and discussions

In this paper, we do a preliminary simulation on the System Error which caused by the require the ratio taken from the set S1 = {1/9, 1/8, ..., 1, 2, ... 9}. From the simulation, we find the System Error may make the rank of the CDP confusable especially when the number of the alternatives increased, but it doesn't make the reverse rank in the rank of the CDP. In order to clear the confusable of the rank of the alternatives, we propose a method which increases the accuracy of the pairwise ratio of those close alternatives. By using the revised method, the number of the confusable rank decrease very fast. Therefore, we can conclude that the revised method is an effective method to solve the confusable rank of alternatives.

There are other problems relative to the System Error. In Trantaphyllou and Mann's paper, they found for the two level of a problem, the System Error causes the reversal problem. Therefore, the further research needed to do on this topic. These research include:

1. Can the revised method be used to solve the reversal problem which mentioned in Trantaphyllou and Mann's paper.
2. How to detect the Judgment Error? First, the model of judge has to be established. The model is decide by the probability of making wrong judgment when the real is *r*. Suppose the wrong ratio will be given in AHP is r, $r\pm1$, $r\pm2$, ..., $r\pm9$. As judge intent to get the correct ratio, the first corresponding probabilities are $9P_{r\pm9}$, $8P_{r\pm9}$, ..., $1P_{r\pm9}$. The $P_{r\pm9}$ is the probability which judge give the $r\pm9$ value. Then $P_{r\pm9} = 1/45$. $9P_{r\pm9} = 20\%$. That means judge will give correct ratio *r* at probability 20%. The second corresponding probabilities are $2^8P_{r\pm9}$, $2^7P_{r\pm9}$, ..., $2^0P_{r\pm9}$. Then $P_{r\pm9} = 1/511$, $2^8P_{r\pm9} = 256/511 = 50.098\%$.
3. How to measure the System Error when we do not know the real weight of the alternatives?
4. The inconsistency of the PCJM is partly caused by system error and the judgment error. Which one causes the inconsistency more? It should be true that Judgment Error more than the System Error?
5. Can the System Error and the Judgment Error be used as another method to determine the consistency ratio?

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Title page:

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