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Abstract

The primary objective of this study is to explore the various ways that Internet advertising provider (IAP) can charge firms to advertise. There are two pricing strategies that can be adopted. The first is called *uniform pricing*: Firms pay a fixed fee, depending on the size and location of the advertisement. The second strategy is known as *two-part tariff*: Firms pay a fixed charge and an additional per-click fee. IAPs may adopt one or both of these pricing strategies. Our model hypothesizes that there are two IAPs offering advertising space. The modeling shows that in cases where a uniform pricing strategy is adopted, the fee that each IAP can attain is monotonically decreasing over their substitutability. That is, IAPs that wish to maximize their revenue have to take measures to distinguish themselves from the competition in order to achieve zero substitutability. In cases where a two-part tariff pricing strategy is adopted, the revenue curve becomes convex. In other words, IAPs may choose to have either full substitutability or zero substitutability in order to maximize their profits.

1. Introduction

The ever-increasing expansion of the Internet has led to the dominance of Internet companies such as Amazon and Yahoo!. These companies offer not only their own services but also advertising space to firms wishing to promote their products or brands. Hereafter, these companies are referred as Internet Advertising Providers (IAPs). Visitors to these sites can access their services and also find themselves directed to other sites by simply clicking on banners on the Internet pages. For the surfer, these banners are no more than just icons offering access to other websites. However, for IAPs that go to great expense to construct and maintain these websites, these banners may be the only source of revenue. In other words, double clicking on those banners has becoming a new medium in advertising. This has led to the creation of a new advertising industry called Internet marketing.

In terms of the significance of Internet marketing, although its popularity began only a few years ago, its potential is tremendous. In the US for example, the revenue from Internet advertising amounted to $2 billion in 1998, according to a report by the Internet Advertising Bureau (IAB, 2000). The same report also estimates this may have exceeded $4 billion in 1999. In Japan, the revenue from Internet advertising is estimated at ¥11.4 billion in 1998. In Taiwan, the Internet advertising revenue is estimated at NT$100 million (Yeh, 2000). Compared to the annual advertising budget of NT$80 billion in Taiwan, this is only a fairly small portion. Nevertheless, with a growth rate that has doubled annually in the past few years, this clearly should not be ignored.

Internet marketing is not only significant in this practical way but is also important in academic terms, with research into areas such as pricing and competition. Generally, IAPs may collect revenue from two sources: Advertisement Impression (AI) and Click Through Rate (CTR). AI represents an advertisement delivered successfully to a qualified visitor. A qualified visitor is one who has the opportunity to view the advertisement. Therefore, this does not include cases where sites bearing the advertisement are simply listed as results of a search. Clearly, visits to those sites by internal users are also excluded. CTR represents the number of times visitors click on an advertisement banner and are then directed to the target site. For payment purposes, firms that wish to have a banner on an IAP's web page are usually asked to pay a lump sum fee. Hereafter this lump sum fee is referred to as the AI fee. If a visitor clicks on their banner and is successfully directed to the target site, the provider may then collect a running payment according to the number of clicks each banner receives. Hereafter, this running payment is referred as the *referral fee*. (In some cases, however, the provider may choose not to charge unless this results in a business for the target site.) An interesting question is how the provider can set those fees. Do pricing strategies exist that can help providers maximize their revenue?

1.1. Internet vs. traditional advertising media

We begin our investigation of those questions by comparing the practice of Internet advertising with other traditional forms of advertising media, such as magazine, TV and radio. It has always been a challenge in measuring the effectiveness of advertising with traditional media. This may be due to the lack of accurate ways to assess the link between traditional advertising and consumer purchasing behavior. Therefore, traditional media can only charge clients according to impression numbers. However, one significant difference we can see in Internet advertising is that CTR tracing technology provides a much better estimate of the power of advertisements. The server proxy offers a detail recording on Internet visitors' clicking actions. Furthermore, if firms can agree to inform providers when visitors make a purchase, then the effectiveness of advertisements can be measured with the utmost accuracy. In this case, firms can pay a referral fee only for "successful" advertising.

1.2. Non-linear pricing

Since the missing link in measuring the advertising effectiveness between advertising and purchase in traditional advertising media can be recovered with the CTR technology via Internet, the next question is what pricing strategy (uniform pricing or two-part tariff) will the IAPs employ to maximize their profit. Is two-part tariff a superior pricing strategy to uniform pricing, which is widely adopted in the area of traditional advertising media? Among the several tariff schemes in nonlinear pricing strategy, two-part tariff is the most widely used pricing strategy. Firms pay a fixed charge and an additional per-click fee. From the provider's viewpoint, the fixed fee is less attractive to firms but results in higher overall revenue (Wilson, 1992). Nahata, et al. (1999) show that "…, a fixed investment that …reduces future transaction cost, a two-part tariff [or multi-part tariff]… becomes more profitable (than uniform price)." Compared with the huge investment involved in website development, the cost of putting a banner on a web page is negligible. In other words, the nature of Internet advertising may favor the two-part (or maybe multi-part) tariff system.

Since the introduction of the Internet, an impression has been created that the cost of doing things through the net is negligible. For example, to the providers the cost of placing advertisement banners on web pages is negligible. However, the space on a web page is limited and quite often for the same page there will be only one type of firm able to advertise. Consequently, firms may have to compete for the space, which drives the price up. Another difficulty is that when the impression rate or the CTR is high, this may use up the bandwidth and slow down the transmission speed. This can cause visitors to lose patience. In this regards, providers do indeed have a variable (opportunity) cost. This gives a reference for us to model the marginal cost in the following sections.

Given that, however, in the Internet's very short history we have observed quite a few cases that uniform pricing (or sometimes referred to as "buffet pricing" in Nahata, et al. (1999)) is more viable than the two-part tariff. A significant example is the case of America Online. America Online is an Internet service provider (ISP). It was the first significant ISP to provide a flat rate for users to connect with the Internet. Other ISPs were then forced to provide a flat rate (uniform pricing) strategy to stay in the market. In Taiwan uniform pricing was implemented on the wide band system put on market in 1999. To some extent, this study explains the pricing behavior above.

The next section shows model without the referral fee, which follows the uniform pricing strategy. Section 3 shows the model that allows the use of referral fee, which follows the two-part tariff strategy. Related issues and future studies are addressed in Section 4.

2. Model without referral fee

For nonlinear pricing strategy, such as the two-part tariff, to be successful firms must be heterogeneous and providers must have monopolistic market power (Wilson, 1992; Oi, 1971). However, in practice very similar oligopolistic competitors may compete using the two-part tariff strategy. For example, China Times and United Daily News both provide web pages for advertisements, and they both price their banners on a two-part tariff basis. Hence, for simplicity, we will setup our model by assuming that the IAPs compete in duopoly and firms are assumed to be either light users or heavy users.[1]

Moving onto the modeling in a fashion of two-part tariff, it is necessary to set the demand profile of the buyers by assuming they are homogenous (Nahata, et al., 1999). Although the homogenous assumption seems unrealistic, the simplified situation may help to clarify this complexity. Furthermore, it may help to justify the results derived from the heterogeneous

[1] If providers have information in regard to the light and heavy usage, they may charge different fixed fees with respect the two types of firms. The fixed fees reflect the consumer surpluses of the firms. However, in most cases, the provider does not have this information. As a result, the provider may charge a fixed fee according to the surplus that the light users have, and set a per-unit price equal to the marginal cost for both types users.

assumption.

Next, according to Wilson (1992), if the following four preconditions are satisfied, sellers may adopt a nonlinear pricing strategy, such as two-part tariff. The four preconditions are as follows (Wilson, 1992, p.10)：

- The seller has a monopoly of power.
- Resale markets are limited or absent.
- The seller can monitor customers' purchases.
- The seller has disaggregated demand data.

Although in the Internet industry, IAPs are far from monopolistic in their areas, as long as the competition is imperfect the first precondition is not completely violated. Resale is also unlikely in the Internet advertising market, because the advertisements have to be placed directly with the provider. The firm is thus unlikely to resell its advertising space to a third party without releasing the resale information to the provider. As mentioned above, existing information technology can allow the provider to record the CTR precisely.[2] That is, the third precondition can at least be partially satisfied. The last precondition is assumed in the present study. Therefore, a nonlinear pricing strategy is justified for our modeling of Internet advertising as follows.

Consider that there are two Internet advertisement providers (IAPs), say $M_i, i = 1,2$. These two providers offer web pages for firms that need to place advertisements to promote their brand or increase sales. The two providers may compete with each other on price ($p_i, i = 1,2$). The price of an advertisement may depend on the web pages ($q_i, i = 1,2$) that are available. There may exist certain substitutability between web pages on the two providers' websites. Set θ the substitutability and assume the demands for the two as

$p_1 = a_1 - b(q_1 + \theta q_2)$, and

$p_2 = a_2 - b(q_2 + \theta q_1)$.

Assume the manufacturing costs are $c_i, i = 1,2$. Then the profits for each provider are

$\pi_i = p_i q_i - c_i q_i$, *where* $i = 1,2$.

This model is no different to that used for traditional advertising media. Hence we call it the benchmark model. Later we will use this benchmark to demonstrate the differences between traditional advertising media and Internet advertising.

The solutions to the benchmark model are

$q_1^N = \frac{1}{b(1-\theta^2)}(p_1 - c_1)$, and

$q_2^N = \frac{1}{b(1-\theta^2)}(p_2 - c_2)$, where the N stands for the Nash equilibrium.

In Appendix 1, we calculate the Nash prices as follows.

$p_1^N = \frac{1}{(4-\theta^2)}\left((2-\theta^2)a_1 - \theta a_2 + 2c_1 + \theta c_2\right)$.

$p_2^N = \frac{1}{(4-\theta^2)}\left((2-\theta^2)a_2 - \theta a_1 + 2c_2 + \theta c_1\right)$..

In addition to those results, we are particularly interested in profit changes over the choice of the level of substitutability (θ). This derivative in the next equation is derived in Appendix 2.

$$\frac{\partial \pi_1}{\partial \theta} = \frac{2(P_1^N - C_1)}{b(1-\theta^2)}\left[\frac{\theta}{1-\theta^2}(P_1^N - C_1) + A_p - B\right]$$

,

where

$$A_p - B = \frac{\partial(P_1^N - C_1)}{\partial \theta}$$

$$= \frac{2(P_1^N - C_1)}{b(1-\theta^2)}\left[\begin{array}{l}\left(\frac{\theta}{1-\theta^2} + \frac{2\theta}{4-\theta^2}\right)\cdot(P_1^N - C_1) + \\ \frac{-1}{4-\theta^2}\left[(2-\theta^2)C_1' + 2\theta(a_1 - C_1) - \theta C_2' + (a_2 - C_2)\right]\end{array}\right]$$

Set $\frac{\partial \pi_1}{\partial \theta} = 0$, and we then have

$$\frac{\theta}{1-\theta^2}(P_1^N - C_1) + \frac{\partial(P_1^N - C_1)}{\partial \theta} = 0 .$$

Assume $A_p - B < 0$, that is $\frac{\partial(P_1^N - C)}{\partial \theta} < 0$, then

$$\frac{\theta}{1-\theta^2}(P_1^N - C_1) + \frac{\partial(P_1^N - C_1)}{\partial \theta} = 0 .$$

Reorganize the equation,

$$\Rightarrow \frac{-\theta}{1-\theta^2} = \frac{\frac{d(P_1^N - C_1)}{d\theta}}{(P_1^N - C_1)} .$$

To solve the equation, let $x = (P_1^N - C_1)$ and $x' = \frac{\partial(P_1^N - C_1)}{\partial \theta}$, and we then have

$$(1-\theta^2)\cdot\frac{dx}{d\theta} + \theta \cdot x = 0 .$$

Through the reorganization of this

[2] Further agreements between the firm and the provider would be needed to share information regarding final purchases if one would like to have a perfect measure of the link in advertising effectiveness mentioned earlier.

equation, we have

$$\frac{dx}{d\theta}\theta^2 - x\theta - \frac{dx}{d\theta} = 0 .$$

The solutions to the above equation are as follows.

$$\theta_1^* = \frac{x+\sqrt{x^2+4x'^2}}{2x'} > 1 \quad , \quad \text{or}$$

$$\theta_2^* = \frac{x-\sqrt{x^2+4x'^2}}{2x'} < 0 \quad .$$

Figure 1 shows a plot of $\frac{\partial \pi_1}{\partial \theta}$ over θ. This graph could be reversed into π over θ as shown in the graph below. Since the θ only makes sense between zero and one, according to the graph above, the provider's profit is maximized if θ is minimized. This result implies that if a referral fee does not exist between the provider and the advertiser, it is better for the provider to fully distinguish its market position from that of its competitors. Nevertheless, it might be interesting to study further the case in which one provider is the anchor and other providers move closer to its market position.

3. Model with referral fee

The next model is set to consider the way in which referral fee can be collected via the application of CTR technology. Typically, the provider may collect a one-off advertising fee at the outset. In addition, the provider may collect a referral fee for each visitor that clicks on the advertiser's banner and is directed to the advertiser's website.

In addition to the assumptions and notations we made in the benchmark model, we add the referral fee to our model as follows.

$$P_1 = a_1 - b[q_1 + \theta\, q_2] \quad (1)$$
$$P_2 = a_2 - b[q_2 + \theta\, q_1] \quad (2)$$

} Macleod model

$$\pi_1 = P_1 q_1 - C_1 q_1 + r_1 k_1 \quad (3)$$

$$\pi_2 = P_2 q_2 - C_2 q_2 + r_2 k_2 \quad (4)$$

$r_i, i = 1,2$ represent the referral fee per click and $k_i, i = 1,2$ represent the number of clicks.

By reorganizing the equations (1) and (2), we have

$$\Rightarrow$$

$$q_1 = \frac{1}{b(1-\theta^2)}[a_1 - \theta\, a_2 + \theta\, P_2 - P_1]$$

$$q_2 = \frac{1}{b(1-\theta^2)}[a_2 - \theta\, a_1 + \theta\, P_1 - P_2]$$

Also, by substituting these equations into equations (3) and (4), we derive the profit functions as follows.

$$\pi_1 = (P_1 - C_1)\cdot q_1 + r_1 k_1$$
$$= \frac{1}{b(1-\theta^2)}\begin{bmatrix}(a_1 - C_1) - \theta(a_2 - C_2) + \\ \theta(P_2 - C_2) - (P_1 - C_1)\end{bmatrix}\cdot(P_1 - C_1) + r_1 k_1$$

$$\pi_2 = (P_2 - C_2)q_2 + r_2 k_2$$
$$= \frac{1}{b(1-\theta^2)}\begin{bmatrix}(a_2 - C_2) - \theta(a_1 - C_1) + \\ \theta(P_1 - C_1) - (P_2 - C_2)\end{bmatrix}\cdot(P_2 - C_2) + r_2 k_2$$

By setting $\frac{\partial \pi_1}{\partial P_1} = 0$, we derive the Nash equilibria as follows.

$$q_1^N = \frac{1}{b(1-\theta^2)}(P_1 - C_1) - \frac{\partial r_1}{\partial P_1}\cdot k_1$$

$$q_2^N = \frac{1}{b(1-\theta^2)}(P_2 - C_2) - \frac{\partial r_2}{\partial P_2}\cdot k_2$$

Substitute the two equilibria into equations (1) and (2), and we get

$$\begin{cases} a_1 - C_1 - \theta\, a_2 + \theta\, C_2 + \theta\, P_2 - \theta\, C_2 - 2P_1 \\ + 2C_1 + b(1-\theta^2)\cdot r_1' \cdot k_1 = 0,\ r_1' = \frac{\partial r_1'}{\partial P_1} \\ \\ a_2 - C_2 - \theta\, a_1 + \theta\, C_1 + \theta\, P_1 - \theta\, C_1 - 2P_2 \\ + 2C_2 + b(1-\theta^2)\cdot r_2'' \cdot k_2 = 0,\ r_2' = \frac{\partial r_2'}{\partial P_2} \end{cases}$$

The Nash price equilibria are then arrived at as follows.

$$P_1^N = \frac{1}{4-\theta^2}\begin{bmatrix}(2-\theta^2)a_1 - \theta\, a_2 + 2C_1 + \theta\, C_2 \\ + b(1-\theta^2)(2r_1' k_1 + \theta\, r_2' k_2)\end{bmatrix}$$

$$P_2^N = \frac{1}{4-\theta^2}\begin{bmatrix}(2-\theta^2)a_2 - \theta\, a_1 + 2C_2 + \theta\, C_1 \\ + b(1-\theta^2)(2r_2' k_2 + \theta\, r_1' k_1)\end{bmatrix}$$

As in the benchmark model, we are interested in the profit change over the substitutability. That is,

$$\frac{\partial \pi_1}{\partial \theta} = \frac{\partial}{\partial \theta}\left[\frac{1}{b(1-\theta^2)}(P_1 - C_1)^2 - r_1' k_1 (P_1 - C_1) + r_1 k_1\right]$$

.

Let $x = (P_1 - C_1)$, $B = b\, r_1' k_1 x'$, $r_1' = \frac{dr_1}{dP_1}$, and $x' = \frac{d(P_1 - C_1)}{d\theta}$. Note that, if $r_1' < 0$ and $x' < 0$ then it is also true that $B > 0$. The intuition for $r_1' < 0$ is that the referral fee is compensatory to the advertising charge. If the provider charges the advertiser a high price for placing an advertisement on its website, the provider may have to reduce the referral fee as an enticement. The intuition for $x' < 0$ is that the price the provider can charge increases proportionate to the reduction in substitutability.

$$\frac{\partial \pi_1}{\partial \theta} = \frac{1}{b(1-\theta^2)^2}\begin{bmatrix} 2x^2 \cdot \theta + 2xx' - 2xx'\theta^2 \\ - B + 2B\theta^2 - B\theta^4 \end{bmatrix}$$

Set $\frac{\partial \pi_1}{\partial \theta} = 0$, we have

$$-B\theta^4 + (2B - 2xx')\theta^2 + 2x^2\theta + 2xx' - B = 0$$

.

Since $0 < \theta < 1$, the solutions to the complicated situation above may be bounded between two extremes: $-B\theta^4 + (2B - 2xx')\theta^2 + 2x^2 + 2xx' - B = 0$ and

$$-B\theta^4 + (2B - 2xx')\theta^2 + 2xx' - B = 0 .$$

Let Y denote $-B\theta^4 + (2B - 2xx')\theta^2 + 2xx' - B$.

Thus the two extremes above are equivalent to $Y + 2x^2 = 0$ and $Y = 0$. The former case gives

$$\theta_1^2 = \left(1 - \frac{xx'}{B}\right) \pm \sqrt{\left(\frac{xx'}{B}\right)^2 + \frac{2x^2}{B^2}} \quad ,$$

whereas the latter case gives

$$\theta_2^2 = \left(1 - \frac{xx'}{B}\right) \pm \sqrt{\left(\frac{xx'}{B}\right)^2} \; .$$

Figure 1 (see below) shows the three cases: $Y + 2x^2 = 0$, $Y + 2x^2\theta = 0$ and $Y = 0$. It reveals the interesting finding that there exists a unique point where the provider's profit is minimized. A move in substitutability away from this point toward either zero or one increases profitability.

4. Discussions

The primary objective of this study is to explore the non-linear pricing strategy that Internet advertising provider (IAP) can charge firms to advertise. A simple model is build to take into account IAP's strategic behavior and firm's advertising demands. The strategy of *uniform pricing* is modeled, which is used as a benchmark. This benchmark is then compared to a model built with the consideration of the strategy of *two-part tariff*. The modeling shows that in cases where a uniform pricing strategy is adopted, the fee that each IAP can attain is monotonic decreasing over their substitutability in market. That is, IAPs that wish to maximize their revenue have to take measures to distinguish themselves from the competition in order to achieve zero substitutability. In cases where a two-part tariff pricing strategy is adopted, the revenue curve becomes convex. In other words, IAPs may choose to have either full substitutability or zero substitutability in order to maximize their profits.

Our theoretical propositions can also be further examined with field data. At present, only a few providers, such as China Times, post their pricing schedules on their websites in Taiwan. To conduct an empirical study of pricing strategies it is necessary to ask providers directly for information on their pricing schedules. If this survey were undertaken, we would expect to find that providers use both uniform pricing and two-part tariff strategies. If further information from those providers were also available, then we would be able to perform econometric analysis on the data. This analysis could be helpful to understand the factors behind the choice between the two pricing strategies. It could also help to measure the relative impact of each factor.

Although for Internet advertising existing information technology can precisely record the advertisement impression and click through rate, opportunistic behavior may remain. On the one hand, the provider may not keep the web site sufficiently attractive to achieve the promised hit rate. On the other hand, the firm's service may disappoint the click through customers and therefore result in a decreasing click through rate. In other words, it takes effort from both sides to ensure success. Where a referral fee is involved, matters can become more complicated. It is not in the firm's interest to inform the provider when a purchase is made.[3] Without this information it

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[3] Sometimes, contract may be designed to allow the collection of referral fee. For example, a

is not possible to collect the referral fee. This may be one reason why the referral fee system is rarely employed. However, if there is technology that can tell if a purchase is made, then the referral fee system is actually superior to the click through rate from the viewpoint of performance-based pricing.

Perhaps the most significant case for using the two-part tariff strategy is in the area of franchising. According to the literature on franchise, the widely employed payment strategy between franchiser and franchisee includes the franchise fee (a fixed fee) and running royalty (per unit charge). This reflects an important consideration that the running royalty provides incentives for the franchisee to work diligently without close monitoring by the franchiser (Bhattacharyya and Lafontaine, 1992; Lal, 1990). In addition, this payment strategy can encourage franchisers to do their best to promote their trademark and products to increase franchisees' sales volume. For all these reasons, this may be why the two-part tariff is widely adopted in franchising. Nevertheless, the popularity of the uniform pricing strategy by ISPs provides a good reason for us to investigate this strategy in future study.

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Appendix 1

The price-reaction functions are as follows.

$$\left(a_1 - C_1\right) - \theta\left(a_2 - C_2\right) + \theta\left(P_2 - C_2\right)$$
$$-2\left(P_1 - C_1\right) = 0$$
$$\left(a_2 - C_2\right) - \theta\left(a_1 - C_1\right) + \theta\left(P_1 - C_1\right)$$
$$-2\left(P_2 - C_2\right) = 0$$
$$\Rightarrow$$
$$a_1 - C_1 - \theta\, a_2 + \theta\, C_2 + \theta\, P_2 - \theta\, C_2 - 2P_1 + 2C_1 = 0$$
$$a_2 - C_2 - \theta\, a_1 + \theta\, C_1 + \theta\, P_1 - \theta\, C_1 - 2P_2 + 2C_2 = 0$$
$$\Rightarrow$$
$$\theta\, P_2 - 2P_1 = \theta\, a_2 - a_1 - C_1 --(a)$$
$$\theta\, P_1 - 2P_2 = \theta\, a_1 - a_2 - C_2 --(b)$$

If we take $(a)\times 2 + (b)\times\theta$, then arrive

$$\Rightarrow P_1^{Nash} = \frac{1}{4-\theta^2}\left[\left(2-\theta^2\right)a_1 - \theta\, a_2 + 2C_1 + \theta\, C_2\right]$$

.

Same for

$$P_2^{Nash} = \frac{1}{4-\theta^2}\left[\left(2-\theta^2\right)a_2 - \theta\, a_1 + \theta\, C_1 + 2C_2\right]$$

.

Appendix 2

Following our definitions and the results in Appendix 1, we have the following in profit, Nash price and Nash quantity.

$$\pi_1 = \frac{1}{b\left(1-\theta^2\right)}\left(P_1^N - C_1\right)^2$$

$$P_1^N - C_1 = \frac{1}{4-\theta^2}\left[\left(2-\theta^2\right)\left(a_1 - C_1\right) - \theta\left(a_2 - C_2\right)\right]$$

$$q_1^N = \frac{1}{b\left(1-\theta^2\right)}\left(P_1^N - C_1\right)$$

Firstly, the derivative of the price to the substitutability is

$$\frac{\partial\left(P_1^N - C_1\right)}{\partial\theta} = \frac{2\theta}{\left(4-\theta^2\right)^2}\left[\left(2-\theta^2\right)\left(a_1 - C_1\right) - \theta\left(a_2 - C_2\right)\right] + \frac{1}{4-\theta^2}\begin{bmatrix} -\left(2-\theta^2\right)C_1{}' + \left(-2\theta\right)\left(a_1 - C_1\right) \\ -\left(a_2 - C_2\right) + \theta C_2{}' \end{bmatrix}$$

$$= \frac{2\theta}{4-\theta^2}\cdot\left(P_1^N - C_1\right) + \frac{-1}{4-\theta^2}\begin{bmatrix} \left(2-\theta^2\right)C_1{}' + 2\theta\left(a_1 - C_1\right) \\ -\theta\, C_2{}' + \left(a_2 - C_2\right) \end{bmatrix}$$

and the derivative of the quantity to the substitutability is

business is attained within IAP's websites. This design enables the IAP a perfectly monitoring whether or not a business is attained. Nevertheless, it will become a contractual problem, which is not a consideration of the present study.

$$\frac{\partial q_1^N}{\partial \theta}=\frac{\partial}{\partial \theta}\left[\frac{1}{b\left(1-\theta^2\right)}\cdot\left(P_1^N-C_1\right)\right]$$

$$=\frac{2\theta}{b\left(1-\theta^2\right)^2}\left(P_1^N-C_1\right)+\frac{1}{b\left(1-\theta^2\right)}\left[\frac{\partial P_1^N-C_1}{\partial \theta}\right]$$

$$=\frac{1}{b\left(1-\theta^2\right)}\left[\frac{2\theta}{1-\theta^2}\left(P_1^N-C_1\right)+\frac{2\theta}{4-\theta^2}\cdot\left(P_1^N-C_1\right)+\frac{-1}{4-\theta^2}\left[\left(2-\theta^2\right)C_1'+2\theta\left(a_1-C_1\right)-\theta C_2'+\left(a_2-C_2\right)\right]\right]$$

$$=\frac{1}{b\left(1-\theta^2\right)}\left[\left(\frac{2\theta}{1-\theta^2}+\frac{2\theta}{4-\theta^2}\right)\left(P_1^N-C_1\right)+\frac{-1}{4-\theta^2}\left[\left(2-\theta^2\right)C_1'+2\theta\left(a_1-C_1\right)-\theta C_2'+\left(a_2-C_2\right)\right]\right]$$

The two jointly deliver the profit to substitutability derivative as follow.

$$\frac{\partial \pi_1}{\partial \theta}=\frac{\partial}{\partial \theta}\left[\frac{1}{b\left(1-\theta^2\right)}\left(P_1^N-C_1\right)^2\right]$$

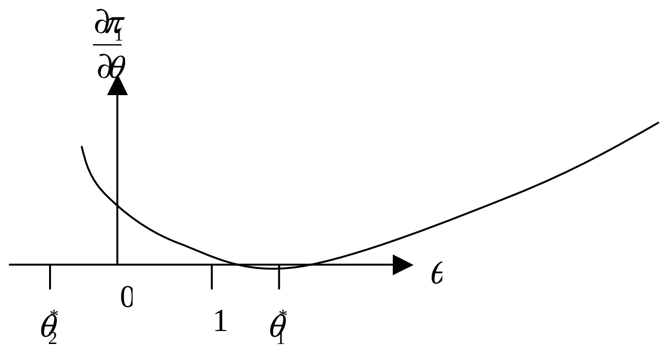
$$=\frac{2\theta}{b\left(1-\theta^2\right)^2}\left(P_1^N-C_1\right)^2$$

$$+\frac{1}{b\left(1-\theta^2\right)}\left[2\left(P_1^N-C_1\right)\cdot\frac{\partial\left(P_1-C_1\right)}{\partial \theta}\right]$$

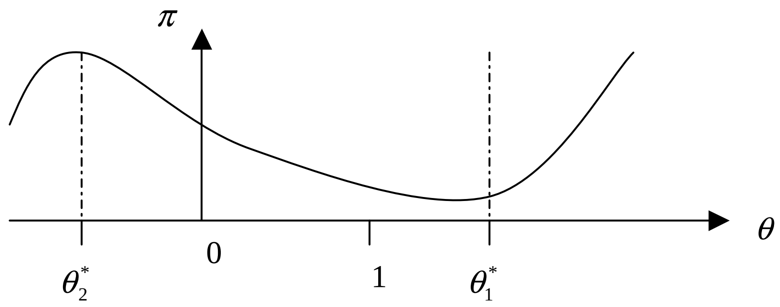
$$=\frac{2\left(P_1^N-C_1\right)}{b\left(1-\theta^2\right)}\left[\frac{\theta}{1-\theta^2}\cdot\left(P_1^N-C_1\right)+\frac{\partial\left(P_1^N-C_1\right)}{\partial \theta}\right]$$

$$=\frac{2\left(P_1^N-C_1\right)}{b\left(1-\theta^2\right)}\left[\begin{array}{l}\frac{\theta}{1-\theta^2}\left(P_1^N-C_1\right)+\frac{2\theta}{4-\theta^2}\left(P_1^N-C_1\right)\\+\frac{-1}{4-\theta^2}\left[\begin{array}{l}\left(2-\theta^2\right)C_1'+2\theta\left(a_1-C_1\right)\\-\theta C_2'+\left(a_2-C_2\right)\end{array}\right]\end{array}\right]$$

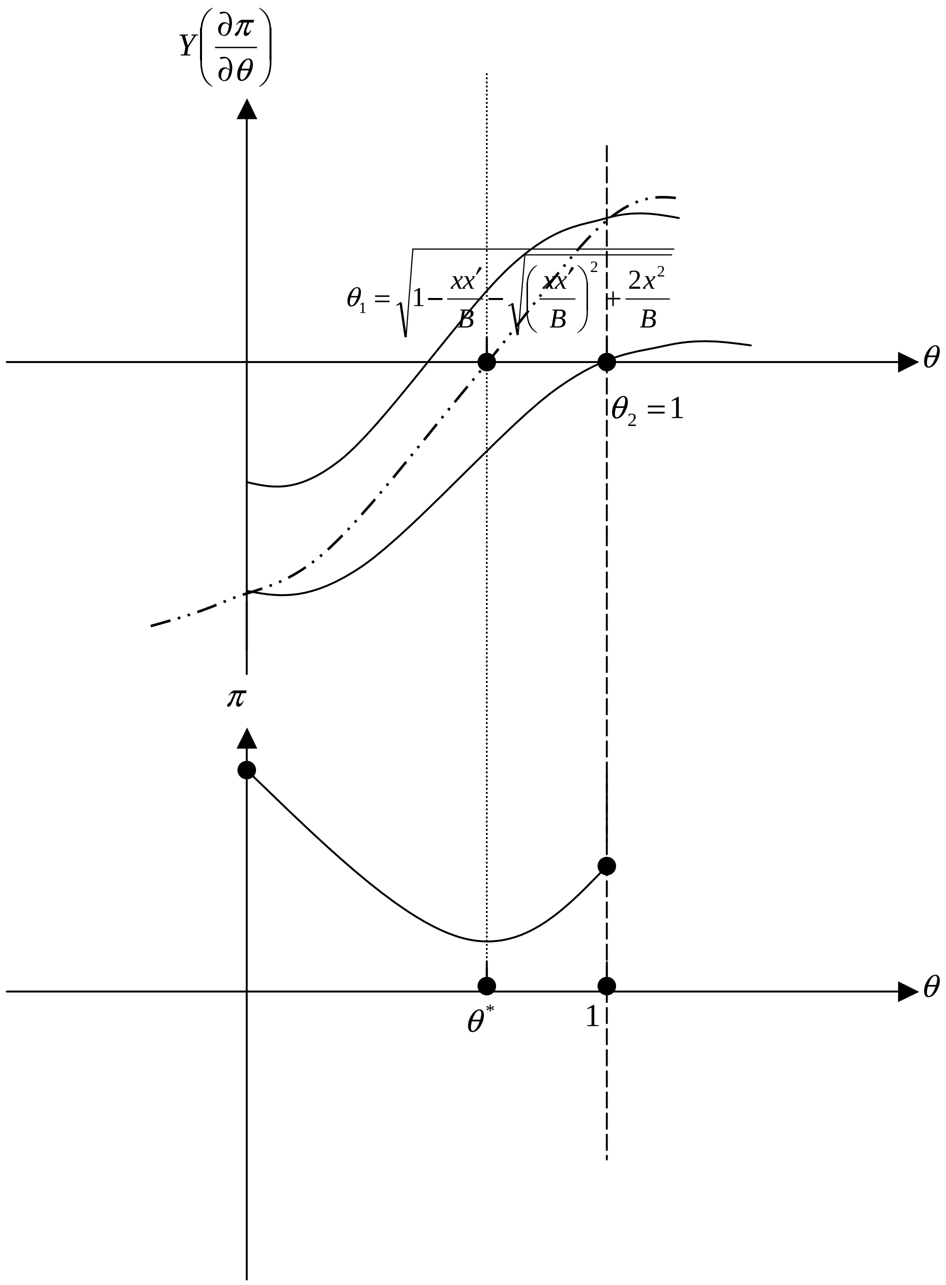
$$=\frac{2\left(P_1^N-C_1\right)}{b\left(1-\theta^2\right)}\left[\begin{array}{l}\left(\frac{\theta}{1-\theta^2}+\frac{2\theta}{4-\theta^2}\right)\cdot\left(P_1^N-C_1\right)\\+\frac{-1}{4-\theta^2}\left[\begin{array}{l}\left(2-\theta^2\right)C_1'+2\theta\left(a_1-C_1\right)\\-\theta C_2'+\left(a_2-C_2\right)\end{array}\right]\end{array}\right]$$



∂π
∂θ
0
1
θ



π
θ
0
1



$Y\left(\frac{\partial \pi}{\partial \theta}\right)$
$\theta_1 = \sqrt{1 - \frac{xx'}{B} - \sqrt{\left(\frac{xx'}{B}\right)^2 + \frac{2x^2}{B}}}$
θ
$\theta_2 = 1$
π
θ
θ^*
1