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The System Simulation with Optimization Mechanism for Option Pricing

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Abstract

The Monte Carlo approach is a valuable and flexible computational tool in modern finance, and is one of numerical procedures used for solving option valuation problems. In recent years the complexity of numerical computation in financial theory and practice has increased and require more computational power and efficiency. Monte Carlo simulation is one of the numerical computation methods used for financial engineering problems.

The drawback of Monte Carlo simulation is computationally intensive and time-consuming. In attempt to tackle such an issue, many recent applications of the Monte Carlo approach to security pricing problems have been discussed with emphasis on improvements in efficiency. This paper presents a novel approach combining system simulation with GA-based optimization to pricing options. This paper shows how the proposed approach can significantly resolve the option pricing problem.

Keywords: Option Valuation, Financial Engineering, Monte Carlo Simulation, Genetic Algorithms

1. Introduction

As pointed out by Hull and White [19, p.237], many option pricings appear to present intractable pricing problems. Therefore, most of them lack straightforward closed form solutions. The various approaches including analytic approximations [3] [22] and numerical procedures [5] [9] [10] [27] have been suggested for calculating option prices when there is no closed form solutions. In recent years the complexity of numerical computation in financial theory and practice has increased enormously, putting more demands on computational speed and efficiency. One of the numerical computation procedures, the Monte Carlo approach, was suggested by Boyle [5], and has proved to be a valuable and flexible computational tool in modern finance. The method simulated the process generating the returns on the underlying asset and invokes the risk neutrality assumption to derive the value of the option. Numerical methods are used for a variety of purposes of finance. The Monte Carlo approach is a useful tool for many of numerical calculations, evidenced in part by the voluminous literature of successful applications, such as the stochastic volatility applications [11] [18] [20] [29], the valuation of mortgage-backed securities [28], the valuation of path-dependent options [21], the portfolio optimization [25], and the valuation of interest-rate derivative claims [8].

The Monte Carlo approach is flexible and easy to implement and modify. In addition, the increased availability of powerful computers has enhanced the attractiveness of the approach. Anyhow, there are some disadvantages of the approach but in recent years progress has been made in overcoming them. One drawback is that for very complex problems a large number of replications may be required to obtain precise results. Different variance reduction techniques have been developed to increase precision. Two classical variance reduction techniques are the control variate approach and the antithetic variate approach. The introduction of an appropriate control variate provides a very efficient variance reduction technique. However, in some problems it may be difficult to find a suitable control variate. Another alternative approach, the antithetic variate is often easier to apply since it concentrates on the procedure used for generating the random deviates. Essentially this technique relies on the introduction of negative correlation between two estimates. The performance of antithetic variate is better than Monte Carlo approach, but not good as control variate approach.

All these numerical approach have a dual objective of accuracy and computation speed. Therefore, improvements in numerical efficiency are of interest in solving option pricing problems for existing numerical procedures. The main purpose of this paper is to show how system simulation with optimization mechanism can be used to improve the efficiency of the numerical approach. And the result of our experiments has proved that our proposed approach has the better performance either in accuracy or speed of computation than Monte Carlo approach and antithetic variate approach.

2. Methods for Option Evaluation

2.1 System Simulation

Real-world problems always contain too many uncertain factors to be simply described by decision models. Simulation is one of the possible ways to be used to model real-world problems. Many aspects are concerned when adopting simulation approach to a complex problem [26].

(1) It can be used to experiment with a new design or scheme before implementing it.
(2) It can be used to enable the study of the internal interactions of a complex system or subsystem within a complex system.
(3) It provides the analyst with a tool to conduct various experiments that can be done in real time.
(4) Organizational and environmental changes can be simulated and the effect of these changes on the model's behavior can be observed.
(5) It can be used as a tool to validate analytic results.
(6) Simulation provides a flexible mean to experiment with the system or its design. Such experiments can reveal and predict valuable information to the designer, user, manager and purchaser.
(7) Simulation is a cost-effective tool for capacity planning and tuning of system or subsystems.

In addition, system simulation has several major advantages as listed as follows [26]：

(1) Flexibility: It permits controlled experiments.
(2) Speed: It permits time compression operation of a system over extended period of time.
(3) It permits sensitivity analysis.

However, system simulation has some disadvantages. These are listed below [26]：

(1) It may become expensive in terms of computer time and manpower.
(2) There are some hidden critical assumptions that may affect the credibility of the simulation outputs.
(3) It may encounter extensive development time.
(4) It may encounter difficulties in model's parameters initialization.
(5) Not all of parameters are considered when developing system simulation.

Although system simulation has those shortcomings as mentioned above, there are still some ways to improve these drawbacks. GAs can be adopted to enhance its lack of search efficiency to bring better outcomes. For input of parameters, the previous fixed parameters can be replaced with different statistic distribution to more closely reflect real world situations. The process of system simulation is shown in Fig. 1.

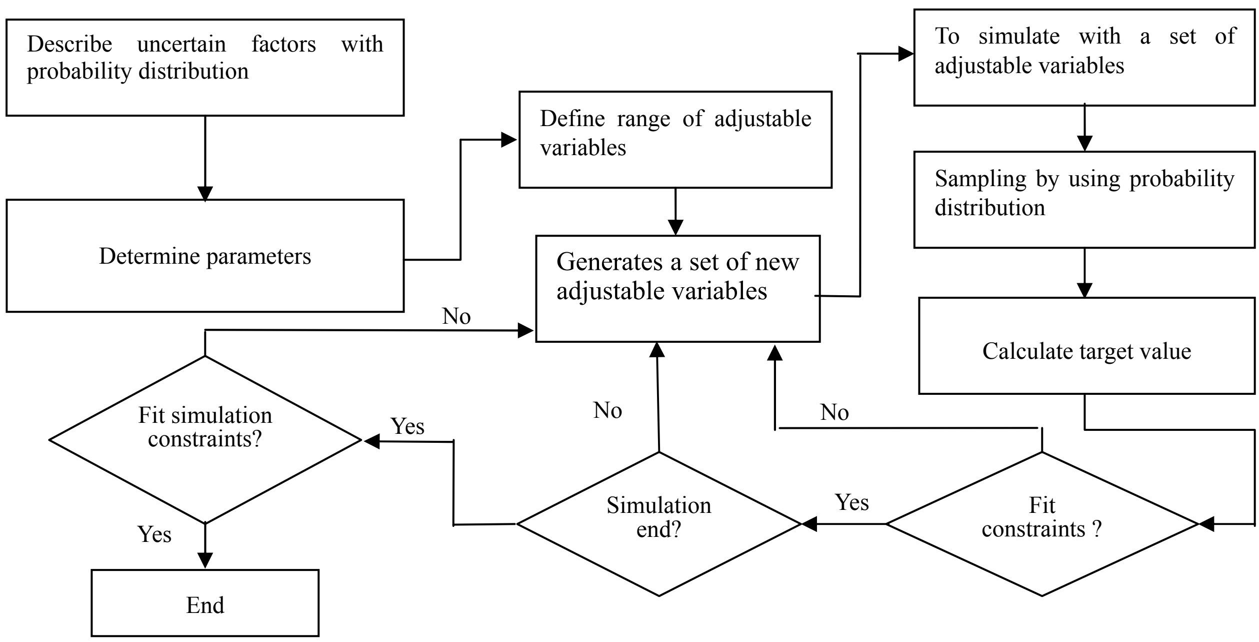

Fig. 1 The Concept of System Simulation

2.2 Monte Carlo Simulation

System simulation takes different environmental factors into consideration to produce different outcomes. We can observe the pattern of environmental changes by system simulation technique.System simulation provides an elastic environment and system to show the results that is valuable information for managers [2] [6].

There are three main methods for system simulation technique. They are Monte Carlo simulation [1] [2] [29] [31], trace-driven simulation [24] and discrete event simulation [24]. Monte Carlo simulation is a static system simulation technique and dose not include time factor. It's widely used in statistic models whose property will not shift as time goes. It can also be used in non-probability expression (by statistic technique). It allows users to set up statistic distribution or object function along with randomly generated values and iterative computation to figure out all possible solutions. In this way, computers will test every set of inputs to achieve optimization.

System simulation technique can reflect various kinds of solutions or projects under uncertainty [24]. Its solutions, however, are not necessarily the optimized ones. Combining with optimization techniques is then becoming worth developing. Some lectures in addition reveal that using GAs when optimizing can speed up the time to search for solution [31].

In the iterative simulative process, the statistic distributions in variables will be sampled with one new value during every system simulation. This action is a method to simulate risks. After all variables are given values, they will be the inputs of the optimization process. Monte Carlo simulation can be applied to optimization problems with uncertainty and huge search space. When combining system simulation and GAs, we can solve problems with uncertainty quickly by guiding the direction of expected solutions. This has been proposed in [30][32].

2.3 Genetic algorithms

Optimization is a process of finding out an optimum solution from a problem where the searching space is likely enormous. Most of these problems contain many variables, which are restricted by some given constraints [13] [14] [22].

Genetic algorithm (GAs) is one of optimization algorithm [12] [25]. It uses fitness function to determine the direction of searching and does organism-like computation. It converts "The fittest survives" from Darwin to simulate reproduction, crossover and mutation of chromosomes. Through the above computation process, it selects out successful evolving chromosome, that is, the desired chromosome [16]. There are three basic operators in GAs computation process:

1.Reproduction: it duplicates one chromosome directly.

2.Crossover: it takes two chromosomes to exchange their genes to produce the other two chromosomes.

3.Mutation: some genes in a single chromosome may change to produce the other different chromosome.

After generations of reproduction, crossover and mutation, genes in chromosomes change and the chromosome is selected with the highest fitness value. It can promise a chromosome with better genes than before. In this way, GAs is a method to search for optimized solution and can be applied to problems with enormous solution space. The concept of GAs process is shown in Fig. 2. It uses a loop from fitness evaluation to stop condition along with mechanisms of reproduction, crossover and mutation to make a new solution (new generation) evolve.

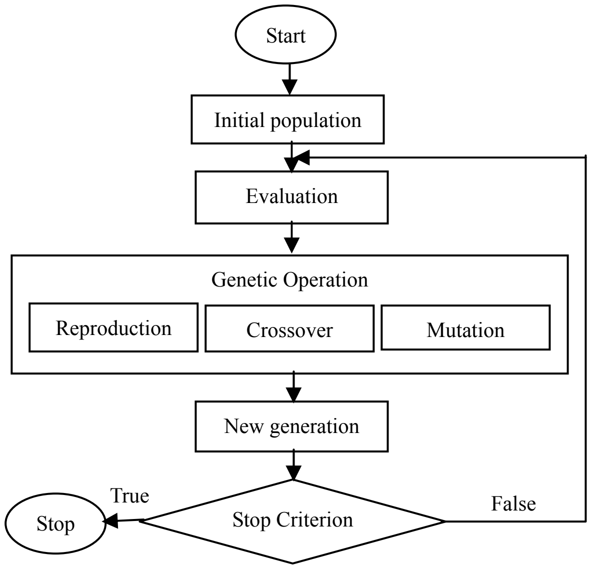

Fig. 2 The Genetic Algorithm Process

2.4 Simulation Optimization

Simulation optimization is an optimization in itself; however, it requires additional mechanism to make the entire optimization process much closer to real world [1]. The concept of simulation optimization can be conceptualized in Fig. 3.

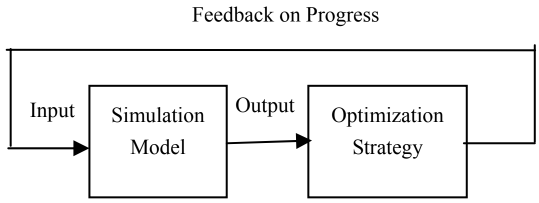

Fig. 3 The Concept of Simulation-Optimization abstracted from [34]

The optimization strategy is improved through generations. The improving process is, however, slightly different from other original GAs methods. It is influenced by the simulation model that iteratively sample values from statistic distributions of the optimization model. The iterative samplings are called iterations in this paper, and one single simulation consists of iterations. The amount of iterations in one single simulation can be arbitrary, however, the more the better. That's because from statistic viewpoint, more sampling values can represent more statistic population and can reflect much closer to the real-world situation.The concept of simulation combined with GAs is illustrated in Fig. 4.

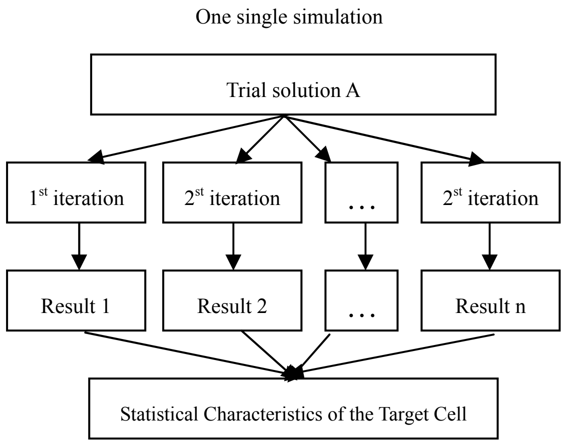

Fig. 4 The Iteration Concept

It can be briefly described that one single simulation equals to one chromosome in GAs, but simulation consists of additional computations, that is, iterations. As a result, the fitness value for each generation is no longer one value but statistical characteristics, such as mean, standard deviation, variation, skewness, kurtosis and so on. Usually the mean is a good choice for its stable property to stand for population. Therefore, the proposed methodology attempts to employ simulation-optimization concept in the option pricing problem.

3. Methodology

3.1 European Call Option Evaluation Model

Consider European Call Option [17] dependent on a single market variable S (stock price) that provides a payoff at time *T.* Assume that interest rates are constant, and we can value the derivative as follows:

(1) Sample a random path for S in a risk-neutral world. Using the properties of the lognormal distribution that we can generate the distribution of stock prices one period hence by forming the random variables,

$$S_{t+1} = S_t \, exp\left(r - 0.5\sigma^2\right)\Delta t + \sigma\varepsilon\sqrt{\Delta t} \quad (1)$$

where ε is a normally distributed random variable with zero mean and unit variance. S_t represents the current stock price and Δt is the stock price changes in a small time.

(2) Calculate the payoff from the European Call Option (i.e. Max [S_T -*E*, 0]). *E* represents exercise price.

(3) Repeat steps 1 and 2 to get many sample values of the payoff from the derivative in a risk-neutral world.

(4) Calculate the mean of the sample payoffs to get an estimate of the expected payoff in risk-neutral world.

(5) Discount the expected payoff at the risk-free rate to get an estimate of the value of the derivative.

3.2 Simulation Optimization

In this study we adopt simulation optimization method in order to research European call option pricing problems. Although system simulation has some advantages such as flexibility, speed, and sensitivity analysis etc., it may encounter difficulties in model's parameters initialization and not all of parameters are considered when developing system simulation. Genetic algorithms could provide strong multi-dimensions search ability to find fitness parameters and an optimal combination from different parameters. Therefore, system simulation could integrate with genetic algorithms, not only to make up genetic algorithms without simulation framework, but also to express problems with high uncertainty to raise system fault tolerance ability. System simulation integrated with genetic algorithms will bring both sides advantages, not only to construct models more fit in real world, but also to find better decision information.

The integration of system simulation and genetic algorithms procedure framework is shown in Fig. 5, and the execution steps are illustrated as blow:

(1) To describe uncertain factors characters, then search fitness probability distribution, and determine simulation statistics parameters (ex. a mean value、standard deviation).

(2) Determine input variables, and settle adjustable variable's range of value.

(3) Generate a set of new adjustable variables with genetic algorithms.

(4) To estimate new population's fitness function.

(5) To test if new population meet conditions? if not matched, then repeats step 4.

(6) To simulate with a set of new adjustable variables.

(7) Sampling with probability distribution and compute target value.

(8) If target value out of range of adjustable variables, then repeat steps 3~8, otherwise go to the next step.

(9) To check if simulation procedure is completed or not? If yes, then go to next step, otherwise repeat step 6~9.

(10) To check if target value is within the scope of simulation? If yes, then go to the next step, otherwise repeat step 3~10.

(11) If the value is optimal, then end, otherwise repeat step 3~10 until the value is optimal.

The simulation optimization methodology as mentioned above uses system simulation to do sampling and the genetic algorithms to find adjustable variables, which procedure is shown in Fig. 6. It could help to understand how system simulation integrated genetic algorithms method finds the best value. Here, the system simulation technique is Monte Carlo simulation. Its main concept is to sample very large size of data from the given statistic distribution (that is, stock price here) and they apply these values to the optimization solution mentioned before. Doing so will make the optimization process of simulation much closer to the real world situation.

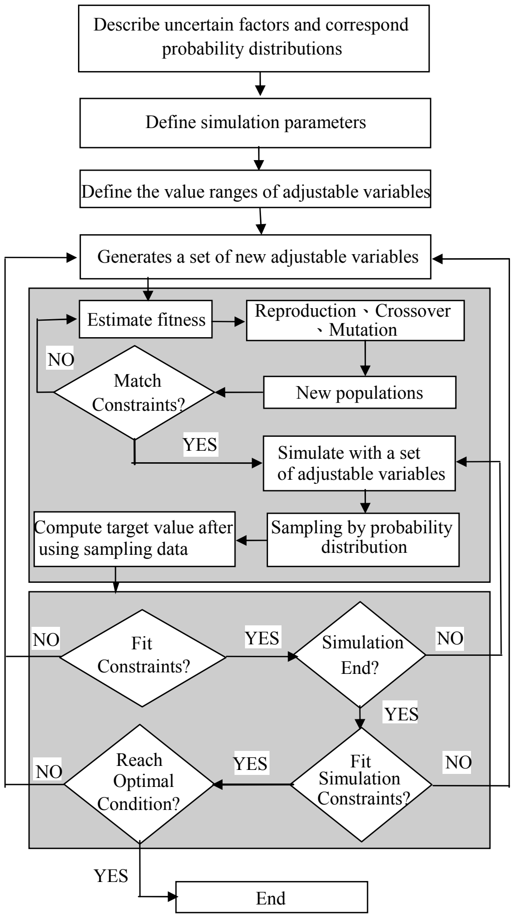

Fig. 5 The System Simulation Optimization Process

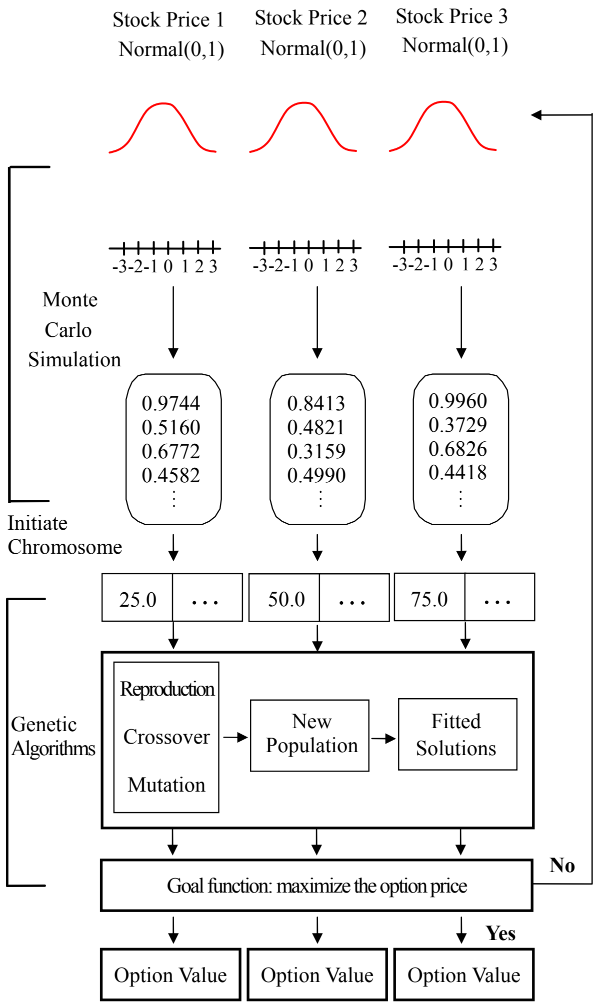

Fig. 6 The Process by Monte Carlo Integrated Genetic Algorithm

4. The Experiments

4.1 Data Sets

Eq.1 is used to obtain the estimates of European call options values on dividend paying stocks.

Let

S_t = the current stock price at time t (S = 25, 50, 75 respectively)

r = the risk-free rate per quarter compounded continuously (r = 0.015),

σ^2 = the (assumed constant) variance rate per quarter on the underlying stock (σ^2=0.025)

D_t = the dividend payable at time t (D = 0.25),

E = the exercise price of the option (E= 50),

T = the expiration date of the option.

In this case, the assumption used is same as that mentioned in the Boyle's article [5, p328-329]. It is assumed that time is measured in the unit of one quarter. Assume that S_t represents the stock price just after the quarterly dividend D_t has been paid. To set up the simulation method in this case a value of S_{t+1} is generated. If this value is greater than D_{t+1} then (S_{t+1} - D_{t+1}) is used as the initial value in the beginning of the second period and the procedure continues until a value of S_T is obtained. If at some stage S_{t+m} (m = 1,2,…(T-t-1)) is less than or equal to the corresponding dividend payment D_{t+m}, then the process stops. In this case another simulation trial is started again from time t. A series of simulation trials is carried out in this way and the expected value of Max [S_T – E, 0] is found. This quantity is then discounted at the risk-free rate to yield an estimate of the option value [5].

4.2. Optimization Model Parameters Settings

The proposed optimization model applies GAs evolutionary mechanisms; as a result, some parameters in a formal GAs model must be determined first. They are population size, crossover rate and mutation rate respectively. Because the model is a hybrid of GA and simulation techniques, the iteration number is a required parameter to be defined. All settings are specified as follows:

Table 1 Model Parameter Settings

Population size	50
Crossover rate	0.5
Mutation rate	0.06
Iteration number	500

The iteration number here is set to 500; however, the number can be variable. That is the optimization model which will stop the iterations when the expected statistic characteristics converge. Assigning a fixed value here is to make further comparison easier later on.

4.3 The Comparison

A comparison was made among the performance of the Monte Carlo approach, control variate, antithetic variates, and GA-Based optimization to pricing options. For this comparison, the estimates of option values on dividend paying stocks in the case of European call options were obtained. The data we used is same as that given in the Boyle's article [5]. The results we obtained are compared with those results as listed in the article.

Table 2 indicates the results of the simulation method for selected values of the underlying parameters. It is assumed that r = 0.015, σ^2 = 0.025 per period, D = 0.25 per period and E = 50. The estimates of options with even maturities ranging from 2 to 20 are provided.

From the results as shown in Fig. 7, 8, 9, we can see the best performance is control variate, which provides a very efficient variance reduction technique in this problem. In some problems it may be difficult to find a suitable control variate. The alternative method, the antithetic variate method, is often easier to be applied since it concentrates on the procedure used for generating the random deviates, but its performance is not good as the proposed GA-Based methodology.

5. Conclusion

Although Monte Carlo simulation model provides a flexible tool to obtain numerical estimates of a European call option on a stock which pays discrete dividends, it is very time-consuming. Based on the example given in the Boyle's article [5], when the current stock price is 50 and option has 20 periods to maturity, the option value of the crude Monte Carlo estimate (5000 trials) is 17.190 with standard deviation 0.479. Therefore, the 95 percent confidence limits are 17.190±0.958. Reducing the range of these confidence limits to ±0.05 would require increasing the number of trials from 5000 to 1,835,500. A number of effective techniques for reducing the variance of the estimates have been developed. One of the most effective methods is control variate. However, it is very difficult to find a suitable control variate in real case.

In this paper, we propose the powerful approach based upon the coupling of a Monte Carlo simulation with a genetic algorithms-optimization procedure for solving complex option pricing problems. The genetic algorithm considers a population of chromosomes, each one encoding a different alternative design solution. For a given design solution, the Monte Carlo simulation allows us to evaluate the system performance over a specified mission time, in terms of a pre-defined net profit function. This latter constitutes the objective function to be maximized by the genetic algorithm through the evolution of the successive generations of the population.

In order to avoid an explosion of Monte Carlo simulation runs and an overwhelming use of computer time, each potential solution proposed by the genetic algorithm is explored only by few hundreds Monte Carlo simulation. Due to the fact that during the genetic evolution the superior chromosomes appear repeatedly many times, statistically significant results for the solutions of interest (i.e. the best ones) are obtained. This approach coupled with the 'evolutionary guidance' in the search procedure by the genetic algorithms allows one to efficiently perform the analysis of a realistic system in reasonable computing times.

In the future, we try to apply simulation optimization methodology to handle American call options problems. The problems of this kind of option, which can be exercised at any time during its life, would become more complicated and challenging.

Table 2 Option values using Crude Monte Carlo , control variate, antithetic variate method, and GA-Based simulation; 5000 trial per estimate.

S/E	Number of periods to maturity	Crude Monte Carlo estimate	Standard deviation of crude Monte Carlo estimates	Control variate method	Standard deviation of using control variate method	Anti-thetic variate method	Standard deviation of using Anti-thetic variate method	GA-Based optimization	Standard deviation of using GA-Based optimization
0.50	2	0.004	0.002	0.003	0.00004			0.154433	0.001248
	4	0.087	0.014	0.075	0.0002			0.730537	0.009494
	6	0.273	0.031	0.266	0.001			1.404183	0.015707
	8	0.539	0.047	0.542	0.001			2.247438	0.024833
	10	0.879	0.066	0.869	0.002	N/A		3.304752	0.033606
	12	1.202	0.081	1.222	0.003			4.26557	0.040313
	14	1.547	0.094	1.591	0.004			5.180937	0.050361
	16	1.897	0.108	1.964	0.006			6.046652	0.06431
	18	2.281	0.130	2.330	0.007			6.586973	0.072767
	20	2.873	0.157	2.689	0.010			8.116041	0.085043
1.0	2	5.121	0.114	5.028	0.0003	5.093	0.06	7.797639	0.060528
	4	7.427	0.170	7.251	0.001	7.254	0.092	11.037971	0.0814082
	6	9.247	0.215	9.000	0.002	9.101	0.126	14.11116	0.100583
	8	10.793	0.258	10.494	0.003	10.609	0.150	17.25313	0.12242
	10	11.923	0.297	11.815	0.004	11.716	0.170	19.61279	0.151521
	12	13.135	0.330	13.006	0.006	12.974	0.194	22.59007	0.188225
	14	14.281	0.361	14.094	0.007	14.050	0.215	24.24747	0.208428
	16	15.079	0.389	15.106	0.009	15.013	0.231	26.21128	0.224026
	18	15.945	0.428	16.038	0.010	16.018	0.260	28.80928	0.25622
	20	17.190	0.479	16.903	0.013	17.030	0.287	30.61221	0.284947
1.5	2	26.445	0.240	26.369	0.0004			29.55248	0.127712
	4	28.250	0.333	27.818	0.001			29.43402	0.156068
	6	29.832	0.405	29.299	0.002			35.55066	0.207981
	8	31.257	0.470	30.715	0.003			37.45568	0.235312
	10	32.319	0.528	30.048	0.005	N/A		39.86465	0.256224
	12	33.491	0.578	33.298	0.006			41.93361	0.283278
	14	34.835	0.624	34.465	0.008			45.51148	0.30051
	16	35.503	0.666	35.578	0.009			45.68854	0.347882
	18	36.382	0.722	36.616	0.011			47.46086	0.372783
	20	37.989	0.798	37.582	0.014			49.56996	0.418978

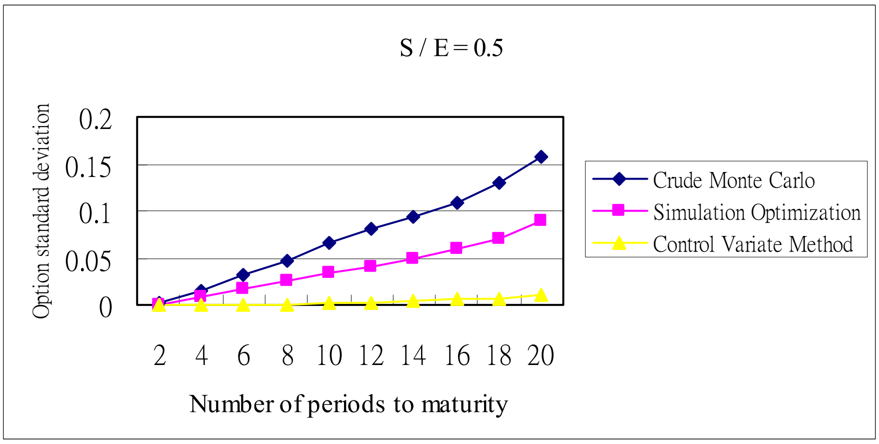

Fig. 7 The comparison of Standard deviation(S/E=0.5)

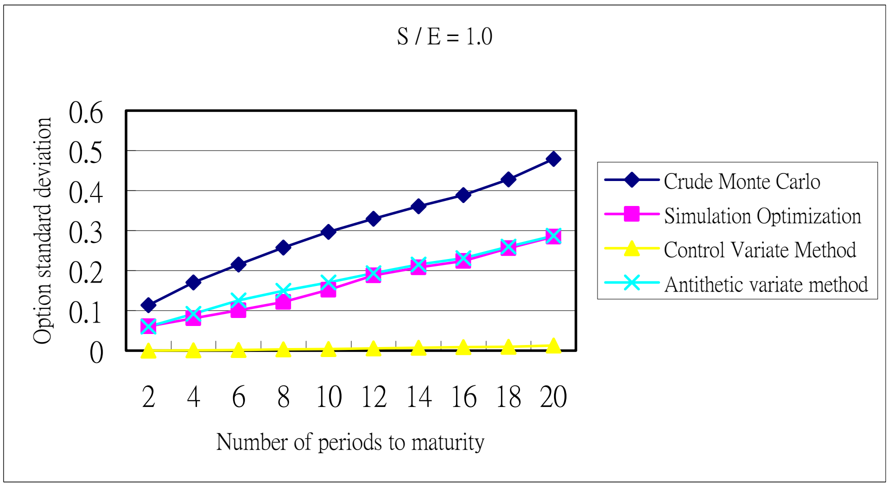

Fig. 8 The comparison of Standard deviation(S/E=1.0)

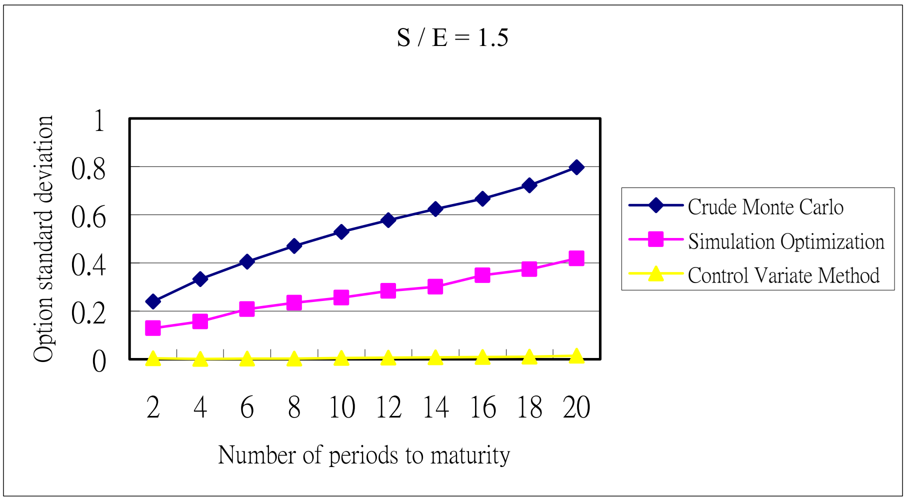

Fig. 9 The comparison of Standard deviation(S/E=1.5)

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