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# A New Approximate Algorithm for Solving Multiple Objective Linear Programming with Fuzzy Parameters

Fengjie Wu

Jie Lu

Guangquan Zhang

# A New Approximate Algorithm for Solving Multiple Objective Linear Programming with Fuzzy Parameters

Fengjie Wu, Jie Lu and Guangquan Zhang

Faculty of Information Technology,
University of Technology, Sydney
PO Box 123, Broadway, NSW 2007,
Australia
{fengjiew, jielu, zhangg}@it.uts.edu.au

## Abstract

Many business decisions can be modeled as multiple objective linear programming (MOLP) problems. When formulating a MOLP problem, objective functions and constraints involve many parameters which possible values are assigned by the experts who are often imprecisely or ambiguously known. So, it would be certainly more appropriate to interpret the experts' understanding of the parameters as fuzzy numerical data which can be represented by fuzzy numbers. This paper focuses on fuzzy multiple objective linear programming (FMOLP) problems with fuzzy parameters in any form of membership function in both objective functions and constraints. Based on the related results of fuzzy linear programming (FLP) and linear programming problems with fuzzy equality and inequality constraints proposed by Zhang et al, this paper firstly proposes related definitions and concepts about FMOLP problems with fuzzy parameters. It then proposes a new approximate algorithm developed for solving the corresponding MOLP problems and the FMOLP problems. Finally, the use of related concepts, theorems, and the proposed approximate algorithm is illustrated by an example involving different cases which include setting various iterate steps, tolerances, weights, and satisfaction levels.

*Keywords*: Optimization; Decision support technology; Multiple objective linear programming; Fuzzy multiple objective linear programming, Approximate algorithm.

## 1. Introduction

Most of decision-making problems in the real world take place in an environment in which the goals, the constraints and consequences of possible actions are not known precisely [1]. In order to deal with such decision situations, fuzzy set theory has been applied into decision-making models.

Multiple objective linear programming (MOLP) is one of the popular methods to deal with complex and ill-structured decision-making. When formulating a MOLP problem, various factors of the real world system should be reflected in the description of the objective functions and the constraints. Naturally, these objective functions and constraints involve many parameters which possible values are assigned by the experts who are often imprecisely or ambiguously known [2] [8]

With this observation, it would be certainly more appropriate to interpret the experts' understanding of the parameters as fuzzy numerical data which can be represented by fuzzy numbers. The fuzzy multiple objective linear programming (FMOLP) problems involving fuzzy parameters would be viewed as a more realistic version than the conventional one [2] [8].

Various kinds of FMOLP models have been proposed to deal with different decision-making situations which involve fuzzy values in objective function parameters, constraints parameters, or relationships. Transforming a FMOLP problem into a crisp programming problem is still employed by researchers [3] including methods proposed by Tanaka and Asai [9], Lai and Hwang [4], Luhandjula [5], Wierzchon [10]. Although various approaches have been made to solve a FMOLP problem with fuzzy parameter, however, most of works listed above are dealt with the fuzzy parameters with symmetric triangular, trapezoidal, or some kind of symmetric membership functions. Zhang et al [11] [12] proposed one method to solve a fuzzy linear programming (FLP) problem by transforming it into a corresponding four-objective constrained optimization problems and another method to formulate linear programming problems with fuzzy equality and inequality constraints. Based on that, this paper develops an approximate algorithm for solving the proposed FMOLP problems with fuzzy parameters in any form of membership function in both objective functions and constraints..

This paper is organized in five sections. The introduction of the related concepts about fuzzy set theory is made in Section 2. A review of related definitions and theorems for solving FMOLP problems with fuzzy parameters in objective functions and constraints are described in Section 3. The core idea of the approximate algorithm proposed for solving FMOLP problems is described in Section 4. An example is presented in Section 5 and aimed at demonstrating the concepts and methods for solving a FMOLP problem.

## 2. Related Definitions Used in the Paper

**Definition 2.1** Let $X$ denotes a universal set. Then a fuzzy subset $\tilde{A}$ of $X$ is defined by its membership function

$$\mu_{\tilde{A}} : X \to [0,1], \tag{1}$$

which assigns to each element $x \in X$ a real number $\mu_{\tilde{A}}(x)$ in the interval [0, 1] where the value of $\mu_{\tilde{A}}(x)$ at $x$ represents the grade of membership of $x$ in $\tilde{A}$ [6] [11].

**Definition 2.2** A fuzzy set $\tilde{A}$ is convex if and only if

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)) \tag{2}$$

for every $x_1, x_2 \in X$ and $\lambda \in [0,1]$ [6] [11].

**Definition 2.3** A fuzzy number is a convex normalized fuzzy set of the real line $R^1$ whose membership function is piecewise continuous [6] [11].

Let $F(R)$ and $F^*(R)$ be the set of all fuzzy numbers and the set of all finite fuzzy numbers on $R$ respectively. According to the decomposition theorem of fuzzy set, we have

$$\tilde{\alpha} = \bigcup_{\lambda \in [0,1]} \lambda [\alpha_\lambda^L, \alpha_\lambda^R] \tag{3}$$

$$\tilde{\alpha} = \bigcup_{\lambda \in R_0} \lambda [\alpha_\lambda^L, \alpha_\lambda^R] \tag{4}$$

for all $\tilde{\alpha} \in F(R)$, where $R_0$ is all rational numbers in (0, 1] [11].

**Definition 2.4** Let $\tilde{\alpha}, \tilde{\beta} \in F(R)$ with the membership functions $\mu_{\tilde{\alpha}}(x)$ and $\mu_{\tilde{\beta}}(x)$ respectively, and $0 \leq \lambda \in R$. The addition of two fuzzy number $\tilde{\alpha} + \tilde{\beta}$ and the scalar product of $\lambda$ and $\tilde{\alpha}$ are defined by the membership functions

$$\mu_{\tilde{\alpha}+\tilde{\beta}}(z) = \sup_{z=x+y} \min\{\mu_{\tilde{\alpha}}(x), \mu_{\tilde{\beta}}(y)\}, \tag{5}$$

$$\mu_{\lambda\tilde{\alpha}}(z) = \max\left\{0, \sup_{z=\lambda x} \mu_{\tilde{\alpha}}(x)\right\}, \tag{6}$$

Form the decomposition theorem of fuzzy set and the Definition 2.4, for any two fuzzy number $\tilde{\alpha}$ and $\tilde{\beta}$, and $0 \leq \alpha \in R$, $0 \leq \beta \in R$

$$\tilde{\alpha} + \tilde{\beta} = \bigcup_{\lambda \in [0,1]} \lambda [\alpha_\lambda^L + \beta_\lambda^L, \alpha_\lambda^R + \beta_\lambda^R] \tag{7}$$

$$\alpha\tilde{\alpha} = \bigcup_{\lambda \in [0,1]} \lambda [\alpha\alpha_\lambda^L, \alpha\alpha_\lambda^R] \tag{8}$$

$$\alpha\tilde{\alpha} + \beta\tilde{\beta} = \bigcup_{\lambda \in [0,1]} \lambda [\alpha\alpha_\lambda^L + \beta\beta_\lambda^L, \alpha\alpha_\lambda^R + \beta\beta_\lambda^R] \tag{9}$$

Let $F(R^n)$ and $F^*(R^n)$ be the set of all $n$-dimensional fuzzy numbers and the set of all $n$-dimensional finite fuzzy numbers on $R$ respectively [11].

**Definition 2.5** For any two $n$-dimensional fuzzy numbers $\tilde{\alpha}, \tilde{\beta} \in F(R^n)$, we define

(1) $\tilde{\alpha} \succeq \tilde{\beta}$ iff $\alpha_{i\lambda}^L \geq \beta_{i\lambda}^L$ and $\alpha_{i\lambda}^R \geq \beta_{i\lambda}^R$, $i = 1,\ldots,n$, $\lambda \in [0,1]$,

(2) $\tilde{\alpha} \succeq \tilde{\beta}$ iff $\alpha_{i\lambda}^L \geq \beta_{i\lambda}^L$ and $\alpha_{i\lambda}^R \geq \beta_{i\lambda}^R$, $i = 1,\ldots,n$, $\lambda \in [0,1]$,

(3) $\tilde{\alpha} \succ \tilde{\beta}$ iff $\alpha_{i\lambda}^L > \beta_{i\lambda}^L$ and $\alpha_{i\lambda}^R > \beta_{i\lambda}^R$, $i = 1,\ldots,n$, $\lambda \in [0,1]$ [11].

## 3. Fuzzy Multiple Objective Linear Programming and Related Definition and Theorems

In this paper, we now consider the situation that all coefficients of the objective functions and constraints are fuzzy numbers parameters in any form of membership function, then such FMOLP problems can be formulated as follows:

$$\text{(FMLOP)} \begin{cases} \text{Maximize} & \tilde{f}(x) = \tilde{C}x \\ \text{s.t.} & x \in X = \{x \in R^n \mid \tilde{A}x \precsim \tilde{b}, x \geq 0\} \end{cases} \tag{10}$$

where $\tilde{C}$ is an $k \times n$ matrix, each element of which $\tilde{c}_{ij}$ is fuzzy number represented by membership function $\mu_{\tilde{c}_{ij}}(x)$, $\tilde{A}$ is an $m \times n$ matrix coefficients of the constraints, each element of which $\tilde{a}_{ij}$ is fuzzy number represented by membership function $\mu_{\tilde{a}_{ij}}(x)$, $\tilde{b}$ is an m-vector of the rhs, each element of which $\tilde{b}_i$ is fuzzy number represented by membership function $\mu_{\tilde{b}_i}(x)$, and $x$ is an $n$-vector of decision variables, $x \in R^n$.

In the proposed FMOLP problems, for each $x \in X$, the values of objective function $\tilde{f}(x)$ are fuzzy numbers. According to the definition 2.5, we have the following definitions about FMOLP problems.

**Definition 3.1** $x^*$ is said to be a complete optimal solution, if and only if there exists $x^* \in X$ such that $\tilde{f}_i(x^*) \succeq \tilde{f}_i(x)$, $i = 1,\ldots,k$, for all $x \in X$.

**Definition 3.2** $x^*$ is said to be a Pareto optimal solution, if and only if there does not exists another $x \in X$ such that $\tilde{f}_i(x) \succeq \tilde{f}_i(x^*)$, for all $i$.

**Definition 3.3** $x^*$ is said to be a weak Pareto optimal solution, if and only if there does not exists another $x \in X$ such that $\tilde{f}_i(x) \succ \tilde{f}_i(x^*)$, for all $i$.

Associated with the FMOLP problems, let's consider the following multiple objective linear programming ($\text{MOLP}_\lambda$) problems:

$$(\text{MOLP}_\lambda) \begin{cases} \text{Maximize} & \left(\langle C_{\lambda_i}^L, x\rangle, \langle C_{\lambda_i}^R, x\rangle\right)^T, \forall \lambda \in [0,1] \\ \text{s.t.} & x \in X = \left\{ \begin{array}{l} x \in R^n \mid A_\lambda^L x \leq b_\lambda^L, A_\lambda^R x \leq b_\lambda^R, \\ x \geq 0, \forall \lambda \in [0,1] \end{array} \right\} \end{cases} \tag{11}$$

where

$$C_\lambda^L = \begin{bmatrix} c_{11\lambda}^L & c_{12\lambda}^L & \cdots & c_{1n\lambda}^L \\ c_{21\lambda}^L & c_{22}^L & \cdots & c_{2n\lambda}^L \\ \vdots & \vdots & \ddots & \vdots \\ c_{k1\lambda}^L & c_{k2\lambda}^L & \cdots & c_{kn\lambda}^L \end{bmatrix}, C_\lambda^R = \begin{bmatrix} c_{11\lambda}^R & c_{12\lambda}^R & \cdots & c_{1n\lambda}^R \\ c_{21\lambda}^R & c_{22}^R & \cdots & c_{2n\lambda}^R \\ \vdots & \vdots & \ddots & \vdots \\ c_{k1\lambda}^R & c_{k2\lambda}^R & \cdots & c_{kn\lambda}^R \end{bmatrix}$$

$$A_\lambda^L = \begin{bmatrix} a_{11\lambda}^L & a_{12\lambda}^L & \cdots & a_{1n\lambda}^L \\ a_{21\lambda}^L & a_{22}^L & \cdots & a_{2n\lambda}^L \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1\lambda}^L & a_{m2\lambda}^L & \cdots & a_{mn\lambda}^L \end{bmatrix}, A_\lambda^R = \begin{bmatrix} a_{11\lambda}^R & a_{12\lambda}^R & \cdots & a_{1n\lambda}^R \\ a_{21\lambda}^R & a_{22}^R & \cdots & a_{2n\lambda}^R \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1\lambda}^R & a_{m2\lambda}^R & \cdots & a_{mn\lambda}^R \end{bmatrix}$$

,

$b_{\lambda}^{L} = \left[b_{1\lambda}^{L}, b_{2\lambda}^{L}, \cdots, b_{m\lambda}^{L}\right]^{T}$, $b_{\lambda}^{R} = \left[b_{1\lambda}^{R}, b_{2\lambda}^{R}, \cdots, b_{m\lambda}^{R}\right]^{T}$

**Definition 3.4** $x^*$ is said to be a complete optimal solution, if and only if there exists $x^* \in X$ such that $f_i(x^*) \geq f_i(x)$, i = 1, …, k, for all $x \in X$ [7] .

**Definition 3.5** $x^*$ is said to be a Pareto optimal solution, if and only if there does not exists another $x \in X$ such that $f_i(x) \geq f_i(x^*)$, for all *i* and $f_j(x) \neq f_j(x^*)$ for at least one j [7] .

**Definition 3.6** $x^*$ is said to be a weak Pareto optimal solution, if and only if there does not exists another $x \in X$ such that $f_i(x) > f_i(x^*)$, *i* = 1, … , k [7] .

**Theorem 3.1** Let $x^* \in X$ be a feasible solution to the FMOLP problem. Then

1. $x^*$ is a complete optimal solution to the problem if and only if $x^*$ is a complete optimal solution to the $MOLP_{\lambda}$ problem.
2. $x^*$ is a Pareto optimal solution to the problem if and only if $x^*$ is a Pareto optimal solution to the $MOLP_{\lambda}$ problem.
3. $x^*$ is a weak Pareto optimal solution to the problem if and only if $x^*$ is a weak Pareto optimal solution to the $MOLP_{\lambda}$ problem.

Proof. The proof is obvious from Definition 2.5.

In the next section, we will propose an approximation algorithm to solve the $MOLP_{\lambda}$ problem and the FMOLP problem.

## 4. An Approximation Algorithm for Solving FMOLP Problem

Based on the definition of FMOLP problem and $MOLP_{\lambda}$ problem and Theorem 3.1, an approximation algorithm is proposed as following for solving $MOLP_{\lambda}$ problem, the solution of which is equally the solution of FMOLP problem.

For the simplicity in presentation, we define

$$X_{\lambda} = \left\{x \in R^n \mid A_{\lambda}^{L} x \leq b_{\lambda}^{L}, A_{\lambda}^{R} x \leq b_{\lambda}^{R}, x \geq 0\right\} \quad \lambda \in [0,1]$$

The main steps of the approximation algorithm are described as follows:

Let the interval [0,1] be decomposed into *m* mean sub-intervals with $(m+1)$ nodes $\lambda_i (i = 0, \cdots, m)$ which are arranged in the order of $0 = \lambda_0 < \lambda_1 < \cdots < \lambda_m = 1$, then define $X^m = \bigcap_{i}^{m} X_{\lambda_i}$, and denote

$$(MOLP_{\lambda})_m \begin{cases} \text{Max} & \left(\langle C_{\lambda_i}^{L}, x\rangle, \langle C_{\lambda_i}^{R}, x\rangle\right)^T, 0 = \lambda_0 < \ldots < \lambda_m = 1 \\ \text{s.t.} & x \in X^m \end{cases} \tag{12}$$

*Step 1*: Set $m = 1$, then solve $(MOLP_{\lambda})_m$ for $(x)_m$, where $(x)_m = (x_1, x_2, \ldots, x_n)_m$, and the solution is obtained subject to constraint $x \in X^m$.

*Step 2*: Solve $(MOLP_{\lambda})_{2m}$ for $(x)_{2m}$.

*Step 3*: If $\|(x)_{2m} - (x)_m\| < \varepsilon$, the solution $x^*$ of $MOLP_{\lambda}$ problem is $(x)_{2m}$. Otherwise, update *m* to *2m* and go to Step 2.

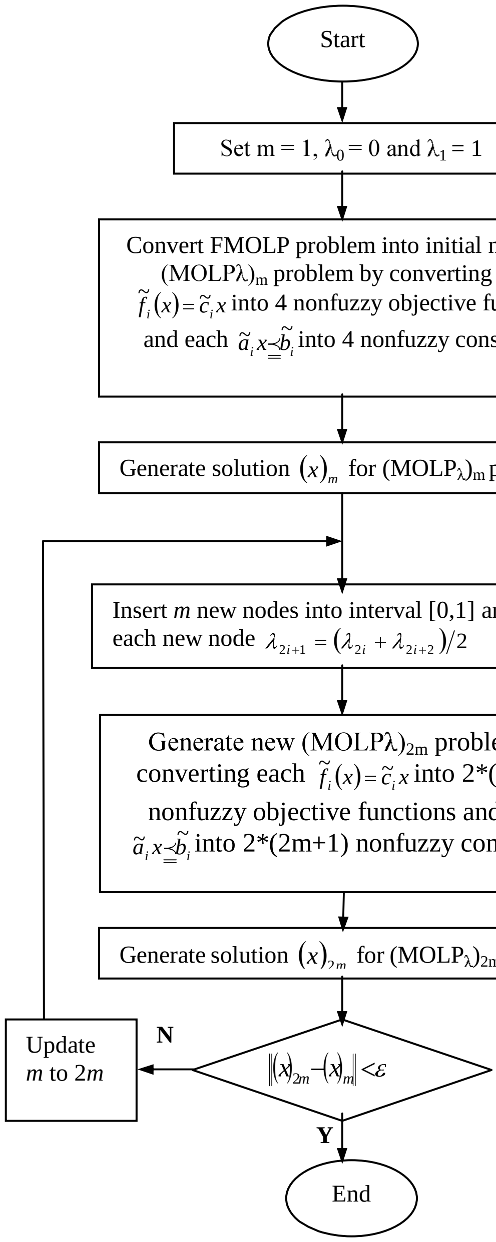


**Diagram 1: The data flow diagram for the approximate algorithm**

Initially, in step 1, the interval [0,1] is not split, only $\lambda_0 = 0$ and $\lambda_1 = 1$ are be considered, then, each fuzzy objective function $\tilde{f}_i(x) = \tilde{c}_i x$ is converted into four corresponding nonfuzzy objective functions $\left(\langle c_{i0}^{L}, x\rangle, \langle c_{i1}^{L}, x\rangle, \langle c_{i0}^{R}, x\rangle, \langle c_{i1}^{R}, x\rangle\right)^T$ $i = 1, \ldots, k$. This conversion also applies on constraints in the same way. The solution $(x)_m$ is based on new nonfuzzy objective functions and constraints.

In step 2, the interval [0,1] is split further. In this step, suppose there are $(m+1)$ nodes $\lambda_i(i=0,2,\ldots,2m)$ in the interval [0,1], and $m$ new nodes are inserted and $\lambda_{2i+1}=(\lambda_{2i}+\lambda_{2i+2})/2$ . Therefore, each fuzzy objective function $\tilde{f}_i(x)=\tilde{c}_i x$ is converted into $2*(2m+1)$ corresponding nonfuzzy objective functions, and the same conversion happens on the constraints. Suppose that the number of fuzzy objective functions and fuzzy constraints are $k$ and $l$ respectively, the total number of nonfuzzy objective functions and constraints are $2*k*(2m+1)$ and $2*l*(2m+1)$ respectively. The solution $(x)_{2m}$ is also based on new nonfuzzy objective functions and constraints.

In step 3, if the difference between two of solutions $(x)_m$ and $(x)_{2m}$ in two consecutive steps is within the preset tolerance, the solution of current step is the final result; otherwise, the algorithm needs more iteration.

We can shift the low bound $\lambda_0$ of interval from 0 to 1 to increase the satisfaction level. When $\lambda_0=1$, the original fuzzy problem will be changed to crisp problem.

The detail data flow diagram is listed in Diagram 1.

# 5. An illustrative example

## 5.1 The description of an example of FMOLP

In this section, let us consider the following FMOLP problem with two objective functions:

$$\max \tilde{f}(x)=\max\begin{pmatrix}\tilde{f}_1(x)\\ \tilde{f}_2(x)\end{pmatrix}=\max\begin{pmatrix}\tilde{c}_{11}x_1+\tilde{c}_{12}x_2\\ \tilde{c}_{21}x_1+\tilde{c}_{22}x_2\end{pmatrix} \tag{13}$$

$$\text{s.t.}\quad\begin{cases}\tilde{a}_{11}x_1+\tilde{a}_{12}x_2\le\tilde{b}_1\\ \tilde{a}_{21}x_1+\tilde{a}_{22}x_2\le\tilde{b}_2\\ \tilde{a}_{31}x_1+\tilde{a}_{32}x_2\le\tilde{b}_3\\ \tilde{a}_{41}x_1+\tilde{a}_{42}x_2\le\tilde{b}_4\end{cases} \tag{14}$$

Where membership functions of coefficients of objectives and constraints are as following:

$$\mu_{\tilde{c}_{11}}(x)=\begin{cases}0 & x<1\\ (x^2-1)/3 & 1\le x<2\\ 1 & 2\le x\le 3\\ (256-x^2)/247 & 3<x\le 16\\ 0 & 16<x\end{cases}$$

$$\mu_{\tilde{c}_{12}}(x)=\begin{cases}0 & x<0\\ x^2 & 0\le x<1\\ 1 & 1\le x\le 2\\ (576-x^2)/574 & 2<x\le 24\\ 0 & 24<x\end{cases}$$

$$\mu_{\tilde{c}_{21}}(x)=\begin{cases}0 & x<-2\\ (4-x^2)/3 & -2\le x<-1\\ 1 & -1\le x\le 0\\ (13-x)/13 & 0<x\le 13\\ 0 & 13<x\end{cases}$$

$$\mu_{\tilde{c}_{22}}(x)=\begin{cases}0 & x<1\\ (x^2-1)/3 & 1\le x<2\\ 1 & 2\le x\le 3\\ (25-x)/22 & 3<x\le 25\\ 0 & 25<x\end{cases}$$

$$\mu_{\tilde{a}_{11}}(x)=\begin{cases}0 & x<-2\\ 2x+4 & -2\le x<-1.5\\ 1 & -1.5\le x\le -0.5\\ 4x^2 & -0.5<x\le 0\\ 0 & 0<x\end{cases}$$

$$\mu_{\tilde{a}_{12}}(x)=\begin{cases}0 & x<0\\ x/4 & 0\le x<2\\ 1 & 2\le x\le 4\\ (e^{12}-e^x)/(e^{12}-e^4) & 4<x\le 12\\ 0 & 12<x\end{cases}$$

$$\mu_{\tilde{a}_{21}}(x)=\begin{cases}0 & x<0\\ 2x & 0\le x<0.5\\ 1 & 0.5\le x\le 1.5\\ (e^{10}-e^x)/(e^{10}-e^{1.5}) & 1.5<x\le 10\\ 0 & 10<x\end{cases}$$

$$\mu_{\tilde{a}_{22}}(x)=\begin{cases}0 & x<0\\ (e^x-1)/(e^2-1) & 0\le x<2\\ 1 & 2\le x\le 4\\ (e^{18}-e^x)/(e^{18}-e^4) & 4<x\le 18\\ 0 & 18<x\end{cases}$$

$$\mu_{\tilde{a}_{31}}(x)=\begin{cases}0 & x<1\\ (e^x-e^1)/(e^3-e^1) & 1\le x<3\\ 1 & 3\le x\le 5\\ (18-x)/13 & 5<x\le 18\\ 0 & 18<x\end{cases}$$

$$\mu_{\tilde{a}_{32}}(x)=\begin{cases}0 & x<0\\ x^2/4 & 0\le x<2\\ 1 & 2\le x\le 4\\ (10-x)/6 & 4<x\le 10\\ 0 & 10<x\end{cases}$$

$$\mu_{\tilde{a}_{41}}(x)=\begin{cases}0 & x<0\\ x/2 & 0\le x<2\\ 1 & 2\le x\le 4\\ (e^{10}-e^x)/(e^{10}-e^4) & 4<x\le 10\\ 0 & 10<x\end{cases}$$

$$\mu_{\tilde{a}_{42}}(x)=\begin{cases}0 & x<0\\ 2x & 0\le x<0.5\\ 1 & 0.5\le x\le 1.5\\ (1000-x^3)/996.625 & 1.5<x\le 10\\ 0 & 10<x\end{cases}$$

$$\mu_{\tilde{b}_1}(x)=\begin{cases}0 & x<18\\ (e^x-e^{18})/(e^{20}-e^{18}) & 18\le x<20\\ 1 & 20\le x\le 22\\ (27000-x^3)/16352 & 22<x\le 30\\ 0 & 30<x\end{cases}$$

$$\mu_{\tilde{b}_2}(x)=\begin{cases}0 & x<24\\ \left(e^{x}-e^{24}\right)/\left(e^{26}-e^{24}\right) & 24\le x<26\\ 1 & 26\le x\le 28\\ (40-x)/12 & 28<x\le 40\\ 0 & 40<x\end{cases}$$

$$\mu_{\tilde{b}_3}(x)=\begin{cases}0 & x<42\\ (x-42)/2 & 42\le x<44\\ 1 & 44\le x\le 46\\ \left(e^{60}-e^{x}\right)/\left(e^{60}-e^{46}\right) & 46<x\le 60\\ 0 & 60<x\end{cases}$$

$$\mu_{\tilde{b}_1}(x)=\begin{cases}0 & x<27\\ \left(x^2-729\right)/112 & 27\le x<29\\ 1 & 29\le x\le 31\\ \left(64000-x^3\right)/34209 & 31<x\le 40\\ 0 & 40<x\end{cases}$$

Associate with the FMOLP problem, the corresponding $MOLP_\lambda$ problem is listed as follows:

$$\max\ f(x)=\max\begin{pmatrix}\sqrt{3\lambda+1}x_1+\sqrt{\lambda}x_2\\ \sqrt{256-247\lambda}x_1+\sqrt{576-574\lambda}x_2\\ \sqrt{4-3\lambda}x_1+\sqrt{3\lambda+1}x_2\\ (13-13\lambda)x_1+(25-22\lambda)x_2\end{pmatrix}$$

st.

$$\begin{cases}(\lambda-4)x_1/2+4\lambda x_2\le\ln\left(\left(e^{20}-e^{18}\right)\lambda+e^{18}\right)\\ \sqrt{\lambda}x_1/2+\ln\left(e^{12}-\left(e^{12}-e^{4}\right)\lambda\right)x_2\le\sqrt[3]{27000-16352\lambda}\\ \sqrt{\lambda}x_1/2+\ln\left(\left(e^{2}-1\right)\lambda+1\right)x_2\le\ln\left(\left(e^{26}-e^{24}\right)\lambda+e^{24}\right)\\ \ln\left(e^{10}-\left(e^{10}-e^{1.5}\right)\lambda\right)x_1+\ln\left(e^{18}-\left(e^{18}-e^{4}\right)\lambda\right)x_2\le 40-12\lambda\\ \ln\left(\left(e^{3}-e^{1}\right)\lambda+e^{1}\right)x_1+2\sqrt{\lambda}x_2\le 42+2\lambda\\ (18-13\lambda)x_1+(10-6\lambda)x_2\le\ln\left(e^{60}-\left(e^{60}-e^{46}\right)\lambda\right)\\ 2\lambda x_1+\lambda x_2/3\le\sqrt{729+112\lambda}\\ \ln\left(e^{10}-\left(e^{10}-e^{4}\right)\lambda\right)x_1+\sqrt[3]{1000-996.6252\lambda}x_2\le\sqrt[3]{64000-34209\lambda}\end{cases}$$

where $\lambda\in[0,1]$.

**Table 1: Summary of the running solution by the approximate algorithm**

| Step | The number of nonfuzzy objective functions | The number of nonfuzzy constraints | $x_1^*$ | $x_2^*$ |
|---|---|---|---|---|
| 1 | 8 | 16 | 1.5179 | 1.3790 |
| 2 | 12 | 24 | 1.5985 | 1.1049 |
| 3 | 20 | 40 | 1.6130 | 1.0296 |
| 4 | 36 | 72 | 1.6161 | 1.0280 |
| 5 | 68 | 136 | 1.6163 | 1.0279 |
| 6 | 132 | 264 | 1.6167 | 1.0263 |
| 7 | 260 | 520 | 1.6166 | 1.0264 |
| 8 | 516 | 1032 | 1.6166 | 1.0264 |

**Table 2: Summary of the running solution by setting different weights of objective functions**

| $w_1$ | $w_2$ | $x_1^*$ | $x_2^*$ | $\tilde{f}_1(x_1^*,x_2^*)$ | $\tilde{f}_2(x_1^*,x_2^*)$ |
|---|---|---|---|---|---|
| 1 | 0 | 3.2260 | 0.1932 | $3.2260\tilde{c}_{11}+0.1932\tilde{c}_{12}$ | $3.2260\tilde{c}_{21}+0.1932\tilde{c}_{22}$ |
| 0.9 | 0.1 | 2.9141 | 0.3543 | $2.9141\tilde{c}_{11}+0.3543\tilde{c}_{12}$ | $2.9141\tilde{c}_{21}+0.3543\tilde{c}_{22}$ |
| 0.8 | 0.2 | 2.5954 | 0.5201 | $2.5954\tilde{c}_{11}+0.5201\tilde{c}_{12}$ | $2.5954\tilde{c}_{21}+0.5201\tilde{c}_{22}$ |
| 0.7 | 0.3 | 2.2721 | 0.6878 | $2.2721\tilde{c}_{11}+0.6878\tilde{c}_{12}$ | $2.2721\tilde{c}_{21}+0.6878\tilde{c}_{22}$ |
| 0.6 | 0.4 | 1.9454 | 0.8568 | $1.9454\tilde{c}_{11}+0.8568\tilde{c}_{12}$ | $1.9454\tilde{c}_{21}+0.8568\tilde{c}_{22}$ |
| 0.5 | 0.5 | 1.6166 | 1.0264 | $1.6166\tilde{c}_{11}+1.0264\tilde{c}_{12}$ | $1.6166\tilde{c}_{21}+1.0264\tilde{c}_{22}$ |
| 0.4 | 0.6 | 1.2872 | 1.1957 | $1.2872\tilde{c}_{11}+1.1957\tilde{c}_{12}$ | $1.2872\tilde{c}_{21}+1.1957\tilde{c}_{22}$ |
| 0.3 | 0.7 | 0.9589 | 1.3637 | $0.9589\tilde{c}_{11}+1.3637\tilde{c}_{12}$ | $0.9589\tilde{c}_{21}+1.3637\tilde{c}_{22}$ |
| 0.2 | 0.8 | 0.6337 | 1.5296 | $0.6337\tilde{c}_{11}+1.5296\tilde{c}_{12}$ | $0.6337\tilde{c}_{21}+1.5296\tilde{c}_{22}$ |
| 0.1 | 0.9 | 0.3134 | 1.6922 | $0.3134\tilde{c}_{11}+1.6922\tilde{c}_{12}$ | $0.3134\tilde{c}_{21}+1.6922\tilde{c}_{22}$ |
| 0 | 1 | 0 | 1.8506 | $1.8506\tilde{c}_{12}$ | $1.8506\tilde{c}_{22}$ |

According to Theorem 3.1, the solution to the MOLP$_\lambda$ problem is also a solution to the FMOLP problem. Here, we recommend using weighting maxmin method to solve the MOLP$_\lambda$ problem.

### 5.2 Solving MOLP$_\lambda$ problem by the approximate algorithm

Refer to MOLP$_\lambda$ problem, in step 1, the interval [0,1] is not split, only $\lambda_0 = 0$ and $\lambda_1 = 1$ are be considered, then totally 8 nonfuzzy objective functions and 16 nonfuzzy constraints are generated. By weighting maximum method, the solution is $(x_1^*, x_2^*) = (1.5179, 1.3790)$.

In step 2, one node is inserted into the interval [0,1], then totally 12 nonfuzzy objective functions and 24 nonfuzzy constraints are generated. By weighting maximum method, the solution is $(x_1^*, x_2^*) = (1.5985, 1.1049)$. The more detail information about further steps is listed in Table 1.

From Table 1, we can find that if the tolerance $\varepsilon = 10^3$, the approximate algorithm stops at step 5, if the tolerance $\varepsilon = 10^5$, the approximate algorithm will stop at step 8, and the number of nonfuzzy objective functions and constraints will increase dramatically.

When choose different weights $w_1$ and $w_2$ for objective functions, the different results are summarized in Table 2:

It can be found from Table 2, when $w_1 = 1$ and $w_2 = 0$, the solution (3.2260, 0.1932) is only concerned about the first objective function. When $w_1 = 0$ and $w_2 = 1$, the solution (0, 1.8506) is only concerned about the second objective function. When $w_1$ decreases from 1 to 0 and $w_2$ increases from 0 to 1 simultaneously, the solution will move from (3.2260, 0.1932) to (0, 1.8506) gradually.

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