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International Conference on Electronic Business (ICEB)

Winter 12-9-2003

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Abstract

In this paper, we propose a new model for improving lot size as obtained on applying the model given in "An integrated inventory model for a single vendor and multiple buyers with ordering cost reduction" (Int. J. Production Economics 73 203-215) proposed by Woo, Hsu, and Wu (2001) and our model provides a lower or equal joint total relevant cost as compared to Woo, Hsu, and Wu's model. And a numerical study based on the example used by Woo, Hsu, and Wu is presented.

1. Introduction

Reducing inventory levels of raw materials, work-in-process, and finished items simultaneously in different stages has become the major focus for supply chain management. In recent years, there has been a growing trend in both research work and practical applications of VMI (vendor managed inventory) policy for various industries. Goyal (1977) [3] proposes a joint economic lot size (JELS) model where the objective is to minimize the total relevant costs for a single vendor, single buyer system. Banerjee (1986) [1] generalizes Goyal's model by incorporating a finite production rate for the vendor and gives the optimal joint production or order quantity. Goyal (1988) [4] extends Banerjee's model again by relaxing the lot-for-lot production assumption and argues that the economic production quantity will be an integral multiple of the buyer's purchase quantity and shows that its model provides a lower or equal joint total relevant cost as compared to Banerjee's model. Kohli and Park [5] investigate joint ordering policies as a method to reduce transaction costs between a single vendor and a homogeneous group of buyers. They present expressions for optimal joint order quantities assuming all products are ordered in each joint order. Lu [6] considers a one-vendor multi-buyer integrated inventory model and gives a heuristic approach for joint replenishment policy. Banerjee and Banerjee (1992) [2] consider an EDI-based vendor-managed inventory (VMI) system in which the vendor makes all replenishment decisions for his/her buyers to improve the joint inventory cost. Woo, Hsu, and Wu (2001) [7] extend the previous work to three level supply chain in which raw materials and EDI are considered. In the work, they considered that the replenishment cycle is an integral multiple of the common replenishment cycle of the vendor and all buyers. The work also relaxes the lot-for-lot production assumption and performs a good effect since many optimal integer multiple used in the numerical example of Woo, Hsu, and Wu's paper are greater than 1, that is, the replenishment cycle of raw materials is no less than that of finished items. This may be true when ordering cost per raw material order for vendor is quite high, the inventory cost for raw materials is relatively low and the ordering cost for finished items is quite low etc. Otherwise, the replenishment cycle for raw materials may be less than that of finished items. So In this paper, we propose a new model for improving lot size as obtained on applying the model given in "An integrated inventory model for a single vendor and multiple buyers with ordering cost reduction"(Int. J. Production Economics 73 203-215) proposed by Woo, Hsu, and Wu (2001). And a numerical study based on the example used by Woo, Hsu, and Wu is presented.

2. Assumptions and Notations

2.1 Assumptions

1. Shortages are not allowed for the vendor.

2. The information of each buyer's replenishment decision parameters is available to the vendor.

3. The planned ordering cost for each buyer depends on the expenditure incurred per unit time to operate the new ordering system. This expenditure could be the leasing cost of equipment and the operating cost to keep the system working effectively.

4. The vendor purchases raw materials outside to produce finished items. The procurement cycle of raw materials is assumed to be an integral multiple of the common replenishment cycle for finished items or the common replenishment cycle for finished items is assumed to be an integral multiple of the procurement cycle of raw materials.

Assumptions 1-3 follow the assumptions of Woo, Hsu, and Wu (2001), and assumption 4 is a relaxation to that of Woo, Hsu, and Wu (2001).

2.2 Notations

M	Number of buyers considered
i=1,2,…,m	Index of buyers
D_i	Demand rate for buyer i, which is a known constant
n	Integral number or fraction which denominator is an integer and numerator is one, which indicates the production batches per raw material procurement cycle and n is a decision variable
K	Expenditure per unit time to operate the planned ordering system between vendor and all buyers, which is a decision variable
C	Common cycle time for buyers, which is a decision variable
f_i	Fraction of backlogging time in a cycle for buyer i, which is a decision variable
M	Usage rate of raw materials for producing each finished item
A	Ordering cost per raw material order for vendor
S	Setup cost for production run for vendor
x	x=0 if $n \ge 1$ and x=1 if $n < 1$ which is a binary variable
T_{0i}	Original ordering cost per buyer i's order
$T_i(K)$	Planned ordering cost per buyer i's order, which is a strictly decreasing function of K with $T_i(0) = T_{0i}$ and $T_i(K_0) = 0$
H_{vm}	Carrying cost per unit of raw materials held per time for vendor
H_{vp}	Carrying cost per finished item held per unit time for vendor
H_{bi}	Carrying cost per unit held per unit time for buyer i
L_i	Backlogging cost per unit backlogged per unit time for buyer i
TC_{bi}	the buyer i's total inventory cost for finished items per unit time
TC_{vp}	Vendor's total cost per unit time for finished items
TC_p	the inventory cost of finished items for all buyers and the vendor
HC_{vm}	Vendor's carrying cost per procurement cycle for raw materials when $n \ge 1$
TC_{vm}	Vendor's total cost per unit time for raw materials when $n \ge 1$
JTC	Joint total cost per unit time for vendor and all buyers
HC_{vm}'	Vendor's carrying cost per procurement cycle for raw materials when $n < 1$
TC_{vm}'	Vendor's total cost per unit time for raw materials when $n < 1$

3. Model

The behavior of inventory levels for the vendor and all buyers for finished items is illustrated in Fig.1 which is the same as that of Woo, Hsu and Wu's. Therefore, the buyer i's total inventory cost for finished items per unit time is

$$TC_{bi} = \frac{T_i(K)}{C} + \frac{C}{2}[H_{bi}(1-f_i)^2 D_i + L_i f_i^2 D_i] \tag{1}$$

The vendor's inventory for finished items is

$$TC_{vp} = \frac{1}{C}[S + H_{vp}\sum_{i=1}^{m}\frac{D_i^2 C^2}{2P}] = \frac{S}{C} + \frac{CH_{vp}}{2P}\sum_{i=1}^{m} D_i^2 \tag{2}$$

Then the inventory cost of finished items for all buyers and the vendor is

$$TC_p = \frac{\sum_{i=1}^{m} T_i(K)}{C} + \frac{C}{2}\sum_{i=1}^{m}[H_{bi}(1-f_i)^2 D_i + L_i f_i^2 D_i]$$
$$+\frac{S}{C} + \frac{CH_{vp}}{2P}\sum_{i=1}^{m} D_i^2 \ . \tag{3}$$

The inventory levels for raw materials in the paper of Woo, Hsu and Wu (2001) is illustrated in Fig.2. The arrival of each raw material procurement will coincide with the start of a new production run. The procurement lot size of raw materials is equal to n multiple of the usage of each production batch and hence each procurement cycle equals nC time units. The stock of raw materials will be consumed continuously during each production run period $\sum_{i=1}^{m} D_i C / P$, and then be held until the next production run starts.

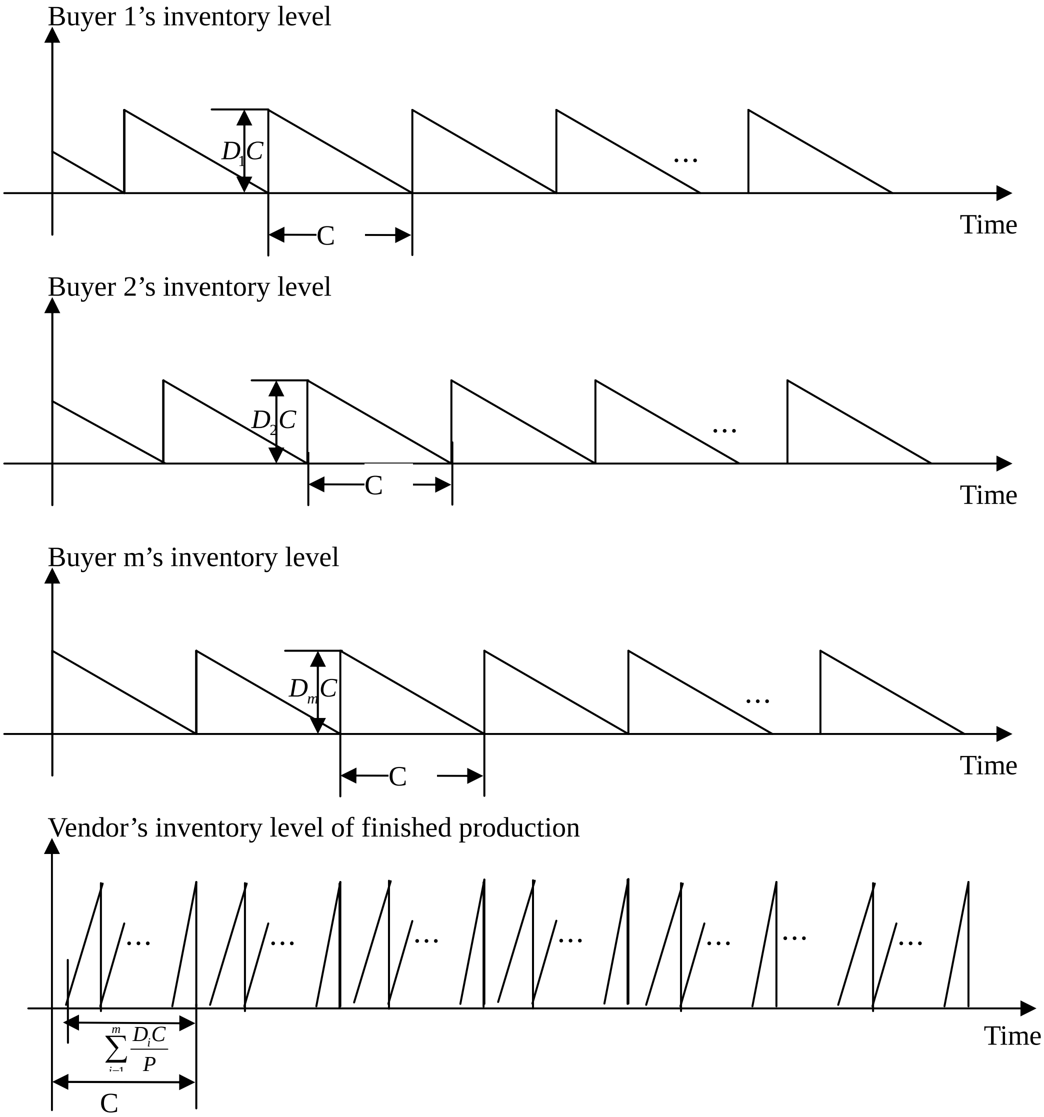

Fig. 1. The inventory level for finished items

Therefore, the vendor's raw materials stock per procurement cycle consists of n triangles and (n-1) rectangles and the carrying cost for raw materials per cycle is

$$HC_{vm} = H_{vm}[\frac{1}{2}(nM\sum_{i=1}^{m} D_iC)(\frac{\sum_{i=1}^{m} D_iC}{P})$$

$$+\sum_{i=1}^{n-1}(jM\sum_{i=1}^{m} D_iC^2]$$

$$=\frac{nMH_{vm}}{2P}(\sum_{i=1}^{m} D_iC)^2+\frac{n(n-1)MH_{vm}}{2}\sum_{i=1}^{m}(D_iC^2)\,. \qquad (4)$$

Thus, the vendor's total inventory cost for raw materials per unit time is

$$TC_{vm}=\frac{1}{nC}(A+HC_{vm})=\frac{A}{nC}+\frac{CMH_{vm}}{2}\sum_{i=1}^{m} D_i(n-1$$

$$+\frac{\sum_{i=1}^{m} D_i}{P})\,. \qquad (5)$$

From the inventory formulation of raw materials above, the least procurement cycle of raw materials is the common replenishment cycle, C. These may be true when the ordering cost per raw material order for the vendor is larger enough than setup cost per production run for the vendor. However, if the ordering cost per raw material order is small, for example, all raw materials being sent by all suppliers of the vendor or in JIT (just in time) setting), the procurement cycle of raw materials may be relatively short comparing with that of the common replenishment cycle for all buyers. In this setting we assume the usage of each production batch is an integral multiple of the procurement lot size of raw materials. That is, the procurement cycle of raw materials is the fraction multiple of the production run of finished items. For the fraction (notified by n), the denominator is an integer and the numerator is one. By this policy, the change of inventory levels of raw materials is shown in Fig. 3.

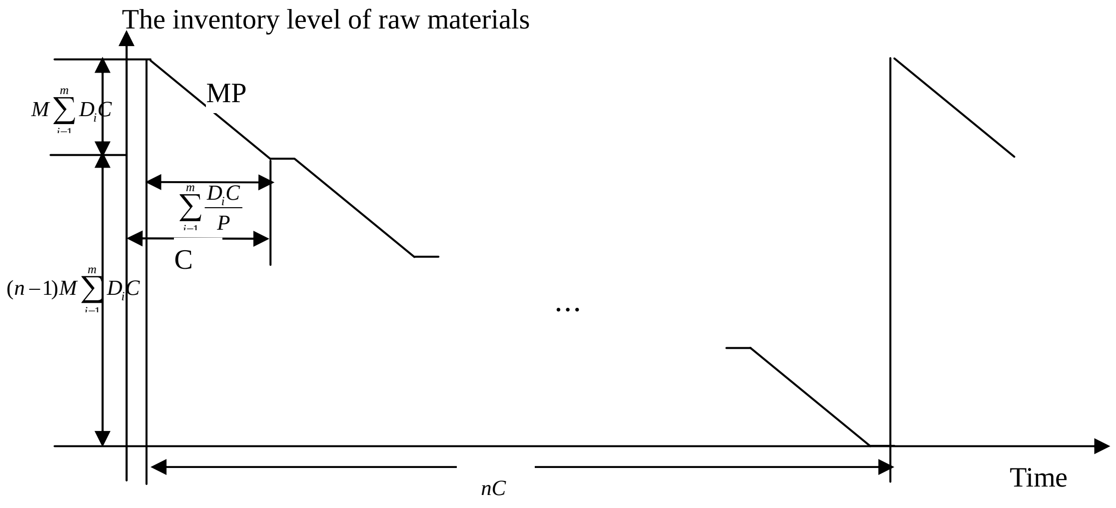

Fig. 2. One kind inventory level for raw materials

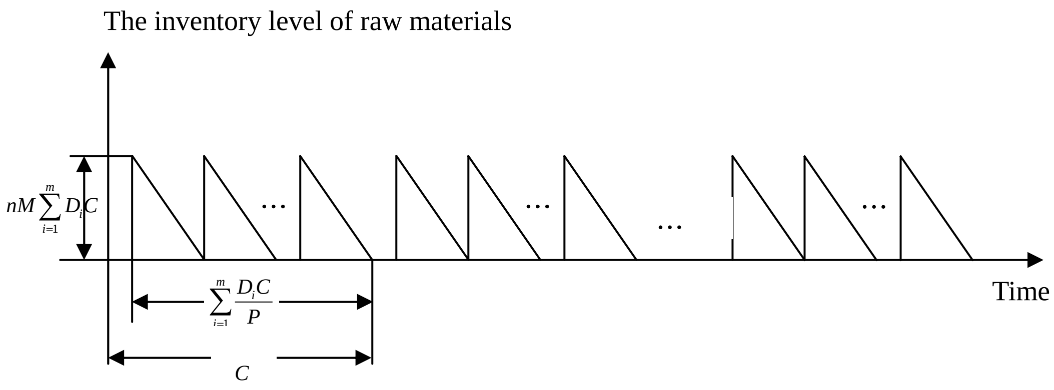

Fig. 3. Another kind inventory level for raw materials

The vendor's raw materials stock per procurement cycle consists of n triangles and the carrying cost of raw materials per cycle is

$$HC'_{vm} = \frac{nMH_{vm}}{2}\sum_{i=1}^{m}(D_i C)\left(\frac{\sum_{i=1}^{m} D_i C}{P}\right)$$

$$= \frac{nMC^2 H_{vm}}{2P}\left(\sum_{i=1}^{m} D_i\right)^2 . \qquad (6)$$

Thus, the vendor’s total inventory cost for raw materials per unit time is

$$TC'_{vm} = \frac{1}{nC}(A + HC'_{vm}) = \frac{A}{nC} + \frac{nMCH_{vm}}{2P}\left(\sum_{i=1}^{m} D_i\right)^2 . \qquad (7)$$

Therefore, comparing with lot-for-lot policy and that of Woo, Hsu and Wu (2001) a more general model (notified by GM) is proposed here.

Model GM:

$$\min_{K,n,C,x,f_i} \text{JTC=K+}TC_p\text{+(1-}x\text{)}TC_{vm} + xTC'_{vm}$$

$$=K + \frac{1}{C}\left[\frac{A}{n} + S + \sum_{i=1}^{m} T_i(K)\right] + \frac{C}{2}\sum_{i=1}^{m}[H_{bi}(1-f_i)^2 D_i$$

$$+L_i f_i^2 D_i] + \frac{CH_{vp}}{2P}\sum_{i=1}^{m} D_i^2 + \frac{CMH_{vm}}{2}[(1$$

$$-x)\sum_{i=1}^{m} D_i (n-1+\frac{\sum_{i=1}^{m} D_i}{P}) + x\frac{n(\sum_{i=1}^{m} D_i)^2}{P}], \tag{8}$$

subject to:

$$1-n \le x(\sum_{i=1}^{m} D_i C), \tag{9}$$

$$\sum_{i=1}^{m} D_i < P, \tag{10}$$

$$T_i(K) = T_{0i}e^{-rK}, \text{ r>0 and i=1,2,...,m} \tag{11}$$

where K, f_i and C are nonnegative variables; x is a binary variable; and $n \in \{\frac{1}{\infty},\ldots,\frac{1}{2},1,2,\ldots,\infty\}$.

4. Analysis of the Model

Model GM gives an alternative for raw material procurement cycle; one is that the procurement cycle is integral times common cycle time and another is that the common cycle time of finished items is integral times the procurement cycle of raw materials. When n>1 the model in this paper is the same as that of Woo, Hsu, and Wu (2001). From the paper of Woo, Hsu, and Wu (2001), the optimal result is derived as:

$$f_i^* = \frac{H_{bi}}{H_{bi}+L_i} \tag{12}$$

$$C^* = \sqrt{\frac{2[A/n + S + \sum_{i=1}^{m} T_i(K)]}{H\sum_{i=1}^{m} D_i}} \tag{13}$$

$$n^*(K) = \begin{cases} n^*(n^*-1) < \dfrac{AG}{MH_{vm}(S+\sum_{i=1}^{m} T_i(K))} \le n^*(n^*+1) & \text{if } G>0, \\ 1 & \text{otherwise,} \end{cases} \tag{14}$$

$$K^* = \begin{cases} \dfrac{1}{r}\ln\dfrac{Hr^2\sum_{i=1}^{m} D_i \sum_{i=1}^{m} T_{0i}}{1+\sqrt{1+2Hr^2\sum_{i=1}^{m} D_i((A/n)+S)}} & \text{if n>N,} \\ 0 & \text{otherwise,} \end{cases} \tag{15}$$

where

$$H = MH_{vm}(n-1+\frac{\sum_{i=1}^{m} D_i}{P}) + \frac{1}{\sum_{i=1}^{m} D_i}[\frac{H_{vp}}{P}\sum_{i=1}^{m} D_i^2$$

$$+\sum_{i=1}^{m}\frac{H_{bi}L_iD_i}{H_{bi}+L_i}], \tag{16}$$

$$G = \frac{1}{\sum_{i=1}^{m} D_i}[\frac{H_{vp}}{P}\sum_{i=1}^{m} D_i^2 + \sum_{i=1}^{m}\frac{H_{bi}L_iD_i}{H_{bi}+L_i}]$$

$$-MH_{vm}(1-\frac{\sum_{i=1}^{m} D_i}{P}), \tag{17}$$

$$N = \frac{2(S+\sum_{i=1}^{m} T_{0i}) - Gr^2\sum_{i=1}^{m} D_i(\sum_{i=1}^{m} T_{0i})^2}{2MH_{vm}r^2\sum_{i=1}^{m} D_i(\sum_{i=1}^{m} T_{0i})^2}$$

$$+\frac{\{[2(S+\sum_{i=1}^{m} T_{0i}) - Gr^2\sum_{i=1}^{m} D_i(\sum_{i=1}^{m} T_{0i})^2]^2}{2MH_{vm}r^2\sum_{i=1}^{m} D_i(\sum_{i=1}^{m} T_{0i})^2}$$

$$+\frac{8AMH_{vm}r^2\sum_{i=1}^{m} D_i(\sum_{i=1}^{m} T_{0i})^2\}^{1/2}}{2MH_{vm}r^2\sum_{i=1}^{m} D_i(\sum_{i=1}^{m} T_{0i})^2}. \tag{18}$$

We have $x=1$ from Model GM when n<1. Model GM can be rearranged as model RGM.

Model RGM:

$$\min_{K,n,C,x,f_i} \text{JTC}' = K + \frac{1}{C}[\frac{A}{n} + S + \sum_{i=1}^{m} T_i(K)]$$

$$+\frac{C}{2}\sum_{i=1}^{m}[H_{bi}(1-f_i)^2D_i + L_i f_i^2 D_i]$$

$$+\frac{CH_{vp}}{2P}\sum_{i=1}^{m} D_i^2 + \frac{nCMH_{vm}(\sum_{i=1}^{m} D_i)^2}{2P}, \tag{19}$$

subject to Equations (10) and (11).

From Equation (19), for any given C, K, and n, the optimal value of f_i can easily be obtained as

$$f_i^* = \frac{H_{bi}}{H_{bi}+L_i} \tag{20}$$

By substituting (20) into (19) and rearranging the result, we can have

$$\text{JTC}' = K + \frac{1}{C}[\frac{A}{n} + S + \sum_{i=1}^{m} T_i(K)] + \frac{C}{2}[\sum_{i=1}^{m}\frac{H_{bi}L_iD_i}{H_{bi}+L_i}$$

$$+\frac{H_{vp}}{P}\sum_{i=1}^{m} D_i^2 + \frac{nMH_{vm}(\sum_{i=1}^{m} D_i)^2}{P}]. \tag{21}$$

From Equation (21), for any given K, and n, the optimal value of C can be obtained as

$$C^* = \sqrt{\frac{2[A/n + S + \sum_{i=1}^{m} T_i(K)]}{H'\sum_{i=1}^{m} D_i}}, \tag{22}$$

where

$$H^{'}=\frac{nMH_{vm}\sum_{i=1}^{m}D_i}{P}+\frac{1}{\sum_{i=1}^{m}D_i}[\frac{H_{vp}}{P}\sum_{i=1}^{m}D_i^2+\sum_{i=1}^{m}\frac{H_{bi}L_iD_i}{H_{bi}+L_i}] \tag{23}$$

Substituting (22) into (21), the joint total cost for any given K and n becomes

$$\text{JTC}^{'}(\text{n,K})=K+\sqrt{2H^{'}\sum_{i=1}^{m}D_i[\frac{A}{n}+S+\sum_{i=1}^{m}T_i(K)]}\ . \tag{24}$$

Theorem 1 For any fixed K, the optimal value of n for (24) can be determined as

$$\frac{1}{n^{*2}}(1-n^*)<\frac{MH_{vm}\sum_{i=1}^{m}D_i[S+\sum_{i=1}^{m}T_i(K)]}{PB}\le\frac{1}{n^{*2}}(n^*+1)\ , \tag{25}$$

where

$$B=\frac{A}{\sum_{i=1}^{m}D_i}[\frac{H_{vp}}{P}\sum_{i=1}^{m}D_i^2+\sum_{i=1}^{m}\frac{H_{bi}L_iD_i}{H_{bi}+L_i}]$$ (see Appendix A for proof).

Lemma 1 With the increasing of K, n^* is decreasing and the minimal n^* is bounded by the case when K=0 (see Appendix B for proof).

Lemma 1 implies that the greater the expenditure in ordering cost reduces the shorter raw materials procurement cycle is. When n^*=1, JIT setting occur. It is intuitively appealing and also the main reason why the integrated inventory system is willing to invest to reduce the ordering cost.

Theorem 2 The optimal K can be determined by

$$K^*=\begin{cases}\frac{1}{r}\ln\frac{H^{'}r^2\sum_{i=1}^{m}D_i\sum_{i=1}^{m}T_{0i}}{1+\sqrt{1+2H^{'}r^2\sum_{i=1}^{m}D_i((A/n)+S)}} & \text{if } n>N^{'},\\ 0 & \text{otherwise,}\end{cases} \tag{26}$$

where $N^{'}=$

$$\frac{P(2(S+\sum_{i=1}^{m}T_{0i})-Br^2(\sum_{i=1}^{m}T_{0i})^2)+\{8PAMH_{vm}r^2(\sum_{i=1}^{m}T_{0i})^2}{2MH_{vm}r^2(\sum_{i=1}^{m}T_{0i})^2}$$

$$+\frac{[P(2(S+\sum_{i=1}^{m}T_{0i})-Br^2(\sum_{i=1}^{m}T_{0i})^2]^2\}^{1/2}}{2MH_{vm}r^2(\sum_{i=1}^{m}T_{0i})^2}\ . \tag{27}$$

(see Appendix C for proof)

5. Algorithm Steps

Through the above analysis, we present an algorithm for solving Model GM by the following steps:

Step 1: Give the all solutions by Equations (14)-(15) which are given by *the* paper of Woo, Hsu, and Wu (2001). If n>1, these solutions are considered as the optimal solutions of Model GM, set n=0 and go to Step 3; else, go to Step 2.

Step 2: For each $n\in\{n|n(K=0)\le n\le 1\}$, calculate the corresponding optimal K by Equation (26), and give the optimal K and n. Then gives the other optimal solutions f_i and C by Equations (20) and (22), set n=1 and go to Step 3.

Step 3: substitute the optimal n, K, f_i and C into Equation (8), and get minimal JTC.

6. Numerical Analysis

This section presents numerical example to Model GM. Some of related input parameters which no change come from the paper of Woo, Hsu, and Wu (2001) are given in Table 1. The other parameters changed are shown in Table 2. These parameters are changed five times and the corresponding results are shown in Table 2 too for comparing with the policy of Woo, Hsu, and Wu (2001). The unit time is one year and the monetary unit is U.S. dollar.

Table 1 Values of input parameters

Parameters	Value	Parameters	Value
D_i (i $\in\{1,2,3\}$)	1000	P	60000
M	1	R	0.01
H_{bi} (i $\in\{1,2,3\}$)	8	L_i (i $\in\{1,2,3\}$)	20
H_{vm}	2	H_{vp}	4

Table 2 Five selected points and the corresponding results

Selected point	1	2	3	4	5
Parameters	A=20 S=1000 T_{0i} =200	A=20 S=300 T_{0i} =200	A=200 S=300 T_{0i} =200	A=200 S=200 T_{0i} =100	A=600 S=200 T_{0i} =100
Results 1*	n^*=1 K^*=413 C^*=0.096 f_i^*=0.29 JTC^*=21767	n^*=1 K^*=471 C^*=0.054 f_i^*=0.29 JTC^*=12476	n^*=1 K^*=449 C^*=0.068 f_i^*=0.29 JTC^*=15429	n^*=2 K^*=417 C^*=0.047 f_i^*=0.29 JTC^*=13512	n^*=3 K^*=412 C^*=0.049 f_i^*=0.29 JTC^*=17039
Results 2**	n^*=1/3 K^*=406 C^*=0.10 x^*=1 f_i^*=0.29 JTC^*=21171	n^*=1/2 K^*=464 C^*=0.055 x^*=1 f_i^*=0.29 JTC^*=12412	n^*=1 K^*=449 C^*=0.068 x^*=0 or 1 f_i^*=0.29 JTC^*=15429	n^*=2 K^*=417 C^*=0.047 x^*=0 f_i^*=0.29 JTC^*=13512	n^*=3 K^*=412 C^*=0.060 x^*=0 f_i^*=0.29 JTC^*=17039

*The results with the policy of Woo, Hsu, and Wu (2001). ** The results with Model GM

From Table 2, we have two interesting findings:

1. When x^*=1 the joint total cost with Model GM is less than that of Woo, Hsu, and Wu (2001), such as points 1 and 2. In these settings, n^*=1 with the policy of Woo, Hsu, and Wu (2001). However since in these points A is lower and S and T_{0i} are higher than those at point 3, the vendor prefers more ordering times in a common replenishment cycle C, and we have a lower joint total cost than the results with the policy of Woo, Hsu, and Wu (2001). For example, at point 3, n^*=1 and JTC^*=\$21767 in results 1, however n^*=1/3 and JTC^*=\$21171 in results 2. Model GM adopts the policy of ordering three times in one common replenishment cycle while lot-for-lot policy is implemented with the policy of Woo, Hsu, and Wu (2001) and the joint cost is reduced from \$21767 to \$21171 by \$596
2. When x^*=0, n>1 the results with Model GM are the same as those with the policy of Woo, Hsu, and Wu (2001) where $n^* \geq 1$, such as at point 3, 4 and 5.

7. Conclusion

In this paper, a general model and the corresponding algorithm are proposed. From the last numerical example, it can be seen that Model GM provides a lower or equal joint total relevant cost as compared to that of Woo, Hsu, and Wu's model.

Acknowledgement

This paper is supported by 863 Program of China, NO: 2002AA41361 and Graduate special finance for innovation study of the Chinese Academy of Sciences.

Appendix A:

For any fixed K, the minimization of $JTC'(n,K)$ in Equation (24) is equivalent to

$$\min_n \{\frac{nMH_{vm}\sum_{i=1}^m D_i}{P} + \frac{1}{\sum_{i=1}^m D_i}[\frac{H_{vp}}{P}\sum_{i=1}^m D_i^2 + \sum_{i=1}^m \frac{H_{bi}L_iD_i}{H_{bi}+L_i}]\}[\frac{A}{n} + S + \sum_{i=1}^m T_i(K)]$$

$$= \min_n \frac{AMH_{vm}\sum_{i=1}^m D_i}{P} + n\frac{MH_{vm}\sum_{i=1}^m D_i[S+\sum_{i=1}^m T_i(K)]}{P}$$

$$+\frac{A}{n}\frac{1}{\sum_{i=1}^m D_i}[\frac{H_{vp}}{P}\sum_{i=1}^m D_i^2 + \sum_{i=1}^m \frac{H_{bi}L_iD_i}{H_{bi}+L_i}]$$

$$+\frac{1}{\sum_{i=1}^m D_i}[\frac{H_{vp}}{P}\sum_{i=1}^m D_i^2 + \sum_{i=1}^m \frac{H_{bi}L_iD_i}{H_{bi}+L_i}][S+\sum_{i=1}^m T_i(K)]$$

. (A-1)

Thus $\min_n JTC'(n,K)$ is equivalent to

$$\min_n\{n\frac{MH_{vm}\sum_{i=1}^m D_i[S+\sum_{i=1}^m T_i(K)]}{P}$$

$$+\frac{A}{n}\frac{1}{\sum_{i=1}^{m}D_i}[\frac{H_{vp}}{P}\sum_{i=1}^{m}D_i^2+\sum_{i=1}^{m}\frac{H_{bi}L_iD_i}{H_{bi}+L_i}]\} . \qquad \text{(A-2)}$$

And let

$$Y(n^{'})=\{\frac{MH_{vm}\sum_{i=1}^{m}D_i[S+\sum_{i=1}^{m}T_i(K)]}{n^{'}P}$$

$$+\frac{n^{'}A}{\sum_{i=1}^{m}D_i}[\frac{H_{vp}}{P}\sum_{i=1}^{m}D_i^2+\sum_{i=1}^{m}\frac{H_{bi}L_iD_i}{H_{bi}+L_i}]$$

and $n^{'}=\frac{1}{n}$, we have

$$Y(n^{'}+1)-Y(n^{'})=$$

$$-\frac{1}{n^{'}(n^{'}+1)}\frac{MH_{vm}\sum_{i=1}^{m}D_i[S+\sum_{i=1}^{m}T_i(K)]}{P}$$

$$+\frac{A}{\sum_{i=1}^{m}D_i}[\frac{H_{vp}}{P}\sum_{i=1}^{m}D_i^2+\sum_{i=1}^{m}\frac{H_{bi}L_iD_i}{H_{bi}+L_i}] \qquad \text{(A-3)}$$

and

$$[Y(n^{'}+1)-Y(n^{'})]-[Y(n^{'})-Y(n^{'}-1)]=$$

$$\frac{2}{n^{'}(n^{'2}-1)}\frac{MH_{vm}\sum_{i=1}^{m}D_i[S+\sum_{i=1}^{m}T_i(K)]}{P}>0 . \qquad \text{(A-4)}$$

So $Y(n^{'})$ is a strictly convex function and the sufficient and necessary condition for $Y(n^{'})$ to be minimal at n is therefore

$Y(n^{'}+1)-Y(n^{'})\geq 0$ and $Y(n^{'}-1)-Y(n^{'})>0$.

That is,

$$n^{'}(n^{'}-1)<\frac{MH_{vm}\sum_{i=1}^{m}D_i[S+\sum_{i=1}^{m}T_i(K)]}{PB}\leq n^{'}(n^{'}+1) .$$

Therefore,

$$\frac{1}{n^2}(1-n)<\frac{MH_{vm}\sum_{i=1}^{m}D_i[S+\sum_{i=1}^{m}T_i(K)]}{PB}\leq\frac{1}{n^2}(n+1) .$$

Appendix B:

When n<1,

$$\frac{\partial}{\partial n}[\frac{1}{n^2}(1-n)]=\frac{n-2}{n^3}<0 \quad \text{and}$$

$$\frac{\partial}{\partial n}[\frac{1}{n^2}(1+n)]=\frac{-n-2}{n^3}<0 .$$

So we have the minimal n when

$\frac{MH_{vm}\sum_{i=1}^{m}D_i[S+\sum_{i=1}^{m}T_i(K)]}{PB}$ get its maximum.

Since

$$\frac{\partial}{\partial K}[\frac{MH_{vm}\sum_{i=1}^{m}D_i[S+\sum_{i=1}^{m}T_i(K)]}{PB}]$$

$$=\frac{MH_{vm}\sum_{i=1}^{m}D_i}{PB}\sum_{i=1}^{m}\frac{\partial T_i(K)}{\partial K}$$

$$=\frac{MH_{vm}\sum_{i=1}^{m}D_i}{PB}\sum_{i=1}^{m}\frac{\partial T_i(K)}{\partial K}$$

$$=-\frac{MH_{vm}\sum_{i=1}^{m}D_i}{PB}\sum_{i=1}^{m}T_{0i}re^{-rK}<0.$$

$\frac{MH_{vm}\sum_{i=1}^{m}D_i[S+\sum_{i=1}^{m}T_i(K)]}{PB}$ is a decreasing function with respect to K.

So when K=0, n^* get its minimum.

That is,

$n(K=0)\leq n\leq 1$.

Appendix C:

From Lemma 1, we know that $n\in\{1,\frac{1}{2},\frac{1}{3},...,n^*(K=0)\}$ for any n. that is, n is numerical. So we can calculate optimal K to minimize JTC$^{'}$(n,K) for any fixed n. So for any fixed n, from

$$\frac{\partial}{\partial K}\text{JTC}^{'}(n,K)=$$

$$1+\sqrt{\frac{H^{'}\sum_{i=1}^{m}D_i}{2[A/n+S+\sum_{i=1}^{m}T_i(K)]}}\sum_{i=1}^{m}\frac{\partial T_i(K)}{\partial K}$$

$$=1-\sqrt{\frac{H^{'}\sum_{i=1}^{m}D_i}{2[A/n+S+\sum_{i=1}^{m}T_{0i}e^{-rK}]}}\sum_{i=1}^{m}T_{0i}re^{-rK}=0, \qquad \text{(C-1)}$$

the solution of Equation (C-1), notified by K_1 can be obtained as

$$K_1 = \frac{1}{r}\ln\frac{H^{'}r^2\sum_{i=1}^{m}D_i\sum_{i=1}^{m}T_{0i}}{1+\sqrt{1+2H^{'}r^2\sum_{i=1}^{m}D_i((A/n)+S)}}, \quad \text{(C-2)}$$

and

$$\sum_{i=1}^{m}\frac{\partial T_i(K_1)}{\partial K_1} = -\sqrt{\frac{2[A/n+S+\sum_{i=1}^{m}T_i(K_1)]}{H^{'}\sum_{i=1}^{m}D_i}}. \quad \text{(C-3)}$$

Since

$$\frac{\partial^2 \mathrm{JTC}^{'}(\mathrm{n,K})}{\partial \mathrm{K}^2} = \sqrt{\frac{H^{'}\sum_{i=1}^{m}D_i}{2[A/n+S+\sum_{i=1}^{m}T_i(K)]}}\{\sum_{i=1}^{m}\frac{\partial^2 T_i(K)}{\partial K^2}$$

$$-\frac{\sum_{i=1}^{m}(\partial T_i(K)/\partial K)^2}{2[A/n+S+\sum_{i=1}^{m}T_i(K)]}\} \quad \text{(C-4)}$$

, substituting (C-3) to Equation (C-4), we have

$$\frac{\partial^2 \mathrm{JTC}^{'}(\mathrm{n,K_1})}{\partial \mathrm{K_1}^2} = \sqrt{\frac{H^{'}\sum_{i=1}^{m}D_i}{2[A/n+S+\sum_{i=1}^{m}T_i(K_1)]}}\{\sum_{i=1}^{m}\frac{d^2 T_i(K_1)}{dK_1^2}$$

$$-\frac{1}{H^{'}\sum_{i=1}^{m}D_i}\}. \quad \text{(C-5)}$$

Since

$$\sum_{i=1}^{m}\frac{d^2 T_i(K_1)}{dK_1^2} = -\frac{1+\sqrt{1+2H^{'}r^2\sum_{i=1}^{m}D_i(A/n+S)}}{H^{'}\sum_{i=1}^{m}D_i}$$

$$> \frac{1}{H^{'}\sum_{i=1}^{m}D_i}, \quad \text{(C-6)}$$

we have

$$\frac{\partial^2 \mathrm{JTC}^{'}(\mathrm{n,K_1})}{\partial \mathrm{K_1}^2} > 0. \quad \text{(C-7)}$$

That is, $\mathrm{JTC}^{'}(\mathrm{n,K_1})$ is a strictly convex function and have only a unique solution in Equation (C-1).

So if $K_1>0$, the optimal K is K_1.

Note that $K_1>0$ if and only if

$$\frac{H^{'}r^2\sum_{i=1}^{m}D_i\sum_{i=1}^{m}T_{0i}}{1+\sqrt{1+2H^{'}r^2\sum_{i=1}^{m}D_i((A/n)+S)}} > 1.$$ That is,

$$H^{'}r^2\sum_{i=1}^{m}D_i\sum_{i=1}^{m}T_{0i}$$

$$> 1+\sqrt{1+2H^{'}r^2\sum_{i=1}^{m}D_i((A/n)+S)}. \quad \text{(C-8)}$$

By rearranging the inequality and taking square for both sides, we can have

$$(H^{'}r^2\sum_{i=1}^{m}D_i\sum_{i=1}^{m}T_{0i}-1)^2$$

$$> 1+2H^{'}r^2\sum_{i=1}^{m}D_i((A/n)+S). \quad \text{(C-9)}$$

That is,

$$((\frac{nMH_{vm}\sum_{i=1}^{m}D_i}{P}+B)r^2\sum_{i=1}^{m}D_i\sum_{i=1}^{m}T_{0i}-1)^2$$

$$> 1+2(\frac{nMH_{vm}\sum_{i=1}^{m}D_i}{P}+B)r^2\sum_{i=1}^{m}D_i((A/n)+S)$$

$$(n+\frac{BP}{H_{vm}M})\{MH_{vm}r^2\sum_{i=1}^{m}D_i(\sum_{i=1}^{m}T_{0i})^2n^2-P(2(S$$

$$+\sum_{i=1}^{m}T_{0i})-Br^2(\sum_{i=1}^{m}T_{0i})^2)n-2PA>0. \quad \text{(C-10)}$$

that is, $(n+\frac{BP}{H_{vm}M})$

$$\times(n-\frac{P(2(S+\sum_{i=1}^{m}T_{0i})-Br^2(\sum_{i=1}^{m}T_{0i})^2)+\sqrt{8PAMH_{vm}r^2(\sum_{i=1}^{m}T_{0i})^2+[P(2(S+\sum_{i=1}^{m}T_{0i})-Br^2(\sum_{i=1}^{m}T_{0i})^2]^2}}{2MH_{vm}r^2(\sum_{i=1}^{m}T_{0i})^2})$$

$$\times(n-\frac{P(2(S+\sum_{i=1}^{m}T_{0i})-Br^2(\sum_{i=1}^{m}T_{0i})^2)-\sqrt{8PAMH_{vm}r^2(\sum_{i=1}^{m}T_{0i})^2+[P(2(S+\sum_{i=1}^{m}T_{0i})-Br^2(\sum_{i=1}^{m}T_{0i})^2]^2}}{2MH_{vm}r^2(\sum_{i=1}^{m}T_{0i})^2}) > 0.$$

(C-11)

Therefore, $K_1>0$ if and only if

$$n > \frac{P(2(S+\sum_{i=1}^{m}T_{0i})-Br^2(\sum_{i=1}^{m}T_{0i})^2)+\sqrt{8PAMH_{vm}r^2(\sum_{i=1}^{m}T_{0i})^2+[P(2(S+\sum_{i=1}^{m}T_{0i})-Br^2(\sum_{i=1}^{m}T_{0i})^2]^2}}{2MH_{vm}r^2(\sum_{i=1}^{m}T_{0i})^2}.$$

Else, there is no solution in Equation (C-1). The optimal K is zero since $JTC'(n,+\infty)=+\infty$.

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