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Abstract: For the online problem of Internet Search Engines' Keyword Advertisement-place auction, two online models are present in this paper. The number of the auction phases is known or unknown beforehand respectively in the two models. For the first model we propose the Multi-Level of Accepted Price Strategy (abbr. MLAP) and prove it to be $O(\Phi^{1/4})$-competitive, where Φ is the ratio of the highest bid to the lowest one in the auction. For the second model a strategy similar to the first one (abbr. MLAP1) is put forward and proved to be $(2\sqrt{\Phi}-1)$-competitive. Furthermore, the experimental results show that in both models the experimental ratios between what the optimal strategy and the online strategies gain are much better than the theoretical ones, indicating that the two strategies perform much better in practice than in theory. In practice, these strategies may help to realize the automatic auction of heterogeneous objects that have similar traits to advertisement-places.

Keywords: Advertisement-Place Auction; Online Strategy; Competitive Ratio.

I. Introduction

Since 1993, the advertisement-place auction has brought Chinese CCTV great revenue each year. On Nov. 18, 2004, it obtained a revenue of 5.482 billion yuan by auctioning some golden period advertisement-places of year 2005. Recently many Internet Content Providers (Abbr. ICP) adopt to price their advertisement-place by auction gradually [1]. In an auction, the highest bid wins the first advertisement-place, the second highest bid wins the second place, and so on. This is a kind of heterogenous object auction in which each bidder demands one unit of object.

In benchmark auction model, the auctioneer auctions one object and each bidder demands one. The auctioneer designs an auction mechanism to maximize his revenue. In the online model, however, we use the competitive ratio to gauge the efficiency of an online strategy or mechanism. The competitive ratio is equals to the revenue obtained by the optimal offline strategy to that by the online strategy [2].

There are lots of literature on online auction. Ziv Bar-Yossef et al. [3] presented an online model of digital object auction and studied the competitiveness of an online strategy in the condition of compatibility. They gave a $O(e^{\sqrt{loglogh}})$-competitive randomized strategy where h is the ratio between the highest and the lowest estimated values by bidders. Avrim Blum et al. [4] studied a digital multi-object online model. A randomized online learning strategy, called WM, was given and proved to be $O(e^{\sqrt{loglogh}})$-competitive. Both Ziv Bar-Yossef and Avrim Blum assumed that the estimated values of all the objects is in $[1, h]$ and each bidder can only select one of the l bids. That is, the bid is discrete. Thus, an interesting question is what the competitive ratio will be if a bid may be an arbitrary value in $[1, h]$. In this paper we will focus on the continuous case of bid price.

The rest of this paper is organized as follows. Section 2 discusses the first model where the auctioneer knows the number of auction phases at the beginning of the auction. An effective strategy is put out and proved to be $O(\Phi^{1/4})$-competitive. The second model of unknowing the number of auction phases at the beginning is analyzed in section 3, and a competitive strategy is proposed. In section 4 some simulation results of the two strategies are presented. Finally, section 5 concludes this paper.

II. The Number of the Auction Phases Is Known Beforehand

II.1 Model Description

In this section we will discuss the case that the auctioneer know the number of bidders or the auction phases at the beginning. For expressional convenience, we call advertisement places heterogeneous objects. We first give the following four assumptions.

a) There are totally $m(\geq 2)$ heterogeneous objects. The price of each bid is in $[\underline{v}, \overline{v}]$ satisfying $\frac{\overline{v}}{\underline{v}} = \Phi$. The above information is the common knowledge.

b) The auctioneer obtains no value from a heterogeneous object if he keeps it. That is, the auctioneer gets a profit from the object only when it is sold successfully.

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c) There are n bidders in the auction, only one of which arrives and bids at each phase. Denote by $\sigma = \{b_1, \ldots, b_n\}$ the whole auction process where b_i is the bid of bidder i ($1 \le i \le n$). n is known by the auctioneer beforehand.

d) The auctioneer must decide whether to accept bid b_i on its arrival since b_i will leave the system if it has not been accepted before the arrival of the next bid.

By the assumption b), the auctioneer shall try to sell as many heterogeneous objects as possible, and for each object the price shall be as high as possible. The objective is to maximize the total revenue of accepted bids. More precisely, $max \sum_{j=1}^{m} p_j$ where $p_j \in [\underline{v}, \overline{v}] \bigcup \{0\}$ is the accepted price for the jth heterogeneous object. $p_j = 0$ if the auctioneer keeps the jth object.

II.2 The Multi-Level of Accepted Price Strategy

According to the model given above, when a bid arrives, the auctioneer decides whether to accept the bid but not to assign an object to the bid immediately. The auctioneer also knows n and $[\underline{v}, \overline{v}]$ at the beginning of the auction. Thus, we give an effective strategy called Multi-Level of Accepted Price (Abbr. MLAP). The main idea of MLAP is to divide the bidding support $[\underline{v}, \overline{v}]$ into several sub-supports and accept a fixed number of bids in each sub-support.

MLAP is described as follows. Assume that there are w bids accepted at the beginning of the ith phase. If $n - i < m - w$, that is, the number of the rest phases is less than the number of acceptable bids, MLAP accepts all the bids arriving, including b_i. Otherwise, MLAP accepts x bids satisfying

$$\alpha\underline{v} \le b_i < \sqrt{\phi}\underline{v}, \tag{1}$$

y bids satisfying

$$\sqrt{\phi}\underline{v} \le b_i < \beta\underline{v}, \tag{2}$$

z bids satisfying

$$\beta\underline{v} \le b_i < \phi\underline{v}, \tag{3}$$

and $(m - x - y - z)$ bids satisfying

$$b_i = \phi\underline{v} = \overline{v}, \tag{4}$$

respectively, where x, y, z $1 \le \alpha < \sqrt{\Phi}$ and $\sqrt{\Phi} \le \beta < \Phi$ are some positive constants determined later. The total number of accepted bids is at most m. In special, if b_i satisfies (4) and MLAP has accepted $(m - x - y - z)$ bids satisfying (4), then MLAP will accept b_i if only the total number of accepted bids is less than m. Similarly, if b_i satisfies (3) and MLAP has accepted $(m-x-y-z)$ bids satisfying (4) and z bids satisfying (3), then MLAP will accept b_i if only the total number of accepted bids is less than m. If b_i satisfies (2) and MLAP has accepted y bids satisfying (2) but less than x bids satisfying (1), then MLAP will accept b_i and count the number of accepted bids by (1) plus 1.

II.3 Competitive Analysis

In this section we will analyze the competitiveness of MLAP. We first give the following theorem and the rest of this section is contributed to the proof of the theorem.

Theorem 1 *MLAP is $O(\Phi^{1/4})$-competitive.*

Proof. We utilize the adversary strategy and worst case analysis to prove the theorem. Assume that an adversary always tries to design the worst bid input sequence to make MLAP behave worst, making the optimal strategy OPT obtain the most revenue and MLAP obtain the least. More precisely, the adversary will make MLAP accept the lowest bids as many as possible and OPT accept the highest bids as many as possible. For the whole auction process $\sigma = \{b_1, \ldots, b_n\}$, denote by $M(\sigma)$ and $O(\sigma)$ the total revenues obtained by MLAP and OPT respectively. According to the construction of MLAP, there are four possible bid input worst cases.

Case 1. The highest bid in σ is less than $\alpha\underline{v}$. The adversary try to make MLAP accept the lowest bids as many as possible and to make OPT accept bids with value a little less than $\alpha\underline{v}$. In the first $n - m$ phases, all the bids equal $\alpha\underline{v} - \epsilon$ ($\epsilon \to 0^+$), and the rest m bids equal $\underline{v}$. OPT will accept m bids with value $\alpha\underline{v} - \epsilon$ but MLAP will accept the last m bids with value $\underline{v}$ for all the bids do not satisfy the inequalities in the construction of MLAP. Thus, in this case the ratio of what OPT obtains to what MLAP obtains equals $\frac{O(\sigma)}{M(\sigma)} = \frac{m(\alpha\underline{v}-\epsilon)}{m\underline{v}} < \alpha$.

Case 2. The highest bid in σ is less than $\sqrt{\Phi}\underline{v}$. The adversary releases the first x bids with value $\alpha\underline{v}$, the following m bids with value $\sqrt{\Phi}\underline{v}-\epsilon$ and the last $n-x-m$ bids with value $\underline{v}$. MLAP will accept the first x bids and the last $m - x$ bids due to the construction of MLAP. However, OPT will accept the m bids with value $\sqrt{\Phi}\underline{v} - \epsilon$. Thus, in this case the ratio between what OPT and MLAP obtain equals $\frac{O(\sigma)}{M(\sigma)} = \frac{m(\sqrt{\Phi}\underline{v}-\epsilon)}{x\alpha\underline{v}+(m-x)\underline{v}} < \frac{m\sqrt{\Phi}}{x\alpha+m-x}$. Let $\frac{m\sqrt{\Phi}}{x\alpha+m-x} = \alpha$. We have that

$$x = \frac{(\sqrt{\Phi} - \alpha)m}{(\alpha - 1)\alpha}. \tag{5}$$

Thus, if the above equation holds the ratio between what OPT and MLAP obtain is at most α in this case.

Case 3. The highest bid in σ is less than $\beta\underline{v}$. The adversary releases the first x bids with value $\alpha\underline{v}$, the following y bids with value $\sqrt{\Phi}\underline{v}$ and then m bids with value $\beta\underline{v} - \epsilon$, and the last $n - x - y - m$ bids with value $\underline{v}$. MLAP will accept the first $x + y$ bids and the last

$m-x-y$ bids due to the construction of MLAP. However, OPT will accept the m bids with value $\beta\underline{v}-\epsilon$. Thus, in this case the ratio between what OPT and MLAP obtain equals $\frac{O(\sigma)}{M(\sigma)} = \frac{m(\beta\underline{v}-\epsilon)}{x\alpha\underline{v}+y\sqrt{\Phi}\underline{v}+(m-x-y)\underline{v}} < \frac{m\beta}{x\alpha+y\sqrt{\Phi}+m-x-y}$. Let $\alpha = \frac{m\beta}{x\alpha+y\sqrt{\Phi}+m-x-y}$. Together with equation (5), we have that

$$y = \frac{(\beta-\sqrt{\Phi})m}{(\sqrt{\Phi}-1)\alpha}. \tag{6}$$

Thus, if equations (5,6) hold the ratio between what OPT and MLAP obtain is at most α in this case.

Case 4. The highest bid in σ is less than $\overline{v}$. The adversary releases the first x bids with value $\alpha\underline{v}$, the following y bids with value $\sqrt{\Phi}\underline{v}$ and z bids with value $\beta\underline{v}$, and then m bids with value $\overline{v}-\epsilon$, and the last $n-x-y-z-m$ bids with value $\underline{v}$. MLAP will accept the first $x+y+z$ bids and the last $m-x-y-z$ bids due to the construction of MLAP. However, OPT will accept the m bids with value $\overline{v}-\epsilon$. Thus, in this case the ratio between what OPT and MLAP obtain equals $\frac{O(\sigma)}{M(\sigma)} = \frac{m(\overline{v}-\epsilon)}{x\alpha\underline{v}+y\sqrt{\Phi}\underline{v}+z\beta\underline{v}+(m-x-y-z)\underline{v}} < \frac{m\Phi}{x\alpha+y\sqrt{\Phi}+\beta z+m-x-y}$. Let $\frac{m\Phi}{x\alpha+y\sqrt{\Phi}+\beta z+m-x-y} = \alpha$. Together with equations (5) and (6), we have that

$$z = \frac{(\Phi-\beta)m}{(\beta-1)\alpha}. \tag{7}$$

Thus, if equations (5,6,7) hold the ratio between what OPT and MLAP obtain is at most α in this case.

Based on the above argument, if x, y and z satisfy the equations (5,6,7) respectively and inequality $x+y+z \leq m$, MLAP is α-competitive. Combining $x+y+z \leq m$ with equations (5,6,7), we have that

$$\frac{(\sqrt{\Phi}-\alpha)m}{(\alpha-1)\alpha} + \frac{(\beta-\sqrt{\Phi})m}{(\sqrt{\Phi}-1)\alpha} + \frac{(\Phi-\beta)m}{(\beta-1)\alpha} \leq m. \tag{8}$$

The rest problem is to judge the existence of such an α that inequality (8) holds and $1 \leq \alpha \leq \sqrt{\Phi}$. Let $B = \frac{(\beta-\sqrt{\Phi})}{(\sqrt{\Phi}-1)} + \frac{(\Phi-\beta)}{(\beta-1)}$, inequality (8) turns out to be $\alpha^2 - \alpha B + B - \sqrt{\Phi} \geq 0$. Solving the inequality we have that $\alpha \leq \frac{B-\sqrt{B^2-4(B-\sqrt{\Phi})}}{2}$ or $\alpha \geq \frac{B+\sqrt{B^2-4(B-\sqrt{\Phi})}}{2}$. The solution $\alpha \leq \frac{B-\sqrt{B^2-4(B-\sqrt{\Phi})}}{2}$ is deleted since it does not satisfy $\alpha \geq 1$. Thus, $\alpha \geq \frac{B+\sqrt{B^2-4(B-\sqrt{\Phi})}}{2}$. Since the competitive ratio is an increasing function of α, when $\alpha = \frac{B+\sqrt{B^2-4(B-\sqrt{\Phi})}}{2}$, MLAP reaches the optimal competitive ratio. Let $F(\beta) = \frac{B+\sqrt{B^2-4(B-\sqrt{\Phi})}}{2}$. Taking the differential of β and equating it with zero, we get that $B' = 0$ or $-1 = \frac{B-2}{\sqrt{B^2-4(B-\sqrt{\Phi})}}$. The latter solution is deleted for $\Phi \geq 1$. By $B' = 0$, together with the definition of B, we have that $\beta = 1 + (\sqrt{\Phi}-1)\sqrt{\sqrt{\Phi}+1}$ and then $B = 2\sqrt{\sqrt{\Phi}+1} - 2$. Thus, $\alpha = \frac{B+\sqrt{B^2-4(B-\sqrt{\Phi})}}{2} = (\sqrt{2}+1)(\sqrt{\sqrt{\Phi}+1}-1)$ and MLAP is $O(\Phi^{1/4})$-competitive. The relevant values of x, y and z can be obtained by equations (5), (6) and (7). We complete the proof.

III. The Number of the Auction Phases Is Unknown Beforehand

III.1 Model Description

In this section we will discuss the case that the auctioneer does not know the number of the auction phases at the beginning. The assumptions of this model are basically the same as those of the previous model. The only difference lies in assumption c) where the auctioneer does not know n beforehand in this model. The objective is to maximize the total revenue of accepted bids.

III.2 The Multi-Level of Accepted Price-1 Strategy

Since the auctioneer does not know when the auction activity will end, we give an effective strategy called Multi-Level of Accepted Price-1 (Abbr. MLAP1). The main idea of MLAP1 is that no matter at which phase the auction ends, MLAP1 ensures that what it has obtained is at least a fixed factor of what OPT has obtained.

MLAP1 is described as follows. MLAP1 accepts $\frac{m}{\alpha}$ bids in $[\underline{v}, \sqrt{\phi}\underline{v})$, $\frac{(1-1/\sqrt{\Phi})m}{\alpha}$ bids in $[\sqrt{\phi}\underline{v}, \overline{v})$ and $\frac{[a-(2-1/\sqrt{\Phi})]m}{\alpha}$ bids with value $\overline{v}$ respectively. The total number of accepted bids is at most m. In special, if MLAP1 has accepted $\frac{(1-1/\sqrt{\Phi})m}{\alpha}$ bids in $[\sqrt{\phi}\underline{v}, \overline{v})$ but less than $\frac{m}{\alpha}$ bids in $[\underline{v}, \sqrt{\phi}\underline{v})$, then a new bid with value in $[\sqrt{\phi}\underline{v}, \overline{v})$ will be accepted and count the number of the total accepted bids plus 1. Similarly, if MLAP1 has accepted $\frac{[a-(2-1/\sqrt{\Phi})]m}{\alpha}$ bids with value $\overline{v}$ but less than $\frac{(1-1/\sqrt{\Phi})m}{\alpha}$ bids in $[\sqrt{\phi}\underline{v}, \overline{v})$ or $\frac{m}{\alpha}$ bids in $[\underline{v}, \sqrt{\phi}\underline{v})$, then a new bid with value in $[\sqrt{\phi}\underline{v}, \overline{v})$ or $[\underline{v}, \sqrt{\phi}\underline{v})$ will be accepted and count the number of the total accepted bids plus 1.

III.3 Competitive Analysis

In this section we will analyze the competitiveness of MLAP1. We give the following theorem and the rest of this section is contributed to the proof of the theorem.

Theorem 2 *MLAP1 is* $(2\sqrt{\Phi}-1)$*-competitive.*

Proof. The proof idea is similar to that of MLAP. For the whole auction process $\sigma = \{b_1, \ldots, b_n\}$, denote by $M1(\sigma)$ and $O(\sigma)$ the total revenues obtained by MLAP1 and OPT respectively. There are two possible worst cases for MLAP1.

Case 1. The highest bid is less than $\sqrt{\Phi}\underline{v}$. The adversary will release the first $\frac{m}{\alpha}$ bids with value $\underline{v}$ and the following m bids with value $(\sqrt{\Phi}-\epsilon)\underline{v}$ and no more bid arrives. In this case, MLAP1 will only accept the first $\frac{m}{\alpha}$ bids and reject the following m bids due to its construction. OPT will accept the m bids with value $(\sqrt{\Phi}-\epsilon)\underline{v}$. Thus, the ratio between what OPT and MLAP1 obtain equals $\frac{O(\sigma)}{M1(\sigma)}=\frac{(\sqrt{\Phi}-\epsilon)\underline{v}m}{(m/\alpha)\underline{v}}<\alpha\sqrt{\Phi}$.

Case 2. The highest bid is less than $\Phi\underline{v}=\overline{v}$. The adversary will release the first $\frac{m}{\alpha}$ bids with value $\underline{v}$ and the following $\frac{(1-1/\sqrt{\Phi})m}{\alpha}$ bids with value $\sqrt{\Phi}\underline{v}$, and m bids with value $(\Phi-\epsilon)\underline{v}$ and no more bid arrives. In this case, MLAP1 will only accept the first $\frac{(2-1/\sqrt{\Phi})m}{\alpha}$ bids and reject the following m bids due to its construction. OPT will accept the m bids with value $(\Phi-\epsilon)\underline{v}$. Thus, the ratio between what OPT and MLAP1 obtain equals $\frac{O(\sigma)}{M1(\sigma)}=\frac{(\Phi-\epsilon)\underline{v}m}{(m/\alpha)\underline{v}+\frac{(1-1/\sqrt{\Phi})m}{\alpha}\sqrt{\Phi}\underline{v}}<\frac{\alpha\Phi}{1+(1-1/\sqrt{\Phi})\sqrt{\Phi}}=\alpha\sqrt{\Phi}$.

Based on the analysis of the above two cases, MLAP1 is $\alpha\sqrt{\Phi}$-competitive. Moreover, the total accepted bids shall be no more than m, that is, $\frac{m}{\alpha}+\frac{(1-1/\sqrt{\Phi})m}{\alpha}\leq m$. Solving the inequality we have that $\alpha\geq 2-1/\sqrt{\Phi}$. Since the competitive ratio is an increasing function of α. Let $\alpha=2-1/\sqrt{\Phi}$, then MLAP1 is $(2\sqrt{\Phi}-1)$-competitive. Hence, the proof is completed.

IV. Experiment

In the previous two sections we proposed two effective strategies for two models respectively and proved their competitiveness theoretically. In this section we will give some simulations and compare the experimental results to the theoretical ones.

The following is the simulation for both models. The original data are from www.baidu.com website. There are 83 bids for the keyword "Flower" on Feb. 12, 2005. The highest bid is 6.75 yuan and the lowest one is 0.3 yuan. Thus, for the model of knowing the auction phases beforehand, the bid phases $n=83$ and the ratio between the highest bid and the lowest one $\Phi=22.5$. The theoretical value of the competitive ratio of MLAP can be obtained by Theorem 1, equaling $(\sqrt{2}+1)(\sqrt{\sqrt{\Phi}+1}-1)=3.37$. On the other hand, in the experiment, the bid arrival sequence is equal to the data list in the website. some of the experiment results by MLAP are shown in form 1. $m=20,30,40$ are the numbers of heterogeneous objects. For each m, the x,y,z values are figured out by equations (5), (6) and (7). We can see that the highest value of the competitive ratio equals 1.96 in the form, which is much less than the theoretical value. The reason is that the theoretical result is proved by worst case analysis. However, in practice, the worst case rarely happens.

Form 1 Some simulation results of the first model.

	m=20			m=30			m=40		
Assigned values	x, y, z=4,8,8			x, y, z=6,12,12			x, y, z=7,17,26		
Online Revenue	90.11	79.28	64.65	125.56	118.33	98.88	168.37	156.56	109.37
Optimal Revenue	124.74	155.3	91.66	160.06	123.24	109.95	182.24	174.49	120.04
Competitive ratio	1.38	1.96	1.42	1.27	1.04	1.11	1.08	1.11	1.1

Form 2 Some simulation results of the second model.

	m = 20				m = 40			
n	Online Revenue	Offline Revenue	E-ratio	T-ratio	Online Revenue	Offline Revenue	E-ratio	T-ratio
50	43.01	93.1	2.16	4.89	81.84	114.23	1.40	5.02
17	38.05	53.55	1.41	4.81	38.05	53.55	1.41	5.53
73	43.01	113.36	2.64	6.24	81.84	155	1.89	7.37
32	43.01	73.25	1.70	4.62	64.58	78.08	1.21	4.98
42	43.01	88	2.05	4.25	81.54	101.9	1.25	5.31
31	43.01	71.48	1.66	4.02	62.06	75.56	1.22	4.35
46	43.01	92.73	2.16	4.98	81.84	109.97	1.34	4.58
51	43.01	55.01	1.28	5.54	81.84	100.87	1.23	6.02
55	43.01	97.33	2.26	5.31	81.84	120.44	1.47	6.47

For the model of unknowing the auction phases beforehand, the original data are the same as those of the previous model. The number of auction phases n are produced randomized in $[1,83]$. For each n, we randomly select n numbers from the 83 bids, and the bid arrival sequence is randomized produced by the n bids. For each experiment, the relevant theoretical competitive ratio can be figured out by the n bids and Theorem 2, as shown in Form 2 below T-ratio. We set $m=20,40$ and obtain the experiment results by MLAP1. The optimal revenue, MLAP1 revenue and the competitive ratio below E-ratio in the experiment are shown in Form 2. Obviously, all the values below E-ratio are much less than those below T-ratio. The reason is the same as that of the previous model.

V. Conclusion

In this paper we analyze the online problem of network advertisement-place auction. We present two online models where the number of auction phases are known or unknown beforehand respectively. Two deterministic strategies are proposed and proved to be $O(\Phi^{1/4})$ and $(2\sqrt{\Phi}-1)$-competitive respectively where Φ is the ratio between the highest and the lowest bids in the auction. In practice, these strategies may help to realize the automatic auction of heterogeneous objects that have similar traits to advertisement-places. There are still some problems that can be further analyzed. For example, whether there exist better deterministic strategies for the two models or what is the lower bound of deterministic strategies? Moreover, if a bidder can submit a vector of bids for different objects, then whether there exists an efficient strategy for this case?

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