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ABSTRACT

This paper considered a single-item, three-echelon (supplier, retailer, and customer) inventory problem. At random times, the supplier offers the retailer a discount. The inter-arrival times of discounts are exponentially distributed. For the retailer, whether or not to take a discount offer depends on its inventory level. If inventory is below threshold level S, the retailer will order to replenish inventory up to S+Q units. Otherwise, the retailer will pass. At the regular purchase price, the retailer uses the order-up-to reorder point inventory system. The reorder point is r, and the order-up-to quantity is R. The demand of the customer is price-elastic. Depending on its inventory level, the retailer may discounts its selling price to boost up demand. The decision variables are S, Q, R, r, and the selling prices (regular and discount selling price). The simulation experiment covered reasonable range of values for each decision variable. Each run represented one scenario, i.e., one combination of the decision variables.

Keywords: demand management, price discount, multi-echelon inventory system, simulation

1. INTRODUCTION

Consider a three echelon (supplier, retailer, and customer) inventory system. At random times, the supplier offers the retailer a price discount. The inter-arrival time of discounts is exponentially distributed with expected value of 100 days.

For the retailer, whether or not to take the discount offer depends on its inventory level. If inventory is below threshold S, the retailer will order to replenish inventory up to S+Q units. Otherwise, the retailer will pass.

At the regular purchase price, the retailer uses the order-up-to reorder point inventory system. The reorder point is r, and the order-up-to quantity is R. The demand of the customer is price-elastic. Depending on its inventory level, the retailer discounts its selling price to boost up demand.

In this paper, we used Monte Carlo simulation to investigate the following scenarios: (1) the basic model, (2) the limited-time offer model, (3) stochastic extension of the limited-time offer model, (4) pricing to clear inventory, and (5) stochastic extension of pricing to clear inventory.

The decision variables are S, Q, R, r, and the selling prices (regular and discount selling price). The simulation experiment covered reasonable range of values for these decision variables. Each run represented one scenario, i.e., one combination of the decision variables.

In the simulation analysis, the ranges of values for the decision variables were:

- $1{,}000 \le S+Q \le 7{,}000$
- $50 \le R \le 3000$
- $50 \le S$ 500
- \$7.00 ≤ regular selling price ≤ 10.00
- \$5.00 ≤ discount selling price ≤ 8.00

The cost parameters used in the simulation were:

- Unit Cost (Regular) C_L=\$5.00/unit
- Unit Cost (Discount) C_D=\$4.00/unit
- Holding Cost/Year 15% of cost
- Ordering Cost \$50
- Backorder Cost \$10 /unit

Each combination of the decision variables was simulated for 365 days, and replicated as many times as necessary for making meaningful statistical inferences. Descriptive statistics on the contribution margin for each scenario was generated. The contribution margin is the difference between annual revenue and annual relevant cost. The annual revenue equals daily sales in units * selling price for the day, for 365 days. The annual relevant costs include item cost, ordering cost, holding cost, and shortage cost. The item cost is based on the weighted average unit cost.

Clearly, the weighted average unit cost is lower if the retailer orders more often at the discount price.

The results of the simulation were plotted. On each plot, a polynomial trend line was fitted based on the points on the graph.

All computations were made using Microsoft Excel on a Pentium 4 2.6 GHz computer.

2. THE BASIC MODEL

The assumptions for the basic model are:

- Demand rate d is deterministic, i.e. $d = \alpha p^{-e}$, where α=1,000 and ε=3.

- At random times, the supplier offers the retailer a discount. The inter-arrival times of discount offers are exponentially distributed with mean = 100 days. **The discount offer is good until the retailer takes the offer.**
- The retailer offers discounts to its customers whenever the inventory level is above the threshold level S. Below S the selling price reverts to the regular price. Since there are two selling prices, there are two levels of demand rate (demand is price elastic)

Each run consisted of 1,000 replications. Each 1,000 replication took less than 3 seconds. The results of the simulation are summarized in Charts 1-1 to 1-5 below:

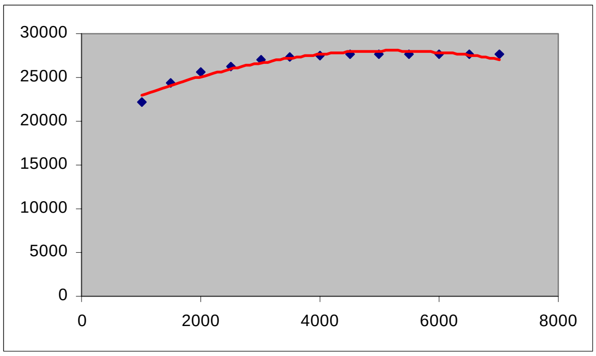

Chart 1-1 Contribution Margin vs. S+Q

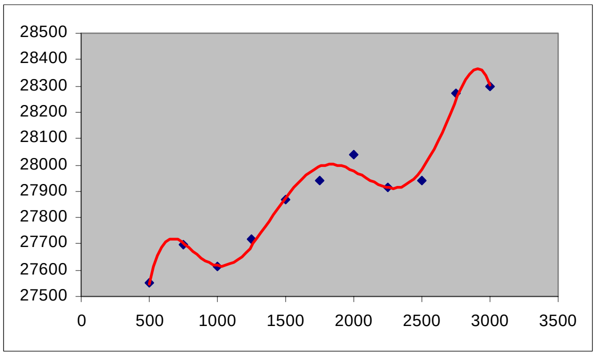

Chart 1-2 Contribution Margin vs. R

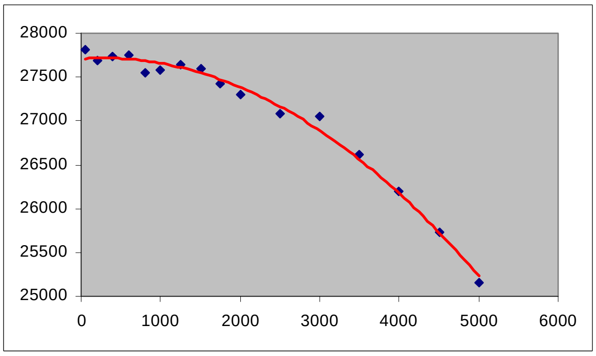

Chart 1-3 Contribution Margin vs. S

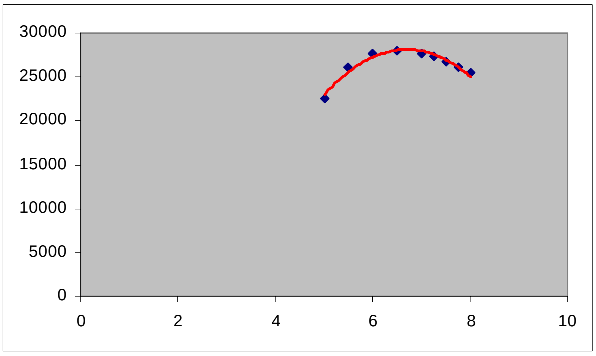

Chart 1-4 Contribution Margin vs. Discount Price

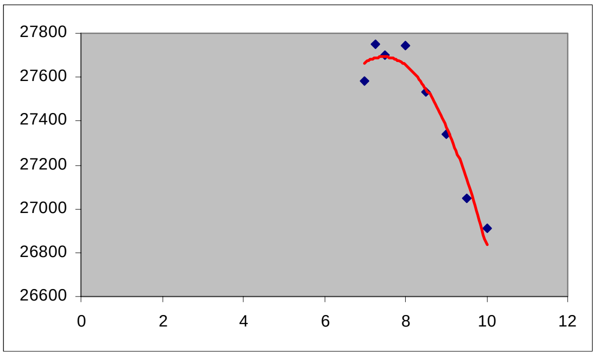

Chart 1-5 Contribution Margin vs. Regular Price

The contribution margin appears to be rather flat in the vicinity of S+Q=6,000. The trend-line for the Contribution Margin vs. R chart (Chart 1-2) appears to be roller-coaster-like. However, from R=500 to R=3,000, the difference in contribution is less than $750. The trend-line in Chart 1-3 shows the contribution margin dropping very fast beyond S=1,000. The trend-lines in Charts 1-4 and 1-5 suggest a regular selling price of $7.50 and a discount selling price of $6.50.

3. LIMITED-TIME-OFFER MODEL

In the basic model, each discount offer from the supplier is valid until the retailer takes the offer. This is not common in practice. In the ensuing models, we considered the more realistic practice of “limited-time-offer.” The Limited-time-offer Model retained all other assumptions in the Basic Model, except that each offer of discount is good for 5 days only.

The following charts show the results of the simulation for the Limited-time-offer model:.

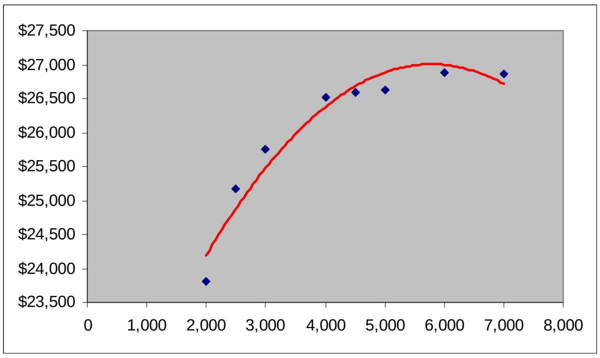

Chart 2-1 Contribution Margin vs. S+Q

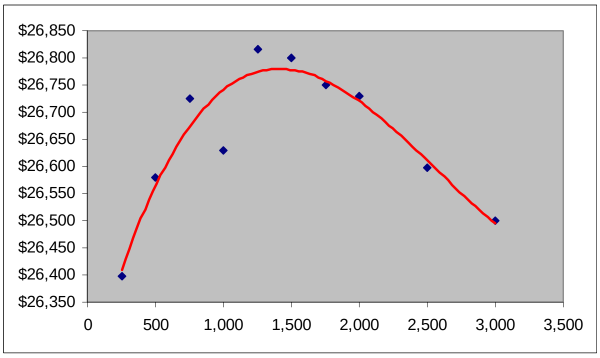

Chart 2-2 Contribution Margin vs. R

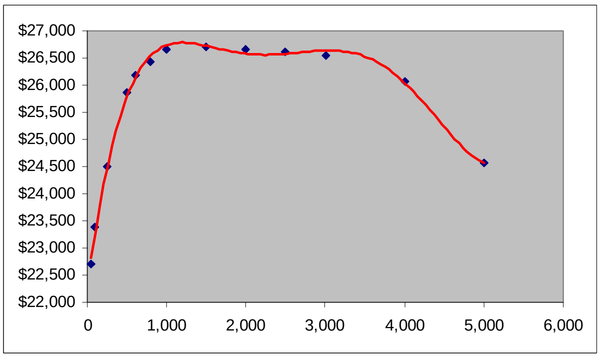

Chart 2-3 Contribution Margin vs. S

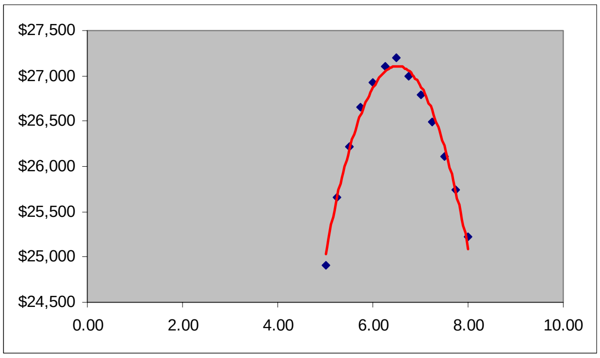

Chart 2-4 Contribution Margin vs. Discount Price

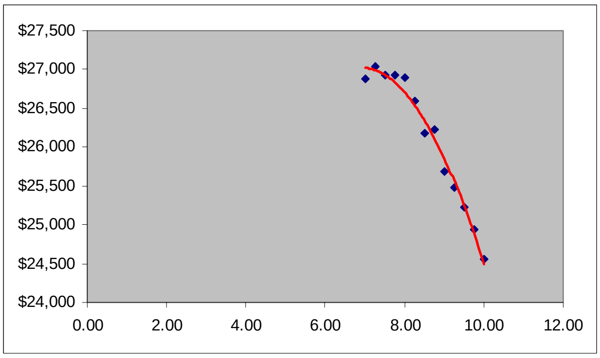

Chart 2-5 Contribution Margin vs. Regular Price

The trend-lines appear to be unimodal. The table below shows the summary statistics, based on the input parameters suggested by the charts above:

- Threshold Level S=3,000
- Order Quantity R=1,500
- Max Inventory S+Q=6,000
- Nominal Price $7.50
- Discount Price $6.50

Contribution Margin	
Mean	27,583
Standard Error	63
Median	28,151
Mode	20,817
Standard Deviation	2,002
Sample Variance	4,009,841
Kurtosis	0.545
Skewness	-0.858
Range	9,375
Minimum	20,817
Maximum	30,192
Sum	27,583,363
Count	1,000
Confidence Level (95.0%)	124

4. STOCHASTIC EXTENSIONS

In this model, we extended the above scenario and considered stochastic demand. During the lead-time, inventory level may drop down to zero before replenishment arrives (a shortage situation), or the level may be positive when replenishment arrives. Thus the reorder point r is another decision variable of interest. We examined two alternative probability distributions – (a) Truncated Normal (i.e., demand is non-negative) and (b) Exponential.

The trend-line charts for this scenario are very similar to the ones shown above, and will not be reproduced here.

5. DISCOUNT TO CLEAR EXCESSIVE INVENTORY.

Since demand is price-elastic, the retailer can clear excessive inventory by adjusting the selling price accordingly. In this model, we analyzed a scenario where the retailer tweaks its selling price periodically, based on inventory level. This model is very common among sellers of commodity items or perishable goods. For example, a bumper crop of grape leads to overproduction of wine, bringing down the price of wine in retail stores. Overproduction of oil and petroleum drives down prices. As reserves are depleted, the prices slowly rise upwards. This is also very common among grocers.

This model retained all other assumptions in Limited-time-offer model except that retailer adjusts the selling price based on beginning inventory. The % discount is selling price is $I^{(1-\delta)}/100$, where I = beginning inventory, and $\delta = 0.6250$.

The following charts show the results of the simulation for the Limited-time-offer model:

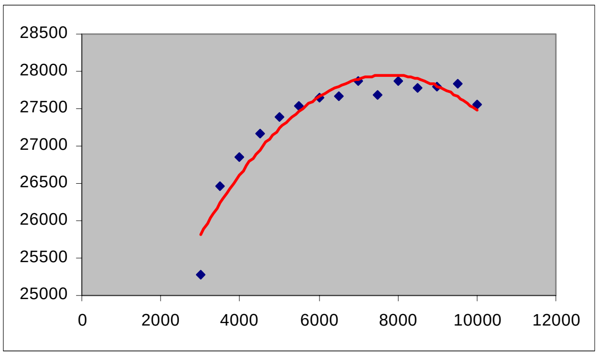

Chart 3-1 Contribution Margin vs. S+Q

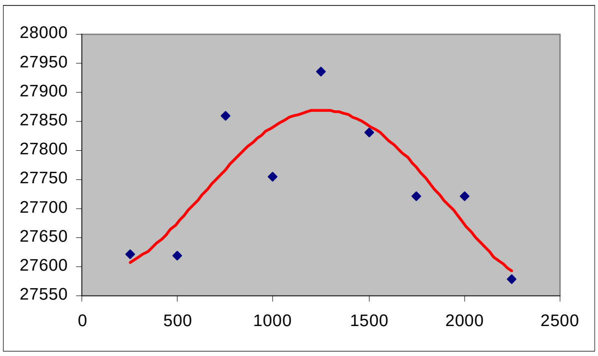

Chart 3-2 Contribution Margin vs. R

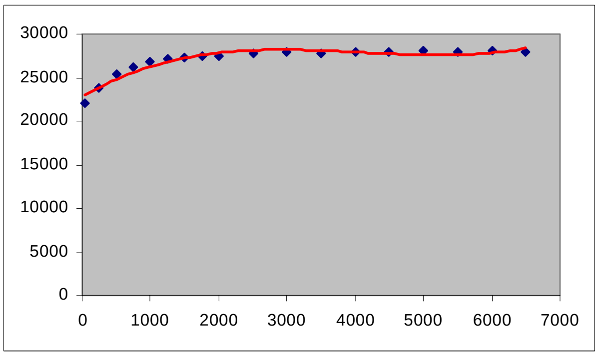

Chart 3-3 Contribution Margin vs. S

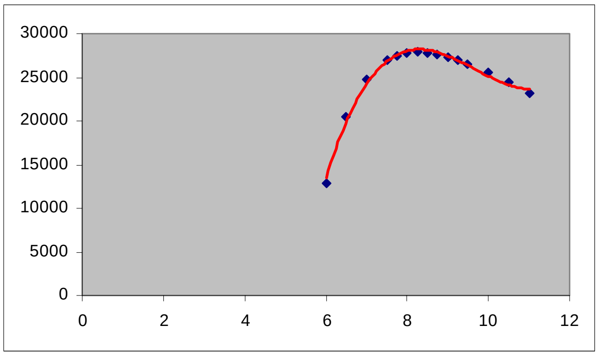

Chart 3-4 Contribution Margin vs. Regular Price

The trend-lines appear to be unimodal. The table below shows the summary statistics, based on the input parameters suggested by the charts above:

- Threshold Level S 3,000
- Reorder Point r 60
- Order Quantity R 1,500
- Max Inventory' (S+Q) 7,000
- Nominal Selling Price $8.00

Contribution Margin	
Mean	27,839
Standard Error	74
Median	28,094
Mode	20,389
Standard Deviation	2,352
Sample Variance	5,530,168
Kurtosis	0.835
Skewness	-0.898
Range	10,673
Minimum	20,389
Maximum	31,062
Sum	27,838,767
Count	1,000
Confidence Level (95.0%)	146

6. DISCOUNT TO CLEAR EXCESSIVE INVENTORY: STOCHASTIC EXTENSIONS

In this model, we extended the preceding scenario for exponentially-distributed demand. The expected demand d = αp-e, where α=1,000 and ε=3.

The trend-line charts for this scenario are very similar to the ones shown above, and will not be reproduced here.

7. CONCLUDING REMARKS

This paper demonstrated the application of Monte Carlo simulation to analyze the problem of supply management under inducement of price discounts. We investigated the three echelon (supplier, retailer, and customer) inventory problem under the following scenarios: (1) the basic model, (2) the limited-time offer model, (3) stochastic extension of the limited-time offer model, (4) pricing to clear inventory, and (5) stochastic extension of pricing to clear inventory.

The decision variables are S, Q, R, r, and the selling prices (regular and discount selling price). The simulation experiment covered reasonable range of values for each decision variable. Each run represented one scenario, i.e., one combination of the decision variables. The results of the simulation were plotted. On each plot, a polynomial trend line was fitted based on the points on the graph.

Examining the trend-lines, we were able to identify appropriate values for the decision variables. The author is currently working on an analytical approach to determine the optimal values of these decision variables.

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