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APPLYING PARTICLE SWARM OPTIMIZATION TO SOLVE PORTFOLIO SELECTION PROBLEMS

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ABSTRACT

Particle swarm optimization (PSO), introduced by Kennedy and Eberhart in 1995, is a social population-based search algorithm and is generally similar to the evolutionary computation techniques that have been successfully applied to solve various hard optimization problems. The standard Markowitz mean-variance approach to portfolio selection involves tracing out an efficient frontier, a continuous curve illustrating the tradeoff between return and risk. In this paper we applied the particle swarm approach to find an efficient frontier associated with the classical and general (unconstrained and constrained) mean-variance portfolio selection problem. The OR library data sets were tested in our paper and computational results showed that the PSO found better solutions when compared to genetic algorithm (GA), simulated annealing(SA), and tabu search(TS).

Keywords: Particle swarm optimization, Portfolio selection, Mean-variance approach, Efficient frontier

INTRODUCTION

In the portfolio selection problem, given a set of available securities or assets, we want to find out the optimum way of investing a particular amount of money in these assets. Each one of the different ways to diversify this money between the several assets is called a portfolio. For solving this portfolio selection problem, Markowitz' mean –variance model of portfolio selection is one of the best known models which assumes that the total return of a portfolio can be described using the mean return of the assets and the variance of return (risk) between these assets [3, 11]. In it basic form, this model requires to determine the composition of a portfolio of assets which minimizes risk while achieving a predetermined level of expected return what it is called the efficient frontier. For every level of desired mean return, this efficient frontier gives us the best way of investing our money.

However, the standard mean-variance model has not got any cardinality constraint ensuring that every portfolio invests in a given number of different assets, neither uses any bounding constraint limiting the amount of money to be invested in each asset. This sort of constraints are very useful in practice. In order to overcome these inconveniences, the standard model can be generalized to include these constraints. In this paper we focus on the problem of tracing out the efficient frontier for the general mean-variance model with cardinality and bounding constraints. In previous work, some heuristic methods have been developed for the portfolio selection problem. There are some methods that use evolutionary algorithms [6], tabu search (TS) [1] simulated annealing (SA) [1, 2, 5, 10] and neural networks [5]. Here we present a different heuristic method based on PSO. The results obtained are compared to those obtained using three representative methods from [1] based on genetic algorithms (GA), TS and SA.

Following this introduction, in Section2, we present the model formulation for the portfolio selection problem and describe the PSO as well as the way to use it for solving this problem. Section 3 design two computational experiments to evaluate the PSO model. In Section 4, we present some experimental results and, in Section 5, we finish with some conclusions and future work.

LITERATURE REVIEW

The problem of optimally selecting a portfolio among n assets was formulated by Markowitz in 1952 as a constrained quadratic minimization problem [2]. In this model, each asset is characterized by a return varying randomly with time. The risk of each asset is measured by the variance of its return. If the component w_i of the N-vector w represents the proportion of an investor's wealth allocated to asset I, then the total return of the portfolio is given by the scalar product of w by the vector of individual asset returns.

Portfolio Selection

In this section we first display the standard (unconstrained) Markowitz portfolio model and illustrate the way to calculate the efficient frontier. Second we showed the optimal Markowitz portfolio model that we want solve. Finally we show the general Markowitz constrained optimal model that we want solve.

Standard (Unconstrained) Markowitz Portfolio Model

Let:
N be the number of assets available,

u_i be the expected return of asset *i* (*i*=1,… ,*N*),
σ_i be the covariance between assets *i* and *j* (i=1,… , *N*; j=1, …, *N*),
R_{exp} be the desired expected return.
Then the decision variables are:
w_i the proportion ($0 \leqq w_i \leqq 1$) held of asset i (i=1,… , *N*) and using the standard Markowitz mean –variance approach [1] we have that the unconstrained portfolio optimization problem is:

$$Min \sum_{i=1}^{N}\sum_{j=1}^{N} w_i \cdot w_j \cdot \sigma_{ij} \tag{1}$$

subject to

$$\sum_{i=1}^{N} w_i u_i = R_{exp} \tag{2}$$

$$\sum_{i=1}^{N} w_i = 1 \tag{3}$$

$$0 \le w_i \le 1 \quad , \quad i = 1,..., N$$

Eq.(1) minimizes the total variance(risk) associated with the portfolio whilst Eq.(2) ensures that the portfolio has an expected return of R_{exp}. Eq.(3) ensures that the proportions add to one.
The optimal Markowitz portfolio model that we adopt is as follow Eq.(4).

$$Min \ \lambda\left[\sum_{i=1}^{N}\sum_{j=1}^{N} w_i \cdot w_j \cdot \sigma_{ij}\right] - (1-\lambda)\left[\sum_{i=1}^{N} w_i \cdot u_i\right]$$

subject to

$$\sum_{i=1}^{N} w_i = 1$$

$$0 \le w_i \le 1 \quad , \quad i = 1,..., N \tag{4}$$

The General Markowitz Constrained Optimal Model

In order to extend our formulation to the cardinality constrained case let:
K be the desired number of assets in the portfolio, ε_i be the minimum proportion that must be held of asset *i* (*i*=1, …, *N*) if any of asset *i* is held, δ_i be the maximum proportion that can be held of asset *i* (*i*=1,… , *N*) if any of asset *i* is held, where we must have $0 \leqq \varepsilon_i \leqq \delta_i \leqq 1$ (*i*=1,… , *N*). In practice ε_i represents a "min-buy" or "minimum transaction level" for asset *i* and δ_i limits the exposure of the portfolio to asset *i*. Introducing zero-one decision variables:

$$z_i \begin{cases} 1 & \text{if any of asset } i \ (i=1,\& ,N) \text{ is held,} \\ 2 & \text{otherwise} \end{cases}$$

The cardinality constrained portfolio optimization problem is

$$Min \ \lambda\left[\sum_{i=1}^{N}\sum_{j=1}^{N} w_i \cdot w_j \cdot \sigma_{ij}\right] - (1-\lambda)\left[\sum_{i=1}^{N} w_i \cdot u_i\right] \tag{5}$$

subject to

$$\sum_{i=1}^{N} w_i = 1 \tag{6}$$

$$\sum_{i=1}^{N} z_i = K \tag{7}$$

$$\varepsilon_i z_i \le w_i \le \delta_i z_i \quad , \quad i = 1,..., N \tag{8}$$

$$z_i \in [0, 1] \quad , \quad i = 1,..., N \tag{9}$$

We use the same way like Ref[1] that applying to a weighting parameter(λ) to combine both minimizes the total variance (risk) associated with the portfolio and ensures that the portfolio has an expected return of R_{exp}. Eq.(6) ensures that the proportions add to one whilst Eq.(7) ensures that exactly *K* assets are held. Eq.(8) ensures that if any of asset *i* is held (z_i =1) its proportion w_i must lie between ε_i and δ_i, whilst if none of asset *i* is held (z_i =0) its proportion w_i is zero. Eq.(9) is the integrality constraint.

Particle Swarm Optimization

Particle swarm optimization (PSO), introduced by Kennedy and Eberhart in 1995, is generally similar the evolutionary computation techniques (Trelea, 2003). PSO is a social population-based search algorithm of social influence and learning from it's neighborhood. In PSO, a swarm resembles a population and a particle resembles an individual, is initialized with a swarm of particles, and position of each particle represents a possible solution. the particles fly through the multidimensional search space by dynamically adjusting velocities according to it's own experience and neighbors [4, 7, 8, 12].

$$x_{id}(t+1) = x_{id}(t) + V_{id}(t+1) \quad (10)$$

Where $x_{id}(t)$ denote the position of particle *i* in the *d* dimension search space at time step *t*; unless otherwise stated , *t* denotes discrete time steps. The position of the particle is changed by adding a velocity $v_{id}(t+1)$ to the current position. The $v_{id}(t+1)$ calculating is as following:

$$V_{id}(t+1) = w \times V_{id}(t) + c_1 \times rand() \times (p_{id} - x_{id}(t)) + c_2 \times rand() \times (p_{gd} - x_{id}(t)) \quad (11)$$

Where $V_{id}(t)$ is the velocity of particle *i* in dimension *d*=1, …,*n* at time step *t*, $x_{id}(t)$ is the position of particle *i* in dimension *d* at time step *t*, and c_1, c_2 are positive acceleration constants used to scale the contribution of the cognitive and social components respectively, and *rand*() is random value in the range [0, 1], sampled a uniform distribution. These random values introduce a stochastic element to the algorithm.

The personal best position, p_{id} associated with particle *i* in dimension *d* is the best position the particle has visited since the first time step. The global best position, p_{gd}, at the time step *t*, is the best position discovered by all of particles found since the first time step.

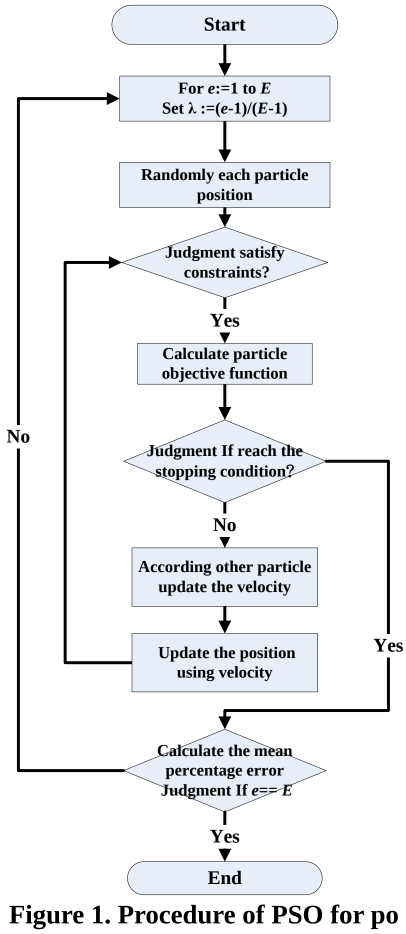

Figure 1. Procedure of PSO for po

COMPUTATIONAL EXPERIMENTS

In this section, we definite our experiments as searching the general efficient frontier that solves the problem formulated in last two optimal models. We employ the five sets of benchmark data that have been already used in [4, 7, 11] These data correspond to weekly prices from March 1992 to September 1997 and they come from the indices: Hang Seng in Hong Kong, DAX100 in Germany, FTSE100 in UK, S&P100 in USA and Nikkei225 in Japan. The number N of different assets considered for each one of the test problems is 31, 85, 89, 98 and 225, respectively. The mean returns and covariances between these assets have been calculated for the data. The sets of mean returns and covariances are publicly available at http://people.brunel.ac.uk/ mastjjb/jeb/orlib/portinfo.html.

All the results presented here have been computed, as in Ref. [2], using the values K =10, ε_i =0.01 and δ_i = 1 for the problem formulation. So we have tested 51 different values for the risk aversion parameter λ.

In this paper, we used the variance(standard deviation) and return of the best solution for each $\lambda$to compare to standard efficient frontiers to measure percentage error respectively, and took the minimum value from variances error and mean returns error as our the percentage error associated with a portfolio.

Let the pair (v_i ,r_i) represent the variance and mean return of a point in a heuristic efficient frontier. Let also v_i^* be the variance corresponding to r_i according to a linear interpolation in the standard efficient frontier. We define the variance of return error e_i for any heuristic point (v_i ,r_i) as the value 100 $(v_i^* - v_i)/ v_i^*$ (note that this quantity will always be nonnegative). In the same way, using the return r_i^* corresponding to v_i according to a linear interpolation in the standard efficient frontier, we define the mean return error η_i as the quantity $100(r_i - r_i^*)/ r_i^*$.

The error measure was also defined in [2]. It is calculated averaging the minimums between the mean return errors e_i and the standard deviation of return errors η_i.

IMPLEMENTATION RESULTS

We present the values correspondent to the minimum error measure in Table1 to

To evaluate the performance of PSO, we first applies the PSO for five test data sets on unconstrained portfolio problem in Ref.[1]. Table 1 shows the comparable results on mean percentage error and to benchmark with three other heuristic algorithms (genetic algorithm (GA), simulated annealing(SA), and tabu search(TS)) developed by [1]. In this evaluation, the parameter settings of the PSO algorithm are as follows: positive acceleration constants c_1 and c_2 are set to (2, 2), and weight of velocity is set to 0.2. Table 2 presents the comparison of five test data sets for constrained portfolio problem. It is obvious that PSO does a better job in finding the standard efficient frontiers with the lowest overall mean percentage error.

Table 1 Result for unconstrained portfolio problem (Mean percentage error)

Index	Number of assets(N)	GA	SA	TS	PSO
Hang Seng	31	0.0202%	0.1129%	0.8973%	0.0198%
DAX	85	0.0136%	0.0394%	3.5645%	0.0094%
FTSE	89	0.0063%	0.2012%	3.2731%	0.0064%
S&P	98	0.0084%	0.2158%	4.4280%	0.0071%
Nikkei	225	0.0085%	1.7681%	15.9163%	0.0082%
	Average	0.0114%	0.4675%	5.6158%	0.0102%

Table 2 Result for constrained portfolio problem (Mean percentage error)

Index	Number of assets(N)	GA	SA	TS	PSO
Hang Seng	31	1.0974%	1.0957%	1.1217%	1.0554%
DAX	85	2.5424%	2.9297%	3.3049%	2.1231%
FTSE	89	1.1076%	1.4623%	1.6080%	1.0028%
S&P	98	1.9328%	3.0696%	3.3092%	1.5699%
Nikkei	225	0.7961%	0.6732%	0.8975%	0.7756%
	Average	1.4953%	1.8461%	2.0483%	1.3100%

CONCLUSION

In this paper we have applied a PSO heuristic algorithm to solve portfolio selection problem based on both standard Markowitz mean-variance model and this model added to cardinality and bounding constraints. Dealing with this kind of constraints, the portfolio selection problem turn into a mixed quadratic and integer programming problem for which no exactly and efficiently optimal algorithms exit. Performance evaluation of our PSO under the same settings of error computational way has been implemented in five benchmark datasets and compared our results to three other heuristic algorithms that included genetic algorithms, simulated annealing, and tabu search.

The results of the performance evaluation indicate that our basic PSO has outperformed the other method for the most part.

Therefore, we can conclude that the PSO is an effective and efficient heuristic algorithm to solve constrained portfolio selection problems.

Future work

In this paper we investigate the ability of the PSO heuristic to deliver high quality solutions for the mean-variance model enriched by additional constraints but from a practical point of view, however, the Markowitz model may often be considered too basic, as it ignores many of the constraints faced by real; world investors: trading limitations, size of the portfolio, etc. Including such constraints in the formulation results in a nonlinear mixed integer programming problem which is considerably more difficult to solve than the original model. In the future we plan to investigate more complex portfolio selection problems that include real constraints.

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