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ABSTRACT

Free open-source operating system (OS) has driven both ad revenue and Internet traffic from smartphones to the historically high levels. Though achieving huge market share, there is no record showing that the profit of a handset maker from the smartphone market can surpass the firm implementing vertical integration ever since the first smartphone has been launched in the market. In this study, we consider that an open-source OS provider free offers its operating system but gain mobile advertising revenue by demanding the hardware maker to install preload apps. After trading off between homogenizing each open-source smartphone but gaining more "eyeball shares" on advertising via handsets due to more preload apps, we examine how the OS provider's optimal agreement with hardware makers changes with market factors. In addition, we also compare and analyze the profit of the OS provider between launching its own brand of smartphones and receiving advertising revenue only. Our results suggest that the OS provider should enhance the quality of its own product when entering the smartphone market. Moreover, if outsourcing the production work to a hardware maker can achieve the economies of scale and increase its profit, our analytic results suggest that the OS provider shouldn't partner with the hardware maker dominating the smartphone market.

Keywords: in-app ads, vertical integration, operating system, outsourcing, production differentiation

INTRODUCTION

In the first quarter of 2015, market surveys have reported that the traffic volume and advertising revenue contributed by Android has first time surpassed iOS. For increasing its market share rapidly to grow mobile ad revenue, Google, in control of developing and maintaining Android, doesn't charge any hardware makers installing the open-source operating system (OS). However, hardware makers have to follow certain requirements, known as Mobile Application Distribution Agreement (MADA), regulated by Google; otherwise, some popular services and apps which are viewed as key components (such as APIs and Google Play) in the ecosystem of Android cannot be offered in their smartphones. In recent years, both mobile search results and YouTube ad revenue have surged up because mobile platforms have become a major channel to connect the Internet. The requirements which specify how many Google apps have to be preloaded and where these gadgets are placed significantly link to the open source OS provider's profit.

To incentivize hardware makers to follow the placement standard of apps and create more ad revenue form preloaded gadgets, Google has also shared a part of its advertising revenue with hardware makers. The details of sharing ratios are often not disclosed, but many news reports are available to shed light on the contexts. Intuitively, the hardware maker having high market share should be able to demand more from advertising revenue. Moreover, from the perspective of providing consistent experience in the manipulation of software and raising more advertising revenue, an open-source OS provider may want to require more pre-load apps on the home screen of smartphones; however, such a requirement may reduce the hardware makers' profits because product differentiation is more difficult to be achieved as the open-source OS provider increase the number of pre-load apps and demand more prominent position for them. Once these smartphone products become more homogeneous, less differentiation will lead to fierce competition in price among hardware makers, which may decline the open-source OS provider's mobile ad revenue eventually.

Launching its own brand of smartphone to acquire higher profit from the smartphone market could be a lucrative alternation for an open-source OS provider. In addition, this means can offer software developers a platform featuring an updated version of operating system for testing, encourage hardware makers to adopt the newest standards, and gain more "eyeball shares" on mobile advertising by making the open source OS become universally accessible. However, such a strategy of releasing its own product, achieved by either outsourcing or vertical integration, may endanger the partnership between the open-source OS provider and hardware markers, and even intensify the competition among hardware makers. Interestingly, Google's smartphone and tablet are also co-developed and manufactured by the hardware makers in the market, as shown in Table 1. Moreover, though the market share of iOS is less than 15% in recent years, it is reported that Apple's revenue from iPhone accounted for 89% of all smartphone profits. Considering such a marvelous instance and the fact that the market share of Android has increased to more than 80%, building its own brand of smartphone is a "worthy of imitation" strategy for Google to expand its profit other than mobile advertising revenue.

From the perspective of mobile advertising and smartphone sales, there are two research questions worthy to be further analyzed. First, how would an open-source OS provider adjust the number of pre-loaded apps to enhance its profit in a fairly competitive environment? We aim to investigate the impact of advertising revenue and the number of hardware makers on the optimal agreement concerning pre-load apps. Second, we prescribe the condition under which it is profitable for an open-source OS provider to make its own brand of smartphone. Finally, we analyze the open source OS provider's decision of outsourcing from the viewpoint of co-competition relationship between the open-source OS provider and the hardware maker delegated to manufacture the open source OS provider's product.

Table 1. The history of Nexus

Brand Name	Product Type		Release Date	Manufacturer
	Smartphone	Tablet		
Nexus One	★		2010	HTC
Nexus S	★		2010	Samsung
Galaxy Nexus	★		2011	Samsung
Nexus 4	★		2012	LG
Nexus 7		★	2012	ASUS
Nexus 10		★	2012	Samsung
Nexus 5	★		2013	LG
Nexus 7 (2nd Generation)		★	2013	ASUS
Nexus 6	★		2014	Motorola Mobility
Nexus 9		★	2014	HTC

LITERATURE REVIEW

The quality of Apps and the content presented by them are the most critical in shaping the satisfaction of smartphone users [6]. In fact, apps and web services themselves are even more important than the operating systems on smartphones because they may replace the home screen of an operating system or offer a direct connection through the Internet [10]. Though granting a high profit sharing ratio can encourage more app developers to join the platform and then leads to more sales, Fukawa and Zhang [3] show that the strategy may be helpless and even harmful to the open source OS provider when its market share is higher than a certain threshold. From the perspective of Google, it will be a top priority that ensuring high ad click-through rate by requiring the number of preload apps on each android handset, which are able to put users on its search interface [5].

From the perspective of methodology used in this study, differentiated Cournot model has been widely applied to various different contexts concerning the competition between firms selling substitutes. For instance, Wang [13] considers a duopolistic setting to prescribe the conditions under which the firm with a cost-reducing innovation gains more from royalty licensing than a one-off payment. Milliou and Petrakis [8] examine how the date of adopting a new technology changes with the level of product differentiation. Tang et al. [9] indicate that high production cost and the advantage of scope economies are major driving forces for outsourcing production work to third parties.

Strategic outsourcing is a global tendency in the manufacturing industry because this strategy can save cost and achieve higher performance by delegating the manufacturer with comparative advantage against in-house production. Shy and Stenbacka [12] find that economies of scale can be achieved in an oligopolistic input-producing industry structure because firms will outsource to the same manufacturers. Xia and Gilbert [14] consider the scenario in which a supplier delegates a retailer to sell two substitutable products and the supplier can provide demand-enhancing services by itself or outsources the service to the retailer. Their results indicate that the decision of outsourcing is linked to the firm acting as a Stackelberg leader. Rothaermel et al. [11] highlight the importance of balancing vertical integration and strategic outsourcing because their empirical results show that achieving a balance in taper integration can increase competitive advantage and improve firm performance.

From the perspective of market structure, our research model is similar to the one analyzed by Foros [2], which investigates the competition between two Internet service provider and shows that a vertical integrated ISP controlling the access network may foreclosure an ISP connecting to the network by overinvesting in value-added services when its R&D in offering the services surpasses the other. From the perspective of advertising, Giri and Sharma [4] consider the aspect of sharing advertising expenses for enhancing consumer demand in a supply chain consisting of one supplier and two competition retailers. By investigating the supper's decision on whether to share its retailers' advertising costs and whether to levy different wholesale prices from them, their numeral results demonstrate that differentiating the wholesale prices is always beneficial to the supplier. Yao et al. [15] consider two heterogeneous value-adding retailers in a supply chain, both of which pay the same wholesale price to their common supplier and only know their individual cost of value-added service. Their results show that both retailers, which are cost inefficient, are willing to share their cost information with the supplier when the market demand is large enough. On the other hand, the private information will be hidden when both have cost advantage. From the view of oligopolistic competition, Bagchi and Mukherjee [1] study a supply chain with differentiated oligopoly in which a supplier can adopt royalty licensing or auction its licenses to retailers. Their results indicate that the supplier prefers royalty licensing than auction when the number of retailers is not too small and the degree of production differentiation is moderate.

Our model differs from prior studies in that the open source OS provider plays the role of a supplier but only receives advertising revenue from hardware makers, which are similar to the retailers in supply-chain literatures. In addition, the supplier can share its advertising revenue with the retailers and regulate the degree of production differentiation by specifying the number of preload apps and the location where the apps can be accessed. Moreover, we also examine the benefit of strategic outsourcing to one of the retailers rather than outside manufacturers when the supplier establishes its own brand of handset.

THE MODEL

Consider an OS provider and two hardware makers in an open-source smartphone market. According to the agreement with the OS provider, both hardware makers can use its operating system without any charge but have to install the apps appointed by the OS provider on the home screen of their handsets. As a result, the OS provider can make a profit from the in-app ads delivered through its preload apps even if there is no payment from the release of its developed operating system. However, once more preload apps are demanded by the OS provider, the less degree of production differentiation the hardware makers suffer from. Therefore, the OS provider has to trade off its advertising revenue from mobile advertising and the competition between hardware makers.

To model the relation among the OS provider and two duopolistic hardware makers, a stylized Cournot differentiated model is used to analyze how the OS provider base various market factors to adjust the requirement of its preload apps on the home screen of handsets. Though smartphone users can also use the apps provided by the hardware makers or install other apps by searching in an app market, we consider that the probability of using the preload apps on the home screen is higher than others and therefore focus on the impact of the number of preload apps appointed by the OS provider on its profit. For simplicity, we let the ratio of preload apps on the home screen handled by the OS provider is u and the hardware makers $1- u$, respectively. The parameter u not only captures the ratio of advertising revenue from apps between the OS provider and either hardware maker but also represents the degree of production differentiation. Thus, letting q_i, $i = 1,2$, be the quantities produced by hardware markers *i*, the inverse demand function in the open-source smartphone market is given by

$$p_i\left(q_i,q_j\right)= a+q_i - q_i - uq_j \quad \text{for } i \neq j, \tag{1}$$

where a represents market profitability [7] and q_i measures individual maker's product competitiveness [2]. As the ratio of preload apps on the home screen increases, the market becomes more homogeneous because all open-source handsets have similar appearance and functionality on their home screen when consumers make their first glances on the products.

Optimal Production Differentiation

In practice, after following the OS provider's requirement, hardware makers can still preload their apps on the rest of space to gain their own advertising revenue; in addition, the OS provider may consider sharing its advertising revenue with hardware makers to strengthen their partnership and encourage them to strategically accommodate their products to the preload apps required by the OS provider. Therefore, letting g be the ratio of advertising revenue taken by the OS provider and b the marginal advertising revenue per demand, the profits of the hardware makers can be expressed as

$$p_i = (p_i - c)q_i + b\left(1- g\right)u \; q_i \quad b\left(1- u\right) \; q_i, \tag{2}$$

where c is marginal production cost. Though it seems more reasonable that all smartphone makers have different marginal production cost, we simplify the issue because a heterogeneous setting doesn't alter our qualitative results. Here, the value of g is considered as an exogenous variable because the negotiation of how much advertising revenue to be split may involve the setting of bargaining power, which is absent in our model. Moreover, the profit of the OS provider is given by

$$p_g = b g u \sum q_i \tag{3}$$

Based on backward induction and the static analysis on the optimal ratio of preload apps with respect to advertising revenue, we have the following findings:

Proposition 1. (The impact of advertising revenue on the number of preload apps)
When the demand of advertising increases or the hardware maker's share of advertising revenue decreases, the optimal number of the preload apps required by the OS provider decreases. Formally, $\partial u^*/\partial b < 0$ and $\partial u^*/\partial g < 0$.

Our results indicate that the OS provider should consider allowing its partner to preload more itself apps when advertising revenue increases, no matter whether the adverting price increases or the OS provider has a higher share of advertising revenue from a new agreement. Thus, this finding reminds the OS provider of the importance of balancing product differentiation and in-app ad revenue. Recently, Google's advertising revenue declines year by year; however, the intention of gaining more advertising revenue from in-app ads is not just simply to arbitrarily increase the number of preload apps because this blind move without understanding the influence of product differentiation on the smartphone market may erode its profit margins eventually.

Subsequently, we consider there are *n* hardware makers in the smartphone market. Thus, the inverse demand function in the smartphone market is refined as

$$p_i(q_i, q_{-i}) = a + \theta_i - q_i - u\sum q_{-i} \quad , \tag{4}$$

To analyze how the OS provider dynamically update its agreement with hardware markers when the number of hardware makers in the market increases, we consider all the makers have the same product competitiveness (that is $\theta_i = \hat{\theta}$ for all i).

Proposition 2. (The impact of market competition on the number of preload apps)
From the perspective of profitability, as more hardware makers enter this open-source smartphone market, the operating system provider should require the less number of preload apps to allow higher level of production differentiation. Formally, $\partial u^* / \partial n < 0$ given in a homogeneous equilibrium.

In order to grow its user base and increase advertising revenue, this result shows that the OS provider can regulate the level of production differentiation by decreasing the number of preload apps on the home screen when more and more hardware makers launch their products in this market. Though it seems that the strategy will reduce the OS provider's advertising revenue, allowing hardware makers to deploy more apps according to their product positioning can facilitate better production differentiation, which turns out to benefit the OS provider.

CO-COMPETITION

By outsourcing or vertical integration, the OS provider may also launch its brand of handset to grow its business in the smartphone market. In the current version, we assume that all preload apps are regulated by the OS provider and the degree of production differentiation is exogenously given; however, the inconsistency will be improved in the future revision. Therefore, following the same inverse demand curve given in (1), if the OS provider chooses not to launch its own brand of smartphone, its profit and the profits of the hardware makers are

$$\pi_{g,N} = \beta\gamma \sum q_i \text{ and } \pi_i = (p_i - c)q_i + \beta(1-\gamma)\cdot q_i \tag{5}$$

If the OS provider decides to launch its own smartphone in the market, the new inverse demand curve is given by

$$p_i(q_i, q_{-i}) = a + \theta_i - q_i - u\sum q_{-i} \tag{6}$$

As a result, the profit of the OS provider can be expressed as

$$\pi_{g,Y} = \beta\cdot(q_g + \gamma\cdot(q_1 + q_2)) + (p_g - c)q_g \tag{7}$$

Comparing $\pi_{g,Y}$ with $\pi_{g,N}$, we have the following findings:

Proposition 3. (The condition on production competitiveness for vertical outsourcing)
When the product competitiveness of the OS provider's smartphone is not too weak, releasing its own brand of handset to the market is better than serving as a pure OS provider. Formally, $\pi_{g,Y} > \pi_{g,N}$ when $\theta_1 = \theta_2 \leq \theta_g$.

When the OS provider's production cost is the same as the other hardware makers, our result shows that the OS provider should always launches its product to the market as long as its product quality is not too low as compared with the products launched by the others. Even if the strategy may annoy its downstream partners and lead to more intensive competition, our finding helps explain why Google still releases its own brand of smartphone and tablet in the market. In the following, the static analysis is implemented to investigate the impact of market factors and the share of advertising revenue on the OS provider's profit.

Proposition 4. (The comparison in profit between vertical specification and vertical outsourcing)

(1) When a is smaller than a certain threshold, $\partial\pi_{g,Y}/\partial\gamma < \partial\pi_{g,N}/\partial\gamma$

(2) When $\theta_1 \approx \theta_2 \approx \theta_g$, $\partial\pi_{g,Y}/\partial a > \partial\pi_{g,N}/\partial a$ and $\partial\pi_{g,Y}/\partial c < \partial\pi_{g,N}/\partial c$

Our results indicate that low production cost and high market profitability are two important factors of supporting the decision of vertical outsourcing. Though releasing its own brand of smartphone makes each firm gain less profit from the consumers, gaining the dual profit made up of the sales of its product line and advertising revenue maintains comparable advantage against receiving mobile ad revenue only as the produce cost decreases or market profitability increases, as shown in Figure 1 and 2. However, when the market profitability is not high enough and all smartphones have symmetric product competitiveness, we find that the OS provider should not enter the smartphone market when the share of the hardware makers' advertising revenues from the OS provider's preload apps is too low, as shown in Figure 3. The results remind the OS provider of the importance of enhancing its operating system and the quality of preload apps to attract more potential buyers because a small market cannot feed so many hardware makers in a highly competitive ecosystem. Finally, though we cannot directly examine the impact of marginal advertising revenue on the OS provider's profit when it launches or not, the numerical result shown in Figure 4 is consistent with our intuitive perspective that high advertising revenue motivates the OS provider to release its own brand of handset in the market.

Outsourcing Strategy

Subsequently, we consider the scenario in which the OS provider has decided to launch its product and outsource the production work to one of the hardware makers due to cost consideration. The hardware maker having the order from the OS provider may leverage economies of scale to reduce production cost and even earn addition profit from the OS provider. In such a complicated relationship, the OS provider doesn't only directly compete with the hardware markers in the smartphone market but also become an important tactic partner from the perspective of outsourcing. For simplicity, the hardware maker taking the order from the OS provider may charge w_i per quantity and its marginal production cost can be reduced to $c - e$ due to the benefit resulting from large-scale production. We consider that the OS provider can always delegate an original equipment manufacturer (OEM) to produce its smartphone and the OEM charges *c* per quantity, which is a very competitive price because it is not higher than other hardware maker's marginal production cost. Accordingly, in the scenario of outsourcing, the profit of the hardware maker *i* receiving the order is given by $p_i = p_i q_i + b(1- g)?q_i \quad (c- e)?(q_i \quad q_{g,i}) + w_i q_{g,i}$, where $q_{g,i}$ is the quantities of the OS provider's smartphone when outsourcing its order to hardware maker *i*. As for the OS provider, its profit in the scenario of outsourcing can be expressed as $p_{g,i} = (p_g - w_i)q_{g,i} + b\,g\,((q_i + q_j) + q_{g,i})$.

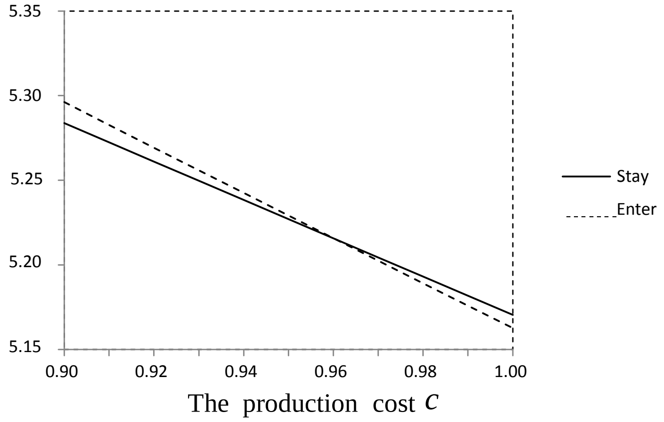

Figure 1. The impact of production cost on the OS provider's profit

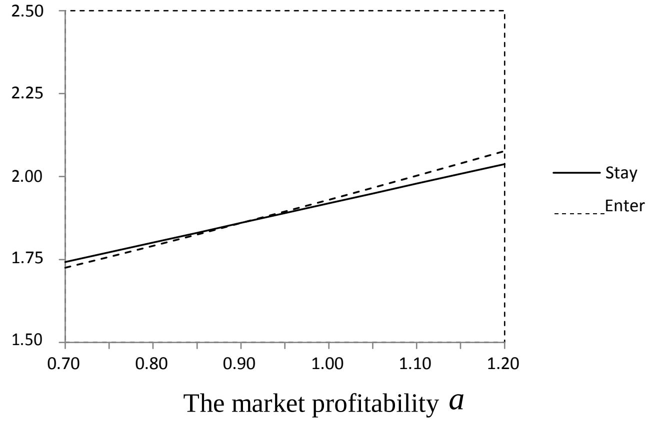

Figure 2. The impact of market profitability on the OS provider's profit

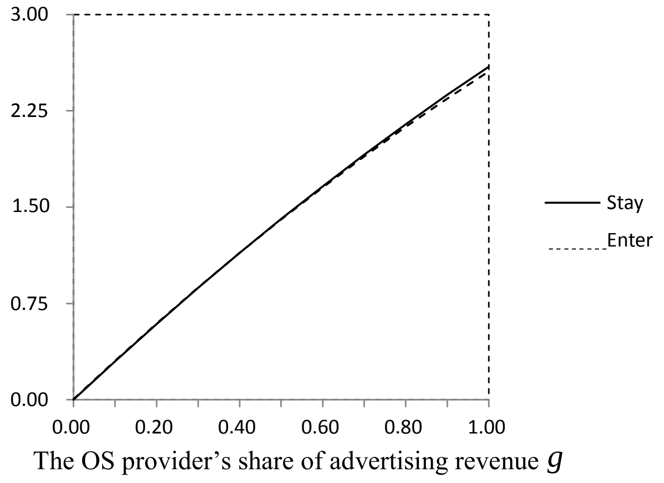

Figure 3. The impact of advertising sharing ratio on the OS provider's profit

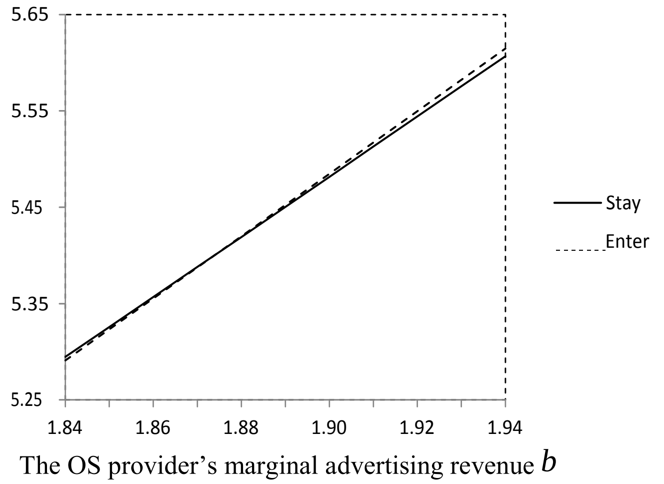

Figure 4. The impact of marginal advertising revenue on the OS provider's profit

The outsourcing process of the OS provider is as follows:
(i) The OS provider outsources its production work to either hardware maker 1 or 2.
(ii) The hardware maker decides the outsourcing cost per quantity w_i.
(iii) The OS provider accepts or rejects the offer. If the negotiation has been broken, the OS provider outsources the production work to an OEM.

Proposition 5. (*The decision of vertical outsourcing for open-source operating system provider*)
The OS provider should delegate the hardware marker with less product competitiveness to make its own brand of smartphone if outsourcing to either of the hardware makers can be better off. Formally, when $Min\{p_{g,i}, p_{g,j}\} > p_{g,Y}$, $p_{g,i} > p_{g,j}$ if $q_i < q_j$.

Our result shows that the OS provider shouldn't outsource its production work to the hardware maker dominating the market when its smartphone business can be more profitable by outsourcing the production work to any hardware maker than an OEM. From the analysis of the outsourcing cost per quantity, we find that a hardware maker with relatively weak product competitiveness is willing to charge a lower fee for the manufacturing work than the one with strong product competitiveness. This finding reminds the OS provider of the possible outcome that delegating a big hardware maker to produce its handsets could not gain the highest profit. To ensure the better profitability of the OS provider, an sensible alternative for achieving a low outsourcing cost is to notify the hardware maker that the negotiation could be broken if the manufacturing cost is too high and its major rival will be considered in the next run of negotiation.

CONCLUSION

In this study, we consider a supply chain consisting of an open-source OS provider and two hardware makers. Both hardware makers can have their own advertising revenue from in-app ads and share a portion of ad revenue from preload apps specified in the agreement with the OS provider. The OS provider can require more number of preload apps to increase its mobile ad revenue but such a strategy may intensify the competition in the smartphone market due to less production differentiation, which may not always benefit its profit in the end. Moreover, we also examine the decision of making the OS provider's own brand of smartphones. To leverage the advantage from economies of scale, the OS provider may outsource its production work to a suitable hardware maker. As a result, the relationship between the OS provider and the hardware maker having the production order could be both partners and competitors. By applying a styled Cournot differentiation model, several helpful findings are discovered.

First, the OS provider should reduce the number of reload apps if the demand of advertising market is so high or more firms decide to launch their own brand of handsets. The purpose of preloading less number of apps is to facilitate product differentiation so that it will soften the intensive competition among hardware makers and incentivize them to create more market demand. Second, we prescribe the condition under which the strategy of making the OS provider's own brand of smartphones will take more advantage than receiving mobile adverting revenue only when market factors such as production cost or market profitability changes. Finally, in the scenario of vertical outsourcing, we find that the OS provider may gain more by contracting with a "relatively weak" hardware maker because the other with strong product competitiveness can manipulate the whole market so that it will charge a higher outsourcing fee.

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APPENDIX

Proof of Proposition 1

Given $i = 1, 2$, solving $\partial \pi_i / \partial q_i = 0$ where simultaneously $q_i^*(\upsilon) = \frac{(2-\upsilon)\left(a-c+(1-\upsilon\gamma)\beta\right)+2\theta_i-\upsilon\theta_j}{4-\upsilon^2}$ where $i \neq j$.

Next, after incorporating $q_i^*(\upsilon)$ into (3), we have $\upsilon^* = Min\left\{1, \sqrt{4+\frac{(2a+2\beta-2c+\theta_1+\theta_2)}{\gamma\beta}}-2\right\}$ by solving $\partial \pi_g / \partial \upsilon = 0$. Accordingly, we have $\frac{\partial \upsilon}{\partial \beta} = -\frac{2a+\theta_1+\theta_2-2c}{2\beta\sqrt{\gamma\beta(4\gamma\beta+2a+2\beta+\theta_1+\theta_2-2c)}} < 0$ and

$$\frac{\partial \upsilon}{\partial \gamma} = -\frac{2a+2\beta+\theta_1+\theta_2-2c}{2\gamma\sqrt{\gamma\beta(4\gamma\beta+2a+2\beta+\theta_1+\theta_2-2c)}} < 0.$$

Proof of Proposition 2

Note that $\partial \pi_i / \partial q_i = \left(a+\hat{\theta}-2q_i-\upsilon\sum_{j\neq i} q_j - c\right)+\beta(1-\gamma)\upsilon - \beta(1-\upsilon)$. In a homogenous equilibrium, $q_1 = q_2 = \cdots = q_n$ so that we have $q_i^* = \frac{(a+\hat{\theta}-c)+\beta(1-\upsilon\gamma)}{\upsilon(n-1)+2}$. Subsequently, after incorporating $q_i^*(\upsilon)$ into (3), solving $\partial \pi_g / \partial \upsilon = 0$ can yield υ^* so that we may straightforwardly have

$$\frac{\partial \upsilon^*}{\partial n} = \frac{\left(\sqrt{2\beta\gamma\left(\left(a+\beta+\hat{\theta}-c\right)(n-1)+4\beta\gamma\right)}-\sqrt{2\left(\left(a+\beta+\hat{\theta}-c\right)(n-1)+4\beta\gamma\right)^2}\right)}{2\sqrt{\beta\gamma\left(\left(a+\beta+\hat{\theta}-c\right)(n-1)+2\beta\gamma\right)}(n-1)^2}$$

. As a result, it is intuitive to verify $\partial\upsilon^*/\partial n < 0$ from the above equation.

Proof of Proposition 3

$$\pi_{g,Y} - \pi_{g,N} = \frac{L_1 \cdot L_2}{4(2-\upsilon)^2(\upsilon+1)^2(\upsilon+2)}, \text{ where}$$

$$L_1 \equiv (\upsilon+2)\left(\upsilon(\theta_1+\theta_2)+2\upsilon\beta\gamma-(2+\upsilon)\theta_g\right)-(a+\beta-c)\left(4-\upsilon^2\right)-4\upsilon^3\beta\gamma$$

$$L_2 \equiv \upsilon(\theta_1+\theta_2)-(2-\upsilon)(a+\beta-c)-(2+\upsilon)\theta_g-2\upsilon\beta\gamma$$

Note that $\dfrac{\partial L_1 \cdot L_2}{\partial \theta_g} > 0$. Letting $\theta_1 = \theta_2 = \theta_g$, we can show the following results because $\upsilon\left(4-4\upsilon^2+3\upsilon\right) < 4$ can be verified in a numerically approach.

$$L_1 = \left(\upsilon^2-4\right)\left(\theta_g + a + \beta - c\right) + \upsilon\left(4-4\upsilon^2+2\upsilon\right)\beta\gamma$$

$$= \left(\upsilon^2-4\right)\left(\theta_g + a - c\right) + \left(\upsilon^2-4\right)\beta + \upsilon\left(4-4\upsilon^2+2\upsilon\right)\beta\gamma$$

$$\leq \left(\upsilon^2-4\right)\left(\theta_g + a - c\right) + \left(\upsilon\left(4-4\upsilon^2+3\upsilon\right)-4\right)\beta$$

$$\leq \left(\upsilon^2-4\right)\left(\theta_g + a - c\right) < 0$$

$$L_2 = -\left(2-\upsilon\right)\left(\theta_g + a + \beta - c\right) - 2\upsilon\beta\gamma < 0$$

Proof of Proposition 4

(1) $\dfrac{\partial \pi_{g,Y}}{\partial \gamma} = \dfrac{\partial \pi_{g,N}}{\partial \gamma} = -4\upsilon^2\beta\left(\left(2\upsilon\left(1-2\upsilon\right)+4\right)\beta\gamma + \upsilon\left(\upsilon\left(\theta_1+\theta_2-\theta_g\right)-2\theta_g\right) - \left(2-\upsilon\right)\upsilon\left(a+\beta-c\right)\right)$

Therefore, $\dfrac{\partial \pi_{g,Y}}{\partial \gamma} < \dfrac{\partial \pi_{g,N}}{\partial \gamma}$ when $a < \dfrac{\left(2\upsilon\left(1-2\upsilon\right)+4\right)\beta\gamma + \upsilon\left(\upsilon\left(\theta_1+\theta_2-\theta_g\right)-2\theta_g\right)}{\left(2-\upsilon\right)\upsilon} - \beta + c$

(2) $\pi_{g,Y} - \pi_{g,N} = \dfrac{L_1 \cdot L_2}{4\left(2-\upsilon\right)^2\left(\upsilon+1\right)^2\left(\upsilon+2\right)}$

$$\frac{\partial L_1 \cdot L_2}{\partial a} = -\frac{\partial L_1 \cdot L_2}{\partial c} = 2\left(2-\upsilon\right)\left(2\upsilon^3\beta\gamma + 4\left(a+\beta-c+\theta_g\right) - \upsilon^2\left(a+\beta-c+\theta_1+\theta_2-\theta_g\right) - 2\upsilon\left(\theta_1+\theta_2-2\theta_g\right)\right)$$

Therefore, $\dfrac{\partial \pi_{g,Y}}{\partial a} > \dfrac{\partial \pi_{g,N}}{\partial a}$ when $\theta_1 \approx \theta_2 \approx \theta_g$ because $\dfrac{\partial L_1 \cdot L_2}{\partial a} \approx 2\left(2-\upsilon\right)\left(2\upsilon^3\beta\gamma + \left(4-\upsilon^2\right)\left(a+\beta-c+\theta_g\right)\right) > 0$

Proof of Proposition 5

$$p_{g,i} = \left(q_{g,i} + 2bg\right)q_{g,i} + b\left(\frac{\theta_i + \theta_j - 2\theta_g - 2bg + 2?w_i \quad 2c + e}{2 - u}\right)$$

, where $q_{g,i} = \dfrac{\left(a+\beta\right)\left(2-u\right) + \left(2+u\right)\left(\theta_g - w_i\right) - u\left(\theta_i + \theta_j - 2c + e\right) + 2u\beta g}{2\left(2-u\right)\left(1+u\right)}$. Note that $\dfrac{\partial^2 p_{g,i}}{\partial w_i^2} > 0$; in addition, $\left(q_{g,i} + 2bg\right)q_{g,i}$ is a quadratic function of w_i and $b\left(\dfrac{\theta_i + \theta_j - 2\theta_g - 2bg + 2?w_i \quad 2c + e}{2-u}\right)$ is an increasing function of w_i. Therefore, solving $p_{g,i} = p_{g,Y}$ yields two roots. When the root of w_i^* is the smaller one, $q_{g,i} > 0$ and $\partial p_{g,i}/?w_i \quad 0$; on the other hand, $q_{g,i} < 0$ and $\partial p_{g,i}/?w_i \quad 0$ when the root of w_i^* is the larger one. Therefore, we consider the case in which the interior solution of w_i is the smaller one for $p_{g,i} = p_{g,Y}$; otherwise, only the boundary solution exists. Note that the formulations of $p_{g,1}$ and $p_{g,2}$ are the same other than w_1 and w_2; thus, we aim to find the smallest outsourcing fee, which enables the OS provider to gain more. Solving $p_{g,i} = p_{g,Y}$, we have

$$w_i^* = \frac{\begin{array}{l} 2\left(2+u-2u^2\right)\theta_g + u^3\left(q_i - bg\right) + \left(\left(u^2+u\right)\left(4-3u\right) + \left(4+2u\right)\right)\left(c-e\right) \\ + \left(\left(2-u\right)\left(a+b\right) + u\left(\theta_g - \theta_j + bg + e\right)\right)\left(2+2u-u^2\right) \end{array}}{8+8u-2u^3-3u^2}$$

W.L.O.G., we assume $\theta_2 > \theta_1$. As a result, the following inequalities complete the proof.

$$w_1^* < w_2^* \ ? \quad u^3 q_1 \quad u\theta_2\left(2+2u-u^2\right) < u^3 q_2 - u\theta_1\left(2+2u-u^2\right)$$

$$? \quad u^2 q_1 \quad \theta_2\left(2+2u-u^2\right) < u^2 q_2 - \theta_1\left(2+2u-u^2\right)$$

$$? \quad -\theta_2(2 \quad 2u) < -\theta_1(2+2u)$$

$$\hat{\mathrm{U}} \quad \theta_2 > \theta_1$$