

Association for Information Systems

AIS Electronic Library (AISeL)

ICEB 2016 Proceedings

International Conference on Electronic Business
(ICEB)

Winter 12-4-2016

Optimal Outsourcing Strategy: a Stochastic Optimization Approach

Ming-Tao Chung

National Chengchi University, 102356509@nccu.edu.tw

Yan-Ping Chi

National Chengchi University, ypchi@mis.nccu.edu.tw

Ming-hua Hsieh

National Chengchi University, mhsieh@nccu.edu.tw

Follow this and additional works at: <https://aisel.aisnet.org/iceb2016>

Recommended Citation

Chung, Ming-Tao; Chi, Yan-Ping; and Hsieh, Ming-hua, "Optimal Outsourcing Strategy: a Stochastic Optimization Approach" (2016). *ICEB 2016 Proceedings*. 80.

<https://aisel.aisnet.org/iceb2016/80>

This material is brought to you by the International Conference on Electronic Business (ICEB) at AIS Electronic Library (AISeL). It has been accepted for inclusion in ICEB 2016 Proceedings by an authorized administrator of AIS Electronic Library (AISeL). For more information, please contact elibrary@aisnet.org.

OPTIMAL OUTSOURCING STRATEGY: A STOCHASTIC OPTIMIZATION APPROACH

Ming-Tao Chung, National Chengchi University, 102356509@nccu.edu.tw

Yan-Ping Chi, National Chengchi University, ypchi@mis.nccu.edu.tw

Ming-hua Hsieh, National Chengchi University, mhsieh@nccu.edu.tw

ABSTRACT

As the production capacity of a company over a certain period of time is limited, enterprises must carefully consider product line development or outsourcing options. Unlike traditional studies that use static or comparative static analyses to determine optimal production strategies, this paper proposes a stochastic optimization model that can be used to determine optimum quantities of multiphase development or outsourcing. The proposed model can be used as a decision framework for future production allocation in high-tech industries that face uncertain demands. It can also be used as a financial projection tool.

Keywords: Production decision, outsourcing, stochastic models, stochastic optimization

INTRODUCTION

Ever since the Industrial Revolution, production manufacturing constituted an important subject of management theory and practice, with the earliest investigations focusing on ways to maximize production capacities or minimize costs of in-house production. This initial decision-making model laid the foundations for production decision-making studies that did not seriously consider supply and demand statuses or assume that demands are greater than supplies (namely, by “producer orientation” and not by “demand orientation”). Since the 1970s, however, as international market demands have become increasingly volatile and as the international production system moves towards the division of labor and specialization, production management research continues to expand. Outsourcing (production outsourcing) constitutes a major important shift characteristic of these accelerating trends.

Though outsourcing has been in practical operation for a long time, it only became well known after Prahalad and Hamel (1990) proposed the concept of corporate core competence. The purpose of outsourcing is to allow a company to subcontract non-core, auxiliary functions or operations to external specialized firms through the signing of business contracts to use these firms’ expertise and strengths to improve the overall firm efficiency and competence. Via outsourcing, a company can not only reduce operating costs, focusing resources on the development of core strengths, on the fulfillment of customer demands, and on increasing competitiveness in the market. Rather, a firm can also fully use external resources to compensate for shortcomings in its own capabilities. Meanwhile, outsourcing can also allow a company to maintain management and business flexibility and diversity. The latter is especially instrumental in allowing a company to adapt to today’s ever-unpredictable business demands (Kremic, Tukel, and Rom, 2006; Moon, 2010; Razzaque and Sheng, 1998; Saouma, 2008).

Courtney, Kirkland and Viguerie (1997) identified four levels of environmental uncertainty (“a clear-enough future,” “alternate futures,” “a range of futures,” and “true ambiguity”) and corresponding decision-making techniques. Information that can be obtained at Level One is relatively adequate and comprehensive and uncertainties are at the lowest levels, and thus, the manager can use traditional analytical tools (for example: market research, five forces analysis, value chain analysis) to

determine tactical strategies. At Level Two, it is understood that several possible outcomes may result, but there is no way to identify which condition will occur in the future. Therefore, various value models are designed for each possibility, and the decision analysis framework is used to assess the risks and benefits of different plans. Level Three uses several key variables to determine feasible future ranges, though the feasible range is too wide and tends require more comprehensive definition for tactical strategies to be determined. Aside from analysis tools used for Level Two conditions, there is also a need to plan for each scenario to supplement demand forecasts and analyses. Finally, as Level Four is characterized by mutual effects of numerous sources of uncertainty—creating the most ambiguous and unpredictable environment of all the levels—and presents the highest levels of uncertainty, system simulation methods must be employed to simulate possible decision schemes.

From 2000 onwards, the high-tech industry's global supply chain system has entered Level Four's "true ambiguity" phase. Over the past five years, market demands have become more unpredictable, and product life cycles have shortened rapidly. These circumstances have driven the high-tech industry to cite more rigorous mathematical models to attempt to address the challenges of a highly uncertain environment. The purpose of this research paper is to employ a system stochastic simulation technique to optimally solve the high-tech industry's optimal allocation ratio for in-house production or outsourcing and to further use sensitivity analysis and scenario simulation methods to examine effects of changes in each random variable on enterprise profit maximization.

The remainder of the paper is organized as follows. Section Two presents a review of the literature on production decision-making. Section Three describes stochastic programming specifications of the optimal in-house production and outsourcing method proposed in this study. Conclusions and recommendations are presented at the end of the paper.

LITERATURE REVIEW

Traditional economics classifies the market and further investigates decision-making behaviors produced in different markets. The market is typically separated into two main categories: a complete competitive market and an incomplete competitive market. The latter can be subdivided into an oligopolistic market and monopolistic market. As complete competitive and monopolistic markets belong to the two extremes of the market, fewer subjects can be examined, and thus, only production behaviors found in an oligopolistic market garner special concern.

Oligopolistic markets include a small number of companies, so few that each company's decisions have an effect on the other companies. The three main oligopolistic enterprise competition model include the Cournot, Bertrand, and Stackelberg models. These models typically assume that the market includes two representative companies. Namely, by first simplifying the market as a duopoly to investigate competition and production decisions, models can then be applied to study several companies. The Cournot model assumes that the duopolistic market includes two companies of equal status that produce homogeneous products. Both parties face the same demand curve. When the companies determine their own outputs, they both naively believe that the competitor will not change production quantities and will pursue the goal of profit maximization, though the market price of the product is still determined based on the combined outputs of both companies. The Bertrand model assumes that two companies in a duopolistic market produce homogeneous products and face the same demand curve. The company that enters the market first sets the price according to its production capacities and profit maximization goals. The second company that enters soon after only needs to set a slightly lower price and waits to make a clean sweep of the entire market. As a result, the two companies compete on prices until the profits of both parties are zero. The Stackelberg model assumes that the leading company in a duopolistic market knows that the other company will engage in production according to the Cournot model. As a result, the leading company uses a naïve company output as a given in its own output

decisions and then determines its own output level based on the principle of profit maximization.

The aforementioned traditional production models examine enterprise competition and production decisions primarily from the perspective of static analysis or comparative static analysis. Primary variables considered include demand, price, and cost. These models also use optimization mathematics to derive a perfect closed solution form under conditions of maximum profit. Though these traditional models are elegant and well formulated, perspectives used in these models and factors under consideration appear to extend beyond one's reach due to processes of dynamic evolution. This paper builds on traditional analysis models by employing dynamic analysis perspectives and by considering the evolution of each variable at different times. Namely, variables considered in the model have been afforded additional fitness via stochastic processes or time series models. Variable dynamic behavior paths can then be further generated via the Monte Carlo simulation method. Finally, companies can determine optimal production allocation status levels for a dynamic environment using the stochastic programming method and can then project expected profit levels. The next section provides a detailed description of the stochastic model setup, estimation, and simulation and the optimization model approach.

MODEL

System Setting

It is assumed that a high-tech company implements just-in-time (JIT) manufacturing producing two types of products (Assumes that this company produces only after receiving an order, and therefore only an inventory system that can be ignored exists in the system (Hutchins, 1999)): high-end products and low-end products. The primary investment limits on these two products are the operating hours of the machinery and equipment. To better conform this based on real phenomena, assume that the demands, prices, and costs of both types of products are uncertain. Furthermore, the company's production capacity is limited. The company may be unable to simultaneously meet demands for the two types of products, or the production costs of outsourcing may be cheaper than those of in-house production. Therefore, production decisions include two alternative plans for in-house production or outsourcing. However, due to the presence of trade secret factors, only low-end products are outsourced. Thus, as a rational decision-maker, we expect to be able to arrive at the optimal production allocation combination in the preceding scenarios using initial profit maximization as a guideline for future production over a certain period.

To obtain the optimal production allocation level, we must first define variables and parameters of the system to facilitate subsequent mathematical formula display, simulation, and solution. We assume that there are n high-end products and m low-end products, and variables in the system include seven uncertainty vectors: market demand for the high-end product ($Q_H(t) = [Q_H^1(t), \dots, Q_H^n(t)]$), market demand for the low-end product ($Q_L(t) = [Q_L^1(t), \dots, Q_L^m(t)]$), market prices for the high-end products ($P_H(t) = [P_H^1(t), \dots, P_H^n(t)]$), market prices for the low-end products ($P_L(t) = [P_L^1(t), \dots, P_L^m(t)]$), costs of in-house high-end product production ($V_H(t) = [V_H^1(t), \dots, V_H^n(t)]$), costs of in-house low-end product production ($V_{L1}(t) = [V_{L1}^1(t), \dots, V_{L1}^m(t)]$), and low-end product outsourcing payment costs ($V_{L2}(t) = [V_{L2}^1(t), \dots, V_{L2}^m(t)]$). We assume that the company uses activity-based costing (ABC) and can effectively sum up the cost driver. Therefore, there is no need to differentiate between changing and fixed costs (Cooper and Kaplan, 1991, 1992; Kaplan and Anderson, 2004). Furthermore, system parameters include the known fixed number of hours spent on in-house high-end product productions ($T_H = [T_H^1, \dots, T_H^n]$), hours spent on in-house low-end product productions ($T_{L1} = [T_{L1}^1, \dots, T_{L1}^m]$), and the upper limit of company employee, machinery and equipment operating hours (T_E). Finally, decision variables include the company's in-house high-end products production quantities ($Q_H^S(t) = [Q_H^{S1}(t), \dots, Q_H^{Sn}(t)]$), the company's in-house low-end product production quantities

($Q_{L1}^S = [Q_{L1}^{S1}(t), \dots, Q_{L1}^{Sm}(t)]$), and the company's outsourced low-end product production quantities ($Q_{L2}^S = [Q_{L2}^{S1}(t), \dots, Q_{L2}^{Sm}(t)]$).

The equation for the profit $\pi_H(t)$ at time t of in-house high-end product production is:

$$\pi_H(t) = [P_H(t) - V_H(t)] Q_H^S(t)^T \quad (1)$$

The equation for the profit $\pi_{L1}(t)$ at time t when the company decides to engage in in-house low-end product production is:

$$\pi_{L1}(t) = [P_L(t) - V_{L1}(t)] Q_{L1}^S(t)^T \quad (2)$$

The equation for the profit $\pi_{L2}(t)$ at time t when the company decides to engage in outsourced low-end product production is:

$$\pi_{L2}(t) = [P_L(t) - V_{L2}(t)] Q_{L2}^S(t)^T \quad (3)$$

The equation for the overall profit can be determined by adding the three aforementioned equations:

$$\pi(t) = \pi_H(t) + \pi_{L1}(t) + \pi_{L2}(t) \quad (4)$$

With different types of the outsourcing partner, the company may have different constraints. Here we consider three types of outsourcing partners. The first type is most flexible; it can accommodate all of the outsourcing request dynamically. Below is the corresponding formulation for the expected profit with three constraints (production resource boundary equation, high-end product demand boundary equation, low-end product demand boundary equation)

$$\max_{\{Q_H^S(t), Q_{L1}^S(t), Q_{L2}^S(t)\}} E_0 \left[\sum_{t=1}^T \pi(t) \right]$$

Subject to:

$$\begin{aligned} Q_H^S(t) T_H^T + Q_{L1}^S(t) T_{L1}^T &\leq T_E \\ Q_H^S(t) &\leq Q_H(t) \\ Q_{L1}^S(t) + Q_{L2}^S(t) &\leq Q_L(t) \\ Q_H^S(t), Q_{L1}^S(t), Q_{L2}^S(t) &\geq 0 \end{aligned} \quad (5)$$

The second type is that the outsourcing partner shares 100α percent of the market demands of the low-end products for the planning periods. Where $\alpha = (\alpha_1, \dots, \alpha_m)$. The following is the corresponding formulation, which is similar with above, but change the decision variables:

$$\max_{\{Q_H^S(t), \alpha\}} E_0 \left[\sum_{t=1}^T \pi(t) \right]$$

Subject to:

$$\begin{aligned} \alpha Q_L(t) &= Q_{L2}^S(t), && [\text{constant percent of outsourcing}] \\ Q_H^S(t) T_H^T + Q_{L1}^S(t) T_{L1}^T &\leq T_E \\ Q_H^S(t) &\leq Q_H(t) \\ Q_{L1}^S(t) + Q_{L2}^S(t) &\leq Q_L(t) \\ Q_H^S(t), Q_{L1}^S(t) &\geq 0 \\ 0 &\leq \alpha \leq 1 \end{aligned} \tag{6}$$

Finally, we consider that outsourcing partner has limited production capacity. They only can produce constant amount β of the low-end products. Where $\beta = (\beta_1, \dots, \beta_m)$. The third type of the equation may be the following:

$$\max_{\{Q_H^S(t), \beta\}} E_0 \left[\sum_{t=1}^T \pi(t) \right]$$

Subject to:

$$\begin{aligned} Q_{L2}^S(t) &= \beta, \quad \text{for all } t = 1, \dots, T && [\text{constant amount of outsourcing}] \\ Q_H^S(t) T_H^T + Q_{L1}^S(t) T_{L1}^T &\leq T_E \\ Q_H^S(t) &\leq Q_H(t) \\ Q_{L1}^S(t) + \beta &\leq Q_L(t) \\ Q_H^S(t), Q_{L1}^S(t) &\geq 0 \end{aligned} \tag{7}$$

The first type is more flexible so that the company can make dynamic decision in different time period. However, the others' outsourcing strategies are determined at the beginning of production. In section 4, we will compare the numerical results of these 3 formulations.

STOCHASTIC MODELS

Stochastic Processes

To portray (adapt) the dynamic behaviors of the N stochastic variables in the system, where $N = 3n + 4m$, we assume that $Y(t) = (Q_H(t), Q_L(t), P_H(t), P_L(t), V_H(t), V_{L1}(t), V_{L2}(t))^T$ obey a multivariate diffusion process:

$$\frac{dY(t)}{Y(t)} = \mu dt + \sigma dZ \tag{8}$$

where parameters $\mu = (\mu_1, \dots, \mu_N)^T$ and $\sigma = (\sigma_1, \dots, \sigma_N)^T$ are the corresponding stochastic variables average and fluctuation parameter, respectively. dZ is a multivariate correlated Brownian motion with a correlation matrix ρ , where

$$\rho = \begin{bmatrix} 1 & \rho_{12} & \dots & \dots & \rho_{1N} \\ \rho_{21} & 1 & \ddots & \ddots & \vdots \\ \rho_{31} & \rho_{32} & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \rho_{N1} & \dots & \dots & \dots & 1 \end{bmatrix}$$

Then, the covariance matrix of dZ for any $t > 0$ is

$$\begin{bmatrix} \sigma_1 & 0 & \dots & 0 \\ 0 & \sigma_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & \sigma_N \end{bmatrix} \begin{bmatrix} 1 & \rho_{12} & \dots & \rho_{1N} \\ \rho_{21} & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ \rho_{N1} & \dots & \dots & 1 \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 & \dots & 0 \\ 0 & \sigma_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & \sigma_N \end{bmatrix} dt \equiv \Sigma dt.$$

we define X_i as the difference between log of Y_i at time $t + \Delta t$ and t :

$$X_i = \log(Y_i(t + \Delta t)) - \log(Y_i(t)) \tag{9}$$

Assume that $X = (X_1, \dots, X_N)^T$, By Ito lemma and Euler scheme, we may find out that

$$X \sim N(a\Delta t, \Sigma\Delta t) \tag{10}$$

And

$$Y_i(t + \Delta t) = Y(t)e^{X_i}, \text{ where } i = 1, \dots, N \tag{11}$$

where $a(t) = [(\mu_1(t) - \sigma_1^2(t)/2), \dots, (\mu_N(t) - \sigma_N^2(t)/2)]^T$. These can be estimated through a maximum likelihood estimation (MLE) of historical data.

NUMERICAL EXAMPLE

Summary of the Data

In the numerical examples, all model parameters are calibrated to XYZ company’s historical data. XYZ is a listed high-tech company in Taiwan stock exchange. We consider six products: three high-end products and three low-end products. The dataset contains monthly data for 3 and half years from January 2013 to June 2016. In table 1, basic statistics of high-end products are shown (selling volumes, unit costs of products and outsourcing, unit prices of products, and gross margins). Table 2 shows basic statistics of the low-end products. The planning period is usually three to six months. Therefore, only the most recent data are used to calibrate model parameter.

Table 1: Basic statistics of high-end products

| HIGH-END | 2016 JAN | 2016 FED | 2016 MAR | 2016 APR | 2016 MAY | 2016 JUN | mean |
|------------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| Product A | | | | | | | |
| selling volumes | 497212456 | 471055544 | 495395000 | 620313000 | 364258000 | 486444000 | 312750238 |
| unit prices | 0.03434 | 0.03352 | 0.03659 | 0.03004 | 0.04825 | 0.03735 | 0.07402 |
| unit costs | 0.02652 | 0.02559 | 0.02798 | 0.02373 | 0.03571 | 0.02597 | 0.06170 |
| gross margin | 22.79% | 23.65% | 23.53% | 21.00% | 25.99% | 30.48% | 17.50% |
| Product B | | | | | | | |
| selling volumes | 5276093 | 5385626 | 7600268 | 6155204 | 7456890 | 6146433 | 5946211 |
| unit prices | 1.03006 | 0.73952 | 0.72842 | 0.86031 | 0.78104 | 0.73905 | 0.94160 |
| unit costs | 0.95548 | 0.69017 | 0.63963 | 0.78175 | 0.69383 | 0.61868 | 0.80909 |
| gross margin | 7.24% | 6.67% | 12.19% | 9.13% | 11.17% | 16.29% | 13.89% |
| Product C | | | | | | | |
| selling volumes | 3930000 | 3156000 | 4176000 | 3146000 | 4805500 | 3888000 | 4818109 |
| unit prices | 0.26841 | 0.26301 | 0.23723 | 0.24257 | 0.22261 | 0.21209 | 0.23945 |
| unit costs | 0.18091 | 0.19818 | 0.17038 | 0.17146 | 0.16185 | 0.16210 | 0.18401 |
| gross margin | 32.60% | 24.65% | 28.18% | 29.32% | 27.30% | 23.57% | 23.07% |

Table 2: Basic statistics of the low-end products

| LOW-END | 2016 JAN | 2016 FED | 2016 MAR | 2016 APR | 2016 MAY | 2016 JUN | mean |
|-------------------|----------|----------|----------|----------|----------|----------|----------|
| Product D | | | | | | | |
| selling volumes | 5416052 | 6202618 | 7826216 | 6372328 | 7088606 | 7317137 | 6946402 |
| unit prices | 8.02337 | 7.97404 | 7.68956 | 8.20980 | 8.39786 | 7.82231 | 7.144754 |
| unit costs | 7.13934 | 6.99579 | 6.76713 | 7.16182 | 7.33053 | 6.77423 | 6.464520 |
| outsourcing costs | 7.49318 | 6.73784 | 5.91800 | 8.26479 | 8.72830 | 7.69675 | 6.651563 |
| gross margin | 11.02% | 12.27% | 12.00% | 12.76% | 12.71% | 13.40% | 9.45% |
| Product E | | | | | | | |
| selling volumes | 36748470 | 33385188 | 53915405 | 44602348 | 47309328 | 56189450 | 34239668 |
| unit prices | 0.15031 | 0.15463 | 0.15248 | 0.14292 | 0.14759 | 0.13732 | 0.162635 |
| unit costs | 0.13741 | 0.14119 | 0.13975 | 0.13150 | 0.13375 | 0.12708 | 0.147037 |
| outsourcing costs | 0.14761 | 0.11099 | 0.13554 | 0.11299 | 0.15670 | 0.12679 | 0.151876 |
| gross margin | 8.58% | 8.69% | 8.35% | 7.99% | 9.38% | 7.46% | 9.47% |
| Product F | | | | | | | |
| selling volumes | 1385000 | 3023150 | 3370320 | 1659858 | 2496750 | 3716600 | 3740527 |
| unit prices | 1.94627 | 1.85428 | 1.80117 | 1.78164 | 1.85138 | 1.54654 | 2.59394 |
| unit costs | 1.83381 | 1.70948 | 1.69750 | 1.68682 | 1.75340 | 1.47279 | 2.46617 |
| outsourcing costs | 1.64386 | 1.58941 | 1.75173 | 1.76445 | 1.45952 | 1.72184 | 2.49815 |
| gross margin | 5.78% | 7.81% | 5.76% | 5.32% | 5.29% | 4.77% | 5.12% |

CONCLUSIONS AND RECOMMENDATIONS

This paper introduced stochastic process models and a time series model for measuring production decision-making variables and proposed a stochastic programming model that can determine optimal multiphase in-house production or outsourcing quantities. Decision-makers can establish parameters of system modeling by estimating historical data or by making subjective judgments of trends. Then, to go one step further, this approach combined with Monte Carlo simulation can generate multiple dynamic routes that can measure the variables. In the end, by substituting these routes into the stochastic optimization model proposed in this paper, optimal quantities of in-house production or outsourcing can be obtained.

As the high-tech industry has faced extremely uncertain demands since the end of the 20th century, traditional static and comparative static production decision models can no longer effectively serve as a basis for production decisions on allocation. Therefore, the model proposed in this paper can be used to improve the quality of high-tech company decision-making by allowing for the determination of production allocation levels and can serve as the basis for future financial forecasts.

Extensions of this research may add matrix decomposition mathematical principles (e.g., Cholesky decomposition or singular value decomposition (SVD)) to the methods described here and may further transform the stochastic variables that decisions must address into linked stochastic variables to better reflect conditions of practical decision making.

REFERENCES

- [1]. Box, G. E. P. and G.M. Jenkins, 1976. Time series analysis: forecasting and control, Holden-Day.
- [2]. Cooper, R. and R.S., Kaplan, 1991. Profit priorities from activity-based costing. *Harvard Business Review*, 69(3): 130-135.
- [3]. Cooper, R. and R.S., Kaplan, 1992. Activity-based systems: measuring the costs of resource usage. *Accounting Horizons*, 6(3): 1-13.
- [4]. Courtney, H., J. Kirkland and P. Viguerie, 1997. Strategy under uncertainty. *Harvard Business Review*, 75(6): 67-79.
- [5]. Hutchins, D., 1999. Just in time, Gower.
- [6]. Kaplan, R.S. and S.R., Anderson, 2004. Time-driven activity-based costing. *Harvard Business Review*, 82(11): 131-140.
- [7]. Kou, S.G., 2002. A jump-diffusion model for option pricing. *Management Science*, 48(8): 1086-1101.
- [8]. Kremic, T., O.I., Tukul, and W.O. Rom, 2006. Outsourcing decision support: a survey of benefits, risks, and decision factors. *Supply Chain Management: an international journal*, 11(6): 467-482.
- [9]. Moon, Y., 2010. Efforts and efficiency in partial outsourcing and investment timing strategy under market uncertainty. *Computers & Industrial Engineering*, 59(1): 24-33.
- [10]. Nelson, D.B., 1991. Conditional heteroskedasticity in asset returns: A new approach. *Econometrica: Journal of the Econometric Society*, 59(2): 347-370.
- [11]. Prahalad, C. K. and G. Hamel, 1990. The core competence of the corporation. *Harvard Business Review*, 68(3): 79-91.
- [12]. Razzaque, M.A., and C.C. Sheng, 1998. Outsourcing of logistics functions: a literature survey. *International Journal of Physical Distribution & Logistics Management*, 28(2): 89-107.
- [13]. Redner, R.A. and H.F. Walker, 1984. Mixture densities, maximum likelihood and the EM algorithm. *SIAM review*, 26(2): 195-239.
- [14]. Saouma, R., 2008. Optimal second-stage outsourcing. *Management Science*, 54(6): 1147-1159.