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Pricing or Advertising? A Game-Theoretic Analysis of Online Retailing

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Abstract

How should online retailers attract customers? Should they advertise intensively to attract online traffic, or should they simply price lower than their competitors? To answer these questions, we develop a game-theoretic model of two firms choosing advertising levels and prices strategically. We find that only asymmetric equilibria exist, where e-tailers choose different strategies along both advertising and pricing dimensions. When market mobility is low (i.e., the majority of buyers have high search costs), firms engage in fierce competition in advertising, and the firm with a higher advertising level charges a higher price and earns higher profits. When market mobility is high (i.e., the majority of buyers have zero search costs) or medium, one firm may choose to advertise intensely while the other may choose to charge a lower price and not advertise at all. In such cases, either firm may make higher profits. We also compare the market outcome in our model to the case in which firms do not have the option of advertising and find that the option to advertise leads to higher expected prices. We further extend the model to consider e-tailers choosing advertising levels sequentially.

Keywords: Advertising, Electronic Commerce, Online Retailing, Price Dispersion, Search Costs

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1 Introduction

How should online retailers (e-tailers) compete for customers? Should they advertise intensively to direct online traffic to their websites, hoping clicks will lead to purchases? Or should they simply price lower than their competitors, counting on consumers insatiably seeking better deals to locate them? We summarize these two approaches as "competing for attention" and "competing in price," respectively. Prior research has generally studied competition along these two dimensions separately. In this paper, we consider firms' optimal decisions concerning both advertising expenditure and product pricing and further characterize the market outcome when firms compete in terms of both advertising and price.

These two dimensions of e-commerce: pricing and advertising, are rooted deeply in the literature and reflect key insights about how the Internet changes business practices. Early literature on e-commerce posits that the use of the Internet will make price and product information increasingly available and transparent to consumers, leading to fierce price competition among sellers and the "law of one price" ruling the online marketplace (see, among others, Bakos, 1997). The emergence of price comparison sites makes information in an already competitive online market even more accessible. This has motivated numerous empirical studies to attempt to prove whether the law of one price holds (see, e.g., Ghose & Yao, 2011) with many studies finding that price dispersion does exist (see, among others, Brynjolfsson & Smith, 2000; Clemons, Hann, & Hitt, 2002; Baye, Morgan, & Scholten, 2004; Li, Gu, & Liu, 2013), which has, in turn, spawned further research devoted to finding possible explanations (e.g., Pan, Ratchford, & Shankar, 2002; Venkatesan, Mehta, & Bapna, 2007; Ba, Stallaert, & Zhang, 2012).

Another equally influential notion is that attention has become a scarce commodity on the Internet and can even be regarded as "the hard currency of cyberspace" (der Leun & Mandel, 1996; Goldhaber, 1997). Herbert A. Simon predicts that in an information-rich world, "a wealth of information creates a poverty of attention" (Simon, 1971). Online advertising has become one of the fastest growing industry segments enabled by the

Internet, and e-tailers make substantial marketing expenditures to attract buyers (Hoffman & Novak, 2000). Internet advertising revenues in the United States reached $72.5 billion in 2016, increasing 21.8% over 2015 (PwC, 2017). Online retailers are advertising intensively to attract buyers' attention—hoping to convert attention into website traffic and, eventually, purchases (Falkinger, 2003); Huberman & Wu, 2007). According to a survey of 221 retailers conducted by Shop.org, customer acquisition costs are more than twice as high for pure-play e-tailers as for brick-and-mortar retailers. Taobao.com, the world's largest online platform for e-tailers, does not charge sellers for commission; instead, it adopts the business model of charging advertising fees to sellers, some of which report spending over a third of their revenues on buying customer traffic (Chen, Fan, & Li, 2016).

We believe that both advertising and pricing are key issues faced by e-tailers. E-tailers must decide how much they should spend on advertising and what price they should charge, given their competitors' behavior. We study how firms make these decisions as well as how these two decisions affect each other. We are also interested in the overall market outcomes. If all e-tailers advertise to attract buyers, will e-tailers find themselves in a classic prisoner's dilemma in equilibrium? How do such decisions affect price levels in the market and the e-tailers' profitability? This study attempts to answer these questions.

We develop a game-theoretical model in which two e-tailers compete for buyers using two mechanisms: advertising and price. One key feature of our model is that consumers are heterogeneous when searching for products online. Following the convention of the literature, consumers who have zero search cost are called "shoppers" while those with positive search costs are called "high-search-cost buyers" (e.g., Xu, Chen, & Whinston, 2011). Intuitively, a low-price strategy may be effective for shoppers who tend to search for deals on the Internet, while advertising is essential for attracting high-search-cost buyers. The relative proportion of these two types of consumers captures the overall market mobility: the larger the shopper segment is, the more mobile the market is. The e-tailers' optimal decisions on prices and advertising expenditures depend on market mobility.

Our model provides a framework for understanding the interaction of attention competition, pricing, and consumer search behavior. Our research yields some interesting results. First, we find that only asymmetric equilibria exist, which means that e-tailers choose different strategies along both advertising and pricing dimensions. When the market mainly consists of high-search-cost buyers (low market mobility), firms engage in fierce competition in advertising, and the firm that advertises at a higher level also charges higher prices, yielding higher profits. In other words, firms are in a situation akin to the prisoner's dilemma in the advertising game (though this is an asymmetric equilibrium).

However, when the market mobility is high or medium, differentiated strategies are more effective: one firm may choose to advertise to attract high-search-cost buyers, while another may choose not to advertise at all—instead attracting buyers simply by charging a lower average price. Furthermore, in such cases, no strategy is superior and either firm may make higher profits depending on the parameters. Interestingly, firms are most differentiated in their advertising levels when market mobility is at a medium level.

Following a general discussion of our model, we then compare the market outcomes in our model with the scenario in which firms do not have the option to advertise. The option to advertise leads to higher expected prices for any given market composition (i.e., mobility level), which implies that the existence of advertising is worse for shoppers. Interestingly, when market mobility is high, both types of e-tailers make higher profits when the option to advertise exists—even the firm that advertises intensively and bears the extra cost. This is because the option to advertise allows firms to differentiate along two dimensions, which attenuates the price competition.

We also discuss the effects of other market characteristics. A novel finding is that even when advertising is less expensive, firms do not necessarily lower their prices. When market mobility is high, firms adopt differentiated strategies, and thus when advertising costs less, the firm that focuses on attracting high-search-cost buyers can intensify advertising, thus ameliorating the price competition (i.e., by charging higher prices). We also find that search cost may influence the market outcome only under certain circumstances—namely, when buyers have a high willingness to pay for a product and the market is reasonably mobile. Only under these conditions are firms' optimal strategies constrained by buyers' search cost. This is because a firm considers buyers' search behavior in relation to their willingness to pay as well as in relation to the overall market composition.

We further extend the model to consider e-tailers that choose advertising levels sequentially. We find that the follower's decision variable is a strategic complement to that of the leader: the leader sets its advertising intensity at a lower level, compared with scenarios in which firms set advertising levels simultaneously, which results in higher profits for both firms. In essence, moving sequentially provides a cooperative mechanism for firms to collectively reduce advertising levels. Buyers in this scenario benefit as well because market prices are also lower.

Our research contributes to the literature on consumer search and online advertising. First, unlike previous research, in our model, consumers' search is neither uniformly random nor does it occur in a predetermined order (see the next section for detailed discussions of the related literature). Instead, we allow consumers' search order to vary from one individual to another, with each individual following a pattern. Second, in this paper, advertising changes the likelihood of a firm being sampled first, which is a form of saliency- enhancing advertising. There has been limited research on saliency-enhancing advertising, as compared to persuasive advertising, which addresses consumers' willingness to pay, or informative advertising, which contains information about a product (see Bagwell, 2007). One exception is Haan and Moraga-González' (2011) study, which examines firms' decisions about advertising as a means of enhancing prominence. We depart from Haan and Moraga-González in that we find that advertising can be used as a differentiation strategy, whereas in Haan and Moraga-González's study, firms choose the same advertising level in equilibrium. Lastly, we directly address the issue concerning the effectiveness of a low-price strategy vs. an advertising strategy for e-tailers facing heterogeneous consumers.

The rest of the paper is organized as follows: Section 2 explains how our research relates and contributes to the literature, Section 3 sets up the model, Section 4 presents the analysis of the model, Section 5 discusses the effects of key features of the model on the results, Section 6 extends the model to sequential decisions, and Section 7 summarizes and concludes. Most proofs are presented in the Appendix.

2 Relevant Literature

Our research draws on several strands of literature. Researchers have tried to explain the persistence of price dispersion from either the consumer side or the seller side. On the seller side, researchers have discovered that firms may compete along several dimensions (e.g., service quality), which can lead to price dispersion even for identical products (Pan et al., 2002; Ba et al, 2012). On the consumer side, researchers argue that the assumption underlying the "law of one price"—that information is fully and freely accessible to consumers—does not necessarily hold true in online markets. Consumers' information cost, or search cost, is often modeled in one of the following ways. First, consumers can be heterogeneous in their search costs, with some having significantly higher search costs than others (Childers, Carr, Peck, & Carson, 2001; Hong & Shum, 2006; Johnson, Moe, Fader, Bellman, & Lohse, 2004). Second, information is costly when consumers have to search and visit stores sequentially (Diamond, 1971). In both settings, price dispersion occurs, which allows firms to compete in terms of price. In this paper, we adopt the conventional assumption that consumers vary in their search costs, but we depart from the literature by allowing consumers to differ in their order of search. Our results also provide an alternative explanation for price dispersion.

We present a model in which the order of search is neither predetermined nor uniformly random. Classic search models assume that consumer search is uniformly random, which suggests that each seller is equally likely to be sampled (Reinganum, 1979; Salop, 1977; Stahl, 1989; Varian, 1980). In recent years, a small but rapidly growing literature on ordered search has emerged. Perry and Wigderson (1986) consider consumers who sample a finite number of suppliers in a known, predetermined order with uncertain costs and show that the observed prices could be non-monotonic in the search order. Arbatskaya (2007) shows a pure-strategy price equilibrium in a predetermined ordered search model. In the information systems and Internet marketing literature, much research has been conducted on webpage visibility, web location competition, and search patterns. Lohse (1997) and Hoque and Lohse (1999) use experiments to explore factors that affect visibility or the prominence of web locations. Weber and Zheng (2007) study the design of search intermediaries and firms' bidding strategies given consumers' search behavior. Xu et al. (2011) demonstrate pricing patterns in an ordered search framework. Xu, Chen, and Whinston (2012), He and Chen (2006) and Athey and Ellison (2011) study the bidding behavior in monopolistic search advertising market. Common to these studies is the assumption that all users follow the same search order, often prescribed by an advertising intermediary. In reality, e-tailers usually promote across a large number of channels, and any of these channels can be the starting point of a consumer's search. In sum, we differ from both the literature with ordered search and the classic search literature with uniformly random search: in our model, consumers may vary in terms of the first e-tailer they sample, but each individual follows a specific pattern of search depending on his or her own particular type.

Our research is related to the attention competition literature (e.g., Falkinger, 2008; Armstrong, Vickers, & Zhou, 2009; Haan & Moraga-González, 2011). In these studies, the efficacy of ads is decided by signal/advertising strength, which endogenously generates demand functions or search probability. In our model, advertising has similar effects: the more intensely an e-tailer advertises, the more likely the e-tailer is to be visited first by consumers. When buyers view both e-tailers' ads on the same advertising channel, our setting implies that each e-tailer will have a 50% chance being visited first, exhibiting the crowding-out effect. However, our research departs from this literature in several ways. Falkinger (2008) describes economies facing attention scarcity and does not discuss firms' strategies. In Armstrong et al. (2009), the prominence of a firm is exogenously given rather than given by choice. Our model is more closely related to Haan and Moraga-González' (2011) study, but in Haan and Moraga-González' (2011) study, firms are differentiated in their products ex ante but adopt the same advertising strategy ex post; in contrast, in our model, the firms are identical ex ante but choose asymmetric advertising strategies ex post.

Our work is loosely related to the study of Internet advertising intermediaries. This line of research investigates how intermediaries allocate user traffic and price prominent locations (Baye & Morgan, 2001; Chen, Iyer, & Padmanabhan, 2002; Hagiu & Jullien, 2011; Iyer & Pazgal, 2003; Weber & Zheng, 2007; Zettelmeyer, 2000). They find that by allocating buyers only to chosen firms, the monopolistic information intermediary relaxes price competition between firms and extracts surplus (Baye & Morgan, 2001; Chen et al., 2002; Hagiu &Jullien, 2011; Iyer &Pazgal, 2003). In our model, there is no monopolistic buyer allocator and firms endogenously set advertising levels in an attempt to reach consumers. Note that in our model, advertising is assumed to be effective, which implies that the intermediaries, if any, are transparent. Modeling the detailed mechanisms of intermediaries may help firms decide "how to advertise," whereas our research focuses on firms' decision of "to advertise or not to advertise."

3 Model

Consider two e-tailers selling the same product online and facing no capacity constraints. The marginal cost of the product is assumed to be constant and the same for the firms, thus normalized to zero. The market consists of a unit mass of buyers. Each buyer desires, at most, one unit of the product and has the same reservation price r.

We develop a multistage game and the sequence of events is as follows. The number of firms, the production and advertising costs, and the structure of the game are assumed to be common knowledge. Our model uses the classic rational expectation equilibrium concept (Harsanyi, 1967; Salop & Stiglitz, 1982).

First, the two e-tailers simultaneously decide on their advertising levels, with firm i choosing β_i, where β_i represents the percentage of the market that the ads of firm i can reach.

Second, after observing the other's advertising level, the two e-tailers simultaneously decide on their pricing strategy: Firm i chooses a price randomly from $F_i(p)$, where $F_i(\cdot)$ is a cumulative distribution function. This order of events is based on the observation that, in practice, e-tailers usually decide on their advertising budget at a regular interval (e.g., set annually and reviewed/adjusted quarterly) while pricing is adjusted more frequently.

Third, buyers search for a satisfactory deal. We assume that in equilibrium buyers hold correct conjectures about advertising levels and pricing strategies, but buyers do not know the firms' identities—i.e., which firm chooses which advertising level and price. We assume recall is costless. After visiting the first firm, buyers update their beliefs about the identities of the firms and decide whether to continue the search. When a buyer stops searching, he or she makes a purchase if the price is below his or her reservation price.

We first explain firms' advertising decisions. E-tailers may advertise to attract consumers' attention and then direct consumers to their online storefronts. The e-tailers can be interpreted as multiproduct sellers each supplying a large assortment of products in a given category. We assume that ads do not contain price information; instead, the ads make potential buyers aware of the e-tailer and generate traffic to its online store. For example, a camping store may advertise itself as the largest camping equipment supplier online and attract any customer interested in a tent or a battery lantern.

We characterize the advertising function as follows. First, each e-tailer's advertising scale is assumed to be minuscule compared to the overall advertising market, so the e-tailers are ad-price takers, and the advertising cost function can be assumed to be exogenous (readers interested in strategic ads pricing may refer to Chen & He, 2011; Varian, 2007; Edelman et al., 2007; and Athey & Ellison, 2011 for discussions of auction mechanisms used in the pricing of search engine ads). Second, given the competitive advertising market, it is more costly to reach additional buyers, thus the cost of reaching β percentage of buyers should be convexly increasing (Grossman & Shapiro, 1984). Formally, we assume the advertising cost function $A(\beta)$ to be twice continuously differentiable, strictly increasing, and convex. Moreover, we assume that the cost of reaching all buyers, $A(\beta = 1)$, is prohibitively high, so the optimal advertising level is less than 1. No advertising is costless, so $A(0) = 0$; and the marginal cost of starting to advertise is negligible, so $A'(0) = 0$.

Next, we model how buyer traffic is swayed by e-tailers' advertising efforts.[1] The e-tailers are assumed to independently choose their advertising levels, β_i, and the ads are randomly served in media. When a buyer goes online, she has a probability of β_i to be reached by e-tailer i. Therefore, buyers can be divided into four types: (1) a fraction $\beta_1\beta_2$ is reached by both

[1] An e-tailer choosing a higher advertising level is likely to intensify online advertising through any media. Therefore, when an e-tailer advertises more, it can reach more consumers, often in various ways.

firms' advertising; (2) a fraction $\beta_1(1-\beta_2)$ is reached by Firm 1 but not Firm 2; (3) a fraction $\beta_2(1-\beta_1)$ is reached by Firm 2 but not Firm 1; (4) a fraction $(1-\beta_1)(1-\beta_2)$ is reached by neither firm. Naturally type 2 buyers visit Firm 1 first, and type 3 buyers visit Firm 2 first. Type 1 buyers choose to visit one of the firms first with equal likelihood since the two firms appear the same.[2] Type 4 buyers continue to surf the web until they see the link to one of the firms (through third party links or organic search), again with an equal chance for both firms, regarding which the buyers will visit first. We believe the above search pattern captures how typical web users process e-tailer advertising information. This search pattern implies that the number of buyers that first visit firm i is $\frac{\beta_i\beta_j}{2}+\beta_i(1-\beta_j)+\frac{(1-\beta_i)(1-\beta_j)}{2}=\frac{1+\beta_i-\beta_j}{2}$. By increasing its advertising level, a firm can increase traffic to its store and reduce traffic to its competitor's store, which highlights the crowding-out effect of traffic competition. In particular, if two firms choose the same advertising level, each firm will attract half of the buyers and the attention-attracting effect will be completely canceled out.

We denote the number of buyers that first visit firm i by $\alpha_i=\frac{1+\beta_i-\beta_j}{2}$. Note that $\alpha_i+\alpha_j=1$. When $\alpha_i>0.5$, it means that firm i is advertising more aggressively than its competitor and, as a result, attracts more initial traffic to its site. To further simplify notation, we use α to denote the traffic to the more advertised firm, that is $\alpha=\alpha_i=\frac{1+\beta_i-\beta_j}{2}$ if $\beta_i\geq\beta_j$, and $\alpha=\alpha_j=\frac{1+\beta_j-\beta_i}{2}$ if $\beta_i<\beta_j$. Therefore $0.5\leq\alpha\leq1$. α measures the degree that buyer traffic is skewed to the more advertised firm: $\alpha=0.5$ means the two firms choose the same level of advertising and initial traffic is evenly distributed between the two, while $\alpha=1$ means one firm's advertising covers the entire market while the other does not advertise at all and all initial traffic is diverted to the advertised firm.

Buyers' search for an e-tailer is modeled as follows. Assume buyers do not know which firm chooses which advertising level.[3] Buyers' search behavior involves visiting an e-tailer's website and finding out the product and price information. We normalize all buyers' search cost for the first e-tailer to zero, which ensures that all buyers must visit at least one e-tailer. This technical assumption is commonly used in the literature (e.g. Salop & Stiglitz, 1982; Stiglitz, 1987; Stahl, 1989; Kuksov, 2004; Jerath, Ma, Park, & Srinivasan, 2011; Honka & Chintagunta, 2017; Zhang, Chan, & Xie, 2017). Buyers differ in their search costs for the second e-tailer.[4] Assume a proportion γ of buyers has positive search cost c (high-search-cost buyers) and the rest $(1-\gamma)$ are shoppers with zero search cost. The shoppers are akin to the informed buyers in Varian (1980) or the switchers in Narasimhan (1988). The shopper segment captures the notion that some consumers enjoy shopping online or have a very low opportunity cost of doing so. This assumption is widely adopted in the literature (e.g. Stahl, 1989). The parameter γ thus captures the overall mobility of the market: $\gamma=0$ means the market is totally mobile where all consumers search at no cost; $\gamma=1$ means the market is completely immobile where all consumers have positive search costs (Diamond, 1971). Further, among the α_i buyers that first visit firm i, $\alpha_i\gamma$ are high- search-cost buyers and $\alpha_i(1-\gamma)$ are shoppers. Table 1 summarizes the notation, some to be defined later.

4 Analysis

The model is solved using backward induction. First, we examine buyers' behavior, then we discuss the firms' pricing strategies, and, finally, we identify the firms' optimal advertising decisions.

4.1 Buyer Behavior

Buyers search for the product they desire after e-tailers set their advertising levels and prices. Recall that e-tailer i randomly sets a price from $F_i(p)$. Suppose the support of $F_i(p)$ is P_i, with upper bound u_i and lower bound l_i. $F_i(p)$ degenerates into a single mass in the case of a pure strategy. $f_i(p)$ is the probability density function associated with $F_i(p)$. Obviously, $u_i\leq r$, since no firm charges a price that nobody accepts. All buyers are assumed to be risk-neutral and have the same belief regarding β_i and $F_i(\cdot)$; however, they do not know the association between a firm and its strategy before they observe any price.

[2] When consumers are reached by both firms, it can mean that the two firms' ads appear in the same media at the same time or that the two firms are shown in the same search results page. In these cases, we assume that the order of the firms' ads/places is random and thus that the firms have equal chances of being visited by consumers first.

[3] One could argue that buyers could go to many other websites to check the ads listed elsewhere and infer advertising levels. Yet, we believe that since there are many other ads and sources of information and the same e-tailer may design ads differently for different websites, the high mental cost of processing this large amount of information is not commensurate with the benefits associated with knowing the firms' identities.

[4] The search cost we define here can also be understood as marginal search cost; for simplicity, we refer to it as search cost.

Table 1. Notation

β_i	advertising intensity of firm i
β_i^*	the equilibrium advertising intensity of firm i
$A(\beta)$	the advertising cost of intensity β
α_i	the number of buyers that first visit firm i
α	the number of buyers that first visit the more advertised firm
γ	the proportion of high-search-cost buyers
r	the reservation price of buyers
c	the search cost of high-search-cost buyers
$F_i(p)$	the cumulative distribution function of firm i's pricing strategy
$f_i(p)$	the probability density function of firm i's pricing strategy
P_i	the support of firm i's pricing strategy
l_i	the lower bound of P_i
l	the common lower bound of both firms' pricing strategy
u_i	the upper bound of P_i
u	the common upper bound of both firms' pricing strategy
u^*	the upper bound of price due to buyers' search behavior
m	the probability mass at u
π_i	the revenue of firm i
π_i^n	the net profit of firm i
$b_i(\beta_j)$	firm i's best advertising intensity for given intensity β_j
θ	advertising cost parameter
β_{is}^*	the equilibrium advertising level in the sequential game
β_{ic}^*	the equilibrium advertising level in the simultaneous game

We know that both shoppers and high-search-cost buyers get the price information at the first e-tailer they visit at zero cost. Both types maximize their expected payoff by comparing their benefits and the costs of searching for the second e-tailer.

Suppose a buyer first visits firm i and observes price z. She updates her belief about from which firm she observes z according to Bayes' rule,

$$Prob(i = k) = \frac{f_k(z)}{f_1(z)+f_2(z)}, k = 1,2. \qquad (1)$$

Note that if a price is only charged by one firm, a buyer who has observed that price is able to infer the firm's strategy with certainty.

The buyer's expected benefit from searching for the next firm $j, j \neq i$, is

$$S(z) = \sum_{j=1,2} \text{Prob}(i = 3-j) \int_{l_j}^{z} (z - x) dF_j(x) = \sum_{j=1,2} \frac{f_{3-j}(z)}{f_1(z)+f_2(z)} \int_{l_j}^{z} F_j(x) dx. \qquad (2)$$

For a shopper, since the expected benefit of searching for the next e-tailer is positive and the cost is zero, the shopper will search both firms unless the first firm's price is equal to $\min\{l_1, l_2\}$. We shall later show in Lemmas 3 and 4 that $l_1 = l_2$, and that the event that either firm charges the lower bound equals zero in equilibrium. Therefore, a shopper will buy from the firm that offers the lowest price with a probability that equals 1.

For a high-search-cost buyer, an additional search is worthwhile if the expected benefit exceeds the search cost c, that is, $S(z) > c$. We assume high- search-cost buyers choose either firm with equal probability when presented with equal prices and choose to stop the search when indifferent between stopping and continuing to search. Lemma 1 below shows that in equilibrium high- search-cost buyers who happen to draw the highest possible price (i.e., the upper limit of the price range) will not search again (see Appendix for proof).

Lemma 1: $S(u_i) \leq c, i = 1,2$.

We further prove that for any price $z < u_i$, we have $S(z) \leq c$ (see the proof for Lemma 2 in the Appendix), which means that for a high-search- cost buyer, the cost of searching for the second e-tailer always outweighs the benefits, no matter what price is observed at the first e-tailer. Thus, high-search-cost buyers visit and purchase from the first e-tailer, while shoppers keep searching. Lemma 2 summarizes the behavior of both types of buyers.

Lemma 2: High-search-cost buyers buy from the first firm they visit; shoppers buy from the firm that offers the lower price.

The intuition is as follows: if high-search-cost buyers search for the second e-tailer, they will end up buying from the e-tailer with the lower price— just like the shoppers—which means that the firm charging the lower price will always win the entire market, rendering any mixed-strategy pricing suboptimal. Therefore, in equilibrium, the e-tailers will adopt pricing strategies that ensure that high-search-cost buyers purchase at the first e-tailer they visit.

4.2 Pricing Strategies

In this subsection, we discuss the firms' optimal pricing strategies. Recall that firm i's pricing strategy is described by a cumulative distribution function $F_i(p)$, with support $P_i = [l_i, u_i]$. We first characterize the properties that the pricing strategies must satisfy.

Lemma 3 shows that the lower bounds of the price ranges for the two firms are equal. In Lemma 4 we prove that the support of the price range P_i is continuous and that there is no probability mass at any price below $\min\{u_1, u_2\}$, which implies that $F_i(p)$ is continuous.

Lemma 3: The lower bounds of P_1 and P_2 are equal—i.e., $l_1 = l_2 \equiv l$.

Lemma 4: On the interval $[l, \min\{u_1, u_2\})$, the support of the price range P_i and the pricing strategy $F_i(p)$, $i = 1,2$ are both continuous.

Next, we find the relationship between the two firms' pricing strategies. The equilibrium revenue of firm i is denoted by π_i. Given equilibrium strategy $F_j(p)$, firm i's expected revenue is:

$$\pi_i = pq_i = p\left(\alpha_i\gamma + (1-\gamma)\left(1 - F_j(p)\right)\right). \quad (3)$$

The first component in the parenthesis, $\alpha_i\gamma$, is the number of high-search-cost buyers who visit firm i first. The second component, $(1-\gamma)(1-F_j(p))$ is the expected number of shoppers, calculated by the number of all shoppers multiplied by the probability of firm j charging a price higher than p, because shoppers, after visiting both firms, will generally choose to buy from firm i if i charges a lower price.[5]

[5] In this paper, we focus on the price factor in consumer's purchase decions.

Note that firm j does not have a mass at p, so the probability that firm j also charges p is zero.

Solving Equation 3 yields firm j's equilibrium pricing strategy:

$$F_j(p) = \frac{1}{1-\gamma}\left(1 - \gamma + \alpha_i\gamma - \frac{\pi_i}{p}\right), \quad l \le p < \min\{u_1, u_2\}. \quad (4)$$

Note that when $p = l$, $F_j(l) = 0$. Thus, we have $\pi_i = l(1-\gamma+\alpha_i\gamma)$. Therefore, firm j's equilibrium pricing strategy can be written as $F_j(p) = \frac{1-\gamma+\alpha_i\gamma}{1-\gamma}\left(1 - \frac{l}{p}\right)$. Similarly, firm i's strategy is $F_i(p) = \frac{1-\gamma+\alpha_j\gamma}{1-\gamma}\left(1 - \frac{l}{p}\right)$. Obviously, $F_i(p) = F_j(p)\frac{1-\gamma+\alpha_j\gamma}{1-\gamma+\alpha_i\gamma}$. Thus, we have the following lemma:

Lemma 5: When $\alpha_i \ge \alpha_j$, $F_i(p) \le F_j(p)$.

Lemma 5 states that the first-order condition of the price charged by the firm with more initial traffic (i.e., the more advertised firm) stochastically dominates that of the other firm. This means that the more advertised firm is more likely to charge a higher price.

The next lemma shows that the price ranges of the two firms have the same upper bound.

Lemma 6: The upper bounds of P_1 and P_2 are equal (i.e., $u_1 = u_2 \equiv u$).

From Lemmas 3 and 6, we know that the two firms have the same price range (i.e., $P_1 = P_2 = [l, u]$). Since only one firm could have a mass at u, and according to Lemma 5, $F_i(p) < F_j(p)$ when $\alpha_i > \alpha_j$, the firm that has a mass must be the one with more initial traffic. To simplify notation, without loss of generality, we assume firm i to be the more advertised in the rest of this subsection (i.e., $\beta_i \ge \beta_j$ and thus $\alpha = \alpha_i \ge \alpha_j$).

To specify $F_i(p)$, we need to determine l and u. Adopting a mixed strategy for pricing means that the expected revenue remains the same for any price charged. For firm i, charging u yields an expected equilibrium revenue of $\pi_i = u\alpha\gamma$; charging l yields $\pi_i = l(\alpha\gamma + 1 - \gamma)$. Thus, $l = u\frac{\alpha\gamma}{1-\gamma+\alpha\gamma}$. So, we only need to determine u, the upper bound of the price range. There are two mechanisms that limit the highest price a firm can charge. First, a firm cannot charge a higher price than the reservation price, so $u \le r$. Second, a firm cannot charge a price so high that even the high-search-cost buyers would search for a second e-tailer. Formally, define u^*, such that $S_i(u^*) = c$. Since

$$c = S_i(u^*) = \int_l^u F_j(x)dx = \int_l^u \frac{1-\gamma+\alpha\gamma}{1-\gamma}\left(1-\frac{l}{x}\right)dx = u^*\left(1-\frac{\alpha\gamma}{1-\gamma}\ln\left(\frac{1-\gamma+\alpha\gamma}{\alpha\gamma}\right)\right),$$

$$u^* = \frac{(1-\gamma)c}{1-\gamma-\alpha\gamma\ln\left(\frac{1-\gamma+\alpha\gamma}{\alpha\gamma}\right)}. \quad (5)$$

Therefore, the upper bound of the price range should be r or u^*, whichever is lower.

Based on the above analysis, we characterize the equilibrium pricing strategies in Proposition 1.

Proposition 1: Suppose $\beta_i \geq \beta_j$, and thus $\alpha = \alpha_i = \frac{1+\beta_i-\beta_j}{2}$. There is a unique mixed-strategy pricing equilibrium characterized by:

$$F_i(p) = \begin{cases} 1 & p \geq u \\ \frac{1-\alpha\gamma}{1-\gamma}\left(1-\frac{l}{p}\right) & l \leq p < u, \\ 0 & p < l \end{cases} \quad (6)$$

$$F_j(p) = \begin{cases} 1 & p \geq u \\ \frac{1-\gamma+\alpha\gamma}{1-\gamma}\left(1-\frac{l}{p}\right) & l \leq p < u, \\ 0 & p < l \end{cases} \quad (7)$$

where $l = u\frac{\alpha\gamma}{1-\gamma+\alpha\gamma}$, $u = \min\{r, u^*\}$, and $u^* = \frac{(1-\gamma)c}{1-\gamma-\alpha\gamma\ln\left(\frac{1-\gamma+\alpha\gamma}{\alpha\gamma}\right)}$.

Proposition 1 reveals some interesting features of the firms' pricing decisions. First, we find that buyers' search behavior influences the firms' pricing power. In this mixed-strategy pricing equilibrium, the highest price a firm can charge is not only bounded by the buyers' maximum willingness to pay, but may also be restricted by the buyers' search cost. In the literature of pricing games with buyers of different mobility, the upper bound of the price in an asymmetric mixed-strategy equilibrium is commonly shown to be defined by the monopoly price (i.e., the reservation price) r (Narasimhan, 1988; Raju, Srinivasan, & Lal, 1990). In our model, however, the upper bound of the price range is determined by the minimum of r and u^*, the latter being a function of the high-search-cost buyers' search cost c. As long as $r > u^*$, the higher the search cost c, the higher the upper bound of the price range. This implies that when the search cost for the high-search-cost buyers increases, firms can potentially charge a higher price.

The price range also depends on the composition of buyers. When all the buyers are the high-search-cost type (i.e., $\gamma = 1$, then $u = l = r$), both firms charge the reservation price, consistent with Diamond (1971). However, when the proportion of shoppers increases, u^* declines ($u^{*\prime}(\gamma) > 0$), and when u^* is lower than r, the upper bound is determined by u^*. When γ is close to zero (i.e., most buyers are shoppers) the upper bound u has the lowest value, c. The lower bound l also decreases as γ declines. This suggests that when the percentage of shoppers increases, the firms have less pricing power.

Second, for any price below u, the two firms have the same mixed-strategy pricing scheme. We know that as long as $\alpha > 0.5$, the more advertised firm is the only one that has a probability mass at u. Define this probability mass as m. From Proposition 1, we have $m = 1 - F_i^-(u) = \frac{(2\alpha-1)\gamma}{1-\gamma+\alpha\gamma}$. By the Bayesian update rule, if a buyer observes a price of u, she can infer with certainty that the current firm is the more advertised firm. Interestingly, at any other price below u, $F_i(p|p<u) = F_j(p)$, which means that firm i adopts the same pricing strategy as firm j at any price below u. Therefore, if a buyer gets any price $p,\ p < u$, it is equally possible that it comes from either firm.

Based on Proposition 1, we offer the following corollary regarding the expected price, sales quantities, and revenues of the two firms:

Corollary 1. Suppose $\beta_i \geq \beta_j$ and $\alpha = \alpha_i = \frac{1+\beta_i-\beta_j}{2}$. The expected prices, sales quantities, and revenues of firm i and j are given by:

$$Ep_i = \int_l^u p\,f_i(p)dp = u\frac{\alpha\gamma(1-\alpha\gamma)}{(1-\gamma)(1-\gamma+\alpha\gamma)}\ln\left(\frac{1-\gamma+\alpha\gamma}{\alpha\gamma}\right) + u\frac{(2\alpha-1)\gamma}{1-\gamma+\alpha\gamma}, \quad (8)$$

$$Ep_j = \int_l^u p\,f_j(p)dp = u\frac{\alpha\gamma}{1-\gamma}\ln\left(\frac{1-\gamma+\alpha\gamma}{\alpha\gamma}\right), \quad (9)$$

$$Eq_i = \int_l^u \frac{\pi_i}{p} f_i(p)dp = \alpha\gamma + \frac{(1-\alpha\gamma)(1-\gamma)}{2(1-\gamma+\alpha\gamma)}, \quad (10)$$

$$Eq_j = \int_l^u \frac{\pi_j}{p} f_j(p)dp = (1-\alpha)\gamma + \frac{(1-2\gamma+3\alpha\gamma)(1-\gamma)}{2(1-\gamma+\alpha\gamma)}, \quad (11)$$

$$\pi_i = u\alpha\gamma, \quad (12)$$

$$\pi_j = u\alpha\gamma\frac{1-\alpha\gamma}{1-\gamma+\alpha\gamma}, \quad (13)$$

where $l = u\frac{\alpha\gamma}{1-\gamma+\alpha\gamma}$, $u = \min\{r, u^*\}$, and $u^* = \frac{(1-\gamma)c}{1-\gamma-\alpha\gamma\ln\left(\frac{1-\gamma+\alpha\gamma}{\alpha\gamma}\right)}$.

Based on Corollary 1, it can be proven that firm i, the more advertised firm, has a higher expected price, a higher expected quantity, and higher revenues. We can also see that when $\alpha = 1/2$ (i.e., $\beta_i = \beta_j$), $Ep_i = Ep_j = \frac{u\gamma}{2(1-\gamma)}\ln\left(\frac{2}{\gamma}-1\right)$, and $\pi_i = \pi_j = \frac{u\gamma}{2}$. This implies that when the two firms choose the same level of advertising, they will have the same expected price and revenue.

It is instructive to analyze how equilibrium prices are affected by traffic flow. It is easy to verify $\frac{du^*}{d\alpha} > 0$. So, when $u = u^*$, $\frac{du^*}{d\alpha} \geq 0$ (note that u is not differentiable at the kink point $r = u^*(\alpha)$). Similarly, $\frac{dl}{d\alpha} > 0$. Therefore, the upper and lower bounds of the price range increase with the skewness of traffic flow, except when the upper bound is capped by the reservation price. Based on the results in Corollary 1, it can be easily proven that $\frac{dEp_i}{d\alpha} > 0$ and $\frac{dEp_j}{d\alpha} > 0$ when $u = u^*$, which suggests that more unbalanced traffic flow leads to higher expected prices of both firms, relaxing price competition. We offer the following corollary:

Corollary 2: More skewed traffic flow leads to higher expected market prices—i.e., $\frac{dEp_i}{d\alpha} > 0$ and $\frac{dEp_j}{d\alpha} > 0$.

Our model generalizes Arbatskaya (2007) and Xu et al. (2011). In their work, all buyers follow the same search order and the prices decline with the order. We show that the buyers may have different search order, but the firm that more buyers visit first is more likely to set a higher price than the other firm.

It is also worthwhile to analyze the sales quantity. From Corollary 1, we obtain $\frac{dEq_i}{d\alpha} > 0$, which means more skewed traffic leads to more sales for the more advertised firm. We can see that firm i sells to $\alpha\gamma$ high-search-cost buyers and its expected share of shoppers is given by $\frac{(1-\alpha\gamma)(1-\gamma)}{2(1-\gamma+\alpha\gamma)}$. The proportion of high-search-cost buyers in total sales, $\frac{\alpha\gamma}{Eq_i}$, monotonously increases with α, from γ (when $\alpha = 0.5$) to $\frac{2\gamma}{1+\gamma^2}$ (when $\alpha = 1$). This means more unbalanced traffic results in firm i selling more to the high-search-cost buyers and less to the shoppers, and the overall effect is higher sales volume. Also, note $Eq_i < \alpha$; that is, the sales volume is less than the traffic flow initially directed to firm i.

From Corollary 1, we find $\frac{dEq_j}{d\alpha} < 0$. Firm j sells to $(1-\alpha)\gamma$ high-search-cost buyers and its expected share of shoppers. Firm j's share of shoppers consists two parts: the share when firm i prices at u: $m(1-\gamma)$, and the share when firm i prices below u: $\frac{(1-m)(1-\gamma)}{2}$. The proportion of high-search-cost buyers, $\frac{(1-\alpha)\gamma}{Eq_j}$, monotonously decreases with α, from γ (when $\alpha = 0.5$) to 0 (when $\alpha = 1$), which implies that when the advertising (traffic) gap between the two firms widens, the less advertised firm j is less likely to sell to high-search-cost buyers and more likely to sell to shoppers, but the overall effect is lower sales quantity. Note that $Eq_j > 1-\alpha$; that is, the sales volume is greater than the traffic flow initially directed to Firm 2. In summary, we offer the following corollary:

Corollary 3: More skewed traffic flow leads to a higher sales quantity for the more advertised firm and lower sales quantity for the less advertised firm (i.e., $dEq_i/d\alpha > 0$ and $dEq_j/d\alpha < 0$) and makes the buyers of the former firm more skewed to high-search- cost buyers and the buyers of the latter firm more skewed to shoppers.

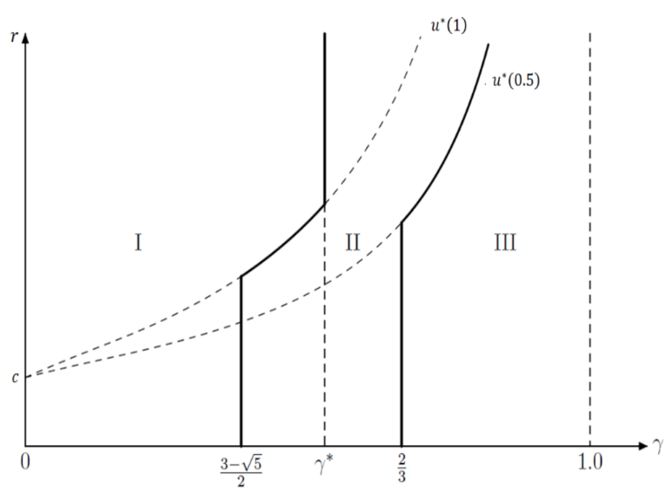


Figure 1. Partition of the Monotonicity of $\pi_2(\alpha)$

We now discuss the revenue of the firms. Since more skewed traffic flow leads to both higher expected price and higher quantity of firm i, the revenue of firm i increases. However, higher skewness of initial traffic (an increase in α) has two countervailing effects on firm j's revenue: the relaxed price competition effect (a higher expected price) and the sales reduction effect (a lower sales quantity). The next corollary states that $\pi_j(\alpha)$ can only be of increasing, unimodal, or decreasing shape, and Figure 1 shows the property of $\pi_j(\alpha)$ for specific parameter ranges (see Appendix for proof):

Corollary 4: $\pi_j(\alpha)$ can only be of increasing, unimodal or decreasing shape.

In Figure 1, the parameter space $\gamma \otimes r$ is divided into three regimes, separated by black solid lines. Regime I is the space where $\gamma \leq \frac{3-\sqrt{5}}{2}$, or $\frac{3-\sqrt{5}}{2} < \gamma \leq \gamma^*$ and $r \geq u^*(1) = \frac{(1-\gamma)c}{1-\gamma+\gamma\ln(\gamma)}$ ($\gamma^* \approx 0.529$). In this regime, $\pi_j(\alpha)$ is an increasing function of α. The intuition is as follows: as the initial traffic gap widens, the sales quantity of firm j decreases, but because there are more shoppers (γ is low), the decrease is relatively modest and the higher price (on average) can more than compensate for the lost sales, resulting in higher revenues. In other words, the relaxed price competition effect outweighs the sales reduction effect.

In Regime III, where $\gamma \geq \frac{2}{3}$ and $r \leq u^*(0.5)$, $\pi_j(\alpha)$ is a decreasing function of α.[6] Here most buyers have high search costs and choose to buy from the first store they visit, thus when significantly more buyers are directed to firm i, the sales reduction effect outweighs the relaxed price competition effect, negatively affecting firm j's revenue.

Regime II is defined by $\frac{3-\sqrt{5}}{2} < \gamma \leq \gamma^*$ and $r < u^*(1)$, or $\gamma^* < \gamma < \frac{2}{3}$, or $\gamma \geq \frac{2}{3}$ and $r > u^*(0.5) = \frac{(1-\gamma)c}{1-\gamma-0.5\gamma\ln(\frac{1-0.5\gamma}{0.5\gamma})}$. In this regime, firm j's revenue first increases and then decreases with more skewed traffic to firm i. This suggests that for parameters in this range, there is an optimal traffic flow that balances the relaxed price competition effect and the sales reduction effect

and yields the maximal revenue for firm j. In other words, the effect of traffic flow pattern on firm j's revenue is moderated by the overall market mobility.

4.3 Advertising Levels

We now discuss firms' advertising decisions. Recall that the number of buyers that first visit firm i is $\alpha_i = \frac{1+\beta_i-\beta_j}{2}$ and that $\alpha_i + \alpha_j = 1$. From Corollary 1, we see that firm i's expected revenues from selling the product is $\pi_i = u\alpha_i\gamma$ when $\beta_i \geq \beta_j$, and $\pi_i = u(1-\alpha_i)\gamma\frac{1-\gamma+\alpha_i\gamma}{1-\alpha_i\gamma}$ when $\beta_i < \beta_j$. A firm's net payoff, or profit, π_i^n is its revenues π_i minus the advertising costs. Thus, the profit of firm i, $i = 1,2$, can be written as:

Equation (14)

$$\pi_i^n(\beta_i;\beta_j) = \begin{cases} u\alpha_i\gamma - A(\beta_i) = u\gamma\frac{1+\beta_i-\beta_j}{2} - A(\beta_i), & \beta_i \geq \beta_j \\ \frac{u(1-\alpha_i)\gamma(1-\gamma+\alpha_i\gamma)}{1-\alpha_i\gamma} - A(\beta_i) = u\gamma\frac{1+\beta_j-\beta_i}{2}\frac{1-\gamma(1+\beta_j-\beta_i)/2}{1-\gamma(1+\beta_i-\beta_j)/2} - A(\beta_i), & \beta_i < \beta_j \end{cases}.$$

Proposition 2: The advertising game does not have a pure strategy symmetric equilibrium.

The intuition behind Proposition 2 is that the payoff function is not quasi-concave at $\beta_i = \beta_j$, so each firm has an incentive to deviate by increasing or decreasing its advertising level at the diagonal of the payoff functions.

To find an asymmetric equilibrium, we first define $b_i(\beta_j)$, the best response of firm i to a given level of β_j, in the following lemma.

Lemma 7. Firm i's best response function to firm j's advertising level β_j, $\beta_i = b_i(\beta_j)$ is defined as follows:

$b_i(\beta_j) = b_i^{(1)}(\beta_j) = argmax_{\beta_i}\, u\gamma\frac{1+\beta_i-\beta_j}{2} - A(\beta_i)$ when $\beta_j < \hat{\beta}$,

$b_i(\beta_j) = b_i^{(2)}(\beta_j) = argmax_{\beta_i}\, u\gamma\frac{1+\beta_j-\beta_i}{2}\frac{1-\gamma(1+\beta_j-\beta_i)/2}{1-\gamma(1+\beta_i-\beta_j)/2} - A(\beta_i)$ when $\beta_j > \hat{\beta}$,

[6] Note $u^*(0.5)$ is a function of γ with $\alpha = 0.5$. If $r < u^*(0.5)$, the equilibrium price must be bounded by r, that is $u = r$. $u^*(1)$ is a function of γ with $\alpha = 1$. If $r > u^*(1)$, the equilibrium price is only bounded by u^* (never by r), that is $u = u^*$. If $u^*(0.5) \leq r \leq u^*(1)$, the equilibrium price is first bounded u^* and then by r as α increases.

where $\hat{\beta}$ is the solution to the equation

$$u\gamma \frac{1+b_i^{(1)}(\hat{\beta})-\hat{\beta}}{2} - A(b_i^{(1)}(\hat{\beta})) =$$
$$u\gamma \frac{1+\hat{\beta}-b_i^{(2)}(\hat{\beta})}{2} \frac{1-\gamma(1+\hat{\beta}-b_i^{(2)}(\hat{\beta}))/2}{1-\gamma(1+b_i^{(2)}(\hat{\beta})-\hat{\beta})/2} - A(b_i^{(2)}(\hat{\beta})).$$

Figure 2 illustrates the best response functions, $b_i(\beta_j)$ (shown in solid lines) and $b_j(\beta_i)$ (dotted lines). We focus on the explanation of $b_i(\beta_j)$, knowing that $b_j(\beta_i)$ is symmetric to $b_i(\beta_j)$. Figure 2a illustrates a typical graph of $b_i(\beta_j)$ and $b_j(\beta_i)$, where $u = u^*$ and $b_i(\beta_j), b_j(\beta_i)$ are not zero. The most prominent feature of $b_i(\beta_j)$ is that it is discontinuous[7] and has two regimes: when $\beta_j < \hat{\beta}$ or firm j advertises moderately, firm i chooses a high advertising level, but when $\beta_j > \hat{\beta}$ or firm j advertises intensely, firm i switches to a much lower advertising level. This means that firms avoid a head-to-head advertising war. Furthermore, when β_j is lower than the threshold level $\hat{\beta}$ and firm i is the one with higher advertising level, firm i gradually advertises less as firm j increases its advertising level ($b'_i(\beta_j) < 0$). The intuition is as follows: in this regime, firm i is advertising intensely, and due to higher marginal advertising cost, it is costly to advertise even more; thus it is in the best interests of firm i to play dovish, advertising less as its competitor advertises more. On the other hand, when $\beta_j > \hat{\beta}$, firm i, the one with a lower advertising level, increases its own advertising level as firm j advertises more, though to a lesser extent than firm j ($0 < b'_i(\beta_j) < 1$). Firm i plays hawkish in this regime because it has lower marginal advertising cost, and thus advertising more is worthwhile to attract more traffic.

Figure 2b illustrates the case where $u = u^*$, but b_i takes the corner solution 0 for some $\beta_j > \hat{\beta}$. In this case, $b_i(\beta_j)$ has three regimes: when $\beta_j < \hat{\beta}$, firm i chooses a high advertising level; when $\hat{\beta} < \beta_j \leq \tilde{\beta}$, firm i does not advertise at all ($b_i(\beta_j) = 0$); and when $\beta_j > \tilde{\beta}$, firm i chooses to advertise at a low, positive level. It is interesting that for certain parameters, in response to the competitor's medium level of advertising, it may be optimal for a firm not to advertise at all.

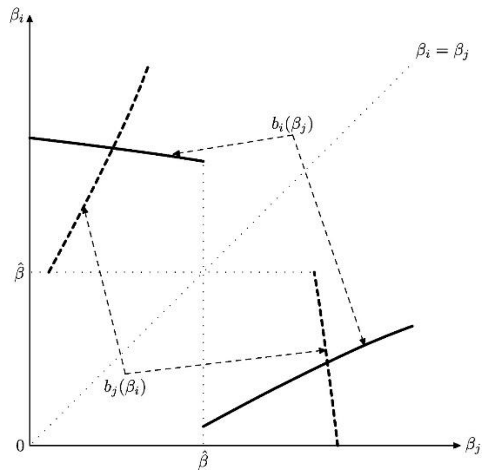
Figure 2c illustrates the case where the upper bound of price range u can be restricted by r. In this chart, when $\beta_j \leq \widetilde{\beta_1}$, $u = r$, and β_i does not change with β_j ($b'_i(\beta_j) = 0$). When $\widetilde{\beta_1} < \beta_j \leq \widetilde{\beta_2}$, it is the case where $u = r = u^*$. In this interval, for an increase in its competitor's advertising level, firm i responds with an equal increase in advertising level ($b'_i(\beta_j) = 1$), resulting in the same traffic flows, hence the same revenues from product sales but lower profits for both, due to higher advertising costs. If $\widetilde{\beta_2} < \beta_j < \hat{\beta}$, then $u = u^*$, and firm i actually reduces its own advertising when firm j increases its advertising level, which is similar to $b_i(\beta_j)$ in Regime I in Figures 2a and 2b. $b_i(\beta_j)$ is also discontinuous at $\hat{\beta}$, which means that firm i switches to a lower advertising level when firm j's advertising is sufficiently high, just like the cases shown in Figures 2a and 2b. In the range $\hat{\beta} < \beta_j \leq \widetilde{\beta_3}$, where $u = u^*$ still holds, $b_i(\beta_j)$ again has similar properties as it does in Regime II in Figure 2a and Regime III in Figure 2b. When $\widetilde{\beta_3} < \beta_j \leq \widetilde{\beta_4}$, we have $u = r = u^*$, and similar to the case where $\widetilde{\beta_1} < \beta_j \leq \widetilde{\beta_2}$, β_i increases with β_j, and their difference remains the same. Finally, when $\beta_j > \widetilde{\beta_4}$, again $u = r$, but now that firm i's advertising level is low, it advertises more in response to firm j's increase in β_j, though to a smaller extent.

We know that each intersection of the best response functions $b_i(\beta_j)$ and $b_j(\beta_i)$ is an equilibrium. In Figures 2a, 2b, and 2c, the intersections of the solid lines and dotted lines are equilibria. We prove that $b_i(\beta_j)$ and $b_j(\beta_i)$ must intersect and thus offer the following proposition.

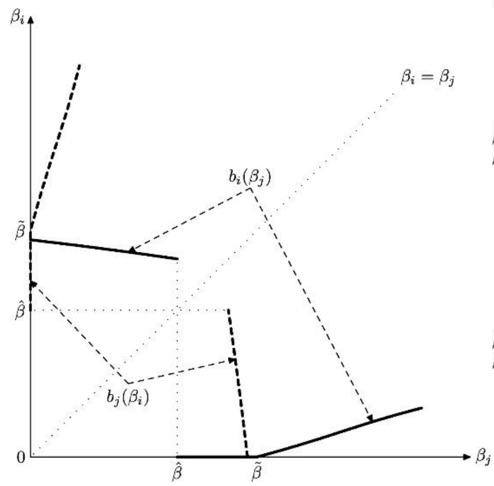
Proposition 3. The advertising game has paired pure strategy asymmetric equilibria. The firms always choose different advertising levels.

Due to the ex ante symmetry of the firms, the best responses of the firms are symmetric. In Proposition 3, we prove that the game has either a pair of pure strategy asymmetric equilibria (i.e., the intersections of $b_i(\beta_j)$ and $b_j(\beta_i)$ in Figures 2a and 2b) or paired continuum asymmetric equilibria—i.e., the two segments where $b_i(\beta_j)$ and $b_j(\beta_i)$ coincide in Figures 2c. We use asterisks to show equilibrium results. Without loss of generality, we label the firm with the higher equilibrium advertising level as Firm 1 and the other as Firm 2 throughout the rest of the paper. Therefore, in equilibrium, $\beta_1^* > \beta_2^*$ and the initial traffic to the more advertised firm is $\alpha^* = \alpha_1^* = \frac{1+\beta_1^*-\beta_2^*}{2}$.

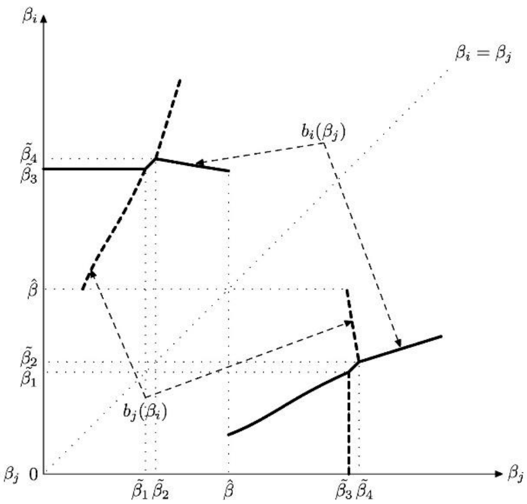
[7] If b_i is continuous, it guarantees a symmetric pure strategy equilibrium.



a. General form of the best response function



b. One optimal advertising level is zero



c. Existence of continuum of equilibria

Figure 2. The Best Response Functions

Figure 3 shows the equilibrium advertising strategies in the space of r and γ. The space is partitioned into six regions. In Regions I and II, one firm chooses not to advertise ($\beta_2^* = 0$). This is because in these regions γ is low (i.e., the market mobility is high), and since there are more shoppers than high-search-cost buyers, the firms choose different strategies: one chooses to advertise and the other chooses not to, focusing on charging a lower average price instead. In Regions III and IV, since market mobility is low, to attract high-search-cost buyers both firms choose to advertise.

In Regions II and III, the reservation price r determines the upper bound of the firms' pricing range. In Regions I and IV, the reservation price r is high; however, the highest price that the firms charge does not equal r, but instead is bounded by u^*. This suggests that buyers' search behavior limits the firms' pricing power.

Regions V and VI, nested between Regions I, II, III, and IV, is the space where there exists a continuum of equilibria. In Region V, where γ is low (i.e., the mobility is high), $\beta_2^* = 0$ is included in the continuum of equilibria, while in Region VI, both firms choose positive advertising levels in any equilibrium of the continuum. We summarize these results in the following corollary:

Corollary 5. When the market mobility is sufficiently high, one firm willingly chooses not to advertise (i.e., $\beta_2^* = 0$); otherwise both firms choose to advertise (i.e., $\beta_1^*, \beta_2^* > 0$). For a narrow area in the space of parameters r and γ, a continuum of asymmetric equilibria β_1^*, β_2^* exist.

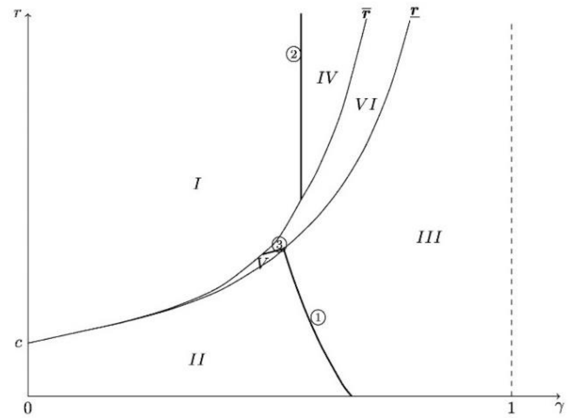


Figure 3. Partition of the Equilibrium Advertising Strategies

One unique aspect of this model is that an asymmetric equilibrium may persist in an ex ante symmetric, static game. In the literature, an asymmetric equilibrium is usually driven by exogenous asymmetric factors, such as store positioning (Rajiv, Dutta, & Dhar, 2002) price formats (Lal & Rao, 1997), and product availability (Janssen & Non, 2009), or exists in a dynamic setting (Doraszelski & Markovich, 2007). Our results show that, in equilibrium, firms can choose different strategies to differentiate.

5 Mobility, Costs and Equilibrium

In this section, we examine how market characteristics and information costs affect firm strategies. We refer to the firm with the higher advertising level in equilibrium as Firm 1 and the other as Firm 2. Recall that in Corollary 5, we show that there is a continuum of asymmetric equilibria for a narrow parameter range. In this section, we assume that in the case of a continuum of equilibria, firms choose the equilibrium that has the lowest advertising levels and hence highest payoffs.

5.1 The Effect of Market Mobility

We first examine the effect of market mobility on the equilibrium advertising levels and, in turn, traffic allocation and market outcomes.

Recall that γ measures the proportion of high- search-cost buyers, and thus higher γ implies lower market mobility. The properties of the comparative statics $\frac{\partial \beta_1^*}{\partial \gamma}$, $\frac{\partial \beta_2^*}{\partial \gamma}$ and $\frac{\partial \alpha^*}{\partial \gamma}$ are summarized in the following proposition.

Proposition 4. β_1^* first increases with γ, then may decrease during a narrow range of γ, and then resumes increasing with γ, albeit at a lower rate than β_2^* does. β_2^* is zero when γ is below a threshold value, beyond which β_2^* is positive and always increases with γ. The traffic to the more advertised firm, α^*, is most skewed at an intermediate value of γ.

Note that $\beta_1^*(\gamma)$ only decreases when (r, γ) is in Region V or VI of Figure 3. Therefore $\beta_1^*(\gamma)$ strictly increases when $r < c$ (the parameters fall in Regions II and III). Figure 4 illustrates $\beta_1^*(\gamma)$, $\beta_2^*(\gamma)$ and $\alpha^*(\gamma)$, where $r > c$, so that $\beta_1^*(\gamma)$ has a decreasing interval.

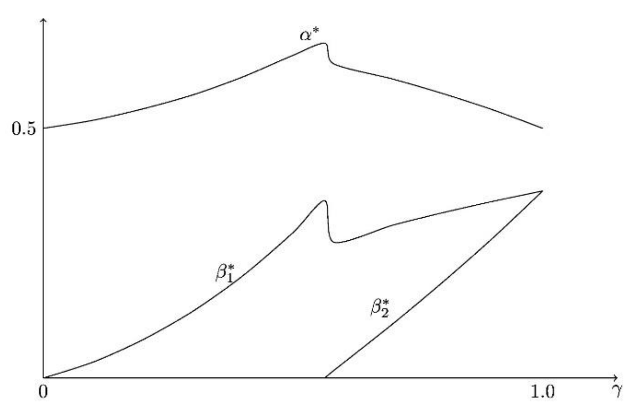
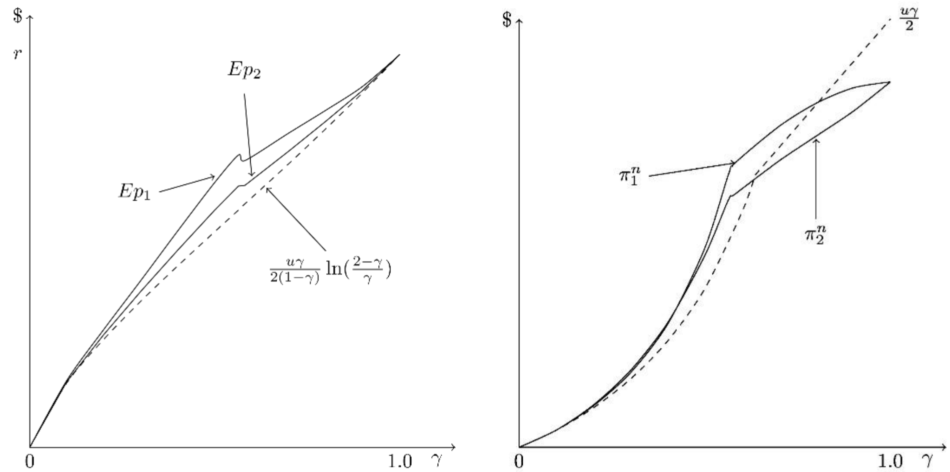


Figure 4. Advertising Levels and Traffic Flow



a. The expected equilibrium prices b. The expected equilibrium profits

Figure 5. The Equilibrium Market Outcome

When $\gamma = 0$, neither firm advertises. As γ increases, one firm starts to advertise more, resulting in more skewed traffic flow. The skewness in advertising levels (traffic flow), represented by α^*, peaks in the medium range of γ. As γ further increases, the other firm starts to advertise, and does so more aggressively, narrowing the gap in the advertising levels. When $\gamma = 1$, both firms choose the same, highest advertising intensity. This shows that the difference in advertising strategy is small when the market is more uniform (γ is either small or large); and the firms have the most unbalanced strategies when the market is more heterogeneous because they can best differentiate by targeting different segments.

Figure 5a depicts how the expected equilibrium prices change with γ and Figure 5b the firms' profits. In Figure 5a, the dashed curve represents each firm's expected equilibrium price, $\frac{u\gamma}{2(1-\gamma)}\ln\left(\frac{2-\gamma}{\gamma}\right)$, when advertising is not available thus neither advertises. Ep_1 and Ep_2 are higher than $\frac{u\gamma}{2(1-\gamma)}\ln\left(\frac{2-\gamma}{\gamma}\right)$, and converge to the monopoly price r as γ approaches 1. The difference between Ep_1, Ep_2, and $\frac{u\gamma}{2(1-\gamma)}\ln\left(\frac{2-\gamma}{\gamma}\right)$ is the largest in the medium range of γ, when price competition is most relaxed. That is because, as shown previously, α^* peaks in the medium range of γ.

In Figure 5b, the dashed curve represents the profit each firm earns, $\frac{u\gamma}{2}$, again when neither firm advertises. $\frac{u\gamma}{2}$ increases monotonically with γ, which implies that without advertising costs, the less mobile the market is, the higher the profits that firms can make. This is because, as the proportion of high-search-cost buyers increases, firms can charge a higher average price and thus earn higher profits. The solid lines represent the firms' net profits. Both firms' profits are higher than $\frac{u\gamma}{2}$ when γ is relatively low. Moreover, it can be proven that when only one firm advertises, both firms earn net profits higher than $\frac{u\gamma}{2}$.

Corollary 6. When market mobility is sufficiently high and in equilibrium and only one firm advertises, both firms' profits are higher than in the case where neither firm has the option to advertise.[8]

Corollary 6 suggests that the opportunity to advertise, even though costly, can benefit firms. Without advertisements, firms must compete in terms of price. The option to advertise allows the firms to differentiate along two dimensions: one firm may choose to advertise and charge a high price, while the other chooses to charge a low price and does not advertise at all (note that such equilibria exist only when the market mobility is high). This results in higher profits for both firms than they would earn in the case of price competition without any advertisement. Another interesting result is that, when γ is low and one firm does not advertise, the firm that advertises more does not necessarily earn higher profits. In Figure 5(b), π_1^n could be lower than π_2^n for some γ when γ is low. This means that under these market conditions, none of the two strategies—low price or more ads—is superior.

[8] The proof of Corollary 6 is straightforward. $\pi_1^n(\beta_1^*;0) > \pi_1^n(0;0) = \pi_1(0;0)$. Since $\beta_2^* = 0$ is a corner solution, $\frac{\partial_+\pi_2^n}{\partial\beta_2^*}|_{\beta_2^*=0} \leq 0$. So $\pi_2^n(0;\beta_1^*) = \pi_2(0;\beta_1^*) > \pi_2(\beta_1^*;\beta_1^*) = \pi_2(0;0)$.

It is interesting to further discuss social welfare. In our model, since consumers always buy, the total social welfare remains unchanged, with or without the option of advertising. Based on Figure 5a, consumers always pay higher prices when advertising is available. Therefore, consumer surplus is reduced by the advertising option. When the market mobility is sufficiently high, firms earn higher profits than they would without any advertising, and thus producer surplus increases. Note that the firms pay to advertise, therefore the reduction in consumer surplus is divided between the firms and the advertising media.

When γ is high, we can prove that the more advertised firm has higher profits than the less advertised firm. In Corollary 1, we show that the more advertised firm has higher expected revenues than its competitor, and as Corollary 7 indicates below, we find that when the market consists of mostly high-search-cost buyers, in equilibrium the higher revenues more than compensate for the higher advertising costs of the more advertised firm.

Corollary 7. When market mobility is sufficiently low, both firms advertise; the firm that advertises more makes a higher profit than the other firm.

Note that when γ is high, both firms' profits are lower than $\frac{u\gamma}{2}$, the profits in the case scenario of no advertising. The firms, in fact, face a form of the prisoner's dilemma: even though both firms could benefit if they both reduced their advertising intensity, neither has the incentive to do so. If the firms coordinate to cut their advertising levels to $\beta_1 = \beta_1^* - \beta_2^*$ and $\beta_2 = \beta_2^* - \beta_2^* = 0$, each would earn a higher profit than earned in equilibrium (β_1^*, β_2^*). However, without a contractual commitment, such a mutually beneficial agreement is not realized, since it does not constitute a Nash equilibrium. In terms of social welfare, since consumers are paying higher prices and firms are making lower profits than in a case scenario of no advertising, both the consumer surplus and the producer surplus are reduced when market mobility is sufficiently low. With the total welfare unchanged, the loss in consumer surplus and producer surplus is appropriated by advertising media.

In sum, the firms' profitability depends on the fierceness of traffic competition. When the mobile segment is relatively large (i.e., γ is small), the competition for high-search-cost buyers is not fierce. With one firm using the low-price strategy and the other using the advertising strategy, both can earn a profit higher than they would in the no advertising scenario. But when the high-search-cost segment is relatively large (i.e., γ is large), the differentiation strategy is less useful and attention competition becomes very fierce and, thus, the advertising costs drive both firms' profits to levels strictly lower than $\frac{u\gamma}{2}$, the profit when neither firm advertises.

5.2 The Effect of Advertising Costs

We consider how a change in advertising costs might affect the equilibrium. Innovations in Internet technologies are constantly creating new approaches to deliver ads. Advertising on new media, including social networking websites, such as Facebook, and social shopping websites, such as Pinterest, has been growing rapidly. These media offer new opportunities to reach consumers at lower costs.

Assume the advertising cost is a function of the parameter θ in the form of $A(\beta;\theta) = \theta\,\widetilde{A}\,(\beta)$, where θ is a multiplicative component independent of the advertising level. $A(\beta;\theta)$ has the following properties: $\frac{\partial A}{\partial\theta} > 0$ and $\frac{\partial^2 A}{\partial\beta\partial\theta} > 0$, which suggests that an increases in θ corresponds to an increase in total and marginal advertising costs. Lower θ may represent shift toward either a more efficient advertising technology or toward more suppliers of advertising services. The following proposition summarizes the results.

Proposition 5. $\frac{\partial\beta_i^*}{\partial\theta} < 0, i = 1,2$, except when (r,γ) is in Region V, where $\frac{\partial\beta_1^*}{\partial\theta} = 0$. $\frac{\partial\alpha^*}{\partial\theta} = 0$ when (r,γ) is in Region V or VI. Otherwise $\frac{\partial\alpha^*}{\partial\theta} < 0$ when γ is low, and the sign of $\frac{\partial\alpha^*}{\partial\theta}$ is determined by the sign of $\frac{\widetilde{A}''(\beta_1^*)}{\widetilde{A}'(\beta_1^*)} - \frac{\widetilde{A}''(\beta_2^*)}{\widetilde{A}'(\beta_2^*)}$ when γ is high.

Proposition 5 shows that lower θ almost always causes firms to advertise more, except in the context of a very narrow parameter space (Region V in Figure 3). However, its effect on the traffic is moderated by the market mobility. When market mobility is high, as long as the advertising equilibrium is not a kink point, lower advertising costs will lead to more skewed traffic and milder price competition, resulting in higher equilibrium prices and higher profits for both firms. This is because the firms adopt differentiated strategies, and since the firm with the advertising strategy now pays less for the same advertising level, price competition is further alleviated and both firms benefit. It should be noted that, in this case, lower advertising costs actually lead to higher expected prices, making buyers worse off.

When market mobility is low, although lower advertising costs still encourage both firms to advertise more, the degree of traffic competition is decided by the relative convexity at the equilibrium levels. For example, if $\widetilde{A}\,(\beta)$ is a polynomial function, $\widetilde{A}\,(\beta) = \beta^t, t > 1$, $\frac{\widetilde{A}''(\beta)}{\widetilde{A}'(\beta)} = \frac{t-1}{\beta}$ is a decreasing function, so $\frac{\widetilde{A}''(\beta_1^*)}{\widetilde{A}'(\beta_1^*)} - \frac{\widetilde{A}''(\beta_2^*)}{\widetilde{A}'(\beta_2^*)} < 0$ and thus $\frac{\partial\alpha^*}{\partial\theta} < 0$, which implies more skewed traffic, milder competition, and higher prices for lower θ. If $\widetilde{A}\,(\beta)$ is a logarithmic function,

$\widetilde{A}(\beta) = -\ln(1-\beta) - \beta$, $\frac{\widetilde{A}''(\beta)}{\widetilde{A}'(\beta)} = \frac{1}{\beta(1-\beta)}$ is not monotonic in interval $(0, 1)$, but increasing in $(0.5, 1)$. So, if $\beta_1^* > \beta_2^* > 0.5$, lower θ will lead to less skewed traffic, more intense competition, and lower prices.

Grossman and Shapiro (1984) find that lower advertising costs encourage firms to advertise more, leading to lower prices and lower profits. Our findings are in line with Grossman and Shapiro, in that decreased advertising costs lead to higher advertising levels, but they diverge from Grossman and Shapiro, in that the effect on price competition is not clear-cut, but moderated by market parameters. Specifically, when market mobility is high, lower advertising costs lead to higher prices.

5.3 The Effect of Search Costs

From Corollary 1, we can see that the search cost c influences the market outcome only when $u = u^*$, which is a function of c, and when $u = r$ the search cost c has no impact on the outcome.

From Figure 3, when parameters fall into Region II, III, V, or VI, we know that $u = r$, which means that the market outcome does not depend on the search cost c. When γ is very high, parameters always fall in Region III or VI. In these regions, the market is dominated by high-search-cost buyers and the firms could charge very high prices if their only concern were buyers shopping around. We can see this from Corollary 1, where u^* can be proven to be increasing in γ. In these regions, it is the buyers' reservation price r that limits the firms' pricing power and determines the overall equilibrium outcomes. In Regions II and V, the market is mobile (γ low), which implies that u^* is low, but the buyers' reservation price r is even lower and as a result $r < u^*$ and thus $u = r$. Therefore, in Regions II and V, again, the search cost has no effect on the outcomes. Note that the value of search cost c does influence the boundaries of Regions II, III, V and VI: the higher the search cost, the larger the space where it is irrelevant.

In Regions I and IV of Figure 3, the upper bound of the price range is determined by u^*, and thus the search cost c influences the outcome. Specifically, in Region I, γ is low, thus u^* is low and the reservation price r is medium or high, which means the condition $r > u^*$ can be easily satisfied; in Region IV, even though a high γ suggests a high u^*, r is so high that $r > u^*$ still holds. This means that when the the reservation price is sufficiently high and the market is not too immobile, the search cost of high-search-cost buyers limits the firms' pricing strategies and the outcome. Specifically, a higher search cost leads to a higher price range and higher expected prices.

6 Extension: Sequential Game

In this section we extend the model to consider the case in which firms set advertising levels sequentially. It is often observed that industry leaders often announce their advertising plans and others tend to adjust theirs thereafter.

Suppose Firm 1 (the leader) first commits to advertising intensity β_1; then Firm 2 (the follower) observes β_1 and sets intensity β_2. Both firms then choose prices simultaneously and buyers behave as described earlier. We solve this game to find the subgame perfect Nash equilibrium. Our purpose is to compare this equilibrium with the simultaneous game equilibrium. We focus on the market of relatively low mobility, where no advertising is never the best response of the follower and the equilibrium is not a kink point.

We first show that the leader will choose to be the more advertised firm. Assume the best response of Firm 2 is $b_2(\beta_1)$. By the first-order condition, $\frac{\partial \pi_2^n}{\partial b_2} = \frac{\partial \pi_2}{\partial b_2} - A'(b_2) = 0$. According to the envelope theorem, $\frac{\partial \pi_2^n}{\partial \beta_1} = \frac{\partial \pi_2}{\partial \beta_1}$. Since $\frac{\partial \pi_2}{\partial \beta_1} = -\frac{\partial \pi_2}{\partial \beta_2}$, $\frac{\partial \pi_2}{\partial \beta_1} = -A'\big(b_2(\beta_1)\big)$. The first-order condition of the leader is $\frac{d\pi_1^n}{d\beta_1} = \frac{\partial \pi_1}{\partial \beta_1} + \frac{\partial \pi_1}{\partial b_2}\frac{\partial b_2}{\partial \beta_1} - A'(\beta_1) = 0$. So $\frac{\partial \pi_1}{\partial \beta_1} + \frac{\partial \pi_1}{\partial b_2}\frac{\partial b_2}{\partial \beta_1} - A'(\beta_1) = \frac{\partial \pi_1}{\partial b_2}(b_2'(\beta_1) - 1) - A'(\beta_1) = 0$, and thus $\frac{\partial \pi_1}{\partial b_2} = \frac{A'(\beta_1)}{b_2'(\beta_1) - 1}$. We show previously $0 < b_2{}'(\beta_1) < 1$ if Firm 2 is the less advertised firm and $b_2{}'(\beta_1) < 0$ if Firm 2 is the more advertised firm. So, $\frac{\partial \pi_1}{\partial b_2} < 0$. Therefore, the equilibrium profit of Firm 1 never increases in the advertising level of the follower, so it is optimal for the leader to be the more advertised firm.

Denote the equilibrium advertising levels in the sequential game by β_{1s}^* and β_{2s}^*, and those in the simultaneous game by β_{1c}^* and β_{2c}^*. The first-order conditions of Firm 1 are is $\frac{d\pi_1^n}{d\beta_{1s}^*} = \frac{\partial \pi_1}{\partial \beta_{1s}^*}\big(1 - b_2'(\beta_{1s}^*)\big) - A'(\beta_{1s}^*) = 0$ and $\frac{\partial \pi_1^n}{\partial \beta_{1c}^*} = \frac{\partial \pi_1}{\partial \beta_{1c}^*} - A'(\beta_{1c}^*) = 0$. Firm 2 is the less advertised firm, so $0 < b_2{}'(\beta_1) < 1$. Therefore $A'(\beta_{1s}^*) < A'(\beta_{1c}^*)$ and $\beta_{1s}^* < \beta_{1c}^*$. By $b'_2(\beta_1) > 0$, $\beta_{2s}^* < \beta_{2c}^*$. We have shown that each firm's optimal profit decreases in the other firm's advertising level, so both firms' profits are higher in the sequential game than in the simultaneous game. We summarize this result in the following proposition.

Proposition 6. The sequential game has a unique asymmetric pure strategy equilibrium where the leader has a higher advertising level than the follower. Both firms choose lower advertising levels and earn higher profits than in the simultaneous game.

The traffic flow is less skewed than that in the simultaneous game since $b'_2(\beta_1) \leq 1$; in this case, buyers benefit as well because market prices are now lower.

Our result runs contrary to the standard Stackelberg production game. In the standard game, quantity is the decision variable and, in equilibrium, only the leader increases the production quantity and has a higher profit than that of the Cournot game while the follower is always worse off. The key insight that drives the difference is that in our model, the follower's decision variable is a strategic complement to that of the leader; while in the production game, the follower's decision variable is a strategic substitute. Therefore, in our setting the leader foresees that the follower will cooperate by cutting its own advertising if the leader limits its own spending to a lower level. Acting sequentially essentially provides a cooperating mechanism that enables the firms to collectively reduce advertising expenditures and increase profits.

7 Concluding Remarks

In this paper, we study how firms make optimal advertising and pricing decisions in face of competition. We find that even for symmetric e-tailers, it is optimal for them to choose differentiated strategies, along both the advertising and the pricing dimensions.

Our results show that market mobility plays an important role in moderating firms' decisions. When the market mobility is low, firms compete fiercely for consumers' attention, and the firm with higher advertising intensity also charges a higher price and earns higher profits. When market mobility is high, however, firms adopt differentiated strategies: one firm advertises, while the other may choose not to advertise at all; furthermore, in such cases, no strategy is superior and either firm may make higher profits, depending on the parameters. Another interesting result is that the firms are most differentiated in their advertising levels when the market mobility is medium. In sum, the two strategies to attract customers—aggressive advertising or low prices—can be both effective when the market composition is balanced between shoppers and high-search-cost buyers, as long as the competing firms adopt differentiated strategies. When the market is dominated by one type of buyers, competition intensifies, in terms of either attention or price.

We also compare the market outcome in our model with the case scenario in which firms do not have the option to advertise. As anticipated, the option to advertise leads to higher expected prices for any given market composition, and therefore consumer surplus is always reduced. It is interesting that the effects on firms are mixed: when market mobility is high, both e-tailers can make higher profits than without the option, even for the firm that advertises intensively; when market mobility is low, however, the option to advertise makes both e-tailers worse off. The advertisers gain from the option to advertise and gain most when the market mobility is low.

This paper shows advertising costs have a different effect on prices and profits than previously shown in the literature. We find lower advertising costs do encourage firms to advertise more, but also lead to higher prices. The effect on profits has mixed results. When market mobility is relatively high, lower advertising costs lead to more skewed traffic, milder price competition, and higher profits. When market mobility level is low, the effect of lower advertising costs depends on the characteristics of the cost function.

Another interesting result is the interaction between consumers' search cost, willingness to pay, and the market composition. When consumers have low willingness to pay and/or the market is immobile, firms consider willingness to pay in their decisions and the search cost does not affect equilibrium outcome. When consumers have high willingness to pay and the market is mobile, search cost limits firms' pricing power and consumers' willingness to pay becomes irrelevant.

In an extension to the model, we find that when firms can choose advertising levels sequentially, they choose lower advertising levels and lower prices and realize higher profits, as compared to the simultaneous game, because advertising levels of the two firms are strategic complements.

In relation to the literature, our research shows that price dispersion can be a result of e-tailers' differentiated strategies. Note that such price dispersion is sustained because advertising is available and the consumers are heterogeneous in search costs. Furthermore, we find that price dispersion is most conspicuous when market mobility is medium (see Figure 5a). This study also confirms that competition for consumers' attention can be fierce, but shows that this is not always the case. When market mobility is low or medium, some e-tailers may not engage in competition for attention.

This work has interesting managerial implications for e-tailers. For e-tailers seeking to outdo their competitors in terms of attracting consumers' attention, it is important to take into consideration both the market segments and their competitors' strategic responses. When buyers are heterogeneous in their search behavior, instead of head-to-head competition in advertising, e-tailers can choose to focus on different segments, by either pricing low or advertising intensively. This can relax price competition and lead to higher profits. With some knowledge of consumers' search behavior, e-tailers that choose to advertise intensively and can afford to price higher—in effect, avoiding unnecessary price wars. E-tailers should also be fully aware of the market composition in order to make advertising and pricing decisions wisely. Lastly, if e-tailers could coordinate to set advertising levels sequentially, they would collectively benefit from lower advertising expenses and higher profits.

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Appendix

Proof of Lemma 1

If $S(u_i) > c$, the buyers that are quoted u_i will definitely continue to search. Firm i charging u_i only makes sense when firm j prices no lower than u_i with positive possibility. Therefore $u_j \geq u_i$.

Assume $u_j = u_i$ and firm j has a mass $w > 0$ at u_j. The buyers that are quoted u_i from firm i only has $w/2$ probability buying from firm i. In comparison, the buyers that are quoted price $u_i - \epsilon$, where ϵ is an arbitrarily small value, have w probability of buying from firm i. Price u_i is strictly dominated by $u_i - \epsilon$, which means u_i cannot be charged with positive probability. The buyers that are quoted u_i from firm j will definitely continue to search and all will purchase from from i. Therefore, it's unwise for firm j to charge u_i, which contradicts with firm j having a mass at u_i. If firm j doesn't have a mass at u_j, the buyers that are quoted u_i from firm i will search and have probability 1 of buying from another firm. So, it is not optimal for firm i to charge u_i and u_i can not be the upper bound.

Assume $u_j > u_i$. If $S(u_j) > c$, the buyers that are quoted u_j will definitely search again and will be certain to find a lower price from firm i and never return to firm j. So, we must have $S(u_j) \leq c$. Then charging any price $p_j \in (u_i, u_j)$ is worse than charging price u_j. It would be optimal for firm j to charge prices in the interval (u_i, u_j) with zero probability. However, it also means that firm i can charge a price higher than u_i but smaller than u_j without losing more sales than charging u_i, which contradicts with u_i being the upper bound.

In summary, we must have $S(u_i) \leq c$.

Proof of Lemma 2

Note that $\int_{l_i}^{z} F_i(x)dx$ increases in z. Suppose $S(z) > c$. There must be a firm i whose pricing strategy satisfies $f_i(z)/(f_1(z) + f_2(z)) > f_i(u_i)/(f_1(u_i) + f_2(u_i))$. However, by reducing the density $f_i(z)$, so that $f_i(z)/(f_1(z) + f_2(z)) = f_i(u_i)/(f_1(u_i) + f_2(u_i))$, firm i can always achieve $S(z) < c$ and higher expected profit (because high-search-cost buyers that are quoted z will not search), which suggests original strategy is not optimal. Therefore, we must have $S(z) \leq c$. This means that for high-search-cost buyers, the expected benefit of additional searching is lower than the search cost, so they buy from the first e-tailer they visit.

Proof of Lemma 3

The proof is straightforward: for firm i, if $l_i < l_j$, there must exist a price p, $l_i \leq p < l_j$, charged with positive density. However, p is strictly dominated by price $\frac{p+l_j}{2}$ and therefore p cannot be charged with positive density. Thus, we prove the lemma by contradiction.

Proof of Lemma 4

The proof of this lemma is similar to Narasimhan (1988) and Jing and Wen (2008). To limit the length of the paper, the proof is omitted.

Proof of Lemma 6

Assume $u_i > u_j$. On interval (u_j, u_i), firm i's expected payoff is $\Pi_i(p) = p\alpha_i\gamma$, which is an increasing function. Therefore, (u_j, u_i) cannot be in support P_i. Since u_i is the upper bound, there must be a mass point at u_i. Also, we must have $S(u_i) = c$, otherwise firm i can slightly increase the upper bound, earning a higher payoff.

Firm i cannot have a mass at u_j. Assume firm i has a mass w at u_j, then firm j can not have a mass at u_j at the same time, since price $u_j - \epsilon$ (ϵ is an arbitrarily small positive number) yields a higher expected payoff than u_j does. If firm j has no mass at u_j, firm i can move the mass w to u_i, earning a higher expected payoff, which violates the equilibrium condition.

Assume $\alpha_i \geq \alpha_j$, then $F_i(p) \leq F_j(p)$. $S(u_j) \leq \int_{l}^{u_j} F_j(x)dx < S(u_i) = c$. Firm j can earn a higher payoff by charging price $u_j + \epsilon$, which violates the condition u_j is the upper bound.

Assume $\alpha_i < \alpha_j$, then $F_i(p) > F_j(p)$. If $S(u_j) < c$, since (u_j, u_i) is not in P_i, by charging any price p, $u_j < p < u_i$, firm j can earn a higher payoff, which violates the equilibrium condition. Therefore, we must have $S(u_j) = c$. So

$c = S(u_j) < \int_l^{u_j} F_i(x)dx = \frac{1-\gamma+\alpha_j\gamma}{1-\gamma}\int_l^{u_j}\left(1-\frac{l}{x}\right)dx < \frac{1-\gamma+\alpha_j\gamma}{1-\gamma}\left(1-\frac{l}{u_j}\right)(u_j-l) = \frac{1-\gamma+\alpha_j\gamma}{1-\gamma}\left(\frac{l}{u_j}-1\right)^2 u_j.$

By previous analysis, $S(u_i) = c$, we have

$c = S(u_i) = u_i - u_j + \int_l^{u_j} F_j(x)dx = u_i - u_j + \frac{1-\gamma+\alpha_i\gamma}{1-\gamma+\alpha_j\gamma}\int_l^{u_j} F_i(x)dx > u_i - u_j + \frac{1-\gamma+\alpha_i\gamma}{1-\gamma+\alpha_j\gamma}c.$

By $\pi_i = l(1-\gamma+\alpha_i\gamma) = u_i\alpha_i\gamma$, $u_i = \frac{(1-\gamma+\alpha_i\gamma)l}{\alpha_i\gamma}$. Combining above formulas, we have

$\left(\frac{1-\gamma+\alpha_i\gamma}{\alpha_i\gamma}l - u_j\right)\frac{1-\gamma+\alpha_j\gamma}{(\alpha_j-\alpha_i)\gamma} < c < \frac{1-\gamma+\alpha_j\gamma}{1-\gamma}\left(\frac{l}{u_j}-1\right)^2 u_j.$

Note $\alpha_j = 1-\alpha_i$. Let $x = l/u_j$, reorganize the above formula, we have

$\Phi(x) \equiv \frac{(2\alpha_j-1)\gamma}{1-\gamma}(x-1)^2 - \frac{1-\alpha_j\gamma}{(1-\alpha_j)\gamma}x + 1 > 0.$

$\Phi(\cdot)$ is a quadratic function with $\Phi(1) = -\frac{1-\alpha_j\gamma}{(1-\alpha_j)\gamma} + 1 = \frac{\gamma-1}{(1-\alpha_j)\gamma} < 0.$

By $\pi_j = l(1-\gamma+\alpha_j\gamma) = u_j(\alpha_j\gamma + (1-\gamma)(1-F_i(u_j)))$, we have $x = \frac{l}{u_j} = \frac{\alpha_j\gamma+(1-\gamma)(1-F_i(u_j))}{1-\gamma+\alpha_j\gamma}$. So, $\frac{\alpha_j\gamma}{1-\gamma+\alpha_j\gamma} < x < 1$. It is easy to verify that $\Phi\left(\frac{\alpha_j\gamma}{1-\gamma+\alpha_j\gamma}\right)<0$. Together with $\Phi(1) < 0, \frac{(2\alpha_j-1)\gamma}{1-\gamma} \geq 0$, we conclude that $\Phi(x) < 0$ for $\frac{\alpha_j\gamma}{1-\gamma+\alpha_j\gamma} < x < 1$. However, it contradicts with previous result $\Phi(x) > 0$.

In summary, assuming $u_i > u_j$ only leads to contradictions. Since i could be either firm, we must have $u_1 = u_2$.

Proof of Proposition 1

Based on Lemma 6, equation 4 and 5, we have the equilibrium prices as defined in the proposition.

Proof of Corollary 4

When $r \leq u^*(0.5)$, the upper bound is restricted by r for all possible α. So, $\pi_j(\alpha) = r\frac{\alpha\gamma(1-\alpha\gamma)}{1-\gamma+\alpha\gamma}$. It can be shown that $\pi_j(\alpha)$ is monotonously increasing in α when $\gamma \leq \frac{3-\sqrt{5}}{2}$; unimodal in α when $\frac{3-\sqrt{5}}{2} < \gamma < \frac{2}{3}$; monotonously decreasing in α when $\frac{2}{3} \leq \gamma < 1$.

When $r \geq u^*(1)$, the upper bound is not restricted by r for all possible α. So, $\pi_j(\alpha) = u^*\frac{\alpha\gamma(1-\alpha\gamma)}{1-\gamma+\alpha\gamma}$. By numerical method, it can be shown that $\pi_j(\alpha)$ is monotonously increasing in α when $\gamma \leq \gamma^* \approx 0.53$; unimodal in α when $\gamma^* < \gamma < 1$.

When $u^*(0.5) < r < u^*(1)$, by the monotonicity of $u^*(\alpha,\gamma)$ on α, there exists a unique $\hat{\alpha}$ such that $u^*(\hat{\alpha},\gamma) = r$. Then $u(\alpha,\gamma) = u^*(\alpha,\gamma)$ if $\alpha < \hat{\alpha}$ and $u(\alpha,\gamma) = r$ if $\alpha \geq \hat{\alpha}$.

If $\gamma \leq \frac{3-\sqrt{5}}{2}$, according to previous discussion, no matter $u(\alpha,\gamma) = u^*(\alpha,\gamma)$ or $u(\alpha,\gamma) = r$, $\pi_j(\alpha)$ is monotonously increasing in α.

If $\frac{3-\sqrt{5}}{2} < \gamma \leq \gamma^*$, we have $u(\alpha,\gamma) = u^*(\alpha,\gamma)$ and $\pi_j(\alpha)$ increasing in α when $\alpha < \hat{\alpha}$; $u(\alpha,\gamma) = r$ and $\pi_j(\alpha)$ unimodal in α when $\alpha \geq \hat{\alpha}$. Connecting these two sections, we have $\pi_j(\alpha)$ is unimodal in α over $0.5 \leq \alpha \leq 1$.

If $\gamma > \gamma^*$, we have $\pi_j(\alpha) = u^*(\alpha,\gamma)\frac{\alpha\gamma(1-\alpha\gamma)}{1-\gamma+\alpha\gamma}$ when $\alpha < \hat{\alpha}$; $\pi_j(\alpha) = r\frac{\alpha\gamma(1-\alpha\gamma)}{1-\gamma+\alpha\gamma}$ when $\alpha \geq \hat{\alpha}$. In both cases, $\pi_j(\alpha)$ is a unimodal function of α. Next, we rule out the possibility that $\pi_j(\hat{\alpha})$ is on the downsloping part of function $u^*(\alpha,\gamma)\frac{\alpha\gamma(1-\alpha\gamma)}{1-\gamma+\alpha\gamma}$ and on the upsloping part of function $r\frac{\alpha\gamma(1-\alpha\gamma)}{1-\gamma+\alpha\gamma}$. The reasoning is simple. If that is so, since $\pi_j(\hat{\alpha})$ is on the downsloping part of function $u^*(\alpha,\gamma)\frac{\alpha\gamma(1-\alpha\gamma)}{1-\gamma+\alpha\gamma}$ and $u^*(\alpha,\gamma)$ is an increasing function of α, we must have $\frac{\partial\frac{\alpha\gamma(1-\alpha\gamma)}{1-\gamma+\alpha\gamma}}{\partial\alpha} < 0$, which means $\frac{\partial r\frac{\alpha\gamma(1-\alpha\gamma)}{1-\gamma+\alpha\gamma}}{\partial\alpha}|_{\alpha=\hat{\alpha}} < 0$.

That contradicts $\pi_j(\hat{\alpha})$ being on the upsloping part of function $r\frac{\alpha\gamma(1-\alpha\gamma)}{1-\gamma+\alpha\gamma}$. After ruling out this possibility, by combining these two sections, we have π_j is unimodal in α over $0.5 \le \alpha \le 1$.

Proof of Proposition 2

Suppose there is an equilibrium $\beta_1^* = \beta_2^* = \beta^*$. β^* can't be zero since $A'(0) = 0$ implies each firm would deviate by increasing its advertising intensity. Equilibrium condition requires that no firm deviates by increasing or decreasing its advertising intensity. By equation 14, at $\beta_i^* = \beta_j^* = \beta^*$, for any γ

$$\frac{\partial_+\pi_i^n}{\partial\beta_i^*} - \frac{\partial_-\pi_i^n}{\partial\beta_i^*} = u\gamma\frac{2(\gamma^2-3\gamma+2)}{(\gamma-2)^2} > 0.$$

Therefore π_i^n is not quasi-concave at $\beta_i^* = \beta_j^*$ and there does not exist a symmetric pure strategy equilibrium.

Proof of Lemma 7

First, we restrict the strategy space to $\beta_i > \beta_j$. If $u = r$, $\pi_i^n(\beta_i)$ is clearly concave. If $u = u^*$, by $A'(0) = 0, A'(1)$ is arbitrarily large, there must be a local maximizer for π_i^n; $\pi_i^n(\beta_i)$ is also concave at the maximizer. If, for certain β_j, there is a value $\beta_i^\dagger$ that satisfies $r = u^*$, $\pi_i^n(\beta_i)$ will not be differentiable at $\beta_i^\dagger$; we can show $\frac{\partial_-\pi_i^n}{\partial\beta_i^\dagger} > \frac{\partial_+\pi_i^n}{\partial\beta_i^\dagger}$; therefore $\pi_i^n(\beta_i)$ is also concave at $\beta_i^\dagger$. The best response $b_i(\beta_j)$ could be either an interior solution or a corner solution (kink point) that satisfies $r = u^*$. If $b_i(\beta_j)$ is an interior maximizer to the profit function with $u = r$, the first-order condition will show that b_i is independent of (β_j); in other words, $b'_i(\beta_j) = 0$ and its graph is horizontal. If $b_i(\beta_j)$ is an interior maximizer to the profit function with $u = u^*$, by the first-order condition and the implicit function theorem, $b'_i(\beta_j) < 0$ and its graph is downward-sloping. If $b_i(\beta_j)$ is the kink-point, $b_i(\beta_j) = 2u^{*-1}(r) + \beta_j - 1$, therefore $b'_i(\beta_j) = 1$. For all above cases, by the envelope theorem, $\frac{\partial\pi_i^n(b_i(\beta_j))}{\partial\beta_j} = -A'(b_i(\beta_j)) < 0$. We represent the best response function derived under condition $\beta_i > \beta_j$ by $b_i^{(1)}(\beta_j)$.

Now consider the strategy space $\beta_i < \beta_j$. For either $u = r$ or $u = u^*$, $\pi_i^n(\beta_i)$ is a unimodal function of β_i. At the kink point $\beta_i^\dagger$ that satisfies $r = u^*\left(\frac{1+\beta_j-\beta_i^\dagger}{2}\right)$, $\frac{\partial_-\pi_i^n}{\partial\beta_i^\dagger} > \frac{\partial_+\pi_i^n}{\partial\beta_i^\dagger}$. So π_i^n must be a unimodal function of β_i in general. If the best response $b_i(\beta_j)$ is an interior maximizer to the profit function with either $u = r$ or $u = u^*$, by the first-order condition and the implicit function theorem, $0 < b'_i(\beta_j) < 1$, so its graph is upward-sloping. If $b_i(\beta_j)$ is the kink-point, $b_i(\beta_j) = 1 + \beta_j - 2u^{*-1}(r)$, $b'_i(\beta_j) = 1$. For above cases, $\frac{\partial\pi_i^n(b_i(\beta_j))}{\partial\beta_j} = -A'(b_i(\beta_j)) < 0$ by the envelope theorem. $b_i(\beta_j)$ has a corner solution 0 when $\frac{\partial\pi_i^n}{\partial\beta_i}|_{\beta_i=0} < 0$. Then $\frac{\partial\pi_i^n(b_i(\beta_j))}{\partial\beta_j} > 0$. We represent the best response function derived under condition $\beta_i < \beta_j$ by $b_i^{(2)}(\beta_j)$.

According to above results, $b_i^{(1)}(\beta_j)$ is always nonincreasing except for an possible interval, so it must intersect the diagonal $\beta_i = \beta_j$ at certain value $\overline{\beta}$, which implies $b_i^{(1)}(\overline{\beta}) = \overline{\beta}$. $b_i^{(2)}(\beta_j)$ never intersects $\beta_i = \beta_j$ for $\beta_j > 0$ since $b_i^{(2)\prime}(\beta_j) < 1$ in general and $b_i^{(2)\prime}(\beta_j) = 1$ just for an possible interval. Therefore $\pi_i^n(b_i^{(2)}(\overline{\beta}); \overline{\beta}) > \pi_i^n(\overline{\beta}; \overline{\beta}) = \pi_i^n(b_i^{(1)}(\overline{\beta}); \overline{\beta})$. It is easy to see that $\pi_i^n(b_i^{(1)}(0); 0) > \pi_i^n(0; 0) = \pi_i^n(b_i^{(2)}(0); 0)$. By continuity, $\pi_i^n(b_i^{(1)}(\beta))$ and $\pi_i^n(b_i^{(2)}(\beta))$ must cross at a point $\hat{\beta}$, such that $\pi_i^n(b_i^{(1)}(\hat{\beta}); \hat{\beta}) = \pi_i^n(b_i^{(2)}(\hat{\beta}); \hat{\beta})$. Moreover, by previous analysis, $\partial\pi_i^n(b_i^{(1)}(\beta_j))/\partial\beta_j < \partial\pi_i^n(b_i^{(2)}(\beta_j))/\partial\beta_j$. So $\hat{\beta}$ must be unique. Therefore, the general best response function is $b_i = b_i^{(1)}$ when $\beta_j < \hat{\beta}$ and $b_i = b_i^{(2)}$ when $\beta_j > \hat{\beta}$.

Proof of Proposition 3

We need to prove that b_i and b_j must intersect, which is equivalent to prove that, suppose $b_i^{(1)}$ intersect $b_j^{(2)}$ at (β_i^*, β_j^*), we must have $\pi_i^n(\beta_i^*; \beta_j^*) > \pi_i^n(b_i^{(2)}(\beta_j^*); \beta_j^*)$.

By Lemma 7, $b_i^{(1)}$ and $b_j^{(2)}$ must intersect. Consider the triangle with vertices (β_j^*, β_j^*), (β_i^*, β_j^*), $(\beta_j^*, b_i^{(2)}(\beta_j^*))$ if $b_i^{(2)}(\beta_j^*) > 0$, and consider the quadrilateral with vertices (β_j^*, β_j^*), (β_i^*, β_j^*), $(\hat{\beta}, 0)$, $(\beta_j^*, 0)$ if $b_i^{(2)}(\beta_j^*) = 0$. In both cases, $\beta_i^* - \beta_j^* \geq \beta_j^* - b_i^{(2)}(\beta_j^*)$ since $0 \leq b_i^{(2)\prime}(\beta_j) \leq 1$ by the proof of Lemma 7.

Define $x, 0 < x \leq \beta_j^* - b_i^{(2)}(\beta_j^*)$. Because $\frac{\partial^2 \pi_i^n}{\partial \beta_i \partial \beta_j} \leq 0$, we have

$$\pi_i^{n\prime}(\beta_j^* + x; \beta_j^*) \geq \pi_{i+}^{n\prime}(\beta_j^* + x; \beta_j^* + x). \tag{15}$$

Because $\pi_{i+}{}'(\beta_j^* + x; \beta_j^* + x) + \pi_{i-}{}'(\beta_j^* + x; \beta_j^* + x) = \frac{u\gamma^2}{2-\gamma} > 0$,

$$\pi_{i+}^{n\prime}(\beta_j^* + x; \beta_j^* + x) > -\pi_{i-}^{n\prime}(\beta_j^* + x; \beta_j^* + x). \tag{16}$$

$\pi_i^n(\beta_i; \beta_j)$ is concave for $\beta_i < \beta_j$. So

$$\pi_{i-}^{n\prime}(\beta_j^* + x; \beta_j^* + x) < \pi_i^{n\prime}(\beta_j^*; \beta_j^* + x). \tag{17}$$

Since $\beta_j^* - (\beta_j^* - x) = (\beta_j^* + x) - \beta_j^*$ and $A'(\beta_j^* - x) < A'(\beta_j^*)$, we have

$$\pi_i^{n\prime}(\beta_j^* - x; \beta_j^*) > \pi_i^{n\prime}(\beta_j^*; \beta_j^* + x). \tag{18}$$

By inequations 15, 16, 17 and 18, $\pi_i^{n\prime}(\beta_j^* + x; \beta_j^*) > -\pi_i^{n\prime}(\beta_j^* - x; \beta_j^*)$. Therefore

$$\frac{\partial \pi_i^n(\beta_j^* + x; \beta_j^*)}{\partial x} > \frac{\partial \pi_i^n(\beta_j^* - x; \beta_j^*)}{\partial x}. \tag{19}$$

By inequation 19, $\pi_i^n(\beta_i^*; \beta_j^*) > \pi_i^n(2\beta_j^* - b_i^{(2)}(\beta_j^*); \beta_j^*) = \pi_i^n(\beta_j^*; \beta_j^*) + \int_0^{\beta_j^* - b_i^{(2)}(\beta_j^*)} \frac{\partial \pi_i^n(\beta_j^* + x; \beta_j^*)}{\partial x} dx > \pi_i^n(\beta_j^*; \beta_j^*) + \int_0^{\beta_j^* - b_i^{(2)}(\beta_j^*)} \frac{\partial \pi_i^n(\beta_j^* - x; \beta_j^*)}{\partial x} dx = \pi_i^n(b_i^{(2)}(\beta_j^*); \beta_j^*)$.

Proof of Corollary 5

In this proof we characterize the equilibrium and the partition of the parameter space.

Define $\overline{\beta_1}(\beta_2) = \text{argmax}_{\beta_1, \beta_1 \in [0,1]}\{r\alpha\gamma - A(\beta_1)\}$, $\overline{\overline{\beta_1}}(\beta_2) = \text{argmax}_{\beta_1, \beta_1 \in [0,1]}\{u^*\alpha\gamma - A(\beta_1)\}$, $\overline{\beta_2}(\beta_1) = \text{argmax}_{\beta_2, \beta_2 \in [0,1]}\{r\phi(\alpha) - A(\beta_2)\}$, and $\overline{\overline{\beta_2}}(\beta_1) = \text{argmax}_{\beta_2, \beta_2 \in [0,1]}\{u^*\, \phi(\alpha) - A(\beta_2)\}$, where $\phi(\alpha) = \frac{\alpha\gamma(1-\alpha\gamma)}{1-\gamma+\alpha\gamma}$.

Define $\beta_1^\dagger$ and $\beta_2^\dagger$ being the kink points that satisfy $r = u^*\left(\frac{1+\beta_1^\dagger - \beta_2^\dagger}{2}\right)$ and $\beta_2^\dagger \geq 0$. When the equilibrium advertising strategy $\beta_1^* = \beta_1^\dagger$ and $\beta_2^* = \beta_2^\dagger$, the left derivative of each firm's profit with regard to its advertising level should be no less than zero, and the right should be no higher. Define the minimal r that induces such an equilibrium by $\underline{r}$. The minimal β_1 should satisfy the first-order condition $\frac{\partial_+ \pi_1^n}{\partial \beta_1^\dagger} = 0$; the maximal β_2 should satisfy $\frac{\partial_- \pi_2^n}{\partial \beta_2^\dagger} = 0, \beta_2^\dagger > 0$ or $\beta_2^\dagger = 0$; and $\underline{r} = u^*\left(\frac{1+\beta_1^\dagger - \beta_2^\dagger}{2}\right)$. Define the maximal r that induces such an equilibrium by $\overline{r}$. The maximal β_1 should satisfy $\frac{\partial_- \pi_1^n}{\partial \beta_1^\dagger} = 0$; the minimal β_2 should satisfy $\frac{\partial_+ \pi_2^n}{\partial \beta_2^\dagger} = 0, \beta_2^\dagger > 0$ or $\beta_2^\dagger = 0$, and $\overline{r} = u^*\left(\frac{1+\beta_1^\dagger - \beta_2^\dagger}{2}\right)$. By above condition, $\overline{r} > \underline{r}$.

The equilibrium advertising strategy (β_1^*, β_2^*) should satisfy both reaction functions, $\beta_1^* = b_1(\beta_2^*), \beta_2^* = b_2(\beta_1^*)$. When $r < \underline{r}$, $r < u^*$ at equilibrium, so $\beta_1^* = \overline{\beta_1}(\beta_2^*)$ and $\beta_2^* = \overline{\beta_2}(\beta_1^*)$. When $r > \overline{r}$, $r > u^*$ at equilibrium, so $\beta_1^* = \overline{\overline{\beta_1}}(\beta_2^*)$ and $\beta_2^* = \overline{\overline{\beta_2}}(\beta_1^*)$. When $\underline{r} \leq r \leq \overline{r}$, $r = u^*$ at equilibrium, so $\beta_1^* = \beta_1^\dagger$, $\beta_2^* = \beta_2^\dagger$, and $r = u^*(\frac{1+\beta_1^* - \beta_2^*}{2})$.

In summary, the equilibrium strategies are

$$(\beta_1^*, \beta_2^*) \quad = \begin{cases} (\overline{\beta_1}(\beta_2^*), \overline{\beta_2}(\beta_1^*)), & r < \underline{r} \\ (\beta_1^\dagger, \beta_2^\dagger), & \underline{r} \le r \le \overline{r}. \\ (\overline{\overline{\beta_1}}(\beta_2^*), \overline{\overline{\beta_2}}(\beta_1^*)), & r > \overline{r} \end{cases} \tag{20}$$

Note that for the equilibrium $\beta_1^* = \beta_1^\dagger$, $\beta_2^* = \beta_2^\dagger$, $r = u^*\left(\frac{1+\beta_1^*-\beta_2^*}{2}\right)$ is an indeterminate equation; so there exist a continuum of equilibria, among which the one of the smallest advertising levels are preferred by both firms for it yields the highest profits for both firms.

We now characterize the parameter partition for each type of equilibrium. The curves $\overline{r}$ and $\underline{r}$ in Figure 3 are defined as above. Line ①, which separates Region II and III, is defined by the boundary condition $\frac{\partial_+\pi_2^n}{\partial\beta_2^*}|_{\beta_2^*=0,\beta_1^*=\overline{\beta_1}(0)} = 0$, and here $u = r$ binds. Line ②, which separates Region I and IV, is also defined by the boundary condition $\frac{\partial_+\pi_2^n}{\partial\beta_2^*}|_{\beta_2^*=0,\beta_1^*=\overline{\overline{\beta_1}}(0)} = 0$. Note here $u = u^*$, so line ② is a straight line. Line ③, which separates Region V and VI, is defined by the boundary condition $\frac{\partial_+\pi_2^n}{\partial\beta_2^*}|_{\beta_2^*=0,r=u^*} = 0$.

Proof of Proposition 4

First, we analyze the sign of $\frac{\partial\beta_i^*}{\partial\gamma}$ in each region of the parameter space.

When γ is small (Region I, II, V in Figure 3), $\beta_2^* = 0, \beta_1^* = b_1(0)$. We only need to analyze $\frac{\partial\beta_1^*}{\partial\gamma}$. By the first-order condition and the implicit function theorem, in Region II, $\frac{\partial\beta_1^*}{\partial\gamma} = -\frac{r}{2} / \frac{\partial^2\pi_1^n}{\partial\beta_1^{*2}} > 0$, and in Region I, $\frac{\partial\beta_1^*}{\partial\gamma} = -\frac{\partial^2 u^*\alpha\gamma}{\partial\beta_1^*\partial\gamma} / \frac{\partial^2\pi_1^n}{\partial\beta_1^{*2}} > 0$. In Region V, $r = u^*(\frac{1+\beta_1^*}{2})$. So $\frac{\partial\beta_1^*}{\partial\gamma} = -\frac{\partial u^*}{\partial\gamma} / \frac{\partial u^*}{\partial\beta_1^*} < 0$.

When γ is sufficiently large (Region III, IV, VI in Figure 3), $\beta_i^* > 0, i = 1,2$. In Region III or IV, by the system of first-order conditions and the implicit function theorem,

$$\begin{pmatrix} \frac{\partial\beta_1^*}{\partial\gamma} \\ \frac{\partial\beta_2^*}{\partial\gamma} \end{pmatrix} = -H^{-1} \begin{pmatrix} \frac{\partial^2\pi_1^n}{\partial\beta_1^*\partial\gamma} \\ \frac{\partial^2\pi_2^n}{\partial\beta_2^*\partial\gamma} \end{pmatrix}. \tag{21}$$

H is the Hessian matrix. $H = \begin{pmatrix} \frac{\partial^2\pi_1^n}{\partial\beta_1^{*2}} & \frac{\partial^2\pi_1^n}{\partial\beta_1^*\partial\beta_2^*} \\ \frac{\partial^2\pi_2^n}{\partial\beta_2^*\partial\beta_1^*} & \frac{\partial^2\pi_2^n}{\partial\beta_2^{*2}} \end{pmatrix}$.

By the optimality condition, $\frac{\partial^2\pi_1^n}{\partial\beta_1^{*2}} < 0$, $\frac{\partial^2\pi_2^n}{\partial\beta_2^{*2}} < 0$, $|H| > 0$.

In Region III, $u = r$. It is easy to verify that $\frac{\partial^2\pi_1^n}{\partial\beta_1^*\partial\beta_2^*} = 0$, $\frac{\partial^2\pi_1^n}{\partial\beta_1^*\partial\gamma} > 0$, and $\frac{\partial^2\pi_2^n}{\partial\beta_1^*\partial\beta_2^*} > 0$, $\frac{\partial^2\pi_2^n}{\partial\beta_2^*\partial\gamma} > 0$. By Equation 21,

$$\begin{aligned} \frac{\partial\beta_1^*}{\partial\gamma} &= -\left(\frac{\partial^2\pi_2^n}{\partial\beta_2^{*2}}\frac{\partial^2\pi_1^n}{\partial\beta_1^*\partial\gamma} - \frac{\partial^2\pi_1^n}{\partial\beta_1^*\partial\beta_2^*}\frac{\partial^2\pi_2^n}{\partial\beta_2^*\partial\gamma}\right)/|H| = -\frac{\partial^2\pi_2^n}{\partial\beta_2^{*2}}\frac{\partial^2\pi_1^n}{\partial\beta_1^*\partial\gamma}/|H| > 0, \\ \frac{\partial\beta_2^*}{\partial\gamma} &= -\left(-\frac{\partial^2\pi_2^n}{\partial\beta_1^*\partial\beta_2^*}\frac{\partial^2\pi_1^n}{\partial\beta_1^*\partial\gamma} + \frac{\partial^2\pi_1^n}{\partial\beta_1^{*2}}\frac{\partial^2\pi_2^n}{\partial\beta_2^*\partial\gamma}\right)/|H| > 0. \end{aligned} \tag{22}$$

Therefore, the advertising levels of both firms increase with γ. On a side note, the limiting values are: $\lim_{\gamma\to 1}\beta_1^* = \lim_{\gamma\to 1}\beta_2^* = A'^{-1}\left(\frac{r}{2}\right)$.

In Region IV, $u = u^*$. The sign of $\frac{\partial\beta_i^*}{\partial\gamma}$ cannot be resolved with general cost functions. It is numerically verified that, with polynomial, exponential, logarithmic convex cost functions, $\frac{\partial\beta_i^*}{\partial\gamma} > 0, i = 1,2$.

In Region VI, $r = u^*$. Since we consider the equilibrium that has the lowest advertising levels, it satisfies $\frac{\partial_+\pi_2^n}{\partial\beta_2^\dagger} = 0$, therefore $\frac{\partial\beta_2^\dagger}{\partial\gamma} = -\frac{\partial_+^2\pi_2^n}{\partial\beta_2^\dagger\partial\gamma} / \frac{\partial_+^2\pi_2^n}{\partial\beta_2^{\dagger 2}} > 0$. By $r = u^*(\alpha^*)$, $\frac{\partial\alpha^*}{\partial\gamma} < 0$. Therefore $\frac{\partial\beta_1^\dagger}{\partial\gamma} < \frac{\partial\beta_2^\dagger}{\partial\gamma}$.

Summarizing above results, $\frac{\partial\beta_1^*}{\partial\gamma} > 0$ except when (r,γ) is in Region V or VI; $\frac{\partial\beta_2^*}{\partial\gamma} > 0$ whenever $\beta_2^* > 0$.

Second, we analyze the sign of $\frac{\partial\alpha^*}{\partial\gamma}$. By above results, in Region I or II, $\frac{\partial\beta_1^*}{\partial\gamma} > 0, \beta_2^* = 0$; so $\frac{\partial\alpha^*}{\partial\gamma} > 0$. In Region V or VI, $\frac{\partial\alpha^*}{\partial\gamma} < 0$. In Region III,

$$\frac{\partial\alpha^*}{\partial\gamma} = \frac{\partial\beta_1^* - \partial\beta_2^*}{2\partial\gamma} = \left(A''(\beta_2^*)\frac{\partial^2\pi_1}{\partial\beta_1^*\partial\gamma} - A''(\beta_1^*)\frac{\partial^2\pi_2}{\partial\beta_2^*\partial\gamma}\right)\frac{1}{2|H|}. \tag{23}$$

The formula $\frac{\partial^2\pi_2}{\partial\beta_2\partial\gamma} = -r\frac{\partial^2\phi(\alpha)}{2\partial\alpha\partial\gamma} = r\frac{-1+\gamma+5\alpha\gamma+3(\alpha-2)\alpha\gamma^2+\alpha(2-3\alpha+\alpha^2)\gamma^3}{2(1-(1-\alpha)\gamma)^3}$ is greater than $\frac{r}{2}$ for any α when $\gamma > 0.6$. So $\frac{\partial^2\pi_2}{\partial\beta_2^*\partial\gamma} > \frac{\partial^2\pi_1}{\partial\beta_1^*\partial\gamma} = \frac{r}{2}$; therefore $\frac{\partial\alpha^*}{\partial\gamma} < 0$.

This result suggests that the advertising intensity of Firm 2 will grow faster than that of Firm 1, and the skewness of the traffic flow will become smaller when γ is sufficiently large. The limiting value is $\lim_{\gamma\to 1}\alpha^* = \frac{1+\lim_{\gamma\to 1}\beta_1^* - \lim_{\gamma\to 1}\beta_2^*}{2} = 0.5$.

Therefore $\frac{\partial\alpha^*}{\partial\gamma} > 0$ when γ is small and $\frac{\partial\alpha^*}{\partial\gamma} < 0$ when γ is large, and the maximum of α^* must be reached at an intermediate γ. Thus, the buyer traffic flow is most skewed at an intermediate proportion of high-search- cost buyers.

Proof of Corollary 7

If b_i is an interior solution to the profit maximization problem, by the envelope theorem, we have $\partial\pi_i^n/\partial\beta_j = -A'(b_i(\beta_j))$. If b_i is a kink point, we still have $\partial\pi_i^n/\partial\beta_j = -A'(b_i(\beta_j))$. Therefore, given β_i is the best response to β_j, π_i^n always decreases in β_j. Hence at equilibrium (β_i^*, β_j^*) with $\beta_i^* > \beta_j^*$, $\pi_i^n(\beta_i^*;\beta_j^*) > \pi_i^n(\beta_j^*;\beta_i^*) = \pi_j^n(\beta_j^*;\beta_i^*)$.

Proof of Proposition 5

When γ is small (Region I, II, V in Figure 3), $\beta_2^* = 0, \beta_1^* = b_1(0)$. We only need to analyze $\frac{\partial\beta_1^*}{\partial\theta}$. By the first-order condition and the implicit function theorem, in Region II, $\frac{\partial\beta_1^*}{\partial\theta} = -\frac{\tilde{A}'(\beta_1^*)}{\tilde{A}''(\beta_1^*)} < 0$, and in Region I, $\frac{\partial\beta_1^*}{\partial\gamma} = -\frac{-\tilde{A}'(\beta_1^*)}{\pi_1^{n\prime\prime}(\beta_1^*)} < 0$. In Region V , $r = u^*(\frac{1+\beta_1^*}{2})$. So $\frac{\partial\beta_1^*}{\partial\theta} = 0$.

When γ is sufficiently large (Region III, IV, VI in Figure 3), $\beta_i^* > 0, i = 1,2$. In Region III or VI, by the system of first-order conditions and the implicit function theorem,

$$\begin{pmatrix} \frac{\partial\beta_1^*}{\partial\theta} \\ \frac{\partial\beta_2^*}{\partial\theta} \end{pmatrix} = -H^{-1}\begin{pmatrix} \frac{\partial^2\pi_1^n}{\partial\beta_1^*\partial\theta} \\ \frac{\partial^2\pi_2^n}{\partial\beta_2^*\partial\theta} \end{pmatrix}. \tag{24}$$

H is the Hessian matrix. $H = \begin{pmatrix} \frac{\partial^2\pi_1^n}{\partial\beta_1^{*2}} & \frac{\partial^2\pi_1^n}{\partial\beta_1^*\partial\beta_2^*} \\ \frac{\partial^2\pi_2^n}{\partial\beta_2^*\partial\beta_1^*} & \frac{\partial^2\pi_2^n}{\partial\beta_2^{*2}} \end{pmatrix}$.

By the optimality condition, $\frac{\partial^2\pi_1^n}{\partial\beta_1^{*2}} < 0$, $\frac{\partial^2\pi_2^n}{\partial\beta_2^{*2}} < 0$, $|H| > 0$.

In Region III, $u = r$ and $\frac{\partial^2\pi_i^n}{\partial\beta_i^*\partial\theta} = -\tilde{A}'(\beta_i^*) < 0$, so

$\frac{\partial \beta_1^*}{\partial \theta} = -(\frac{\partial^2 \pi_2^n}{\partial \beta_2^{*2}} \frac{\partial^2 \pi_1^n}{\partial \beta_1^* \partial \theta} - \frac{\partial^2 \pi_1^n}{\partial \beta_1^* \partial \beta_2^*} \frac{\partial^2 \pi_2^n}{\partial \beta_2^* \partial \theta})/|H| < 0,$
$\frac{\partial \beta_2^*}{\partial \theta} = -(-\frac{\partial^2 \pi_2^n}{\partial \beta_1^* \partial \beta_2^*} \frac{\partial^2 \pi_1^n}{\partial \beta_1^* \partial \theta} + \frac{\partial^2 \pi_1^n}{\partial \beta_1^{*2}} \frac{\partial^2 \pi_2^n}{\partial \beta_2^* \partial \theta})/|H| < 0$.

In Region IV, $u = u^*$. The sign of $\frac{\partial \beta_i^*}{\partial \theta}$ cannot be resolved with general cost functions. It is numerically verified that, with polynomial, exponential, logarithmic convex cost functions, $\frac{\partial \beta_i^*}{\partial \theta} < 0, i = 1,2$.

In Region VI, $r = u^*$. Since we consider the equilibrium that has the lowest advertising levels, it satisfies $\frac{\partial_+ \pi_2^n}{\partial \beta_2^\dagger} = 0$, therefore $\frac{\partial \beta_2^\dagger}{\partial \theta} = -\widetilde{A}'(\beta_2^\dagger) < 0$. By $r = u^*(\alpha^*)$, $\frac{\partial \alpha^*}{\partial \theta} = 0$. Therefore $\frac{\partial \beta_1^\dagger}{\partial \gamma} = \frac{\partial \beta_2^\dagger}{\partial \gamma} < 0$.

Summarizing above results, decreases in advertising costs always induce firms to increase advertising levels except in Region V, where $\frac{\partial \beta_1^*}{\partial \theta} = 0$.

By above results, in Region I or II, $\frac{\partial \alpha^*}{\partial \theta} < 0$; in Region V or VI, $\frac{\partial \alpha^*}{\partial \theta} = 0$. In Region III or IV, $\frac{\partial \alpha^*}{\partial \theta} = \left(\frac{\widetilde{A}''(\beta_1^*)}{\widetilde{A}'(\beta_1^*)} - \frac{\widetilde{A}''(\beta_2^*)}{\widetilde{A}'(\beta_2^*)}\right) \frac{\widetilde{A}'(\beta_1^*)\widetilde{A}'(\beta_2^*)}{2|H|}$. Therefore the sign of $\frac{\partial \alpha^*}{\partial \theta} < 0$ is decided by the sign of $\frac{\widetilde{A}''(\beta_1^*)}{\widetilde{A}'(\beta_1^*)} - \frac{\widetilde{A}''(\beta_2^*)}{\widetilde{A}'(\beta_2^*)}$.

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