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Online Retailing Channel Addition: Risk Alleviation or Risk Maker?

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***Abstract*—The retailing industry traditionally considers the optimal products selection and pricing problem, a complex and challenging one, from marketing and consumer behavior's perspectives. In this study, we take a risk perspective and offer an alternative solution to tackling the problem, echoing the most recent literature that looks at non-risk aspects, such as expected consumer preference, market size and predicted profitability. Adopting a mean-variance framework, our approach explicitly takes into account the interconnectedness of retail products and their impact on risk at the portfolio (retailer) level. Extending the analysis to multiple-channel decisions, our results suggest that the introduction of a new retailing channel (e.g. online shops) can reduce the portfolio risk, whereas a lack of synergy between the new channel and the existing ones may lead to a negative impact on the overall performance. We also provide managerial implications on several conditions when retailers are more economically inclined to introduce more retail channels. Interestingly, our model indicates that larger retailers are less likely to expand their online platform.**

Keywords: Risk Optimization, Multiple Channel Marketing, Capital Asset Pricing Model

I. Introduction

Retailers with multiple distribution channels are often able to give their customers the opportunity to purchase at both physical and virtual transaction platforms, or through a combination of these channels. Due to recent advance of electronic commerce, today's retailing channels might take difference forms. A typical example of mixed channel retailing is that customers can place orders online and drive to the local store (brick & mortar, B&M store) to retrieve the products. The integration of multiple retailing channels has created many advantages and interesting innovative mechanisms over the traditional simple channel, for example, cross promotion, shared sales information, performance leverage, distribution economies, etc. The business model of multiple channel retailing shows some obvious economic incentives due to the channel synergies that help decrease operational costs and increase sales.

Technologically, it's not very difficult for a retailer to adopt multiple channels. Some common requirements of successful deployment include: real time inventory information to keep track of inventories; ability to map outlets within geographical proximity to the customers; ability to make deals with customers accurately and quickly; effective internal cross-channel communication and coordination. Once established, channel synergies can be used to improve operational efficiency and to generate extra revenues by reaching market niches. The common channel synergies include: shared infrastructure; shared operations; shared sales information; enhanced share-service provision; and complementary assets.

For shared infrastructure, traditional retailers can use common logistics for distribution of goods for its online as well as traditional operations. It will ensure no conflicts can occur and also inventory is utilized effectively. Companies can share its IT infrastructure so that IT operations can ensure efficiency in both areas. Also, IT systems can ensure better utilization of resources. For shared operations, companies can have order processing system shared between e-commerce and physical channels. For shared sales information, companies can use Common product catalogue, sales force, promotions and advertisements so as to ensure that one medium supports the other. It should be kept in mind that two mediums should be used as complementary to each other rather than Substitutes. For enhanced share-services provision: using two media in tandem will obviously ensure that better services are given to customers. Consumer can look at online

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portal to have a look at the items available, prices etc. For complementary assets: traditional retailers can always use their intangible assets such as Supplier relations, experience in the field and customer base and can leverage on these to give them advantage in online business as well.

To achieve channel synergies, it's necessary to align channel goals in order to generate cross channel support. Due to its fast development, the electronic channels should be frequently evaluated and informed to make sure that its practice is known to everybody, and accordingly to avoid information/operation discrepancies. In addition, market segmentation should be carefully exercised without introducing much cross channel competition. The second important element to generate channel synergies is having an effective control and coordination mechanism, which keep the cost of cross management at a low level. Thirdly, it's important to have a robust cross channel infrastructure. Benefit of having multiple channel synergy often include lower costs achieved through economies of scale achieved in labor, inventory, marketing/promotion and distribution. Secondly, fine market differentiation can be achieved through value adding services such as pre-orders and just in time orders. Thirdly, such synergies could improve trust by lower perceived risk and leverage brand awareness. Fourthly, companies with multiple retail channel can achieve geographical and product market extension by reaching customers beyond geographic limit. Above benefits result in additional revenue and deduced costs in general.

One notable factor of multiple channel retailing is the ability to better understand consumer behavior. Consumers are very sensitive to trends, and their preferences change in real time to keep up with said trends. So, from a marketing perspective, firms keep an eye on this rapid movement of consumers in order to establish a successful marketing strategy. How do these emerging trends appear? One good example is the "color" trend. Fashion brands launch fashion shows every season and it attracts consumers' attention. Also, it is easy to observe common colors and styles across brands. These facts are directly reflected to both the firm and the consumer. For firms, it could be one way of market analysis and they can then apply a popular trend to the design of new product. For consumers who are aware of this trend, they will choose a product which is in a line with current trends and it could results in the change of a market share through the emergence of a new trend. And we could say that this trend will be applied across industries regardless of the attribute of their products/services. Therefore, it is obvious that there are significant opportunities to share consumer information among different industries. For example, Zara is a popular fashion brand and Target is a well-known retailer. Zara, which is more sensitive to trends relative to Target, would apply emerging trends when they design clothes and set up marketing strategy. By analyzing consumer purchase history and sales data they can observe the reaction of consumers to this specific trend and this becomes very valuable information to Zara. Target, which is slow to react to this change, could increase sales by sharing this information with Zara. On the other hand, Zara benefits from shared local consumer data.

Despite the obvious benefits of multiple channel marketing, it faces challenges such as cross internal competition, much more complicated planning and logistics management, and increased risk of overstock or undersales. In this research we investigate a rarely studied problem that concerns risk management of multiple channel product selection. The retail industry traditionally considers product selection problem from the consumer preference's perspective, based on expected consumer preference, market size and predicted profitability. We in this research argue that the introduction of a product in a local market involves certain degree of uncertainty, which can be the individual risk associated with the product or a systematic risk associated with much broader economic environment. Risks from multiple channel marketing can also be caused by the correlated risk of each other. We investigate these two types of risks of product selection by using a mean-variance analysis model.

The remainder of this paper is organized as follows: In Section 2, we review relevant literature on the different perspectives of multiple channel marketing. We introduce mean-variance model for retail product selection in Section 3. We discuss the managerial implications and suggest an implementational mechanism in Section 4. We conclude the paper in Section 5.

II. Literature Review

A depth Understanding of consumer information provides many advantages to a retailer. By analyzing the large set of data such as consumer preference and purchase history, amongst others, retailers can establish an optimal marketing strategy and are able to target consumer segments in order to increase revenue and reduce cost. Retailers can more precisely predict future sales by aggregating sales data from multiple channels. With a growing usage of an information technology in retailing, the marketeers create and use their databases to achieve various business goals, for example managing a relationship with customers. Consequently, this practice has moved retailers from traditional product-based marketing towards more customer-based marketing Rust and colleagues, 2012). More recently, due to the rapid development of near field communication technologies and other facilitating information communication technologies, database marketing (DBM) systems that are able to simultaneously manage multiple retailing channels have emerged and proliferated across industries. Wright and Fletcher (1998) identified challenges and barriers of adoption to IS/IT through a cross-industry study, by investigating the financial services, travel and retail industries. They observe that cost is a primary barrier, and to setup a sophisticated marketing database system is another.

In general, the recent and fast advance of information technologies and big data analytics have made it much easier to capture marketing trends across retail channels and among various industries. For example, green marketing has gained popularity as concerns about a global environment among consumers have increased. Many retailers and manufacturers alike consider this phenomenon as an important additional business segment/channel and integrate environmentally friendly features to their products/services. The cross channel effects creates interesting trends among different channels. Regarding consumers' preference and shopping behavior across multiple product categories, although it is difficult to capture the distinct feature, the attitude towards a specific trend or consumer preference could captured. Similarity across product categories offers marketers a chance to gain insights for the design of new product by observing consumer preference information from another channel. It also implies that purchasing behavior on products/services reflecting the emerging trend does not vary among categories. [1] explored the similarities in consumer purchase behavior across multiple categories. Although there is a high degree of heterogeneity in sensitivities for marketing mix variables to consumers, high correlations among categories for same consumer have been observed. In other words, if consumer has a high level of sensitivity to marketing mix variables in one category, a similar level of sensitivity is observed regardless of product categories. [14] investigated which determinant is critical to influence consumers' buying decision for environmentally friendly (EF) products by examining two different market conditions. They employed the theory of planned behavior (TPB) to investigate determinants affecting a consumer's intention to buy EF products. From this perspective, the understanding of customers in one industry may have a great implication for other industries although they do not have exactly same consumer segments. Therefore, marketers and producers should pay attention to a change in consumer preference according to emerging trends in a specific industry in order to develop trendy products/services for potential consumers.

Multiple or cross channel retailing have been recently studied. [3] proposes a conceptual framework to explain whether and when the introduction of a new retail store channel helps or hurts sales in existing direct channels. In its model, a conceptual framework separates the short-term and long-term effects by analyzing the capabilities of a channel that help consumers accomplish their shopping goals. A recent literature review by [5] categorizes and defines the Multi-, Cross-, and Omni-Channel Retailing for retailers. In [8] investigates the competition among multiple retail channels and shows that Internet retailers face significant competition from brick-and-mortar retailers when selling mainstream products, but are virtually immune from competition when selling niche products. Furthermore, because the Internet channel sells proportionately more niche products than the catalog channel, the competition between the Internet channel and local stores is less intense than the competition between the catalog channel and local stores.

III. Risk Analysis of Multiple Channel Product Selection

The introduction of a new product in a regional market could generate additional revenues for the retailer, but it can also cause sales decrease for other products. Even for mature products, when the external environment changes (such as a change in the economic or natural environment), it brings dynamics to the demand of those existing products from retail. Substitute products bring competition and their sales are normally negatively correlated. The demand for different products can also be positively correlated. For example, the main stream fashion design can influence the design and sales of products for other industries, such as those in the home improvements department.

Overall, the demand and profitability of local retail products are subject to not only their unique categorical variation but also the covariation amongst each other. For a B&M retailer to maximize the profitability and optimize its risk, it should take careful consideration of the uncertainties not only from selling both new and existing products, but also from the cross marketing influences. In what follows, we investigate this problem by utilizing the mean-variance analysis modeling to measure the return and associated marketing dynamics.

A. Notations & the Mean-Variance Model

We develop a mean-variance model to evaluate the risk issues associated with the additional retail channel. Originally from the financial asset pricing literature, the mean-variance analysis modeling has been widely utilized to measure the risk and return in various industries and business environment. For example, Roques et al (2008) uses the model to analyze the incentives of fuel mix diversification in liberalized electricity markets. We use the following notations:

- x_{ij}: the quantity of product i in channel j
- X_i: a random variable that captures the total demand of product i
- fc_{ij}: fixed cost to include the i^{th} product in channel j
- p_{ij}: sales price of product i
- c_i: acquisition cost of product i
- R: return vector
- $\overline{R}$: the predicted return vector
- Σ: covariance matrix
- σ_{jk}: the covariance of return between product j and product k
- W: the selection weight vector where $w_{ij} = c_{ij}x_{ij}$ represents the weight for the i^{th} product in channel j.

Assumptions:

1. The total number of available merchandise types is n. These products are available for retailing on both channels. The number of sales made in each product type is indicated by x_{i1} and x_{i2} for the b&m channel and online channel, respectively. If $x_{ij} = 0$, it means that the i^{th} product is not for sale in the channel. —

We consider the following setup, including the return vector from N product $R_j = (r_{1j}, r_{2j}, \cdots, r_{Nj})'$, so

$$r_{ij} = \frac{p_{ij} - c_{ij}}{c_{ij}} - \frac{fc_{ij}}{x_{ij}c_{ij}} \tag{1}$$

We assume that x_{ij} follows the lognormal distribution with parameters μ_{ij} and σ_{ij}^2 independently and identically and in that case, $y_{ij} = \frac{FC_{ij}}{C_{ij}x_{ij}}$ follow the lognormal distribution with parameters $-\frac{FC_{ij}\mu_{ij}}{C_{ij}}$ and $\frac{FC_{ij}^2\sigma_{ij}^2}{C_{ij}^2}$. Then the predicted return

$$\overline{R_j} \quad = (\overline{r}_{1j}, \overline{r}_{2j}, \cdots, \overline{r}_{Nj})' \tag{2}$$

where $\overline{r}_{ij} = \frac{P_{ij}-C_{ij}}{C_{ij}} - \exp(-\frac{FC_{ij}}{C_{ij}}\mu_{ij} + \frac{FC_{ij}^2}{2C_{ij}^2}\sigma_{ij}^2)$, and the product selection weight

$$W_j = (w_{1j}, w_{2j}, \cdots, w_{Nj})' \tag{3}$$

where $w_{ij} = C_{ij}x_{ij}$. We use variance matrix to represent the uncertainty of return from each product and the covariance between each others with $\sigma_{ij} = cov(r_i, r_j)$.

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2N} \\ \vdots & & \ddots & \\ \sigma_{N1} & \sigma_{N2} & \cdots & \sigma_N^2 \end{bmatrix}$$

As result, $R_p = W'R = \Sigma_{i=1}^N(P_i - C_i)x_i - \Sigma_{i=1}^N FC_i$ represents the overall return from the N products, $\overline{R_p} = \Sigma_{i=1}^N(P_i - C_i)\exp(\mu_i + \frac{1}{2}\sigma_i^2)$ represents the expected return from the N products, and $\sigma_p^2 = W'\Sigma W = \Sigma_{i=1,j=1}^{N,N}(P_i - C_i)(P_j - C_j)\rho_{ij}exp(\mu_i + \frac{1}{2}\sigma_i^2 + \mu_j + \frac{1}{2}\sigma_j^2)\sqrt{(e^{\sigma_i^2} - 1)(e^{\sigma_j^2} - 1)}$ represents their portfolio variance.

B. Monte Carlo Simulation Results

The investment of additional online retailing channel was simulated for three scenarios.

1) In the first scenario, products sold online or in B&M market are risky but the correlation among them is set at zero. This is a benchmark scenario that would correspond to hypothetical isolated markets.
2) In the second scenario, product sold online or in B&M market are risky and the correlation coefficients are set to affect only the same product between the two channels.

3) In the third scenario, product sold online or in B&M market are risky and the correlation coefficients are set to affect multiple products across channels.

Specifically, in the second scenario, in order to discover the potential relationship between return rates in different channels, we suppose the expected returns of B&M business and online business are $\bar{R}_b$ and $\bar{R}_o$ respectively, and the standard derivations (risk) of the two returns are σ_b and σ_o. The correlation between returns from the two business activities is ρ_{bo} and the covariance is $\sigma_{bo} = \rho_{bo}\sigma_b\sigma_o$. With the knowledge that the quantity of products of same type sold through different channels would affect each other, we specify covariance between B&M and online with the assumption that:

$$x_{ib} = d_{ib} + e_{iob}d_{io} \quad (4)$$

$$x_{io} = d_{io} + e_{ibo}d_{ib} \quad (5)$$

among which, d_{ib} and d_{io} are the demands of the goods in B&M and online separately, $e_{iob} = \frac{\partial x_{ib}}{\partial x_{io}}$ and $e_{ibo} = \frac{\partial x_{io}}{\partial x_{ib}}$ reflect the cross-channel effects.

So we bring (4) into (5) and get $x_{io} - d_{io} = e_{ibo}(x_{ib} - e_{iob}d_{io})$. Owing to $e_{ibo}e_{iob} = 1$, the equation becomes $\frac{x_{io}}{x_{ib}} = e_{ibo} = \frac{\partial x_{io}}{\partial x_{ib}}$, and finally, we get $x_{io} = A_i x_{ib}$.

On the top of that, in the third scenario, we will take Cholesky decomposition method into consideration to simulate the interactions among multiple products, which is commonly used in the Monte Carlo method for simulating systems with multiple correlated variables.The correlation matrix is decomposed to give the lower-triangular matrix A. Applying this to a vector of uncorrelated samples, r, will produce a sample vector with the covariance properties of the system being modeled.[1]

Figure (1), (2) and (3) show successively the single return rate distribution of 1000 products sold on both retailing channels separately in the three scenarios.

[1]See https://en.wikipedia.org/wiki/Cholesky_decomposition

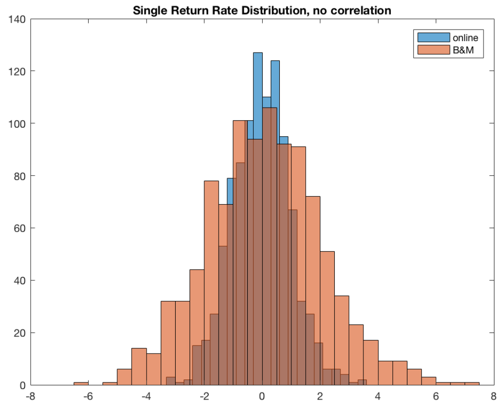

Fig. 1: Single Return Rate Distribution, no correlation

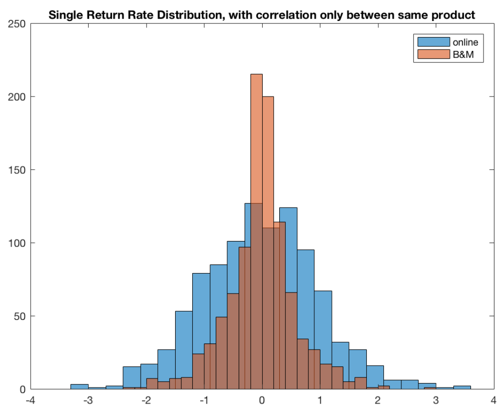

Fig. 2: Single Return Rate Distribution, with correlation only between same product

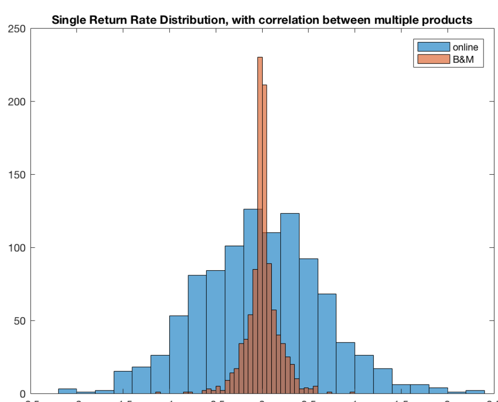

Fig. 3: Single Return Rate Distribution, with correlation between multiple products

In all three scenarios, the expected return rate of products sold B&M has a slight change and is always negative, while the expected return rate of products sold online changes a lot from positive to negative and back to positive again. Some detailed data about these distributions statistics are shown in table I.

TABLE I: Analysis of Single Return Rate Distribution Statistics

Scenario	1^{st} scenario		2^{nd} scenario		3^{rd} scenario	
Statistics	B&M	online	B&M	online	B&M	online
Mean	-0.03	0.07	-0.03	-0.02	-0.02	0.005
Standard Deviation	1	2	0.59	1	0.17	0.67
Minimum	-3.23	-6.14	-3.23	-2.39	-2.17	-1.11
Maximum	3.58	7.14	3.58	2.92	2.40	1
Range	6.81	13.28	6.81	5.31	4.56	2.10

Nevertheless, the distributions of return rate in different channels have various shapes in three scenarios.

1) In the first scenario, with no correlation between products sold online and in B&M market, the range of return rate distribution online is almost twice as much as that of return rate distribution in B&M market, and B&M market appears to be less risky than online.
2) In the second scenario, with correlation between same product sold via two channels, the volatility of return rate of online market decreases faster than that of return rate of online market. Meanwhile, the range of return rate of online market becomes narrower than the range of the return rate of B&M market.
3) In the third scenario, with correlation among multiple products in different channels, B&M market has a less risky distribution of return rate while online market has a less spread distribution of return rate, although both online market and B&M market reduce their volatility and range.

C. *Analysis of Return Correlation*

For the purpose of utilizing the MVP theory to figure out the optimal portfolio of two channels, we would require data about the returns, the risks and the correlations between the returns. The correlation between returns of different channels is determined by running an econometric regression of 1000 simulations of the different channels. The results are presented in Table II for the three scenario described in the previous section.

First of all, it's easy to observe that the correlations between different channels in the second and third scenarios are much higher than that in the first scenario, which is consistent with our settings.

TABLE II: Correlation Coefficients between Two Channels

Correlation of return rates	online/B&M
No correlation	-0.0093
Correlation between same product	0.8971
Correlation between multiple products	-0.7817

Secondly, if we compare the correlation between two channels in the second scenario with that in the third scenario, we will discover the correlation actually decreases when more relationships are considered.

D. *Optimal Portfolio of Two Channels*

Retailers can choose to invest in B&M market and at the same time to exploit virtual market (online). A critical decision to make is whether to go online and if so, how to allocate limited resources among these two activities. From the portfolio management perspective, the objective of retailers is to create a portfolio, consisting of risk-free investment (Treasury Bills), B&M business and online business, which maximizes excess return per unit of risk, e.g. the Sharpe ratio pioneered by Sharpe (1994). Modern mean variance portfolio theory provides a solution to the optimization problem of retailers.

Taking the above analysis into consideration, the covariance between returns from the two business activities is

$$\sigma_{bo} = \Sigma_{i=1,j=1}^{N,N} A_j (P_i - C_i)(P'_j - C'_j) cov(x_{ib}, x_{jb}) \quad (6)$$

E. *The Effects of Product Selection Synergy*

In this section, we will discuss optimal portfolio under two circumstances separately in order to compare the effects of synergy.

1) Case 1: Optimal portfolio without correlations in online and B&M markets: Assuming no synergy between the two channels, then the objective is to maximize the Sharpe ratio:

$$\max \frac{\bar{R_p} - R_f}{\sigma_p} \quad (7)$$

subject to: $\omega_b + \omega_o = 1$, where $\bar{R_p} = \omega_b * \bar{R_b} + \omega_o * \bar{R_o}$ is the expected portfolio return, $\sigma_p = \sigma_p^2 + \sigma_b^2 + 2\rho_{bo}\sigma_b\sigma_o$ is the standard deviation of the portfolio return and

R_f is the risk free rate, e.g. the return of Treasury Bills, ω_b and ω_o are the weights of B&M business and online business, which are the important parameters in our model design that facilitates retailers' decision-making regarding how much online business versus B&M business.

A Lagrangian function is used to solve our constrained optimization problem. It can be shown that the optimization problem is equivalent to solving the following two equations simultaneously:

$$\bar{R_b} - R_f = z_b\sigma_b^2 + z_o\sigma_{ob} \quad (8)$$

$$\bar{R_o} - R_f = z_b\sigma_{ob} + z_o\sigma_o^2 \quad (9)$$

where $z_b = \frac{\bar{R_p}-R_f}{\sigma_p^2} * \omega_b$ and $z_o = \frac{\bar{R_p}-R_f}{\sigma_p^2} * \omega_o$

Finally, $\omega_b = \frac{z_b}{z_b+z_o}$ and $\omega_o = \frac{z_o}{z_b+z_o}$.

We utilize the correlation between the two previous channels, so that the efficient frontier for portfolios of online and B&M market could be illustrated directly and clearly. Figure (4) shows the efficient fronter for portfolios in the first scenario which assumes no correlation in online and B&M market.

Fig. 4: Efficient Fronter for Portfolios of B&M and Online Market, with no correlation

In this case, the correlation between B&M and online market is quite low(0.0093), so entering into online market will produce a significant risk-reducing "portfolio effect". However, the huge differences in risk and return rate between two channels render the investor a good choice to build a diversified portfolio of B&M and online market, and the risk aversion of investor would determine the specific preferred portfolio on the efficient frontier.

2) Case 2: The impact of online and B&M market: In the previous case, we haven't taken the correlation between two channels into consideration. If such correlations are introduced, they would change the risk-return profile of two channels dramatically, which would be shown in Figures (5) and (6).

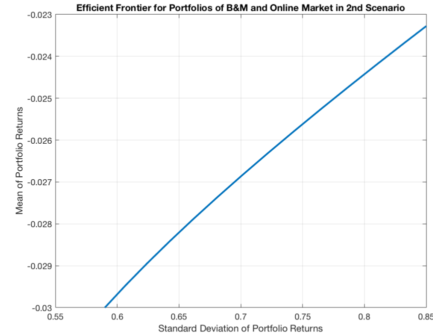

Fig. 5: Efficient Fronter for Portfolios of Online and B&M Market, with correlation between same product

Figure (5) presents the efficient frontier for portfolios of B&M and online market when only correlation between same product has been considered. In the scenario, the correlation between B&M and online market is quite high(0.8971), which makes little sense for the investor to enter into the online market on the purpose of diversifying the risk. In other words, such a relatively high correlation could not produce a effective risk-reducing "portfolio effect".

As for the last scenario, Figure (6) in the following would show the efficient frontier for portfolios of B&M and online market when correlation among multiple products has been introduced. In the scenario, the correlation between B&M and online market is negative and the absolute value is quite high(-0.7917), which indeed has the risk-reducing "portfolio effect" of entering into the online market from B&M for the investor compared with the second scenario. Nevertheless, this effect looks not as significant as that in the first scenario.

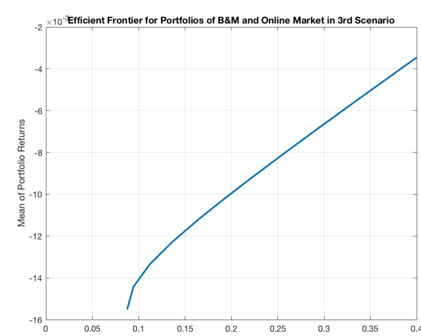

Fig. 6: Efficient Fronter for Portfolios of Online and B&M Market, with correlation among multiple products

One challenge of applying mean-variance theory to retailers' marketing channel decisions is to take into account the synergy effect among different channels, which does not exist among financial assets.

Assume the expected potential synergy benefit of having both online business and B&M business is S and it is equally distributed between the two channels. Our objective is still to maximize the Sharpe ratio (max $\frac{\bar{R_p}-R_f}{\sigma_p}$). However, unlike in Case 1 the expected portfolio return $\bar{R_p}$ should reflect the synergy effect and therefore is expressed as a function of S:

$$\bar{R_p} = \omega_b * (\bar{R_b} + \frac{S}{2} * I_s) + \omega_o * (\bar{R_o} + \frac{S}{2}) \qquad (10)$$

where I_s is an indicator function equal to 1 if $\omega_o > 0$, and 0 otherwise.

The weights of the two channels can be derived, as in Case 1, by solving the constrained optimization problem. Nevertheless, with synergy effect taken into consideration, there is no closed-form solutions for ω_b and ω_o and they have to be calculated numerically.

IV. Discussions

In this section, we analyze the risk impact of additional online retail channel on the perspectives of existing channel size and the channel synergy.

A. The Effects of Existing Channel Size

Without loss of generality, we assume that a retailer considers multiple channels, including an online channel with an expected return $\overline{R_o}$ and a b&m channel with an expected return $\overline{R_b}$. The correlation between the channels is ρ_{ob}. A direct application of our mean-variance theory on existing channel size leads to the following propositions.

Proposition 1: Only when $\frac{R_o-R_f}{\sigma_o} \geq \frac{R_b-R_f}{\sigma_b}$, retailer obtains a higher risk adjusted return by adding the online channel into its existing b&m channel. Only under this condition, the retailer is willing to invest and set up its online platform.

Proposition 2: Larger retailers are less motivated to invest in the online channel due to the diversification benefit they already enjoy through relatively large business scale. As the total number of merchandise N increases, the risk (standard deviation) σ_b decreases. As a result, the right-hand side of the condition equation in proposition 1 increases, which makes the inequation less likely to hold, all else being equal. The managerial implication of this proposition is that it is more rewarding for small retailers with relatively small N to use the online channel together with their b&m channel.

B. The Impact of Synergy

Suppose a retailer, who has already been operating B&M business for many years, is deciding whether or not to go online. The objective is to improve the retailer's mean-variance welfare. Our model suggests that the retailer only starts online business if:

$$\frac{\bar{R_o} - R_f}{\sigma_o} > \frac{\bar{R_b} - R_f}{\sigma_b} * \rho_{bo} \qquad (11)$$

This expression is intuitive in the sense that to improve the retailer's current portfolio (Treasury Bills + B&M business)'s Sharpe ratio $\frac{\bar{R_b}-R_f}{\sigma_b}$ by adding the new investment (online business), the Sharpe ratio of the new investment $\frac{\bar{R_o}-R_f}{\sigma_o}$ must be larger than the Sharpe ratio of the retailer's existing portfolio, taking into account correlation ρ_{bo}. Adding online business is more rewarding (after adjusting risk) if it is less correlated with the current B&M business or/and if its Sharpe ratio is higher than the Sharpe ratio of the current B&M business.

However, retailers' optimization problem is complicated by the potential synergy effects among different B&M products and also between B&M business and online business. It is reasonable to assume that the expected return of B&M business $\bar{R_b}$ is positively related to the number of products it sells (N), namely the scale of the business. In addition, introducing online business will very likely bring a synergy premium ($\bar{R_{sp}}$) to the whole portfolio. As a result, expression (11) becomes:

$$\frac{\bar{R_o} + \bar{R_{sp}} - R_f}{\sigma_o} > \frac{\bar{R_b}(N) - R_f}{\sigma_b} * \rho_{bo} \qquad (12)$$

where $\bar{R_{sp}}$ represents the synergy premium between B&M business and online business. $\bar{R_b}(N)$ is a function of N, which increases with the number of products N, indicating the synergy effects among different B&M products. It is evident from expression (12) that the retailer is more likely to improve its risk-adjusted return by entering into online business if the synergy premium is higher or/and if the existing B&M business scale (N) is smaller.

Our optimization problem is further complicated by diversification effect. For simplicity, we make further assumptions without loss of generality. We assume that returns of all B&M and online products have the same standard deviation (σ) and the correlation between the returns of any two products is the same (ρ). Since there is no (at least much less) space and logistical constraint for online business (compared with B&M business), it is reasonable to assume that the number of products operated online (N_o) is infinity and the number of products operated as B&M (N_b) is constrained to some extent ($N_b = N < \infty$).

Under the above mentioned assumptions, the standard deviation of online business can be calculated as $\sigma_o = \sqrt{\rho}\sigma$ and the standard deviation of B&M business can be expressed as $\sigma_b = \sqrt{\frac{\sigma^2}{N} + (1-\frac{1}{N})\rho\sigma^2}$, which decreases with N. Plug σ_o and σ_b into expression (12) , we have:

$$\frac{\bar{R_o} + \bar{R_{sp}} - R_f}{\sqrt{\rho}\sigma} > \frac{\bar{R_b}(N) - R_f}{\sqrt{\frac{\sigma^2}{N} + (1-\frac{1}{N})\rho\sigma^2}} * \rho_{bo} \tag{13}$$

We observe from expression (13) that as N increases, it is less rewarding to add online business into the existing B&M business, everything else being equal. Put differently, large retailers should be less motived to expand to online business due to the diversification benefit they already enjoy through relatively large business scale (large N). However, our model shows that it is more rewarding for small retailers (small N) to use online business.

Proposition 3: When our model is enriched with a synergy effect, the condition in proposition1 immediately becomes $\frac{R_o - R_f}{\sigma_o} \geq \frac{R_b - R_f}{\sigma_b}$, where $\overline{R_{sp}}$ represents the synergy premium between the b&m channel and the online channel. The intuition is straightforward. The retailer is more likely to engage in online channel if there are positive synergies between the two channels.

Although our model focuses on whether retailers with a traditional bricks and mortar presence should go online and if so to what extent (how to decide ω_o), it can be easily applied to make the product selection decisions of B&M retailers.

V. Concluding Remarks

The retailing industry traditionally considers the optimal products selection and pricing problem, a complex and challenging one, from marketing and consumer behavior's perspectives. In this study, we take a risk perspective and offer an alternative solution to tackling the problem, echoing the most recent literature that looks at non-risk aspects, such as expected consumer preference, market size and predicted profitability. Adopting a mean-variance framework, our approach explicitly takes into account the interconnectedness of retail products and its impact on risk at the portfolio (retailer) level. Extending the analysis to multiple-channel decisions, our results suggest that the introduction of a new retailing channel(e.g. online shop) can reduce the portfolio risk, whereas a lack of synergy between the new channel and the existing ones may lead to a negative impact on the overall performance. We also provide managerial implications on several conditions when retailers are more economically inclined to introduce more retail channels. Interestingly, our model indicates that larger retailers are less likely to expand their online platform.

Optimal retail product selection is a very complex problem and difficult to solve from the optimization problem's perspective. The retailing industry traditionally considers the problem of product selection and pricing from a marketing and consumer behavior's perspective. In most recent literature, the problem of product selection is often considered based on expected consumer preference, market size and predicted profitability. In this research, we argue that the product selection in multiple channel retailing involves uncertainty that can be decomposed as an individual risk and a systematic risk. While most relevant existing literature has a focus on the individual risk, we are keen to investigate the correlational risk of selected products. Our study is based on a mean-variance analysis model. The results show that while the introduction of additional retailing channel can reduce the systematic risk but a lack of channel synergy may have negative impact on the overall performance. We also provide managerial implications on several conditions when retailers are more economically inclined to add more retail channels. Based on our model, we also find that large retailers are less likely to expand their online retail platform.

Appendix A
Derivation of the mean-variance solution

We assume that there doesn't exist an absolutely risk free product with unconstrained demand and supply. And then once we consider the basic scenario that the retailer intends to minimize the risk, given an expected annual return υ, the problem becomes

$$\min_W \frac{1}{2} W' \Sigma W \tag{14}$$

subject to:

$$i'W = 1 \tag{15}$$

$$\overline{R}'W = \upsilon \tag{16}$$

where i is a vector of 1s and υ represents the weighted mean of expected return.

By taking the first order derivation of the Lagrangian formulas of equation 1, 2 and 3, the optimal weight vector

$$W^* = \frac{1}{D}[B\Sigma^{-1}i - A\Sigma^{-1}\overline{R}] + \frac{\upsilon}{D}[C\Sigma^{-1}\overline{R} - A\Sigma^{-1}i] \tag{17}$$

where, (proof in appendix A)

- $A = i'\Sigma^{-1}\overline{R}$,
- $B = \overline{R}'\Sigma^{-1}\overline{R}$,
- $C = i'\Sigma^{-1}i$,
- $D = BC - A^2$.

We define several dummy variables to facilitate the derivation: $A = i'\Sigma^{-1}\overline{R}$, $B = \overline{R}'\Sigma^{-1}\overline{R}$, $C = i'\Sigma^{-1}i$, and $D = BC - A^2$. So, it renders $1 - \gamma C - \lambda A = 0$ and $\upsilon - \gamma A - \lambda B = 0$. Consequently, the optimal product selection strategy follows:

$$\begin{aligned} W^* &= \gamma\Sigma^{-1}i + \lambda\Sigma^{-1}\overline{R} && (18) \\ &= \frac{1}{D}[B\Sigma^{-1}i - A\Sigma^{-1}\overline{R}] + \frac{\upsilon}{D}[C\Sigma^{-1}\overline{R} - A\Sigma^{-1}i] && (19) \end{aligned}$$

and the product selection portfolio risk is

$$\begin{aligned} \sigma_p^2 &= \gamma + \lambda\upsilon && (20) \\ &= \frac{B - 2A\upsilon + C\upsilon^2}{D} && (21) \end{aligned}$$

where $\gamma^* = \frac{B - A\upsilon}{D}$ and $\lambda^* = \frac{C\upsilon - A}{D}$.

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